INTERNAL GEOMETRY OF n-BODY SYSTEMS AND ONE-PARTICLE STATES

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The nonrelativistic (NR) shell model of nuclei based on one-particle states results in a "surprise" pointed out by Peierls, which consists in the accelerated motion of "free" nucleus impossible to eliminate by the standard perturbation theory. The NR mechanics solves the puzzle by indicating a geometrically privileged position of the absolute configuration space generated by the three-dimensional space R_3 spanned on the relative coordinate "y". This converts the "n-body" problem into the "$(n-1)$-relational-object" problem with absolute (frame-independent) structures. The relativistic kinematics shows that the Galilean-absolute characteristics of the system obtained in the $\overbrace{R_3 \times \ldots \times R_3}^{n-1}$ configuration space remain absolute in the true Minkowskian spacetime L_4 of measurement. However, this necessitates abandoning the classical L_4-eventism as the first metrical continuum of physics.

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1. The NR n-body problem and eventism

Let $\boldsymbol{X}$ denote the NR centre-of-mass coordinate of the n-body system and $\boldsymbol{P}$ its total momentum canonically conjugate to $\boldsymbol{X}$ in the space E_3 of an arbitrary inertial reference frame S. On the other hand, let $y_1, \ldots, y_{n-1}$ denote some independent, relative coordinates of that system which would parametrize its internal states in the configuration space $\overbrace{R_3 \times \ldots \times R_3}^{n-1}$ and $q_1, \ldots q_{n-1}$ are the canonically conjugate momenta to $y_1, \ldots, y_{n-1}$. The singularity of symmetry G of the Galilean spacetime G_4 of events X reveals itself in the existence of the absolute "relational" space $R_3(y)$ which makes canonical coordinates $\boldsymbol{y}_s$, $\boldsymbol{q}_s$ ($s = 1, \ldots, n-1$) independent of the reference frame S, *i.e.* form-invariant under the Galilean transformations;

$$\boldsymbol{y}'_s = \boldsymbol{y}_s , \qquad \boldsymbol{q}'_s = \boldsymbol{q}_s . \tag{1.1}$$

The total Hamiltonian H^{G} becomes separated into the external-relative Hamiltonian $\boldsymbol{P}^2/2m$ and the internal-absolute Hamiltonian h^{G},

$$H^{\mathrm{G}} = \frac{\boldsymbol{P}^2}{2m} + h^{\mathrm{G}}(\boldsymbol{y}_1, \ldots, \boldsymbol{q}_{n-1}) . \tag{1.2}$$

Here $m = \sum_{j/1}^{n} m_j$ denotes the total NR mass of the system, where m_j is the mass of its j-th point-constituent A_j. The point is that the above separation of H^{G} provides us with h^{G} parametrized by G-form-invariant dynamical variables (1.1) hence its internal-energy eigenvalues w^{G} are a priori G-absolute. The same concerns classical mechanics where $w^{\mathrm{G}} = h^{\mathrm{G}}$. Note that rotation symmetry contained in the G-symmetry implies that h^{G} depends on the rotation-scalars only. Thus G is the internal symmetry group of the classical as well as quantum equations of motion following from H^{G}, which also reflects the full isolation of our n-body system.

Let us note that the Galilean principle of relativity does not require G to be the internal symmetry group of equations of motion; it is enough that they are G-covariant. This — in Wigner's terminology [1] — means the *"passive"* and not *"active"* role of the symmetry G which guarantees that G_4 is the background of these equations. The "passive" role of symmetry G takes place in the presence of any external field whose analytic representations in various reference frames S are different. In such a case the isolation of the system is broken.

Now let us focus our attention on a fundamental difference between the classical and the quantum description of the n-body system which questions the classical eventism of quantum bound states. By *"eventism"* we mean the hypothesis of the classical and also of the present quantum theory saying that the continuum of directly observable events X (spacetime) is the first (unanalyzable) metrical background of physics.

