PRODUCTION OF HEAVY QUARK–ANTIQUARK PAIR BY DOUBLE POMERON EXCHANGE IN HIGH ENERGY PROTON–PROTON COLLISIONS *

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Using the two-gluon exchange model of the pomeron, the total and differential cross-section for production of heavy quark–antiquark pairs ($c\bar{c}$, $b\bar{b}$, $t\bar{t}$) by double pomeron exchange in high energy proton–proton collisions is calculated. The results for production of heavy quark-antiquark pairs are compared with those for production of the Higgs boson in the same reaction.

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1. Introduction

The recently developed model of the pomeron [1] triggered some renewed interest in the double pomeron exchange reactions [2, 3]. Such reactions are characterized by large rapidity gaps between remnants of colliding hadrons and particles produced in the reaction. It is a rather attractive feature, as it allows a serious reduction of the "minimum bias" background in the process of production of some interesting objects.

It was suggested [2], that double pomeron exchange mechanism can play important role in production of Higgs bosons. In a previous paper [3] we considered contribution of this mechanism to the production of heavy quarks. The present investigation extends the results of [3] to the mass spectra of the produced pairs. Also some details of the calculations are more explicitly shown. The main interest in the process of heavy quarks production stems from the fact that it is expected to be a dominant background for Higgs production. For that, the mass spectrum of the produced

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pairs is of crucial importance. Moreover, our calculations can serve as a check for the model of the pomeron at the energies presently accessible, by comparison of the model prediction for heavy quarks production with the data. It should allow to determine the range of applicability of the model before LHC and SSC energies are available.

2. Discontinuity relations

In the present paper we follow closely Ref. [2] and the model of the pomeron developed by Donnachie and Landshoff [1]. In this model pomeron exchange is described as an exchange of two "nonperturbative" gluons [4]. Thus, to compute in this model the cross-section for reaction $pp \to pp + q\bar{q}$, we have to calculate eight diagrams presented on Fig. 1.

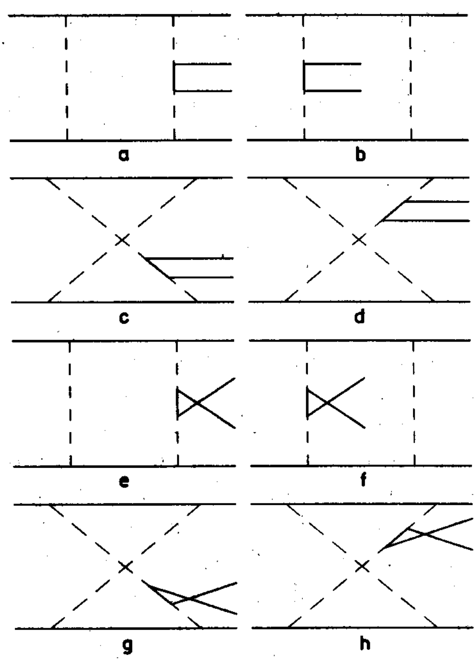

Fig. 1. The diagrams contributing to the process of the heavy quarks production by double pomeron exchange. The dashed lines represent the exchange of non-perturbative gluons.

It follows from general considerations [2] that with accuracy up to the highest powers of s/s_1 and s/s_2 the complete two-pomeron exchange am-

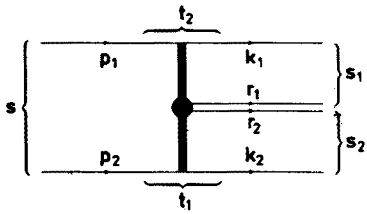

Fig. 2. The process of double pomeron exchange.

plitude is of the form:

$$\left(\frac{s}{s_1}\right)^{\alpha_2-1}\left(\frac{s}{s_2}\right)^{\alpha_1-1} F_{\alpha_1\alpha_2}, \qquad (1)$$

where

$$\alpha_1 = \alpha(t_1)$$
$$\alpha_2 = \alpha(t_2)$$

and $\alpha(t) = 1 + \epsilon + \alpha' t$ is the pomeron Regge trajectory with $\epsilon \approx 0.08$, $\alpha' \approx 0.25\mathrm{GeV}^{-2}$ (s, s_1, s_2, t_1, t_2 — as marked in Fig. 2).

Because $F_{\alpha_1\alpha_2}$ only weakly depends on t_1 and t_2, we have with accuracy to terms neglegible in ϵ

$$M_{fi} = M_{fi}\Big|_{t_1=t_2=0}\left(\frac{s}{s_1}\right)^{\alpha_1-1}\left(\frac{s}{s_2}\right)^{\alpha_2-1}. \qquad (2)$$

Thus, to calculate the double pomeron exchange amplitude, it is enough to evaluate diagrams from Fig. 1 at $t_1 = t_2 = 0$.

It is also a rather difficult task to calculate the diagrams of Fig. 1 at $t_1 = t_2 = 0$, but fortunately there is a way of computing their sum in this limit, without calculating individual diagrams. It follows from analitic properties of these diagrams [5] that at $t_1 = t_2 = 0$, and up to the terms of the order ϵ, the sum of the first four diagrams from Fig. 1 is approximately equal to the discontinuity of diagram 1.a, and the sum of the last four diagrams from Fig. 1 is approximately equal to the discontinuity of diagram 1.e [2].

