GRAVITY AND NON-EXTENSIVE NATURE OF MASS

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Gravitational interaction responsible for stability of large objects like stars and planets results in their mass-defect, thus affecting the gravitational "coupling constants". This double role of gravitation leads to the non-extensive character of mass, which would question the locality of gravitational field equations in the regions filled by matter.

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The gravitation force acting on a bound composite structure must account for its mass-defect, as otherwise the equality of inertial and heavy masses would be violated. Indeed, if heavy mass were insensitive to the mass-defect, then *e.g.* the α-particle composed of four nucleons of masses M would interact with an external gravitational field more strongly than a single nucleon. Its heavy mass would be equal to $4M$, while the inertial mass M_α of α-particle is less than $4M$, as:

$$M_\alpha = 4M - \frac{B}{c^2} = 4M - \Delta M \qquad (\Delta M = \frac{B}{c^2}), \tag{1}$$

where B is its binding energy.

However, the α-particle binding energy is due to the strong and electromagnetic forces and therefore, the α-particle interaction with an external field of gravitation is automatically separated into two uncoupled problems. First, from quantum physics neglecting gravitation, one obtains M_α and next, one puts the corresponding equations of motion of the particle with mass M_α in a gravitational field. However, in the case of astronomical objects like stars and planets, gravitation is responsible for their stability and hence, *via* the mass-defect, for their masses which determine the "coupling constants" of the gravitation itself. This double role of gravitational force makes any local theory of a gravitational field inside matter vague according to the non-extensive nature of masses of these large objects. Let us show this by estimating the rest-mass M of a spherically symmetric "star $\mathcal{M}$" with radius R, under the following simplifying assumptions.

1° Let the static gravitational force between two point-particles of masses m_1, m_2 follow the Newtonian law

$$\vec{f} = -G\frac{m_1 m_2}{r^3}\vec{r} \tag{2}$$

and

2° let the spherically symmetric function $\overset{\circ}{\varrho}(r)$ denote the "proper mass-density" of the star $\mathcal{M}$ built of rigid bodies. Thus we assume that the internal energy of $\mathcal{M}$ is due to the gravitational interaction only. The atomism provides us with a precise definition of the "proper mass-density" of $\mathcal{M}$. Let us suppose for a moment that $\mathcal{M}$ is a neutron star and let $N(r)$ denotes the number of neutrons in a unit volume at a distance r from its centre. Then

$$\overset{\circ}{\varrho}(r) = N(r)M, \tag{3}$$

where M, as before, denotes the rest-mass of a free nucleon.

According to (2), the gravitational potential $\phi(r)$ normalized to zero at infinity ($r \to \infty$) is equal to:

$$\phi(r) = \begin{cases} -\frac{GM(R)}{R} - G\int\limits_r^R dx\frac{M(x)}{x^2} & \text{for } r \leq R \\ -\frac{GM(R)}{r} & \text{for } r \geq R, \end{cases} \tag{4}$$

where the so far unknown function $M(r)$ ($r \leq R$) denotes the heavy (and inertial) mass of the sphere of radius r inside the star $\mathcal{M}$. In agreement with Eq. (2), outside the star ($r > R$) $\phi(r)$ coincides with the Newtonian potential, however, the extensive Newtonian mass $M_0(R)$,

$$M_0(R) = 4\pi\int\limits_0^R dx x^2\,\overset{\circ}{\varrho}(x), \tag{5}$$

is to be replaced by $M(R) < M_0(R)$.

Under the above assumptions, the binding energy of a thin shell of matter covering the sphere of radius r and mass $M(r)$ amounts to:

$$dB(r) = -[4\pi\,\overset{\circ}{\varrho}(r)r^2dr]\phi(r) > 0$$

and so, $M(r)$ must fulfill the following equality:

$$M(r+dr) = M(r) + 4\pi\,\overset{\circ}{\varrho}(r)r^2dr + 4\pi\,\overset{\circ}{\varrho}(r)r^2dr\frac{\phi(r)}{c^2}$$

and hence, the following equations:

$$\frac{dM(r)}{dr} = 4\pi \overset{\circ}{\varrho}(r) r^2 \left[1 - \frac{GM(R)}{Rc^2} - \frac{G}{c^2} \int_r^R dx \frac{M(x)}{x^2} \right] \tag{6.i}$$

and

$$\frac{d}{dr} \left[\frac{1}{\overset{\circ}{\varrho}(r) r^2} \frac{dM(r)}{dr} \right] = 4\pi \frac{G}{c^2} \frac{M(r)}{r^2} . \tag{6.ii}$$

Let us put $\overset{\circ}{\varrho}(r) = \varrho_0 =$ const. which once again oversimplifies real situation but enables one to evaluate $M(r)$ analytically. On introducing the characteristic length

$$a = \frac{c}{(4\pi G \varrho_0)^{1/2}} , \tag{7}$$

Eq. (6.ii) becomes reducible to the confluent hypergeometric equation [1] hence its solution satisfying the boundary condition $M(0) = 0$ takes the form

$$M(y) = C y^3 \mathrm{e}^{-y/2} F(\alpha = 2, \gamma = 4; y) \qquad \left(y = \frac{2r}{a} \right) . \tag{8}$$