The singularity of G_4 makes the configuration space $\overbrace{\mathrm{E}_3 \times \ldots \times \mathrm{E}_3}^{n}$ — where $\mathrm{E}_3(\boldsymbol{X}_j)$ is its subspace of the constituent A_j in some reference frame S — and $\mathrm{E}_3 \times \overbrace{\mathrm{R}_3 \times \ldots \times \mathrm{R}_3}^{n-1}$ of H^{G} from (1.2) coincide. This is so, because the coordinates $\boldsymbol{X}_j$ and $\boldsymbol{X}$, $\boldsymbol{y}_s$ are connected by a point transformation, which means that they represent two different parametrizations of the same configuration space. Consequently, classical equations of motion result in the trajectory which takes the form:

$$\boldsymbol{X} = \boldsymbol{X}(t)\,, \quad \boldsymbol{y}_s = \boldsymbol{y}_s(t) \qquad \text{in} \qquad \mathrm{E}_3 \times \overbrace{\mathrm{R}_3 \times \ldots \times \mathrm{R}_3}^{n-1} \tag{1.3a}$$

or

$$\boldsymbol{X}_j = \boldsymbol{X}_j(t) \qquad \text{in} \qquad \overbrace{\mathrm{E}_3 \times \ldots \times \mathrm{E}_3}^{n}\,. \tag{1.3b}$$

The eventism reflected in the very structure of trajectory (1.3) (which represents the classical state of the system) implies that (1.3) determines the trajectory of A_j at a moment t_j and the trajectory of another constituent A_k at a moment t_k, thus from (1.3) we obtain

$$\boldsymbol{X}_j = \boldsymbol{X}_j(t_j)\,. \tag{1.4}$$

In quantum mechanics, however, the situation is quite different: Let us assume that the system as a whole is in the eigenstate of $\widehat{\boldsymbol{P}}$ to the eigenvalue $\boldsymbol{P}$, while its internal state is a bound eigenstate of h^{G} to the eigenvalue w^{G} (< 0) denoting the internal-absolute energy of the system. Then, in $\mathrm{E}_3 \times \overbrace{\mathrm{R}_3 \times \ldots \times \mathrm{R}_3}^{n-1}$ the quantum state of the whole system takes the form:

$$\Psi = A\mathrm{e}^{i/\hbar(\boldsymbol{P}\boldsymbol{X} - Et)}\psi_{w^{\mathrm{G}}}(y_1, \ldots, y_{n-1})$$

with

$$E^{\mathrm{G}} = \frac{\boldsymbol{P}^2}{2m} + w^{\mathrm{G}}\,. \tag{1.5}$$

Of course, Ψ can be parametrized in the $\boldsymbol{X}_j$ variables when Ψ takes the form $\Psi(\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n; t)$ but cannot be extended onto times t_j different for various constituents A_j. This shows the well known indivisibility of quantum state and, at the same time, the collapse of classical eventism. In fact, if n-constituents are embedded in the pre-existing spacetime continuum, the question about the space localization (not necessarily sharp) of constituent A_j at moment t_j and constituent A_k at moment t_k can be legitimately asked. This suggests that the absolute, relational space $\mathrm{R}_3(y)$ — though admitted by the singularity of symmetry G — precedes the classical eventism.

The four-symmetry of the L_4-eventism excludes any point-transformation that could determine some L-absolute, three-dimensional space R_3. Therefore, the extension of $\mathrm{R}_3(y)$ onto physics of the finite c must explicitly abandon the classical eventism [2, 3] — *cf.* Chapter 4. However, we show [2] that the internal spacetime $\mathrm{R}_4 = R_3(y) \times \mathrm{T}(\tau)$ of directly unobservable "relations" $\boldsymbol{y}$ and the internal time continuum $\mathrm{T}(\tau)$ make L_4 their limiting case. This is conditioned by situations that accompany any direct measurement with one term (particle) of the relation y becoming infinitely heavy and (as such) ceasing to participate in equations of motion, which changes the status of the spacetime of measurement from the eventistic into the relational one.

The fact that symmetry G of the classical G_4-eventism makes the R_4-relationism accessible would explain the tremendous success of NR quantum mechanics. All the same, the transport phenomena in the NR low energy microphysics strongly suggest that the classical spacetime of events X is not their proper background [4, 5]. The same follows from the EPR quantum correlations [6]. They break Bell's inequalities [7] in accordance with non-locality of quantum mechanic and are hardly reconcilable with the locality of the L_4-eventism [8].

