SADDLE POINTS OF STRINGY ACTION

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It is shown that Einstein–Yang–Mills-dilaton theory has a countable family of static globally regular solutions which are purely magnetic but uncharged. The discrete spectrum of masses of these solutions is bounded from above by the mass of extremal Gibbons–Maeda solution. As follows from linear stability analysis all solutions are unstable.

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1. Introduction

In a seminal paper [1] Bartnik and Mckinnon (BM) have discovered a countable family of globally regular static spherically symmetric solutions of the Einstein–Yang–Mills (EYM) equations. A rigorous proof of existence of BM solutions was given by Smoller *et al.* [2]. Recently, Sudarsky and Wald have proposed a heuristic argument [3], in the spirit of Morse theory, which explains existence and properties of BM solutions. This argument exploits the existence of topologically nonequivalent multiple vacua in the SU(2)-YM theory (which is related to the fact that $\pi_3(\mathrm{SU}(2)) \simeq Z$) and is essentially insensitive to the concrete form of the coupling, which suggests that there should exist solutions similar to BM solutions in other theories involving the SU(2)-YM field. Indeed, such solutions were found in YM-dilaton theory [4, 5] and remarkable parallels between these solutions and BM solutions were observed [5].

The YM-dilaton theory and the Einstein–YM theory may be embedded in a single Einstein–YM-dilaton theory governed by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{G}R - 2(\nabla\phi)^2 - \mathrm{e}^{-2a\phi}\mathcal{F}^2\right] , \tag{1}$$

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where R is a scalar curvature, ϕ is a dilaton, $\mathcal{F}$ is a Yang–Mills curvature, G is Newton's constant and a is the dilaton coupling constant. This theory is characterized by a dimensionless parameter $\alpha = G/a^2$. When $\alpha = 0$ the action (1) reduces to the YM-dilaton theory. When $\alpha \to \infty$ the action (1) becomes the Einstein–YM theory (plus trivial kinetic term for the scalar field). Finally, the case $\alpha = 1$ corresponds to the low-energy action of heterotic string theory.

The aim of this paper is to show that the theory defined by the action (1) has (for all values of α) static spherically symmetric globally regular solutions with the following properties:

a) there exist a countable family of solutions X_n ($n \in N$),
b) the total mass M_n increases with n and is bounded from above,
c) the solution X_n has exactly n unstable modes.

The solutions X_n depend continuously on α and interpolate smoothly between the YM-dilaton solutions ($\alpha = 0$) and BM solutions ($\alpha = \infty$). In the limit $n \to \infty$ the solution $X_n(\alpha)$ tends to the abelian extremal charged dilatonic black hole [6, 7], whose mass therefore provides un upper bound for the spectrum M_n. These results were previously announced by the author in [5] and later the case $\alpha = 1$ was analysed in [8].

The paper is organized as follows. In the next Section the field equations are derived and some scaling properties are discussed. In Section 3 the explicit abelian solutions are described. In Section 4 the numerical non-abelian solutions are presented and their qualitative properties are discussed. Section 5 is devoted to linear stability analysis.

2. Field equations

We are interested in static spherically symmetric configurations. It is convenient to parametrize the metric in the following way

$$ds^2 = -A^2 N dt^2 + N^{-1} dr^2 + r^2 (d\vartheta^2 + \sin^2\vartheta d\varphi^2)\,, \tag{2}$$

where A and N are functions of r.

I assume that the electric part of the YM field vanishes (actually this is not a restriction because one can show, following the argument given in [9], that there are no globally regular static solutions with nonzero electric field). The purely magnetic static spherically symmetric SU(2) YM connection can be written, in the abelian gauge, as [10]

$$e\mathcal{A} = w\tau_1 d\vartheta + (\cot\vartheta\tau_3 + w\tau_2)\sin\vartheta d\varphi\,, \tag{3}$$

where τ_i ($i = 1, 2, 3$) are Pauli matrices and w is a function of r. The corresponding YM curvature $\mathcal{F} = d\mathcal{A} + e\mathcal{A} \wedge \mathcal{A}$ is given by

$$e\mathcal{F} = w'\tau_1 dr \wedge d\vartheta + w'\tau_2 dr \wedge \sin\vartheta d\varphi - (1 - w^2)\tau_3 d\vartheta \wedge \sin\vartheta d\varphi\,, \tag{4}$$

where prime denotes derivative with respect to r.