Thus, to calculate invariant matrix element for our reaction, we have to calculate the discontinuity of the sum of two diagrams presented in Fig. 3.

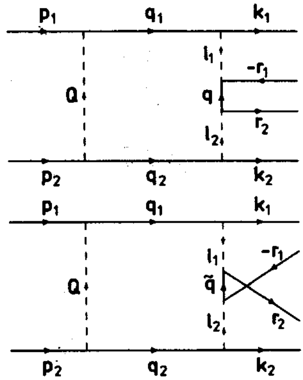

Fig. 3. The discontinuity of these two diagrams gives the amplitude for the process of heavy quarks production by double pomeron exchange at $t_1 = t_2 = 0$.

3. Invariant matrix element

Let the mass of the produced quark be m, and let us put the masses of the light quarks equal to zero. Moreover, let us introduce the following parametrization for momenta marked in Fig. 3 [2].

$$\begin{aligned} Q &= \frac{x}{s} p_1 + \frac{y}{s} p_2 + v\,, \\ k_1 &= x_1 p_1 + \frac{y_1}{s} p_2 + v_1\,, \\ k_2 &= \frac{x_2}{s} p_1 + y_2 p_2 + v_2\,, \\ r_2 &= x_q p_1 + y_q p_2 + v_q\,, \end{aligned} \tag{3}$$

where v, v_1, v_2 and v_q are four-vectors describing the transverse components of the momenta. Other momenta are determined by conservation of energy and momentum.

The invariant matrix element at $t_1 = t_2 = 0$ is given by

$$\begin{aligned} M_{fi} =& g^2 G^4 \int \frac{d^4Q}{(2\pi)^4} \bar{u}(k_1) \gamma^\mu \hat{q}_1 2\pi\delta\left(q_1^2\right) \gamma^\nu u(p_1) \\ & \times \bar{u}(k_2) \gamma^\lambda \hat{q}_2 2\pi\delta\left(q_2^2\right) \gamma_\nu u(p_2) D\left(Q^2\right) D\left(l_1^2\right) D\left(l_2^2\right) \\ & \times \bar{u}(r_2) \left[\gamma_\lambda \frac{\hat{q}+m}{q^2-m^2} \gamma_\mu + \gamma_\mu \frac{\hat{\tilde{q}}+m}{\tilde{q}^2-m^2} \gamma_\lambda\right] v(r_1)\,, \end{aligned} \tag{4}$$

where g, G are the perturbative and non-perturbative quark–gluon coupling constants, and $D\left(p^2\right) = D_0 \exp\left(p^2/\mu^2\right)$ with $\mu \approx 1$ GeV and $G^2 D_0 \approx$

30 GeV$^{-1}\mu^{-1}$ is the non-perturbative gluon propagator. Substituting this form of propagator to the above formula one obtains

$$M_{fi} = \frac{g^2 G^4 D_0^3}{(2\pi)^2} \int d^4Q \delta\left(q_1^2\right) \delta\left(q_2^2\right) \bar{u}\left(k_1\right) \gamma^\mu \hat{q_1} \gamma^\nu u\left(p_1\right)$$
$$\times \bar{u}\left(k_2\right) \gamma^\lambda \hat{q_2} \gamma_\nu u\left(p_2\right) \exp\left(\frac{Q^2 + l_1^2 + l_2^2}{\mu^2}\right)$$
$$\times \bar{u}\left(r_2\right)\left[\gamma_\lambda \frac{\hat{q}+m}{q^2 - m^2}\gamma_\mu + \gamma_\mu \frac{\hat{\bar{q}}+m}{\bar{q}^2 - m^2}\gamma_\lambda\right] v\left(r_1\right) . \quad (5)$$

At $t_1 = t_2 = 0$ we have

$$Q^2 + l_1^2 + l_2^2 = (1 - x_1) y + (1 - y_2) x + 3\frac{xy}{s} - 3\vec{v}^2 . \quad (6)$$

One can see from (6), that integrand in (5) is exponentially damped in $\vec{v}^2$ so, everywhere except in exponent, we can consider $\vec{v}$ as small. In this approximation

$$\int d^4Q \delta\left(q_1^2\right) \delta\left(q_2^2\right) \approx \frac{1}{2s} \int dx\, dy\, d^2\vec{v} \delta\left(x + \vec{v}^2\right) \delta\left(y - \vec{v}^2\right) . \quad (7)$$

Taking into account (7) and keeping only terms of the lowest order in $\vec{v}^2$, (5) takes the form

$$M_{fi} = \frac{g^2 G^4 D_0^3}{2s(2\pi)^2} \int d^2\vec{v} \bar{u}\left(k_1\right) \gamma^\mu \hat{q_1} \gamma^\nu u\left(p_1\right)$$
$$\times \bar{u}\left(k_2\right) \gamma^\lambda \hat{q_2} \gamma_\nu u\left(p_2\right) \exp\left(-\frac{(1 + x_1 + y_2)\vec{v}^2}{\mu^2}\right)$$
$$\times \bar{u}\left(r_2\right)\left[\gamma_\lambda \frac{\hat{q}+m}{q^2 - m^2}\gamma_\mu + \gamma_\mu \frac{\hat{\bar{q}}+m}{\bar{q}^2 - m^2}\gamma_\lambda\right] v\left(r_1\right) . \quad (8)$$