2. The "surprise" of one-particle states

The shell model assumes that n noninteracting, with one another, nucleons interact quasi-elastically with an immobile (external) centre, which results in the nucleon states Φ built from one-particle states. The external centre causes accelerated motion of the centre-of-mass $\boldsymbol{X}$ of "free" nucleus which, according to the Peierls "surprise", cannot be eliminated by standard perturbation methods [9]. Note that since nuclei are composed of n equal-mass nucleons, the shell model does not seem to be realistic at all. That was the reason why we had no proper model of nuclei structure for a long time [10]. Therefore, the success of the shell model offers another surprise explained by Kramer and Moshinsky (KM) [11]. Their model of the nucleus also indicates an essential role of the relational geometry R_4. As we know, the singularity of symmetry G makes us to avoid to explicitly introduce the R_4-hypothesis resorting to a suitable change of parametrization of the system — of its configuration space. Then, let us recall the main points of the KM paper with particular emphasis on its geometrical aspect.

Instead of the external potential of the shell model, KM introduce an internal potential of quasi-elastic pair forces,

$$V = \frac{m_0\omega^2}{4n} \sum_{j,k/1}^{n} \left(\boldsymbol{X}_j - \boldsymbol{X}_k\right)^2 , \tag{2.1}$$

where m_0 is the nucleon mass and thus, $m = nm_0$ denotes the NR mass of the nucleus. Then the total Hamiltonian takes the form:

$$H^{\mathrm{G}} = \sum_{j/1}^{n} \frac{\boldsymbol{P}_j^2}{2m_0} + V\left(\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n\right) \tag{2.2}$$

in $\overbrace{E_3 \times \ldots \times E_3}^{n}$ parametrization, where the internal dynamics (2.1) guarantees free motion of the centre-of-mass $X = (1/n) \sum_{j/1}^{n} X_j$ of the nucleus. A major advantage of the KM model in that it deals with interactions between all nucleons without introducing an artificial "centre" which was justified in the atomic but not in the nuclear physics.

The singularity of symmetry G and a particular symmetry of quasi-elastic interaction (2.1) make the eigenproblem of H^{G} solvable without resorting to perturbation methods. This is due to the change of parametrization of H^{G}, which is also of first importance in constructing the states of composite systems. Such was the main goal of the KM paper [11] and became crucial for determining the structure of any composite system, in particular, of quark model [12]. The Jacobi transformation used by KM which realizes the transition from $\overbrace{E_3 \times \ldots \times E_3}^{n}$ to $E_3 \times \overbrace{R_3 \times \ldots \times R_3}^{n-1}$ parametrization takes the form:

$$
\begin{aligned}
y_s &= [s(s+1)]^{-1/2} \sum_{t/1}^{s} X_t - \left(\frac{s}{s+1}\right)^{1/2} X_{s+1}\,, \qquad (1 \le s \le n-1)\,,\\
\overline{X} &= \left(n^{-1/2}\right) \sum_{j/1}^{n} X_j = \left(n^{1/2}\right) X\,,
\end{aligned} \tag{2.3}
$$

where the letters y and x distinguish between the relational (in R_3) and the relative (in E_3) coordinates.

One easily finds that (2.3) results in the identities:

$$
\begin{aligned}
\sum_{j/1}^{n} X_j^2 &= \overline{X}^2 + \sum_{s/1}^{n-1} y_s^2\,,\\
\sum_{j/1}^{n} X_j^2 &= n\overline{X}^2 = n^2 X^2 = \sum_{j,k/1}^{n} X_j X_k\,,
\end{aligned} \tag{2.4}
$$

which imply that in the new variables H^{G} from (2.2) takes the form (1.2);

$$
H^{G} = \frac{\overline{P}^2}{2m_0} + \left\{ \sum_{s/1}^{n-1} \left(\frac{q_s^2}{2m_0} + \frac{m_0\omega^2}{2} y_s^2 \right) \right\} \equiv \frac{\overline{P}^2}{2m_0} + h^{G}\,. \tag{2.5}
$$

Here free nucleus as a single particle is described by the canonical variables $\overline{X}, \overline{P}$. In the Appendix the whole class of transformations with n arbitrary parameters is presented, which realizes the same symmetries as (2.3).