For these anzätze and for $\phi = \phi(r)$ the action (1) gives the Lagrangian (where $S = 16\pi \int L dt$)

$$L = \int_0^\infty A(m' - U) dr \,, \tag{5}$$

where the mass function $m(r)$ is defined by $N = 1 - 2Gm/r$ and

$$U = \tfrac{1}{2} r^2 N \phi'^2 + \frac{1}{e^2} \mathrm{e}^{-2a\phi} \left[N w'^2 + \frac{(1-w^2)^2}{2r^2} \right] , \tag{6}$$

Hereafter, for convenience, I put the coupling constants $e = a = 1$ which means that I choose a/e as the unit of length and $1/ea$ as the unit of energy. Then the system is characterized by one dimensionless parameter $\alpha = G/a^2$. Variation of L with respect of m, A, w, and ϕ yields the field equations[1]

$$m' = U \,, \tag{7}$$

$$A' = \alpha A (r \phi'^2 + \frac{2}{r} \mathrm{e}^{-2\phi} w'^2) \,, \tag{8}$$

$$(A N \mathrm{e}^{-2\phi} w')' + \frac{1}{r^2} A \mathrm{e}^{-2\phi} w (1 - w^2) = 0 \,, \tag{9}$$

$$(r^2 A N \phi')' + 2A \mathrm{e}^{-2\phi} \left[N w'^2 + \frac{(1-w^2)^2}{2r^2} \right] = 0 \,. \tag{10}$$

Note that L has a characteristic for general relativity "pure constraint" form, namely the integrand in Eq. (5) is the hamiltonian constraint, Eq. (7), multiplied by the lapse function A. Actually L is not differentiable because the variation of L gives an unwanted surface term at infinity. To remedy this one has to add to L the Regge–Teitelboim correction term, $-A(\infty)m(\infty)$. Then $L' = L - A(\infty)m(\infty)$ has a well-defined functional derivative (since I deal with globally regular solutions I assume that $m(0) = 0$). It is convenient to define the energy functional

$$E = -L' = \int_0^\infty (AU + A'm) dr \,, \tag{11}$$

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[1] The principle of symmetric criticality (see, R.Palais, *Comm. Math. Phys.* **69**, 19 (1979)) guaranties that the variation of S within the spherically symmetric ansatz gives the correct equations of motion.

which on shell is equal to the total mass $m(\infty)$ (assuming the boundary condition $A(\infty) = 1$ *i.e.* that t is proper time at spatial infinity).

Eqs (7)–(10) have a scaling symmetry: if w, ϕ, N and A are solutions so are

$$w_\lambda(r) = w(\mathrm{e}^\lambda r)\,, \tag{12a}$$

$$\phi_\lambda(r) = \phi(\mathrm{e}^\lambda r) + \lambda\,, \tag{12b}$$

$$N_\lambda(r) = N(\mathrm{e}^\lambda r)\,, \qquad \left[m_\lambda(r) = \mathrm{e}^{-\lambda} m(\mathrm{e}^\lambda r)\right]\,, \tag{12c}$$

$$A_\lambda(r) = A(\mathrm{e}^\lambda r)\,. \tag{12d}$$

Under this transformation the energy functional scales as follows

$$E_\lambda = \mathrm{e}^{-\lambda} E\,. \tag{13}$$

The existence of this scaling symmetry does not exclude globally regular solutions because for the variation induced by the transformation (12) the energy is not extremized since $\delta\phi(\infty)$ is nonzero. Hereafter, I will assume that all solutions satisfy $\phi(\infty) = 0$, which can always be set by the transformation (12). This choice sets the scale of energy in the theory.

