ON THE SIZES OF OBJECTS WHICH CAN BE MEASURED BY THE STUDY OF CORRELATIONS OF IDENTICAL PARTICLES

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Fundamental limitations of the identical particle correlations method, used for the determination of the source size are discussed. It is shown that the limitations are in each case determined by quantum-mechanical characteristics of the method used for particle registration. For the case of track detectors the upper limit of the source sizes which can be determined by the study of correlations does not exceed the atomic size.

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The analysis of correlations between identical particles is (at present) used as the standard method of studying space-time characteristics of production of such particles. Hundreds of experimental and theoretical papers concerning this approach were published. Nevertheless, there are still some problems, not yet considered, which — although rather simple — are important from the point of view of fundamental ideas underlying the basic principles of correlation analysis. In this paper we consider one such problem.

Since our analysis will concern the general basic ideas in this domain, we can consider the simplest case of independent point-like single particle sources, from which spinless particles (*e.g.* pions) are emitted. The role of final state interactions will be neglected in our considerations.

The elementary theory describing such process is well known. However, for the purpose of further considerations, we shall shortly remind the logical structure of the relevant theoretical approach. Let us denote the positions of the sources by $\vec{r}_1$ and $\vec{r}_2$ and the times corresponding to the generation of two identical pions by t_1 and t_2, respectively. The single-particle amplitude for the process of generation of a pion with the momentum $\vec{p}$ will be written as $U(\vec{p})$. The process of the production of two pions with given momenta and energies $((\vec{p}\,', E')$ and $(\vec{p}\,'', E''))$ may be realized in two ways, schematically shown in Fig. 1.

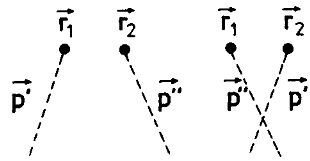

Fig. 1.

In one case the two-particle amplitude should be written as

$$A(\vec{p}\,',\vec{p}\,'') \sim U(\vec{p}\,')U(\vec{p}\,'')\mathrm{e}^{-i(\vec{r}_1\vec{p}\,'-t,E')}\mathrm{e}^{-i(\vec{r}_2\vec{p}\,''-t_2E'')} \tag{1}$$

in the second case — as

$$A(\vec{p}\,',\vec{p}\,'') \sim U(\vec{p}\,''U(\vec{p}\,')\mathrm{e}^{-i(\vec{r}_1\vec{p}\,''-t_1E'')}\mathrm{e}^{-i(\vec{r}_2\vec{p}\,'-t_2E')}\,. \tag{2}$$

Since the emitted pions are identical, the two cases are indistinguishable: they both lead to the same final state. Therefore the full two-particle amplitude for the considered process should be written in the form

$$A(\vec{p}\,',\vec{p}\,'') \sim U(\vec{p}\,')U(\vec{p}\,'')\Big[\mathrm{e}^{-i(\vec{r}_1\vec{p}\,'-t_1E')}\mathrm{e}^{-i(\vec{r}_2\vec{p}\,''-t_2E'')} + \mathrm{e}^{-i(\vec{r}_1\vec{p}\,''-t_1E'')}\mathrm{e}^{-i(\vec{r}_2\vec{p}\,'-t_2E')}\Big] \tag{3}$$

and the corresponding probability function is

$$W(\vec{p}\,',\vec{p}\,'') \sim\mid U(\vec{p}\,')\mid^2\mid U(\vec{p}\,'')\mid^2 \times \left\{1+\cos[(\vec{r}_1-\vec{r}_2)(\vec{p}\,'-\vec{p}\,'')-(t_1-t_2)(E'-E'')]\right\}. \tag{4}$$

The object of our study *i.e.* the correlations of identical particles refer to the second term in Eq. (4): they result from the interference between "straight-like" and "cross-like" (see Fig. 1) amplitudes, defined by Eqs (1) and (2), respectively.