Substituting

$$q_1 = p_1 + \frac{\vec{v}^2}{s}(p_2 - p_1) + v ,$$
$$q_2 = p_2 - \frac{\vec{v}^2}{s}(p_2 - p_1) - v , \quad (9)$$

to spinor expressions for the light quarks lines one obtains

$$\bar{u}(k_1)\gamma^\mu \hat{q}_1 \gamma^\nu u(p_1) = 2p_1^\nu \bar{u}(k_1)\gamma^\mu u(p_1)$$
$$+\bar{u}(k_1)\gamma^\mu \hat{v}\gamma^\nu u(p_1) + \frac{\vec{v}^2}{s}\bar{u}(k_1)\gamma^\mu(\hat{p}_2 - \hat{p}_1)\gamma^\nu u(p_1)$$
$$\bar{u}(k_2)\gamma^\lambda \hat{q}_2 \gamma_\nu u(p_2) = 2p_{2\nu}\bar{u}(k_2)\gamma^\lambda u(p_2)$$
$$-\bar{u}(k_2)\gamma^\lambda \hat{v}\gamma_\nu u(p_2) - \frac{\vec{v}^2}{s}\bar{u}(k_2)\gamma^\lambda(\hat{p}_2 - \hat{p}_1)\gamma_\nu u(p_2)\,, \qquad (10)$$

where $\hat{v} = \sum_{i=2}^{3} v^i\gamma_i$ (Lorentz' indices). Because integral in (8) contains function exponentially damped in $\vec{v}^2$, that restricts $\vec{v}$ to the range of values very small in comparison with p_1 and p_2, $|\vec{v}| \leq 1$ GeV, we can put $\vec{v} \approx 0$ in above formulae

$$\bar{u}(k_1)\gamma^\mu \hat{q}_1 \gamma^\nu u(p_1) \approx 2p_1^\nu \bar{u}(k_1)\gamma^\mu u(p_1)\,,$$
$$\bar{u}(k_2)\gamma^\lambda \hat{q}_2 \gamma_\nu u(p_2) \approx 2p_{2\nu}\bar{u}(k_2)\gamma^\lambda u(p_2)\,, \qquad (11)$$

Now, let us make next approximation by expanding the integral over d^2v in power series

$$\int d^2v\, f(\vec{v}) \exp\left(-\frac{\vec{v}^2}{\alpha}\right) = a\pi\alpha + \sum_i \frac{\pi\alpha^2}{2} a_{ii} + o\left(\alpha^2\right)\,, \qquad (12)$$

where a, a_{ij} are coefficients of expansion of f(v):

$$f(\vec{v}) = a + \sum_i a_i v_i + \sum_{i,j} a_{ij} v_i v_j + o\left(\vec{v}^2\right)\,.$$

In our case $\alpha = \mu^2/1 + x_1 + y_2$ and

$$f(\vec{v}) = \bar{u}(k_1)\gamma^\mu u(p_1)\,\bar{u}(k_2)\gamma^\lambda u(p_2)$$
$$\times\, \bar{u}(r_2)\left[\gamma_\lambda \frac{\hat{q}+m}{q^2-m^2}\gamma_\mu + \gamma_\mu \frac{\hat{\tilde{q}}+m}{\tilde{q}^2-m^2}\gamma_\lambda\right] v(r_1)\,. \qquad (13)$$

It is not difficult to prove that $a = f(\vec{v} = 0) = 0$. Thus in order to compute the leading term in approximation (12) we have to calculate contribution corresponding to the third term of $f(\vec{v})$ series. Carrying out calculations with accuracy up to the terms with highest power in $1/\delta_1$ and $1/\delta_2$ we obtain

$$M_{fi} = -4\sqrt{x_1 y_2}\,\delta^{s_1,s_1'}\delta^{s_2,s_2'}\frac{\pi D_0^3 \mu^4 g^2 G^4}{9(2\pi)^2}\frac{m}{\left(s x_q y_q\right)^2 \delta_1\delta_2}$$
$$\times\, \bar{u}(r_2)\Big[x_q(1 - y_2 - y_q)\hat{p}_1\hat{p}_2 + y_q(1 - x_1 - x_q)\hat{p}_2\hat{p}_1$$
$$+\frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big] v(r_1)\,, \qquad (14)$$

where $\delta_1 = 1 - x_1$, $\delta_2 = 1 - y_2$; s_1, s_2 are the initial spins and s_1', s_2' — final spins of colliding quarks.

Eq. (14) gives invariant matrix element for $t_1 = t_2 = 0$. To obtain M_{fi} for arbitrary values of t_1 and t_2 we have to apply (2) where $(s/s_1) = (1/\delta_2)$, $(s/s_2) = (1/\delta_1)$, $t_1 = -\vec{v}_1^{\,2}$, $t_2 = -\vec{v}_2^{\,2}$. Then we can evaluate the square of the invariant matrix element averaged over initial spins and summed over final spins. This calculation is shown in Appendix A. The result is

$$\overline{|M_{fi}|^2} = \frac{x_1 y_2 H}{(s x_q y_q)^2 (\delta_1 \delta_2)^{1+2\epsilon} \delta_1^{2\alpha' t_1} \delta_2^{2\alpha' t_2}} \left(1 - \frac{4m^2}{s\delta_1\delta_2}\right), \tag{15}$$

where

$$H = 2s \left(\frac{4\pi m D_0^3 \mu^4 g^2 G^4}{9(2\pi)^2}\right)^2 . \tag{16}$$