3. Abelian solutions

In the U(1) sector of the theory (*i.e.* in Einstein–Maxwell-dilaton theory) there are no static globally regular solutions but there are known explicit black hole solutions of Eqs (7)–(10). The uncharged solution is Schwarzschild

$$w = \pm 1\,, \qquad \phi = 0\,, \qquad m = M = \mathrm{const}\,, \qquad A = 1\,. \tag{14}$$

Charged black hole solutions were found by Gibbons and Maeda [6] (and later rediscovered in [7]). In the so called extremal limit the area of the horizon of these charged dilatonic black holes goes to zero and the resulting spacetime has a null naked singularity. This singular extremal solution will play an important role in our discussion of non-abelian solutions. It has a very simple form in isotropic coordinates

$$ds^2 = -\mathrm{e}^{-2\alpha\phi} dt^2 + \mathrm{e}^{2\alpha\phi}(d\rho^2 + \rho^2 d\vartheta^2 + \rho^2 \sin^2\vartheta d\varphi^2)\,, \tag{15a}$$

where

$$\phi = \frac{1}{1+\alpha} \ln\left(1 + \frac{\sqrt{1+\alpha}}{\rho}\right) \tag{15b}$$

and $w = 0$, so the YM curvature is

$$\mathcal{F} = -\tau_3 d\vartheta \wedge \sin\vartheta d\varphi\,, \tag{15c}$$

which corresponds to the Dirac magnetic monopole with unit magnetic charge (where the unit of charge is $1/e$). The total mass is $m(\infty) = 1/\sqrt{1+\alpha}$. For $\alpha = 0$ the solution (15) reduces to the dilatonic magnetic monopole discussed in [5]. For $\alpha = \infty$ the solution (15) reduces to the extremal Reissner–Nordstrom black hole (before taking the limit $\alpha \to \infty$ one has to make rescaling $\rho \to \sqrt{\alpha}\rho$).

4. Non-abelian solutions

In order to construct solutions which are globally regular we have to impose the boundary conditions which ensure regularity at $r = 0$ and asymptotic flatness. The asymptotic solutions of Eqs (7)–(10) satisfying these requirements are

$$\pm w = 1 - br^2 + O(r^4)\,, \tag{16a}$$
$$\phi = c - 2e^{-2c}b^2r^2 + O(r^4)\,, \tag{16b}$$
$$N = 1 - 4\alpha e^{-2c}b^2r^2 + O(r^4)\,, \tag{16c}$$
$$A = d\,(1 + 4\alpha e^{-2c}b^2r^2) + O(r^4)\,, \tag{16d}$$

near $r = 0$, and

$$\pm w = 1 - \frac{B}{r} + O\left(\frac{1}{r^2}\right)\,, \tag{17a}$$
$$\phi = C - \frac{D}{r} + O\left(\frac{1}{r^2}\right)\,, \tag{17b}$$
$$N = 1 - \frac{2\alpha M}{r} + \frac{\alpha D^2}{r^2} + O\left(\frac{1}{r^3}\right)\,, \tag{17c}$$
$$A = 1 - \frac{\alpha D^2}{2r^2} + O\left(\frac{1}{r^4}\right) \tag{17d}$$

near $r = \infty$. Here b, c, d, B, C, D, and M are arbitrary constants. All higher order terms in the above expansions are uniquely determined, through recurrence relations, by b, c and d in (16), and B, C, D and M in (17). The constant D is usually referred to as the dilaton charge while $M = m(\infty)$ is the total mass.

Remark 1. Notice that, for the asymptotic behaviour (17a), the radial magnetic curvature, $\mathcal{B}_r = \tau_3(1-w^2)/r^2$, falls-off as $1/r^3$, and therefore all globally regular solutions have zero YM magnetic charge. In accordance with this, in the asymptotic expansion of $g_{00} = A^2N$ the $1/r^2$ term vanishes.

Remark 2. One can easily show (see [5]) that for global solutions satisfying the above boundary conditions, the function w oscillates around zero between -1 and 1, while ϕ is monotonically decreasing.

Now, let us assume that near $r = 0$ there exist a family of local solutions defined by the expansion (16). Note that this is a nontrivial statement because the point $r = 0$ is a singular point of Eqs (7)–(10), hence the formal power-series expansion (16) may have, in principle, a zero radius of convergence. The initial parameters c and d are irrelevant since they can be chosen arbitrarily by the scaling (12) and by time rescaling, respectively. Thus, effectively we have a one-parameter family parametrized by b. For generic b the solution will not satisfy the asymptotic conditions (17) (in fact, the solution may even become singular at some finite distance). The standard numerical strategy, called the shooting method, is to find an initial value b for which the local solution extends to a global solution with the asymptotic behaviour (17). I have found that for generic orbits with $b < b_\infty(\alpha)$ a function w oscillates finite number of times in the region between $w = -1$ and $w = 1$ and then goes to $\pm\infty$. For $b > b_\infty(\alpha)$ all orbits become singular at a finite distance (in a sense that w' becomes infinite).