The problem of the observation of correlation effects is closely connected with technical characteristics of the experimental setup. If the values of $|\vec{r}_1 - \vec{r}_2|$ and $|t_1 - t_2|$ are very large then the interference peaks become too narrow as compared with the experimental resolution, and the correlations are unobservable. One should stress, however, that in many cases an improvement of the experimental precision may make the observation of correlations feasible.

On the other hand there are situations when the correlations are principally unobservable independently of the level of the experimental resolution. This happens when the generation of the two particles proceeds via one channel only, (for instance "straight"-channel) whereas the other one ("crossed"-channel) does not exist. In such a case we are dealing with only one amplitude (Eq. (1)) and the probability of the considered process is

$$W(\vec{p}\,',\vec{p}\,'') \sim | U(\vec{p}\,')U(\vec{p}\,'')e^{-i(\vec{r}_1\vec{p}\,'-t_1E')}e^{-i(\vec{r}_2\vec{p}\,''-t_2E'')} |^2 \\ = |U(\vec{p}\,')^2|U(\vec{p}\,'')|^2 . \quad (5)$$

As an example one can consider an event in which one of the pions (π^-) is produced together with a Λ hyperon, the latter decaying into a proton and a π^-. Experimentally (*e.g.* in a bubble chamber) we observe a π^- with the momentum $\vec{p}\,'$ emitted from the primary event ($\vec{r}_1$) and another π^- with the momentum $\vec{p}$, from the secondary (Λ decay) event ($\vec{r}_2$). The time of emission of the two pions is, of course, different. In the considered example the particles with momenta $\vec{p}\,'$ and $\vec{p}\,''$ are coupled with positions $\vec{r}_1$ and $\vec{r}_2$ of their production. Consequently, we are dealing with only one ("straight") amplitude. The other one is absent and there is no interference.

Considering such examples leads to many questions;

— on the relations between two kinds of processes (with and without the interference of identical particles),

— on the methods to be used for their theoretical description,

— on the possibility of intermediate situations, *i.e.*, the cases when some interference exists — we can call it a partly-interference case.

We should stress here an important fact, which so far has not been adequately dealt with. In a vast majority of correlation experiments particles are registered by the ionization processes due to their passage through a detecting device[1]. The momentum and energy of the particle is then derived from an ensemble of data on ionization acts, forming the "track" of the particle. An interesting question arises: how does the presence of ionization processes (which carry information on particle characteristics) influence the interference picture of the studied correlations? The importance of this question comes from the fact that — within the framework of the quantum mechanics — any process should be considered globally. It means that the quantum changes of the state of a system should not be considered separately and independently of the methods of their registration.

Let us assume that the particles which have been produced in points $\vec{r}_1$ and $\vec{r}_2$ subsequently were scattered (inelastically — in general) at points $\vec{R}_1$

[1] In the case of neutral particles their registration is also carried out by relevant ionization processes. A detailed study of such cases will not be considered in this work.

and $\vec{R}_2$, respectively. In such a case the points $\vec{R}_1$ and $\vec{R}_2$ may be treated as new sources, from which the particles are emitted with new momenta at times T_1 and T_2 (see for instance [1]). On the other hand each individual ionization act should be considered as an act of inelastic scattering. We can, therefore, ask a question: why the points, in which ionization acts have occurred, are never considered as new sources of the studied particles?

In the search of answers to these questions and in order to achieve a better understanding of their relevance to the problem of studying correlations we shall come back to Eq. (4). If the formula (4) can be applied to all cases, then, in principle, the interference peak should be always present and we should never use the formula (5), corresponding to the presence of only one two-particle amplitude. We have to keep in mind, however, that the derivation of Eq. (4) was based on some assumptions. In the present consideration we have to test whether all the assumptions are valid in all cases.