3. Cross-section

Using (15) one obtains the formula for the cross-section

$$\sigma = \frac{1}{2s(2\pi)^8} \int \overline{|M_{fi}|^2} \left[F(t_1)\, F(t_2)\right]^2 dPH , \tag{17}$$

where $F(t)$ is the nucleon formfactor, approximated by

$$F(t) = \exp(\lambda t) ,$$

with $\lambda = 2\ \mathrm{GeV}^{-2}$ and dPH is the differential phase-space factor

$$dPH = d^4k_1\, \delta\left(k_1^2\right)\, d^4k_2\, \delta\left(k_2^2\right)\, d^4r_1\, \delta\left(r_1^2 - m^2\right)\, d^4r_2\, \delta\left(r_2^2 - m^2\right)$$
$$\times \Theta\left(k_1^0\right)\, \Theta\left(k_2^0\right)\, \Theta\left(r_1^0\right)\, \Theta\left(r_2^0\right)\, \delta^{(4)}\left(p_1 + p_2 - k_1 - k_2 - r_1 - r_2\right) . \tag{18}$$

Integral over r_1 in (17) can be eliminated by means of δ-function expressing the law of conservation of momentum. For other integrals we have the following relations (see Eq. (3))

$$\int d^4k_1 = \tfrac{1}{2} \int dx_1\, dy_1\, d^2\vec{v}_1 ,$$

$$\int d^4k_2 = \tfrac{1}{2} \int dx_2\, dy_2\, d^2\vec{v}_2 ,$$

$$\int d^4r_2 = \tfrac{s}{2} \int dx_q\, dy_q\, d^2\vec{v}_q . \tag{19}$$

Using (19) and performing all integrals except those over x_1 and y_2 in (17) we arrive at

$$\sigma = \frac{\pi^3 H}{4s^3(2\pi)^8} \int dx_1\, dy_2 \frac{1}{(\delta_1\delta_2)^{3+2\epsilon} L_1 L_2} \times t^2 \left(\ln\frac{1+t}{1-t} + \frac{2t}{1-t^2}\right) \Theta\left(1 - \frac{4m^2}{s\delta_1\delta_2}\right) , \quad (20)$$

where

$$L_1 = 2\left(\lambda - \alpha' \ln\delta_1\right) ,$$
$$L_2 = 2\left(\lambda - \alpha' \ln\delta_2\right) ,$$
$$t = \sqrt{1 - \frac{4m^2}{s\delta_1\delta_2}} .$$

As was already mentioned, we calculate the cross-section for a reaction in which initial protons loose only a small part of their energy, thus integrations over x_1 and y_2 in (20) have to be carried out over region, where $\delta_1 \leq \Delta$ and $\delta_2 \leq \Delta$ with $\Delta \leq 0.1$ Consequently, performing change of variables $x_1 \to \delta_1 = 1 - x_1$ and $y_2 \to \delta_2 = 1 - y_2$ we obtain

$$\sigma = \frac{\pi^3 H}{16s^3(2\pi)^8} \int_0^\Delta d\delta_1 \int_0^\Delta d\delta_2 \frac{1}{(\delta_1\delta_2)^{3+2\epsilon}} \Theta\left(1 - \frac{\delta^2}{\delta_1\delta_2}\right) \times \frac{1}{(\lambda - \alpha'\ln\delta_1)(\lambda - \alpha'\ln\delta_2)} t^2 \left(\ln\frac{1+t}{1-t} + \frac{2t}{1-t^2}\right) , \quad (21)$$

where $\delta^2 = 4m^2/s$ and

$$t = \sqrt{1 - \frac{\delta^2}{\delta_1\delta_2}} .$$

After change of variables

$$\delta_2 \to t = \sqrt{1 - \frac{\delta^2}{\delta_1\delta_2}}$$

and integrating over δ_1 we finally have

$$\sigma = C \int_0^{t_0} dt\, t^3 \left(1 - t^2\right)^{1+2\epsilon} \left(\ln\frac{1+t}{1-t} + \frac{2t}{1-t^2}\right) \times \frac{1}{\frac{\lambda}{\alpha'} + \frac{1}{2}\ln\frac{1-t^2}{\delta^2}} \ln \frac{\frac{\lambda}{\alpha'} + \ln\frac{\Delta(1-t^2)}{\delta^2}}{\frac{\lambda}{\alpha'} + \ln\frac{1}{\Delta}} , \quad (22)$$

where

$$t_0 = \sqrt{1 - \frac{\delta^2}{\Delta^2}},$$

$$C = C_F \pi \left(\frac{s}{4m^2}\right)^{2\epsilon} \left(\frac{\pi^2 D_0^3 \mu^4 g^2 G^4}{18 m \alpha' (2\pi)^6}\right)^2 .$$

In the final result we take into account the colour factor for double pomeron exchange $C_F = 1/3$ (see Appendix B).