The numerical results strongly indicate that for all values of α there exist a countable family of initial values b_n ($n \in N$) determining globally regular solutions. Here the index n labels the number of nodes of the function w. In Table I are displayed the initial values and masses of the first five solutions for $\alpha = 1$. The initial value c is determined by the condition that the dilaton vanishes at infinity *i.e.* $C = 0$. The functions w, ϕ and N are graphed in Figs 1–3.

TABLE I

Initial data (b, c) and masses of the first five globally regular solutions for $\alpha = 1$.

n	be^{-c}	c	M
1	0.1666666	0.9311	0.5773
2	0.2318001	1.7925	0.6849
3	0.246861	2.6919	0.7035
4	0.249483	3.5974	0.7065
5	0.249915	4.5043	0.7070

In Figs 4 and 5 are shown the functions w_1 and N_1 for three different values of α. For all α the solutions display three characteristic regions. The energy density is concentrated in the inner core region $r < R_1$, where R_1 is approximately the location of the first zero of w. In the second region,

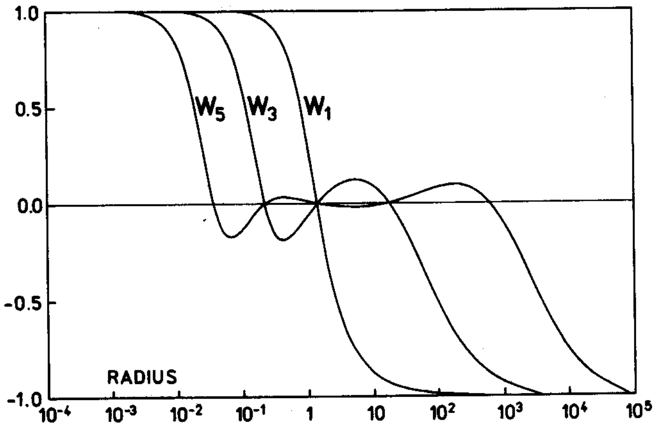

Fig. 1. The YM potential w_n for $n = 1, 3, 5$.

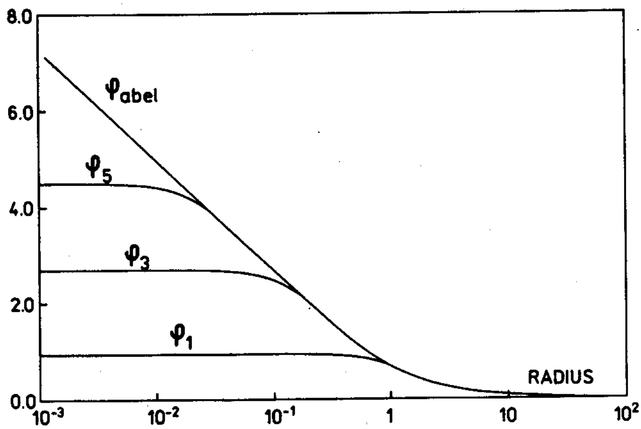

Fig. 2. The dilaton ϕ_n for $n = 1, 3, 5$.

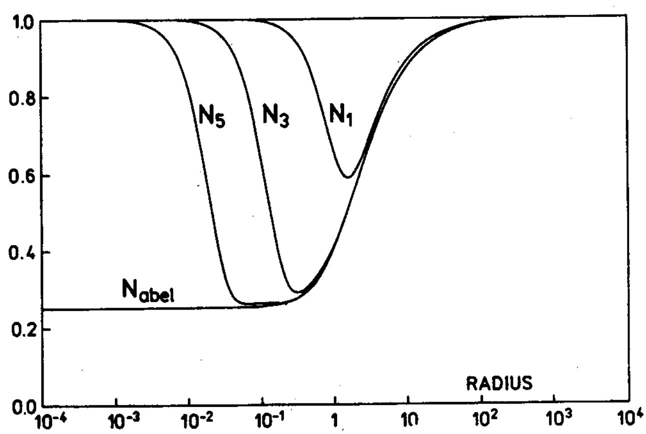

Fig. 3. The metric coefficient N_n for $n = 1, 3, 5$.