First of all we have assumed that single-particle amplitudes are equal for both sources, and, consequently, the modules of the amplitudes (1) and (2) are the same. In a general case we should introduce into our considerations two (not necessarily equal) single-particle amplitudes, $U(\vec{p})$ and $V(\vec{p})$. Instead of Eq. (4) we shall have now

$$A(\vec{p}\,',\vec{p}\,'') \sim U(\vec{p}\,')V(\vec{p}\,'')\mathrm{e}^{-i(\vec{r}_1\vec{p}\,'-t_1E')}\mathrm{e}^{-i(\vec{r}_2\vec{p}\,''-t_2E'')} + U(\vec{p}\,'')V(\vec{p}\,')\mathrm{e}^{-i(\vec{r}_1\vec{p}\,''-t_1E'')}\mathrm{e}^{-i(\vec{r}_2\vec{p}\,'-t_2E')}\,, \qquad (6)$$

where the modules of the two single-particle amplitudes may, in general, differ from each other.

Next, an (rather natural) assumption is usually made, that the single-particle amplitudes practically do not change when the momenta of particles are varied within the region of the interference peak, *i.e.*, $U(\vec{p})$ and $V(\vec{p})$ are replaced by some constant values[2].) Under this assumption the products $U(\vec{p}\,')$. $V(\vec{p}\,'')$ and $U(\vec{p}\,'')$. $V(\vec{p}\,')$ are equal to each other and we obtain again the formula (4).

The situation is completely changed if we decide to reject the assumption concerning slowly varying functions $U(\vec{p})$ and $V(\vec{p})$. Let us assume that the amplitude $U(\vec{p})$ is large in the momentum region $\vec{p} \sim \vec{p}\,'$ and small for $\vec{p} \sim \vec{p}\,''$, whereas the amplitude $V(\vec{p})$ is — *vice versa* — large in the region $\vec{p} \sim \vec{p}\,''$ and small for $\vec{p} \sim \vec{p}\,'$. In such a case two-particle amplitudes

[2] This assumption is usually not valid in real experiments. Consequently the measured value $W(\vec{p}\,',\vec{p}\,'')$ is divided by the so-called "background", where no correlations are present. Within the framework of our analysis this procedure is rather unimportant.

in Eq. (6) are not "equi-important" any more, the module of the first one being much larger than the module of the second one. Consequently, we are dealing with "not-complete" interference. Within such an approach it is possible to consider a continuous change from the "equi-importance" of two two-particle amplitudes (when: $|U(\vec{p}\,')| = |U(\vec{p}\,'')|$ and $|V(\vec{p}\,'')| = |V(\vec{p}\,')|$| to the disappearance of one of them. In the two limiting cases we are dealing either with a "full" interference peak or with a complete disappearance of the interference. In intermediate situations a "not complete" interference takes place. In fact there exist situations in which step-by-step changes from the "maximum" interference effect down to its complete absence are possible. Let us consider, for instance, two sources of particles with a strong overlap of their spectral intervals; in this case the "full" interference occurs. If, on the other hand, spectral ranges of the two sources do not overlap at all — no interference takes place. A partial overlap of spectra corresponds to intermediate case of "non-full" interference.

Within the framework of the model considered at the beginning of this paper, however, it seems that a strong dependence of single particle amplitudes, $U(\vec{p})$ and $V(\vec{p})$ on particle momenta (within the interference peak) is as a rule, impossible. Indeed, the linear sizes of the sources, Δ_r are assumed to be much smaller than the distance between them, $|\vec{r}_1 - \vec{r}_2|$. From the uncertainty principle one expects that single-particle amplitudes may change noticeably only if the momentum change is $\Delta_p \sim \frac{\hbar}{\Delta_r}$ which is much higher than $\frac{\hbar}{|\vec{r}_1 - \vec{r}_2|}$, the latter corresponding to the typical width of the interference peak.

The above argumentation becomes, however, less convincing when the general, quantum-mechanical approach is applied. Indeed, when considering a process one is bound to consider it in a global way, *i.e.* together with the methods of the observation of the process. In the case of our discussion it means that we have to consider not just the amplitude of the process of generation of two pions, (with momenta $\vec{p}\,'$ and $\vec{p}\,''$), but rather the amplitude of the whole process, resulting in formation of two "tracks" of ionized atoms; measurements of these tracks allow to determine the particle momenta $\vec{p}\,'$ and $\vec{p}\,''$ — with some finite precision.