The Hamiltonian H^{G} from (2.5) completes our first thesis, as instead of the "n-body" Hamiltonian (2.2), it accounts for one free body (nucleus) embedded in E_3 and the "$(n-1)$-relational-oscillators" embedded in $\overbrace{\mathrm{R}_3 \times \ldots \times \mathrm{R}_3}^{n-1}$. In our opinion this explains the success of the shell model, provided that the number of "oscillators" is reduced from n to $(n-1)$. However, in order to regain the symmetry of n-oscillators, KM add to H^{G} from (2.5) the external potential

$$\overline{V} = \frac{m_0\omega^2}{2}\overline{X}^2 \qquad \left(= \frac{m\omega^2}{2}X^2\right) \tag{2.6}$$

thus getting the shell-model-like Hamiltonian $\overline{H}^{\mathrm{G}}$ of n (mathematically) identical spherical oscillators,

$$\overline{H}^{\mathrm{G}} = H^{\mathrm{G}} + \overline{V}(\overline{X}) = \left(\frac{\overline{P}^2}{2m_0} + \frac{m_0\omega^2}{2}\overline{X}^2\right) + \sum_{s/1}^{n-1}\left(\frac{q_s^2}{2m_0} + \frac{m_0\omega^2}{2}y_s^2\right). \tag{2.7}$$

As the external potential (2.6) breaks the internal symmetry G of H^{G}, the $\overline{H}^{\mathrm{G}}$ obviously results in the accelerated motion of the centre-of-mass of the nucleus.

Consequently, if one starts with $\overline{H}^{\mathrm{G}}$ as an "unperturbed" Hamiltonian resulting in the n independent spherical oscillators which result in the one-particle states Φ of the nucleus, the "perturbation" of $\overline{H}^{\mathrm{G}}$ which restores free nucleus amounts to:

$$V' = -\overline{V}(\overline{X}). \tag{2.8}$$

The Peierls "surprise" [9] consists just in the fact that the corresponding standard perturbation series are always divergent and so, we can never restore free motion of the nucleus.

On the other hand, if we start with H^{G} from (2.5) then, in order to modify the interaction (2.1) into a more realistic one, we can switch on a suitable perturbing potential of internal forces in the configuration space $\overbrace{\mathrm{R}_3 \times \ldots \times \mathrm{R}_3}^{n-1}$,

$$V'' = V''\left(y_1, \ldots, y_{n-1}\right). \tag{2.9}$$

The interaction V'' of the "$(n-1)$-relational-object" problem obviously does not affect free motion of the nucleus as a whole and, the standard perturbation methods will usually provide us with convergent corrections of the internal-absolute structure and the internal-absolute energy levels w_n^{G} of the nucleus.

From the viewpoint of the R_4-relationism [2], the metrical relations of the "$(n-1)$-relational-object" problem with internal-absolute Hamiltonian $h^G(y_1, \ldots, q_{n-1})$ precede those of the n-particles embedded in the spacetime of events. The priority of directly unobservable relations $\boldsymbol{y}$ in R_3 over the directly observable points $\boldsymbol{X}$ in E_3 (of some S), allowed by quantum symmetry, and the experimentally privileged momentum language p, reflect the priority of dynamics over kinematics. This hierarchy seems reasonable, as physics without dynamics does not exist, and any dynamics (by its very nature) is relational, because it starts with at least two (hypothetical) objects. Within that philosophy, the one-body problem in an external field spanned on the events X represents the limiting case of the two-body problem (elementary in R_4) when one term (particle) of the relation $\boldsymbol{y}$ becomes infinitely heavy and, therefore, ceases to participate in equations of motion in R_4.

The kinematics represents a singular case, because one term of the relation $\boldsymbol{y}$ of two non-interacting particles can always be regarded as infinitely heavy. Therefore, the spacetime of measurement accounts adequately for kinematics, regardless of whether this is the Galilean (G_4) or the Minkowskian (L_4) spacetime. Since quantum physics has eliminated the directly observable classical trajectory (1.3) replacing it by the directly unobservable quantum state (1.5), the "measuring possibilities" make room for the hypothesis of quantum-geometry R_4 which remains consistent with the classical L-symmetry of measurement. The point is that measurement of microprocesses are indirect, *i.e.* performed in the x-non-local momentum-energy language p [13]. This fact is manifest in the theory of the S-matrix whose elements S_{fi} are adequately parametrized by suitable momentum-invariant Mandelstam variables s_J,

$$S_{fi} = \delta^{(4)}\left(P_i - P_f\right) T_{fi}\left(s_1, \ldots, s_K\right) , \tag{2.10}$$

where $P_{i,f}$ denote the initial and final fourmomenta of the isolated system inside which the quantum collision process takes place.