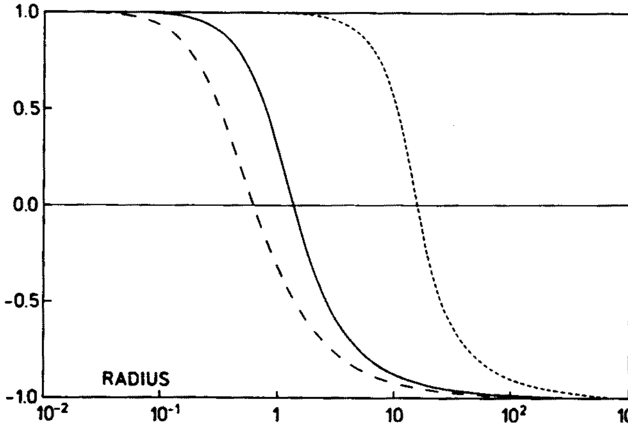

Fig. 4. w_1 for $\alpha = 0.01$ (dashed line), $\alpha = 1$ (solid line) and $\alpha = 100$ (dotted line).

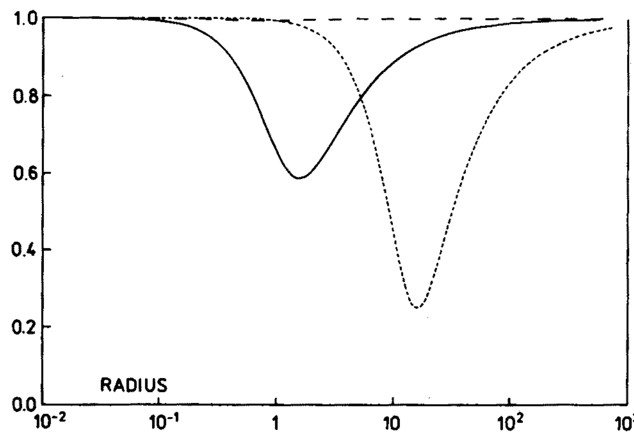

Fig. 5. N_1 for $\alpha = 0.01$ (dashed line), $\alpha = 1$ (solid line) and $\alpha = 100$ (dotted line).

$R_1 < r < R_2$, where R_2 is approximately the location of the last but one zero of w, the function w slowly oscillates around $w = 0$. Finally, in the asymptotic region $r > R_2$, the function w goes monotonically to $w = \pm 1$ (hence the YM magnetic charge is gradually screened) and for $r \to \infty$ the solution tends to the Schwarzschild solution (14).

Let us consider the limit $n \to \infty$. In this limit $R_1 \to 0$ and $R_2 \to \infty$, so the second region covers the whole space. As n grows the amplitude of oscillations of the function w decreases and goes to zero as $n \to \infty$ (see Fig. 1). Thus, for $n \to \infty$ the solution tends (nonuniformly) to the extremal Gibbons-Maeda solution (15). This is clear from Figs 2 and 3, where the solution (15) for $\alpha = 1$, expressed in Schwarzschild coordinates

$$N_{\rm abel} = \left(1 - \frac{1}{1+\sqrt{1+2r^2}}\right)^2 , \tag{18}$$

and

$$\phi_{\rm abel} = \tfrac{1}{2}\ln\left(\frac{1+\sqrt{1+2r^2}}{-1+\sqrt{1+2r^2}}\right) , \tag{19}$$

is included for comparison with non-abelian solutions.

For all n and α the metric coefficient N has one minimum $N_{\rm min}$ located approximately at R_1. For fixed n, $N_{\rm min}$ decreases with α (see Fig. 5). Also for given α, $N_{\rm min}$ decreases with n (see Fig. 3) and $N_{\rm min} \to 1/(1+\alpha)^2$ as $n \to \infty$.