In order to achieve a better understanding of the above problem let us start with a consideration of "ionization track" itself. We shall assume the track to be as long as needed and we shall neglect energy losses and multiple scattering on nuclei. The question is now how precisely one can determine the particle direction and the transverse coordinate of the emission point?

The sizes of bubbles in a bubble chamber or of grains in a developed nuclear emulsion are rather large, but one can use a sufficient length of the track, thus increasing the precision of determination of the direction and transverse coordinate. It is, however, obvious that there is a value, a, limiting the precision of the coordinate determination. This limiting value cannot,by principle,be smaller than the atomic size. In a similar way the precision of the determination of a track direction is also limited. Indeed, each ionization act is accompanied by an uncertainty in the transverse component of the particle momentum

$$\delta p \sim \frac{\hbar}{a}, \tag{7}$$

which results in an uncertainty of the direction

$$\delta\theta \sim \frac{\delta p}{p} \sim \frac{\hbar}{ap} \sim \frac{\lambda}{a}, \tag{8}$$

where λ is the corresponding de Broglie wavelength. In the domain of nuclear physics and high energy physics the value $\delta\theta$ is always very small, nevertheless one should keep in mind the unavoidable presence of this uncertainty. In addition the time of particle emission, determined from ionization data of the "track" is measured with some uncertainty

$$\delta t \sim \frac{a}{v}, \tag{9}$$

where v is the particle velocity. The precision of the determination of the longitudinal coordinate of the source is (as in the case of the transverse coordinate) limited by the atomic size a. In further consideration we shall concentrate mainly on the problem of the transverse coordinate of the source and on the precision of measurement of the transverse component of the particle momentum.

An important conclusion of the above considerations is the following: if the emitted particle is detected by its "ionization track", then the amplitude of this event depends not only on the pure nuclear amplitude, $U(\vec{p})$, but also on — roughly speaking — the distance, $\vec{\rho}$ between the "track axis" and the production point. This fact can be expressed by the form of the amplitude, written now as the product $U(\vec{p})\ \alpha(\vec{\rho})$. The term $\alpha(\vec{\rho}) \approx 1$ for $|\vec{\rho}| < a$ and its value drops down rapidly for $|\vec{\rho}| > a$ [3].

Now we come back to two-particle correlations, and we consider two sources (at points $\vec{r}_1$ and $\vec{r}_2$) and two particles (with momenta $\vec{p}\,'$ and $\vec{p}\,''$). The amplitude for the process should be written in the form

$$A(\vec{p}\,',\vec{p}\,'') \sim U(\vec{p}\,')\alpha(\vec{\rho}\,'_1)e^{-i(\vec{r}_1\vec{p}\,'-t_1E')}V(\vec{p}\,'')\alpha(\vec{\rho}\,''_2)e^{-i(\vec{r}_2\vec{p}\,''-t_2E'')}$$
$$+\,U(\vec{p}\,'')\alpha(\vec{\rho}\,''_1)e^{-i(\vec{r}_1\vec{p}\,''-t_1E'')}V(\vec{p}\,')\alpha(\vec{\rho}\,'_2)e^{-i(\vec{r}_2\vec{p}\,'-t_2E')}, \tag{10}$$

[3] Instead of being rigorously precise we use here qualitative notions and considerations, which — in our opinion — is sufficient to explain main ideas and conclusions of this part of the paper.

where $\vec{\rho}'_1$ and $\vec{\rho}'_2$ denote the distances between the "track axis" of particle of momentum $\vec{p}'$ and points $\vec{r}_1$ and $\vec{r}_2$, respectively, and ρ''_2 and $\vec{\rho}''_2$ are the corresponding distances for the particle of momentum $\vec{p}''$. If the functions $U(\vec{p})$ and $V(\vec{p})$ are (as usually), assumed to be sufficiently slowly changing functions, then Eq. (10) may be written in the form

$$A(\vec{p}',\vec{p}'') \sim \alpha(\vec{\rho}'_1)\alpha(\vec{\rho}''_2)\mathrm{e}^{-i(\vec{r}_1\vec{p}'-t_1E')}\mathrm{e}^{-i(\vec{r}_2\vec{p}''-t_2E'')} + \alpha(\vec{\rho}''_1)\alpha(\vec{\rho}'_2)\mathrm{e}^{-i(\vec{r}\vec{p}''-t_1E'')}\mathrm{e}^{-i(\vec{r}_2\vec{p}'-t_2E')}\,. \tag{11}$$