Variable t in (22) has the close relation to the square of quark–antiquark pair mass $M^2 = (r_1 + r_2)^2$, because the transverse momenta exchanged in the reaction are restricted by the pomeron propagator, and they can be put approximately equal to zero. In this approximation we have $M^2 = s\delta_1\delta_2$. Thus, relation between t and M^2 takes the simple form

$$t = \sqrt{1 - \frac{4m^2}{M^2}} .$$

It allows to calculate immediately from (22) the differential cross-section for production of the quark–antiquark pair of a given square of mass M^2. One has simply to perform change of variables $t \to M^2$. The integrand then becomes the differential cross-section $d\sigma/dM^2$. Carring out above transformations we arrive at the formula

$$\begin{aligned} \frac{d\sigma}{dM^2} = & \frac{C}{M^2} \left(\frac{4m^2}{M^2}\right)^{2(1+\epsilon)} \left(1 - \frac{4m^2}{M^2}\right) \\ & \times \left\{ \ln\left(\frac{M}{2m} + \sqrt{\frac{M^2}{4m^2} - 1}\right) + \frac{M^2}{4m^2}\sqrt{1 - \frac{4m^2}{M^2}} \right\} \\ & \times \frac{1}{\frac{\lambda}{\alpha'} + \frac{1}{2}\ln\frac{s}{M^2}} \ln \frac{\frac{\lambda}{\alpha'} + \ln\frac{s\Delta}{M^2}}{\frac{\lambda}{\alpha'} + \ln\frac{1}{\Delta}}, \end{aligned} \tag{23}$$

where $M^2 \in (4m^2, s\Delta^2)$.

5. Numerical results

Formula for the total cross-section for the heavy quarks production process by double pomeron exchange, derived in Section 4, has the form $\sigma = CI$, where C is a numerical factor and I is an integral depending on few parameters. It is impossible to evaluate this integral analitically, but it is easy to compute it numerically. If we put $\epsilon = 0.08$, $\lambda = 2$ GeV^{-2}, $\alpha' = 0.25$ GeV^{-2} and $\Delta = 0.1$, there remains only one parameter on which I depends, namely $\delta^2 = 4m^2/s$. At high energies this parameter takes small values. In Table I the results of numerical evaluations of σ are given. These evaluations have been performed for three values of quark masses $m = 1.5$ GeV, 4.5 GeV and 140 GeV, which are the masses of quarks c, b and t. The range of s was taken from (0.5 TeV)^2 to (40 TeV)^2.

TABLE I

The numerical results for the cross-section for production of heavy quarks pair by double pomeron exchange.

$\sqrt{s}$ [TeV]	$\sigma_{c-\bar{c}}$ [nb]	$\sigma_{b-\bar{b}}$ [nb]	$\sigma_{t-\bar{t}}$ [pb]
0.5	46.7	1.53	—
1.0	70.0	2.84	—
1.5	86.2	3.75	—
2.0	99.0	4.48	—
3.0	119.0	5.62	—
4.0	135.0	6.52	0.0141
5.0	149.0	7.29	0.0521
6.0	161.0	7.95	0.102
8.0	181.0	9.08	0.207
10.0	198.0	10.0	0.308
15.0	231.0	11.9	0.526
20.0	258.0	13.5	0.704
25.0	281.0	14.7	0.855
30.0	301.0	15.9	0.987
35.0	318.0	16.9	1.1
40.0	334.0	17.8	1.21

The ratio g^2/G^2 was assumed 0.2, 0.17 and 0.1 for $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$ pair production, respectively. In fact, these are the values of perturbative coupling constant itself, not the values of the ratio of perturbative and nonperturbative coupling constants. The value of nonperturbative coupling constant is unknown and here we put it equal to one. This introduces the uncertainty of approximately one order of magnitude. The same uncertainty is present in the results for Higgs production, thus all conclusions concerning the relative magnitude of the heavy quarks and Higgs production processes remain valid.

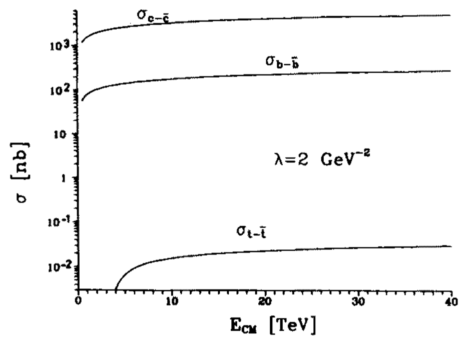

Fig. 4. Double pomeron exchange contribution to the cross-section for the heavy quarks production process versus the center mass energy.

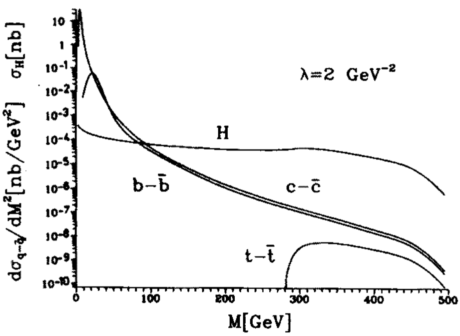

Fig. 5. The differential cross-section for the production of heavy quark–antiquark pair of given square of mass by double pomeron exchange in comparison with the Higgs production at $E_{CM} = 5$ TeV.

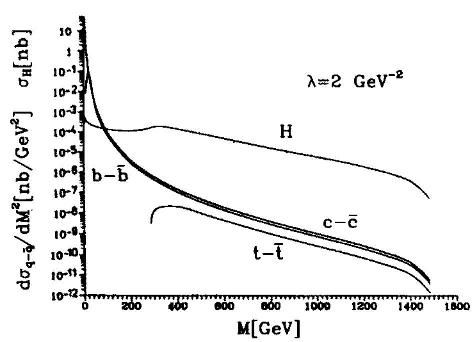

Fig. 6. The differential cross-section for the production of heavy quark–antiquark pair of given square of mass by double pomeron exchange in comparison with the Higgs production at $E_{CM} = 15$ TeV.