The total mass M_n increases with n and for $n \to \infty$ goes to $1/\sqrt{1+\alpha}$. Because our scale of energy breaks down for $\alpha \to \infty$ (*i.e.* $a = 0$), it is more convenient to compare masses of solutions with different values of α using $1/e\sqrt{G+a^2}$ as the unit of energy instead of $1/ea$. This corresponds to rescaling $\widetilde{M} = \sqrt{1+\alpha}M$. Then for all α one has $\lim_{n\to\infty}\widetilde{M}_n = 1$. It turns out that for given n the mass $\widetilde{M}_n$ is increasing approximately linearly with $\beta = \alpha/(1+\alpha)$. This is shown in Fig. 6.

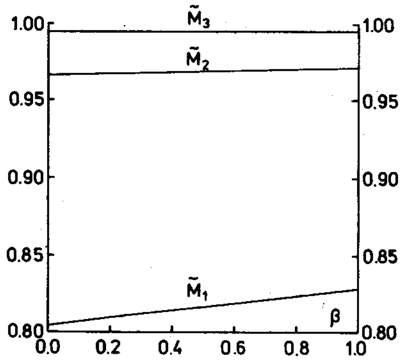

Fig. 6. The total mass $\widetilde{M}_n$ (in units $1/e\sqrt{G+a^2}$) as a function of $\beta = \alpha/(1+\alpha)$.

In [5] it was proven in the case $\alpha = 0$ that the energy is equal to minus dilaton charge D. This property still holds for $\alpha > 0$. To show this I first derive a simple virial identity. Consider a one-parameter family of field configurations defined by

$$w_\lambda(r) = w(\lambda r)\,, \quad \phi_\lambda(r) = \phi(\lambda r)\,, \quad N_\lambda(r) = N(\lambda r)\,, \quad A_\lambda(r) = A(\lambda r)\,.$$

For this family the energy functional (11) is

$$E_\lambda = \lambda^{-1} I_1 + \lambda\, I_2\,, \tag{20}$$

where

$$I_1 = \int_0^\infty \left(\frac{r^2}{2} AN\phi'^2 + A'm\right) dr\,, \tag{21}$$

$$I_2 = \int_0^\infty A e^{-2\phi} \left[N w'^2 + \frac{(1-w^2)^2}{2r^2}\right] dr\,. \tag{22}$$

Since the energy functional is extremized at $\lambda = 1$, it follows from (20) that on shell

$$I_1 = I_2\,. \tag{23}$$

Integrating Eq. (10) one gets $D = -2I_2$, and therefore Eq. (23) yields

$$m(\infty) + D = 0\,. \tag{24}$$

Thus the total mass can be read off from the monopole term in the asymptotic expansion (17b) of the dilaton field. The identity (24) is also true for the limiting solution (15).

5. Stability analysis

In this Section 1 consider the time evolution of linear perturbations about the static solutions described above. I will assume that the time-dependent solutions remain spherically symmetric and the YM field stays within the ansatz (3). This is sufficient to demonstrate instability because unstable modes appear already in this class of perturbations. For radial perturbations the stability analysis is relatively simple, because the spherically symmetric gravitational field has no dynamical degrees of freedom and therefore the perturbations of metric coefficients are determined by the perturbations of matter fields [11]. To see this consider the Einstein equations

$$\dot\lambda = -2\alpha r(\dot\phi\phi' + \frac{2}{r^2}\dot w w') \tag{25}$$

and

$$\nu' - \lambda' = \alpha r\left[\phi'^2 + \frac{2}{r^2}w'^2 + e^{-2\nu}(\dot\phi^2 + \frac{2}{r^2}\dot w^2)\right]\,, \tag{26}$$

where the functions ν and λ are defined by $e^\nu = AN$ and $e^\lambda = N$, and dot denotes the time derivative. Now, take the perturbed fields $\nu(r) + \delta\nu(r,t)$, $\lambda(r) + \delta\lambda(r,t)$, $w(r) + \delta w(r,t)$, and $\phi(r) + \delta\phi(r,t)$, where $(\nu(r), \lambda(r), w(r),$