The correlations will arise provided that each of the two particles may be produced either at the point $\vec{r}_1$ or at the point $\vec{r}_2$. This condition requires that $|\vec{r}_1 - \vec{r}_2| \ll a$; the above inequality — according to our previous considerations — leads to

$$\alpha(\vec{\rho}'_1) \approx \alpha(\vec{\rho}'_2) \approx \alpha(\vec{\rho}''_1) \approx \alpha(\vec{\rho}''_2) \approx 1$$

and, consequently, Eq. (11) becomes identical with the usual expression (3), containing two equi-important two-particle amplitudes[4]. Let us note that the precision of the momentum determination (Eq. (7)), $\delta p \sim \hbar/a$, is — in the considered case — sufficient for the observation of the interference peak, its width being of the order of $\hbar/|\vec{r}_1 - \vec{r}_2|$.

Let us assume now that the distance between points $\vec{r}_1$ and $\vec{r}_2$ is very large, as compared with the atomic size. In this case only one of the two "tracks" (say, that corresponding to the particle of momentum $\vec{p}'$) starts in the vicinity of the point $\vec{r}_1$, whereas the other one — in the vicinity of the point $\vec{r}_2$. Hence, $\alpha(\vec{\rho}'_1) \approx \alpha(\vec{\rho}''_2) \approx 1$ and $\alpha(\vec{\rho}''_1) \approx (\vec{\rho}'_2) = 0$, and, consequently, the expression (11) contains only the first term: the interference disappears completely. In an intermediate case, when the distance between points $\vec{r}_1$ and $\vec{r}_2$ is comparable to the atomic size, a, the factors α fulfill inequalities

$$\alpha(\vec{\rho}'_1) > \alpha(\vec{\rho}''_1) \text{ and } \alpha(\vec{\rho}''_2) > \alpha(\vec{\rho}'_2)\,.$$

Consequently one of the amplitudes in Eq. (11) is higher than the other and we are dealing with a "partial" interference. In addition the interference peak is in this case "smeared out". The latter effect is due to the fact that for $|\vec{r}_1 - \vec{r}_2| \sim a$ and the peak width is comparable to the precision of the momentum determination, $\hbar/a$.

The above considerations are closely connected with the assumed "ionization method" of the detection and measurement. In order to make this statement more clear we shall now consider an experiment in which neutron-neutron correlations are studied. Let us assume that a Λ hyperon (decaying

[4] For a more detailed discussion see Appendix.

via its neutral channel, $\Lambda \to n + \pi^0$) has been produced together with a neutron. Neutron-neutron correlations may be, in principle, observable in spite of the macroscopically large distances between the points of neutron emission. The observation of such correlations would require very high precision momentum measurements (at present unachievable in practice), but at least in principle the study of these effects is possible. When considering the case of charged decay of Λ ($\Lambda \to p + \pi^-$) produced together with a π^-, an attempt to study $\pi^- - \pi^-$ correlations (based on "ionizing tracks") will, unavoidably, be unsuccessful, as shown above. The interference disappears completely.

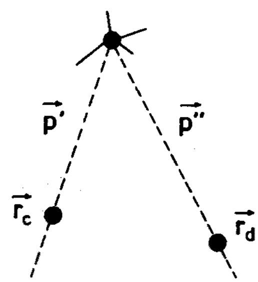

Fig. 2.