The dependence of the total cross-section for production of heavy quarks

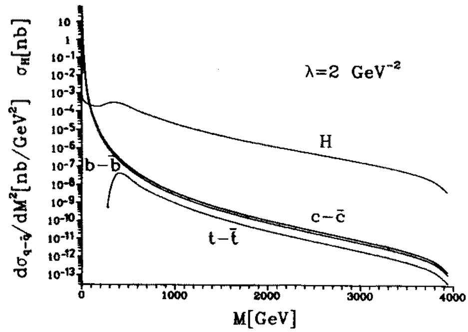

Fig. 7. The differential cross-section for the production of heavy quark–antiquark pair of given square of mass by double pomeron exchange in comparison with the Higgs production at $E_{CM} = 40$ TeV.

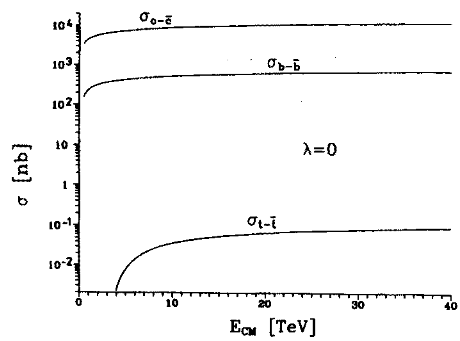

Fig. 8. Double pomeron exchange contribution to the cross-section for the heavy quarks production process versus the center mass energy.

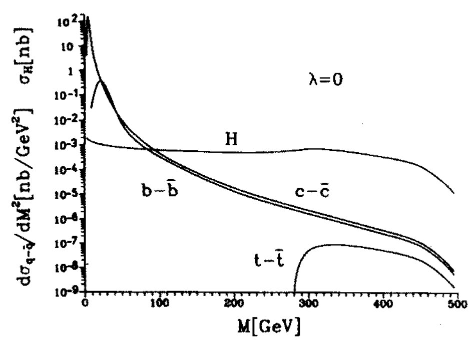

Fig. 9. The differential cross-section for the production of heavy quark–antiquark pair of given square of mass by double pomeron exchange in comparison with the Higgs production at $E_{CM} = 5$ TeV.

c, b and t on the center mass energy $E_{CM} = \sqrt{s}$ is shown in Fig. 4. Figures

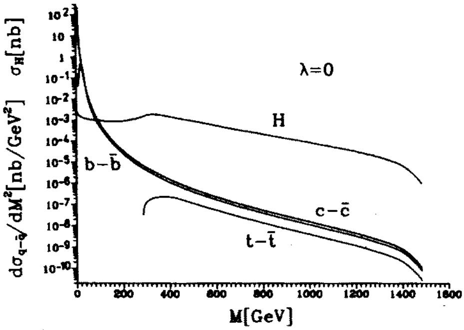

Fig. 10. The differential cross-section for the production of heavy quark–antiquark pair of given square of mass by double pomeron exchange in comparison with the Higgs production at $E_{CM} = 15$ TeV.

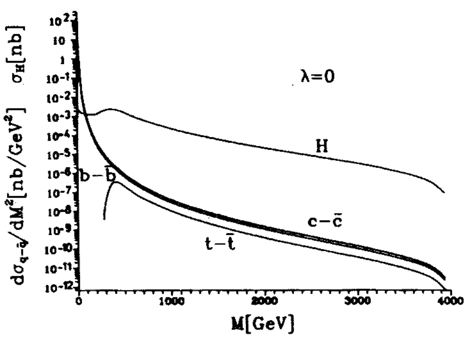

Fig. 11. The differential cross-section for the production of heavy quark–antiquark pair of given square of mass by double pomeron exchange in comparison with the Higgs production at $E_{CM} = 40$ TeV.

5–7 give the dependence of the differential cross-section $d\sigma/dM^2$ on the mass of produced quark–antiquark pair. The dependence of the cross-section for the Higgs production on the assumed mass of Higgs boson [2] is also shown in these figures. If one allows for diffractive dissociation of the incoming protons (*i.e.* one takes $\lambda = 0$) [2, 3], cross-sections increase by about one order of magnitude (see figures 8–11).

6. Conclusions

In the present paper we have calculated the contribution to heavy quarks production coming from double-pomeron exchange and compared it to the cross-section for production of the Higgs boson. The cross-sections for this two processes at three values of E_{CM} : 5 TeV, 15 TeV and 40 TeV are compared in the pictures 5, 6 and 7. One can see, that we can get a clear signal for Higgs production for the Higgs masses above 200 GeV. Below $m_H = 100$ GeV the $c\bar{c}$ and $b\bar{b}$ background can be a serious problem.

I am grateful to Professor Andrzej Bialas for suggesting this investigation and many helpful discussions.