3. Symmetry L of kinematics; momentum and velocity languages

The limit $c \to \infty$ that converts the symmetry L into G exhibits an asymmetry between the "quantum" energy-momentum language p and the "classical" velocity language v. This is due to the fact that transition from L to G symmetries makes the notion of four-momentum P vanish. Together with it goes another discontinuity, as the *one* L-form-invariant four-interval x^2 splits into *two* G-form-invariant intervals $r = |y|$ and Δt.

Let $P = (\boldsymbol{P}; E/c)$ denote the four-momentum P of an isolated n-body system in a fixed but arbitrary reference frame S. Hence

$$P^2 = \boldsymbol{P}^2 - \frac{E^2}{c^2} = -M^2c^2 = -\frac{W^2}{c^2}, \qquad (W = Mc^2),$$

or

$$E = (W^2 + c^2\boldsymbol{P}^2)^{1/2} = W\left(1 + \frac{c^2\boldsymbol{P}^2}{W^2}\right)^{1/2} = W\Gamma. \tag{3.1}$$

Here W is the internal-absolute energy of the system, $M = W/c^2$ is its L-invariant mass in a given internal state, and

$$\Gamma = \left(1 - \frac{\boldsymbol{V}^2}{c^2}\right)^{-1/2} \quad \text{with} \quad \boldsymbol{V} = \frac{\boldsymbol{P}}{\left(M^2 + (\boldsymbol{P}^2/c^2)\right)^{1/2}} \tag{3.2}$$

is the Lorentz factor of the system expressed in the v language.

As seen from (3.1), the total energy E is separated into the internal-absolute energy W and the external-relative term $\boldsymbol{P}^2$. However, the L_4-eventism excludes any absolute dynamical variables such as are $\boldsymbol{y}$, $\boldsymbol{q}$ in the G_4-eventism (1.1), which would provide us with W as an eigenvalue of some operator accounting for the internal-absolute dynamics of the system. Note that W as in (3.1) is determined from the outside of the system by a pure kinematic relation.

The convergence of the series of E from (3.1) expanded into powers of $c^2\boldsymbol{P}^2/W^2 = \Gamma^2 - 1$, required by the NR kinematics expressed in the p-language, demands

$$\frac{c^2\boldsymbol{P}^2}{W^2} < 1, \qquad i.e.: \qquad \boldsymbol{V}^2 < \frac{c^2}{2}. \tag{3.3}$$

Such a limitation of $\boldsymbol{V}^2$ is alien to the classical v-language, as Γ can be expanded into convergent power-series of $\boldsymbol{V}^2/c^2$ for

$$\frac{\boldsymbol{V}^2}{c^2} < 1, \tag{3.4}$$

which is fulfilled automatically. It is true that in the mathematical limit $c \to \infty$ both upper limits of $\boldsymbol{V}^2$, $c^2/2$ and c^2, tend to infinity and thus remain consistent with symmetry G of the NR physics. However, realistically, if one regards the NR theory as a decent one of loosely bound systems, the p-language limitation (3.3) of quantum physics creates an essential asymmetry between the p- and the v-languages.

Since $m = \sum_{j/1}^{n} m_j$ is the sum of the absolute masses of the constituents A_j, which coincides with the NR mass of the system (independently of its internal state), we split W into two absolute (frame-independent) components,

$$W = mc^2 + w\,, \tag{3.5}$$

where for loosely bound "NR" systems there is $|w/mc^2| \ll 1$. Then, E from (3.1) can be rewritten in the form:

$$E = mc^2 \left[\left(1 + \frac{w}{mc^2}\right)^2 + \frac{\boldsymbol{P}^2}{m^2c^2} \right]^{1/2} \tag{3.6}$$

and taking into account the inequality (3.3) we get

$$E = mc^2 + w + \frac{\boldsymbol{P}^2}{2m} + 0\left(\frac{1}{c^2}\right)\,, \tag{3.7}$$

where $0(1/c^2)$ vanishes with $c \to \infty$. Finally,

$$\lim_{c\to\infty} \left(E - mc^2\right) = E^{\mathrm{G}} = \frac{\boldsymbol{P}^2}{2m} + w^{\mathrm{G}} \tag{3.8}$$

which coincides with (1.5), as

$$w^{\mathrm{G}} = \lim_{c\to\infty} w\,. \tag{3.9}$$

The separation of $E = (W^2 + c^2\boldsymbol{P}^2)^{1/2}$ into the absolute W^2 and the relative $\boldsymbol{P}^2$ quantities implies that the limit $c \to \infty$ concerns two, fully independent characteristics of the system: Its external motion can be "ultra-relativistic", but, if the system is loosely bound, the frame-independent (absolute) energy $W = mc^2 + w$ remains very well approximated by $W \cong mc^2 + w^{\mathrm{G}}$, where w^{G} is the eigenvalue of the NR internal Hamiltonian h^{G}. Consequently, the Einsteinian energy-mass relation provides us (for loosely bound systems) with the L-absolute mass defect ΔM equal to

$$\Delta M \cong -\frac{w^{\mathrm{G}}}{c^2} \qquad (w^{\mathrm{G}} < 0)\,. \tag{3.10}$$