Let us now ask the question whether ionization acts due to the particles (produced in an event) may be treated as new sources of particles (see Fig. 2, where these points are denoted as $\vec{r}_c$ and $\vec{r}_d$). The general statement mentioned earlier (p.4) remains, of course, valid, and may be applied to the ionization acts. Indeed, the points $\vec{r}_c$ and $\vec{r}_d$ may be considered as the sources of particles. One has to keep in mind, however, that the determination of momenta $\vec{p}'$ and $\vec{p}''$ requires spatial separation of the measured tracks, *i.e.* $|\vec{r}_c - \vec{r}_d| > a$. As it has been shown this condition results in the disappearance of the interference[5]. On the other hand, if the discussion concerns neutral particles which have undergone nuclear scattering at $\vec{r}_c$ and $\vec{r}_d$, then the corresponding points may be also treated as "sources". In this case a discussion, similar to that carried out earlier in this paper for neutrons (the "direct" ones from an event and the "decay" ones from $\Lambda \to n + \pi^0$), leads to the conclusion that the interference may occur.

[5] At ultrarelativistic energies the angle between $\vec{p}'$ and $\vec{p}''$ may be so small that,initially,the transverse distance between the tracks is smaller than a. In this case a more detailed analysis is needed, in which one should take into account the comparable sizes of "ionization" sources and distances between tracks (see [2]).

In our considerations up to now we have analyzed only the role of "ionization-based" measurements for the determination of the transverse size of the region in which the particles are produced. The analysis of the longitudinal components as well as the analysis of time and energy characteristics, may be carried out in a similar way (keeping in mind Eq. (9)).

In our considerations above we stressed the necessity of taking into account the basic features of the process of ionization of atoms. Indeed, in the experiments devoted to studying correlations of identical particles the measurements are, as a rule, based on ionization acts in detectors. Measurements might be, in principle, based on other physical processes, not involving the ionization of atoms. In such cases an analogous analysis has to be carried out. The approach presented in this paper has a general character, and it may be applied to any particular case.

Let us consider a detection method, which allows to determine the transverse coordinate of the source with some (limiting) precision a [6].

The momentum measurement itself is accompanied by a transverse uncertainty, $\delta p \sim \hbar/a$. If the sources are at points $\vec{r}_1$ and $\vec{r}_2$ then for $|\vec{r}_1 - \vec{r}_2| \ll a$ each of the two particles may be emitted from any of the two sources and, consequently, both two-particle amplitudes exist and they are "equi-important". On the other hand the width of the interference peak $(\hbar/|\vec{r}_1 - \vec{r}_2|)$ is larger than the limiting value of the precision of momentum measurements, $\delta p \sim \hbar/a$. In this situation, interference occurs and two-particle correlation can be measured. The interference (and, consequently, its observation and study) does not take place if $|\vec{r}_1 - \vec{r}_2| \gg a$. Intermediate situations correspond to the condition $|\vec{r}_1 - \vec{r}_2| \sim a$. These considerations can be carried out also in the "opposite direction"; indeed, if the momentum resolution is sufficient to observe the interference peak, then the space resolution of this particular method is not sufficient to localize the particle sources.

Let us note that the above considerations should be treated with some caution, as they are based on quantum–mechanical uncertainty relations, which in fact are inequalities. The corresponding values considered in the discussion may — in particular cases — be rather different from the limiting values.

As an illustration we shall consider the following example: the initial nuclear interaction takes place in a target so thin that secondary particles pass through the target before any ionization act occurs. Particles leaving the target pass through the vacuum and finally enter an "ionization"

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[6] Note that we have in mind the "principal" limit of precision, not a precision achieved in a real experiment. For example the experimentally achieved precision in a bubble chamber depends on bubble sizes and on the track length, whereas we are considering the atomic size.

detector used for registration and measurements. Let the target — detector distance be l. In such a situation the precision of determination of the source transverse coordinate depends on the larger of the two values: a and $\delta x \sim l \cdot \delta\theta$ (the angle uncertainty, $\delta\theta$ is given by Eq. (8)). In our previous considerations we have implicitly assumed that $l \cdot \delta\theta \ll l$, which is usually valid for the case of a thick and sufficiently dense target. Let us assume now that the distance (in vacuum) separating the source and the "ionization" detector is rather long, *i.e.*

$$\delta x \approx l \cdot \delta\theta \gg a\,. \tag{12}$$

Note that the momentum resolution in this case is not limited by $\delta p \sim \hbar/\delta x$, but — as before — by $\hbar/a$, *i.e.* it is much worse. One should add here that the condition (12) is valid quite often, for instance in the cases when interactions in a gas are studied and distances between ionization acts are rather large.