Appendix A

The invariant matrix element at $t_1 = t_2 = 0$ is given by formula (14) derived in Section 3.

$$\begin{aligned} M_{fi} = & -4\sqrt{x_1 y_2}\,\delta^{s_1,s_1'}\delta^{s_2,s_2'}\frac{\pi D_0^3\mu^4 g^2 G^4}{9(2\pi)^2}\frac{m}{(s x_q y_q)^2\,\delta_1\delta_2} \\ & \times \bar{u}(r_2)\Big[x_q(1-y_2-y_q)\hat{p}_1\hat{p}_2 + y_q(1-x_1-x_q)\hat{p}_2\hat{p}_1 \\ & + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1-\delta_2\hat{p}_2)(x_q\delta_2-y_q\delta_1)\Big]v(r_1)\,. \end{aligned} \tag{24}$$

To obtain this element for arbitrary values of t_1 and t_2 we have to apply (2). The result is

$$\begin{aligned} M_{fi} = & -4\sqrt{x_1 y_2}\,\delta^{s_1,s_1'}\delta^{s_2,s_2'}\frac{\pi D_0^3\mu^4 g^2 G^4}{9(2\pi)^2}\frac{m}{(s x_q y_q)^2\,(\delta_1\delta_2)^{1+\epsilon}} \\ & \times \bar{u}(r_2)\Big[x_q(1-y_2-y_q)\hat{p}_1\hat{p}_2 + y_q(1-x_1-x_q)\hat{p}_2\hat{p}_1 \\ & + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1-\delta_2\hat{p}_2)(x_q\delta_2-y_q\delta_1)\Big]v(r_1)\frac{1}{\delta_1^{\alpha' t_1}\delta_2^{\alpha' t_2}}\,. \end{aligned} \tag{25}$$

Averaged matrix element is given by

$$\begin{aligned} \overline{|M_{fi}|^2} = & x_1 y_2\left(\frac{4\pi D_0^3\mu^4 g^2 G^4}{9(2\pi)^2}\right)^2\frac{m^2}{(s x_q y_q)^4\,(\delta_1\delta_2)^{2+2\epsilon}\,\delta_1^{2\alpha' t_1}\delta_2^{2\alpha' t_2}} \\ & \times \mathrm{Tr}\Big\{(\hat{r}_2+m)\Big[x_q(1-y_2-y_q)\hat{p}_1\hat{p}_2 + y_q(1-x_1-x_q)\hat{p}_2\hat{p}_1 \\ & + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1-\delta_2\hat{p}_2)(x_q\delta_2-y_q\delta_1)\Big](\hat{r}_1-m) \\ & \times\Big[x_q(1-y_2-y_q)\hat{p}_2\hat{p}_1 + y_q(1-x_1-x_q)\hat{p}_1\hat{p}_2 \\ & + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1-\delta_2\hat{p}_2)(x_q\delta_2-y_q\delta_1)\Big]\Big\}\,. \end{aligned} \tag{26}$$

Let us denote

$$TR = \mathrm{Tr}\Big\{ (\hat{r}_2 + m)\Big[x_q\eta\hat{p}_1\hat{p}_2 + y_q\xi\hat{p}_2\hat{p}_1 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big] \times (\hat{r}_1 - m)\Big[x_q\eta\hat{p}_2\hat{p}_1 + y_q\xi\hat{p}_1\hat{p}_2 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big]\Big\}, \tag{27}$$

where $\xi = 1 - x_1 - x_q$ and $\eta = 1 - y_2 - y_q$. To evaluate this trace one should commute $\hat{r}_1 - m$ to the left of expression in first square brackets. This gives

$$TR = \mathrm{Tr}\Big\{ (\hat{r}_2 + m)\Big[(\hat{r}_1 - m)(x_q\eta\hat{p}_1\hat{p}_2 + y_q\xi\hat{p}_2\hat{p}_1) + s(x_q\eta - y_q\xi)(\xi\hat{p}_1 - \eta\hat{p}_2) + \frac{ms}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)^2 - (\hat{r}_1 + m)\frac{ms}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big] \times \Big[x_q\eta\hat{p}_2\hat{p}_1 + y_q\xi\hat{p}_1\hat{p}_2 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big]\Big\}. \tag{28}$$

Using relations

$$\Big[x_q\eta\hat{p}_2\hat{p}_1 + y_q\xi\hat{p}_1\hat{p}_2 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big] \times (x_q\eta\hat{p}_1\hat{p}_2 + y_q\xi\hat{p}_2\hat{p}_1) = s^2x_qy_q\xi\eta + \frac{ms}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)(x_q\delta_1\eta\hat{p}_1 - y_q\delta_2\xi\hat{p}_2)$$

$$\Big[x_q\eta\hat{p}_2\hat{p}_1 + y_q\xi\hat{p}_1\hat{p}_2 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big] \times (\xi\hat{p}_1 - \eta\hat{p}_2) = s\xi\eta(x_q\hat{p}_1 - y_q\hat{p}_2) - \frac{m}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)(\xi\delta_2\hat{p}_1\hat{p}_2 + \eta\delta_1\hat{p}_2\hat{p}_1)$$

$$\Big[x_q\eta\hat{p}_2\hat{p}_1 + y_q\xi\hat{p}_1\hat{p}_2 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\Big] \times (\delta_1\hat{p}_1 - \delta_2\hat{p}_2) = s(x_q\delta_1\eta\hat{p}_1 - y_q\delta_2\xi\hat{p}_2) - ms(x_q\delta_2 - y_q\delta_1), \tag{29}$$