Within this approach we have preserved the absoluteness of the R_3-geometry of relations $\boldsymbol{y}$ taken from the singularity of symmetry G and the symmetry L of the relativistic kinematics. This separation of dynamical variables into the internal ($\boldsymbol{y}$) and the external ($\boldsymbol{X}$) certainly conflicts with the L_4-eventism, as it tacitly assumes the hypothesis of quantum-geometry R_4 of the internal-absolute, and directly unobservable, relations y.

4. Relational geometry R_4

Direct non-observability of the internal-absolute quantum structures of bound systems, which are measured indirectly in the "complementary" p-language, enables us to extend the relationism of the R_3-geometry coexisting with the G_4-eventism to physics of finite c [2, 3]. The correspondence between the internal-absolute structures obtained in R_3 and measurement calls for a go-between which is the four-dimensional space $\overline{L}_4(p)$ of the four-momenta p. This correspondence which essentially distinguishes the p-language of measurement of microprocesses — *cf.* (2.10) — yields the definition of the R_3-space.

In fact, if $F(y^2)$ and $\widetilde{F}(q^2)$ represent the same Hilbert vector $|F)$ in $R_3(y)$ and $R_3(q)$, where q is the relational-momentum canonically conjugate to y, we define the manifestly L-form-invariant function $\widetilde{G}(p^2)$ as equal to

$$\widetilde{G}\left(p^2\right) = \widetilde{F}\left(q^2 = p^2 \geq 0\right) , \tag{4.1}$$

which is embedded in $\overline{L}_4(p)$ with the internal symmetry $\overline{L}$; $\overline{L}$ represents the homogeneous group of Lorentz transformations. Indefinite metrics of symmetry $\overline{L}$ calls for the extension of $\widetilde{F}(q^2)$ to the negative q^2's, thus determining $\widetilde{G}(p^2)$ in the whole four-space $\overline{L}_4(p)$. Again, the quantum $x-p$ duality (though now in the four-space $\overline{L}_4$) determines the manifestly L-form-invariant distribution $G(x^2)$ in the four-space $\overline{L}_4(x)$ spanned on the relative four-coordinate x.

The same procedure in G_4 results in the G-form-invariant distribution:

$$G(x) = F(x^2)\delta(\Delta t) = F(y^2)\delta(\Delta t) \tag{4.2}$$

which exhibits the coexistence of the G_4-eventism with the R_3-relationism, *i.e.*, the singularity of symmetry G.

The conclusion is that if we want to extend the NR separability of the internal-absolute from the external-relative degrees of freedom onto measurement of microprocesses, which follows the symmetry L of relativistic kinematics, then we must introduce the space-and-time nonlocality of the L-form-invariant structures $G(x^2)$. Otherwise the form factors of composite particles cannot represent the L-form-invariant shapes $G(x^2)$ separated from external motion of their carriers, which results in the "relativistic distortions" of the particle structures [14]. Let us remember that the locality of the classical L_4-eventism excludes any structure-particle [15] and makes the relativistic equations of Dirac, Klein–Gordon, *etc.*, describe one-particle problem only.

Therefore, if the single-particle Dirac equation of electron in the external Coulomb field is recognized as the theory of the hydrogen atom structure [16], it automatically means that the nucleus is identified with an infinitely heavy, external centre. Of course, the state of isolation of such an atom is broken, much like the state of the nucleus within the standard shell model described by the one-particle states Φ.