APPENDIX

An attempt to obtain the precise form of the function $\alpha(\vec{p})$ is quite a complicated problem. A somewhat more precise approach than that given in the main text (but still semi-qualitative) may be based on the assumption that $\alpha(\vec{\rho}) = e^{-\rho^2/2a^2}$. In order to simplify calculations we assume that axes of the two considered tracks as well as the sources are in the same plane. Let x_3 and x_4 denote the intersections of particle trajectories with the straight line determined by the positions of sources (x_1 and x_2). Under these assumptions the coefficients accompanying the first and the second amplitude in Eq. (11), are

$$\alpha(\rho_1')\alpha(\rho_2'') = e^{-\frac{(x_3-x_1)^2}{2a^2}}\, e^{-\frac{(x_4-x_2)^2}{2a^2}}\,, \tag{A1}$$

and

$$\alpha(\rho_1'')\alpha(\rho_2') = e^{-\frac{(x_4-x_1)^2}{2a^2}}\, e^{-\frac{(x_3-x_2)^2}{2a^2}}\,, \tag{A2}$$

respectively. If $x_1 = x_2 = x$ then the coefficients (A1) and (A2) are equal to each other.

We are dealing, therefore, with the "full-interference" case. The considered events take place at points x, for most of which $|x_3 - x| \lesssim a$ and $|x_4 - x| \lesssim a$. If the sources are very close to each other ($|x_1 - x_2| \ll a$) a similar situation occurs.

Let us consider now the case, when $|x_1 - x_2| \gg a$. There are two possibilities:

(a) The factor (A1) is close to unity, provided that $|x_3 - x_1| \lesssim a$ and $|x_4 - x_2| \lesssim a$; the factor (A2) is then close to zero.

(b) The factor (A2) is close to unity ($|x_3 - x_2| \lesssim a$, $|x_4 - x_1| \lesssim a$) and the factor (A1) is close to zero.

In both configurations ((a) and (b)) no interference occurs. The same conclusion is reached when other positions x_3 and x_4 are considered (they result in the disappearance of both factors ((A1) and (A2)).

If $|x_1 - x_2| \sim a$ then both factors, (A1) and (A2), may not be zero, but their values are different from each other (a rather rare exception corresponds to the situation when $|x_3 - x_4| \ll a$). Let us assume, for instance, that the point x_3 is closer to x_1 than to x_2 and the point x_4 is closer to x_2 than to x_1. Then

$$e^{-\frac{(x_3-x_1)^2}{2a^2}} e^{-\frac{(x_4-x_2)^2}{2a^2}} > e^{-\frac{(x_4-x_1)^2}{2a^2}} e^{-\frac{(x_3-x_2))^2}{2a^2}},$$

and Eq. (A2) may be written in the form

$$A(\vec{p}\,',\vec{p}\,'') \sim e^{-i(\vec{x}_1\vec{p}\,'-t_1E')} e^{-i(\vec{x}_2\vec{p}\,''-t_2E'')} + e^{-\frac{(x_1-x_2)(x_3-x_4)}{a^2}} e^{-i(\vec{x}_1\vec{p}\,''-t_1E'')} e^{-i(\vec{x}_2\vec{p}\,'-t_2E')} \quad \text{(A3)}$$

(the common factor

$$e^{-\frac{(x_3-x_1)^2}{2a^2}} e^{-\frac{(x_4-x_2)^2}{2a^2}}$$

has been taken "outside the bracket" and omitted).

One can see that the relative contribution of the second term in (A3) drops down rapidly when $|x_1 - x_2|$ increases. We are dealing, therefore, with a "partial-interference", decreasing (eventually down to zero) when the distance between the sources increases.

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