one obtains

$$\begin{aligned}
TR = &\mathrm{Tr}\left\{ (\hat{r}_2 + m)(\hat{r}_1 - m)\left[s^2 x_q y_q \xi\eta \right.\right. \\
&\left.\left. + \frac{ms}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)(x_q\delta_1\eta\hat{p}_1 - y_q\delta_2\xi\hat{p}_2)\right]\right\} \\
&+ s(x_q\eta - y_q\xi)\,\mathrm{Tr}\left\{ (\hat{r}_2 + m)\left[s\xi\eta(x_q\hat{p}_1 - y_q\hat{p}_2)\right.\right. \\
&\left.\left. - \frac{m}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)(\xi\delta_2\hat{p}_1\hat{p}_2 + \eta\delta_1\hat{p}_2\hat{p}_1)\right]\right\} \\
&+ \frac{ms}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)^2\,\mathrm{Tr}\left\{ (\hat{r}_2 + m)\left[x_q\eta\hat{p}_2\hat{p}_1\right.\right. \\
&\left.\left. + y_q\xi\hat{p}_1\hat{p}_2 + \frac{m}{\delta_1\delta_2}(\delta_1\hat{p}_1 - \delta_2\hat{p}_2)(x_q\delta_2 - y_q\delta_1)\right]\right\} \\
&- \frac{ms}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)\,\mathrm{Tr}\left\{ (\hat{r}_2 + m)(\hat{r}_1 + m)\right. \\
&\left. \times\left[s(x_q\delta_1\eta\hat{p}_1 - y_q\delta_2\xi\hat{p}_2) - ms(x_q\delta_2 - y_q\delta_1)\right]\right\}. \qquad (30)
\end{aligned}$$

Taking traces we arrive at

$$\begin{aligned}
TR = &2s^2 x_q y_q \xi\eta\left(s\delta_1\delta_2 - 4m^2\right) - \frac{4m^2 s^2}{\delta_1\delta_2} x_q y_q (x_q\delta_2 - y_q\delta_1)^2 \\
&- \frac{2m^2 s^2}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)^2(\xi\delta_2 + \eta\delta_1) \\
&+ \frac{2m^2 s^2}{\delta_1\delta_2}(x_q\delta_2 - y_q\delta_1)^2(x_q\eta + y_q\xi) \\
&- \frac{2m^2 s^2}{(\delta_1\delta_2)^2}(x_q\delta_2 - y_q\delta_1)^4 + 2m^2 s^2 (x_q\delta_2 - y_q\delta_1)^2 . \qquad (31)
\end{aligned}$$

Further simplification gives

$$TR = 2s^2 x_q y_q \xi\eta\left(s\delta_1\delta_2 - 4m^2\right) . \qquad (32)$$

Thus, finally, substituting (32) to (26), we have

$$\begin{aligned}
\overline{|M_{fi}|^2} = &2s x_1 y_2 \left(\frac{4\pi D_0^3 \mu^4 g^2 G^4}{9(2\pi)^2}\right)^2 \left(1 - \frac{4m^2}{s\delta_1\delta_2}\right) \\
&\times \frac{m^2}{(s x_q y_q)^2 (\delta_1\delta_2)^{1+2\epsilon}\, \delta_1^{2\alpha' t_1} \delta_2^{2\alpha' t_2}} . \qquad (33)
\end{aligned}$$

Appendix B

Formula (21) gives the cross-section for the heavy quarks production process in collision of two colourless quarks. To obtain the cross-section for production of heavy quarks in proton–proton collision we have to take into account factors related to colour degrees of freedom and presence of three quarks in each proton.

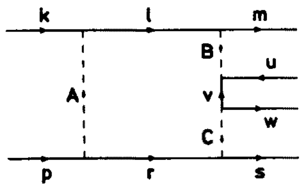

Fig. 12. The colour structure of heavy quark production process by double pomeron exchange. The capital letters denote colour indicies of gluons and small letters — colour indicies of quarks.

The additional factor in the square of the amplitude coming from the colour degree of freedom of quarks can be written as (see Fig. 12)

$$H = \frac{1}{9}\sum\left\{\tfrac{1}{4}\left(\lambda^A_{kl}\lambda^B_{lm}\right)_{\text{singlet}}\tfrac{1}{4}\left(\lambda^A_{pr}\lambda^C_{rs}\right)_{\text{singlet}}\tfrac{1}{4}\lambda^B_{uv}\lambda^C_{vw}\right\}^2,$$

where λ^A are Gell–Mann matrices, $^1/_9$ corresponds to averaging over colours of initial quarks, summing is performed over all gluon and quark indices and

$$\left(\lambda^A_{kl}\lambda^B_{lm}\right)_{\text{singlet}} = \tfrac{1}{8}\delta^{AB}\lambda^C_{kl}\lambda^C_{lm} = \tfrac{2}{3}\delta^{AB}\delta_{km}\,, \tag{35}$$

$$\left(\lambda^A_{pr}\lambda^C_{rs}\right)_{\text{singlet}} = \tfrac{1}{8}\delta^{AC}\lambda^B_{pr}\lambda^B_{rs} = \tfrac{2}{3}\delta^{AC}\delta_{ps}\,. \tag{36}$$

These expressions are related to the fact that pomeron is the colour singlet. Substituting (35) and (36) to (34) and performing all summations we obtain $H = (1/243)$. This should be multiplied by the factor 81 to take into account three quarks in each of the protons, so that the final result is $C_F = {}^1/_3$.

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