In the NR physics the external-centre approximation can be avoided, because in the two-body problem ($n = 2$) the six-dimensional configuration space can be parametrized in

$$\mathrm{E}_3 \times \mathrm{E}_3 = \mathrm{E}_3 \times \mathrm{R}_3 \,. \tag{4.3}$$

In R_3 we deal with one-relational-object which mathematically not geometrically coincides with the one-body problem, provided that the electron mass m_e is replaced by the reduced mass μ equal to $\mu = m_e/(1 + m_e/M_{\mathrm{N}})$, where M_{N} denotes the nucleus mass. The corrections due to $m_e \neq \mu$ are considerable and perfectly testified experimentally in favour of the value μ. This implicitly proves the hypothesis of $\mathrm{R}_3(y)$ relational space which, in physics of finite c, must precede the spacetime of events.

Appendix

The Jacobi transformation (2.3) which changes the parametrization of the same configuration space in G_4 according to the identity

$$\overbrace{\mathrm{E}_3 \times \ldots \times \mathrm{E}_3}^{n} = \mathrm{E}_3 \times \overbrace{\mathrm{R}_3 \times \ldots \times \mathrm{R}_3}^{n-1} \tag{A1}$$

preserving, moreover, the generalized orthogonal relations (2.4) makes room for the construction of states of n-body systems with appropriate internal symmetries. As known, these symmetries are of fundamental importance in deciphering the structures of composite systems, in particular, of quark-structure of hadrons [12]. Therefore it is interesting to point out that besides (2.3), the KM Hamiltonian (2.2) allows a larger class of point transformations that realize (A1) and the separation of $(n-1)$-relational-objects", as in (2.5). For example:

$$\widetilde{\boldsymbol{X}} = \frac{K}{n}\left(\boldsymbol{X}_1 + \boldsymbol{X}_2 + \ldots + \boldsymbol{X}_n\right) ,$$
$$y_1 = a_1 \left[\boldsymbol{X}_1 - \frac{1}{n-1}\left(\boldsymbol{X}_2 + \boldsymbol{X}_3 + \ldots + \boldsymbol{X}_n\right)\right] ,$$

$$y_2 = a_2 \left[X_2 - \frac{1}{n-2} (X_3 + X_4 + \ldots + X_n) \right] ,$$

$$\vdots$$

$$y_{n-2} = a_{n-2} \left[X_{n-2} - \tfrac{1}{2} (X_{n-1} + X_n) \right] ,$$

$$y_{n-1} = a_{n-1} \left[X_{n-1} - X_n \right] , \tag{A2}$$

with arbitrary n-parameters $K, a_1, \ldots, a_{n-1}$ results in the following identities:

$$I = \sum_{j,k/1}^{n} (X_j - X_k)^2 = 2n \sum_{s/1}^{n-1} \left(\frac{n-s}{n+1-s} \right) a_s^{-2} y_s^2 , \tag{A3i}$$

$$J = \sum_{j/1}^{n} \dot{X}_j^2 = nK^{-2} \dot{\tilde{X}}^2 + \sum_{s/1}^{n-1} \left(\frac{n-s}{n+1-s} \right) a_s^{-2} \dot{y}_s^2 . \tag{A3ii}$$

Since the potential (2.1) and the kinetic energy T of the n-body system (nucleus) amount to

$$V = \frac{m_0 \omega^2}{4n} I , \qquad T = \frac{m_0}{2} J , \tag{A4}$$

the Hamiltonian H^{G} from (2.2) takes the form:

$$H^{\mathrm{G}} = \frac{\tilde{P}^2}{2\tilde{m}} + \left\{ \sum_{s/1}^{n-1} \left(\frac{q_s^2}{2\tilde{m}_s} + \tilde{m}_s \omega^2 y_s^2 \right) \right\} \equiv \frac{\tilde{P}^2}{2\tilde{m}} + h^{\mathrm{G}} \tag{A5}$$

with

$$\tilde{m} = mK^{-2} , \qquad \tilde{m}_s = m_0 \left(\frac{n-s}{n+1-s} \right) a_s^{-2} .$$

The mass-parameters $\tilde{m}$, $\tilde{m}_s$ are then dependent on the units in the corresponding configuration sub-spaces E_3, R_3 of the configuration space $E_3 \times \overbrace{R_3 \times \ldots \times R_3}^{n-1}$ and it is for us to decide which units to choose. The essential point is that (A5) realizes the separation of the internal-absolute y from the external-relative X degrees of freedom by realizing the right-hand side of the NR identity (A1). This follows the general philosophy of relationism which, in quantum physics, can be extended to physics of constant c, thus preserving the NR separation of the internal from the external dynamical variables excluded by the L_4-eventism.

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