As

$$F(2,4;y) = 6y^{-2}(\mathrm{e}^y + 1) - 12 y^{-3}(\mathrm{e}^y - 1) ,$$

after inserting $M(y)$ into (6.i), one finds the normalization constant C and

$$M(r) = a \frac{c^2}{G} \left[\cosh\left(\frac{R}{a}\right) \right]^{-1} \left[\frac{r}{a} \cosh\left(\frac{r}{a}\right) - \sinh\left(\frac{r}{a}\right) \right] = M(r;R) . \tag{9}$$

Finally, for $r = R$ we obtain the mass of the star $\mathcal{M}$ equal to:

$$M(R) = a \frac{c^2}{G} (u - \tanh u) = M(R;R) \qquad \left(u = \frac{R}{a} \right) . \tag{10}$$

In the nonrelativistic limit $(c \to \infty)$ $u \to 0$ and, of course, $M(R)$ from (10) coincides with $M_0(R)$,

$$M(R) \underset{c\to\infty}{\longrightarrow} M_0(R) = \frac{4\pi}{3} R^3 \varrho_0 . \tag{11}$$

The same limit (11) also occurs for finite c when $\varrho_0 \to 0$, as then $u \to 0$ too, as $R \sim \varrho_0^{-1/3}$, $a \sim \varrho_0^{-1/2}$, hence $u \sim \varrho_0^{1/6} \underset{\varrho_0\to 0}{\longrightarrow} 0$. In Fig. 1 the ratio

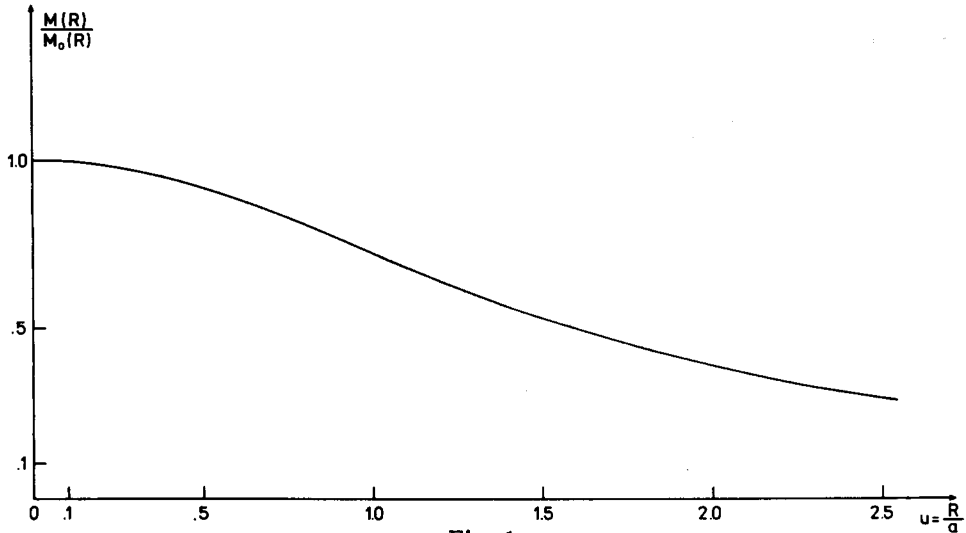

Fig. 1.

$M(R)/M_0(R)$ is presented as a function of $u = R/a$, which also shows the relative mass-defect $\Delta M/M_0$. From (10) and (11) we obtain:

$$\frac{\Delta M(R)}{M_0(R)} = 1 - \frac{M(R)}{M_0(R)}$$
$$= 1 - \frac{3}{u^2}\left(1 - \frac{\tanh u}{u}\right) . \qquad (12)$$

For small values of u ($u \ll 1$), which is usually the case, we get

$$\frac{\Delta M(R)}{M_0(R)} = \frac{2}{5}u^2 - \frac{17}{105}u^4 + \dots , \qquad (13)$$

while for large u ($u \gg 1$)

$$\frac{\Delta M(R)}{M_0(R)} \sim 1 - \frac{3}{u^2} \underset{u\to\infty}{\longrightarrow} 1 . \qquad (14)$$

For earth $\mathcal{E}$ and sun $\mathcal{S}$, although $u \ll 1$, we have:

$$\left(\frac{\Delta M}{M_0}\right)_{\mathcal{E}} \simeq 4 \cdot 10^{-10} , \quad \left(\frac{\Delta M}{M_0}\right)_{\mathcal{S}} \simeq 10^{-6} , \qquad (15)$$

thus the mass-defects are quite large. If $u \gg 1$, from (10) we get

$$M(R) \underset{u\to\infty}{\longrightarrow} \left(\frac{c^2}{G}\right) R \qquad (R \gg a) , \qquad (16)$$

which shows that $M(R)$ increases linearly with R, rather than with the third power of R as in the case of the extensive Newtonians mass $M_0(R)$ from (11).

The non-extensive character of the mass $M(R)$ exhibits its globality, thus calling in question any local theory of gravitation in regions filled by matter. Since $M(R)$ also determines the gravitational coupling constant of $\mathcal{M}$ interacting with another star $\mathcal{M}'$, the detection of the masses of stars from their mutual motion can bring about a considerable underestimation of the number of nucleons constituting heavy stars.

REFERENCES

[1] L.D. Landau, E.M. Lifszyc, *Mechanika Kwantowa*, PWN, Warszawa 1979, pp. 615–619, (in Polish).