$\phi(r)$) is a static non-abelian solution, and insert them into Eqs (25) and (26). Linearizing Eq. (25) one obtains

$$\delta\dot{\lambda} = -2\alpha r(\phi'\delta\dot{\phi} + \frac{2}{r^2}w'\delta\dot{w})\,, \tag{27}$$

which can be integrated to give

$$\delta\lambda = -2\alpha r(\phi'\delta\phi + \frac{2}{r^2}w'\delta w)\,. \tag{28}$$

Linearization of Eq. (26) yields

$$\delta\nu' - \delta\lambda' = 2\alpha r(\phi'\delta\phi' + \frac{2}{r^2}w'\delta w')\,. \tag{29}$$

and therefore using Eq. (28) one gets

$$\delta\nu' = -2\alpha\left[(r\phi'' + \phi')\delta\phi + \frac{2}{r}\left(w'' - \frac{1}{r}w'\right)\delta w\right]\,. \tag{30}$$

The spherically symmetric evolution equations for the matter fields are

$$-(\mathrm{e}^{-\nu-2\phi}\dot{w})^{\cdot} + (\mathrm{e}^{\nu-2\phi}w')' + \frac{1}{r^2}\mathrm{e}^{\nu-\lambda-2\phi}w(1-w^2) = 0\,, \tag{31}$$

and

$$-r^2(\mathrm{e}^{-\nu}\dot{\phi})^{\cdot} + (r^2\mathrm{e}^{\nu}\phi')' + 2\mathrm{e}^{\nu-\lambda-2\phi}\left[\mathrm{e}^{\lambda}w'^2 + \frac{(1-w^2)^2}{2r^2}\right] = 0\,. \tag{32}$$

Multiplying Eqs (31) and (32) by $\mathrm{e}^{-\nu}$, linearizing and assuming harmonic time dependence for the perturbations, $\delta w(r,t) = \mathrm{e}^{i\sigma t}\xi(r)$ and $\delta\phi(r,t) = \mathrm{e}^{i\sigma t}\psi(r)$, one obtains an eigenvalue problem

$$\begin{aligned}
&-\xi'' + (2\phi' - \nu')\xi' + 2w'\psi' \\
&\quad + 2\alpha w'\left[(r\phi'' + \phi')\psi + \frac{2}{r}(w'' - \frac{1}{r}w')\xi\right] - \frac{1}{r^2}\mathrm{e}^{-\lambda}(1-3w^2)\xi \\
&\quad + \frac{2\alpha}{r}\mathrm{e}^{-\lambda}w(1-w^2)(\phi'\psi + \frac{2}{r^2}w'\xi) = \sigma^2\mathrm{e}^{-2\nu}\xi\,,
\end{aligned} \tag{33}$$

$$\begin{aligned}
&-(r^2\psi')' - 4\mathrm{e}^{-2\phi}\Big[w'\xi' - \frac{1}{r^2}\mathrm{e}^{-\lambda}w(1-w^2)\xi \\
&-(w'^2 + \mathrm{e}^{-\lambda}\frac{(1-w^2)^2}{2r^2})\psi + \alpha\mathrm{e}^{-\lambda}\frac{(1-w^2)^2}{4r}(\phi'\psi + \frac{2}{r^2}w'\xi)\Big] \\
&-r^2\nu'\psi' + 2\alpha r^2\phi'\left[(r\phi'' + \phi')\psi + \frac{2}{r}(w'' - \frac{1}{r}w')\xi\right] = \sigma^2 r^2\mathrm{e}^{-2\nu}\psi\,.
\end{aligned} \tag{34}$$

If the perturbations satisfy the boundary conditions $\xi(0) = 0$, $\psi(0) = \text{const}$, $\xi(\infty) = 0$, and $\psi(\infty) = 0$, then the above system is self-adjoint, hence eigenvalues σ^2 are real. Instability manifests itself in the presence of at least one negative eigenvalue.

I have used the generalized rule of nodes (Jacobi criterion) to find that for all values of α the nth solution has exactly n negative eigenvalues (I have checked this up to $n = 4$). This is consistent with the fact that the limiting solution (15) has infinitely many unstable modes which can be seen easily by inserting (15) into Eq. (33) and considering perturbations with $\psi = 0$. The result that the nth solution has n unstable modes is essential for the Sudarsky–Wald argument [3]. However, by considering a restricted class of perturbations some directions of instability might have been suppressed.

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