QUARK IN A MAGNETIC VACUUM*

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It is argued that quarks immersed in the magnetic vacuum, though asymptotically free, are strongly coupled to gluons and share with them appreciable fraction of their momenta. This effect might be qualitatively related to the known nonperturbative property of the nucleon structure function: equal sharing of the nucleon momentum between quarks and gluons.

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1. Introduction

The magnetic vacuum (for a representative sample of references see [1], [2], [3]) is an interesting model of the QCD vacuum: It provides us with an intuitive description of the asymptotic freedom [2] and, also, gets support from calculations on a lattice [3]. It is instructive therefore to look at its behavior when some simple systems are immersed in it.

The simplest possibility is to insert one quark into the magnetic vacuum and see what happens. In the case of a covariant description of this system the quark interacting with the vacuum changes its mass (and this is the

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only possible effect because relativistic covariance demands no difference between a moving quark and a quark at rest, *i.e.* the quark cannot scatter from the vacuum). The situation changes when a piece of the magnetic vacuum is contained in a finite volume, *e.g.* inside of a nucleon. Then, there exists a rest frame of this vacuum and the quark may scatter from it: it may exchange momentum and energy with it.

In our speculations we work in the rest system of the magnetic vacuum, hence we are dealing with a segment of the vacuum confined in space (*e.g.* inside a hadron). Yet, we treat it as a very large object. This is a very drastic simplification. In effect we adopt a "polaron scenario": A single charged particle moving through an ionic crystal and exciting the collective modes of such a medium is the polaron [4]. In our case a single color-charged quark moves through the magnetic QCD vacuum exciting the collective modes of the vacuum.

The first, and as it turned out incomplete, calculations of this kind were given in [5] and here we are going to follow the main steps of [5] introducing, however, explicitly the anisotropy of scattering. Our vacuum is a charged, massless field (bosonic and fermionic) of charge g and spin S, interacting with a homogeneous external magnetic field pointed *e.g.* along the z-axis

$$\vec{H} = \vec{e}_z H\,. \tag{1.1}$$

This system has the following spectrum of modes [2]

$$E(k_z, n, S_z) = \sqrt{k_z^2 + 2|gH|(n+\tfrac{1}{2}) - 2gHS_z}, n = 0,1,2\ldots. \tag{1.2}$$

Here k_z is the z-component of the momentum, n — the harmonic oscillator quantum number, S_z — z-component of the field spin, g — the coupling constant. As in [5] we do the first order perturbation calculations of the scattering rate of the quark interacting with the modes of the vacuum (1.2). The interaction Hamiltonian density operator is

$$\mathcal{H} = \tilde{g}\bar{\psi}_{p'}\gamma_\mu W^\mu \psi_p\,, \tag{1.3}$$

where W^μ is the gluon field operator which we expand into the modes (1.2), ψ_p and $\psi_{p'}$ are the four component spinor plane waves of the initial and final quarks, and $\tilde{g}$ is the quark-gluon coupling constant. Note that we do not have a direct quark-quark coupling, hence, in the first approximation, the quarks moving through the magnetic vacuum interact only with gluons (hence $S_z = \pm 1$).

The transition rate is given by the Fermi Golden Rule ($\hbar = c = 1$)

$$\Gamma_{\rm fi} = 2\pi < |\mathcal{H}_{\rm fi}|^2 > \delta(\varepsilon_{\rm f} - \varepsilon_{\rm i})\,. \tag{1.4}$$

Here $\varepsilon_{\rm f}$ and $\varepsilon_{\rm i}$ are the final and initial energies and $\langle\ldots\rangle$ denotes averaging over the initial and summing over the final quark states.

$$\mathcal{H}_{\rm fi} = \tilde{g} \int \frac{Ldk_y}{2\pi} \frac{Ldk_z}{2\pi} \sum_{n,S_z} \frac{1}{L} \sqrt{\frac{m^2}{V^2 p_0' p_0}} \bar{u}(\vec{p}\,') e_\mu^\star \gamma_\mu u(\vec{p}) (2\pi)^3$$

$$\times \delta(\Delta p_0 - E(k_z, n, S_z) \delta(\Delta p_y - k_y) \delta(\Delta p_z - k_z) \mathrm{e}^{-i(\frac{\Delta p_x k_y}{gH})} \tilde{\Phi}_n(\Delta p_x)\,, \quad (1.5)$$

where L^3 is the normalization volume, and

$$\tilde{\Phi}_n(\Delta p_x) = \int\limits_{-\infty}^{+\infty} \mathrm{e}^{-ix\Delta p_x} \Phi_n(x)\,,$$

$\Phi_n(x)$ being the n-th one-dimensional oscillator wave function.

In the following section we will evaluate $\Gamma_{\rm fi}$ for a general configuration of the vacuum orientation with respect to the initial and final momenta of the quark.

2. Evaluation of the transition rate

The geometry of scattering in the rest frame of the vacuum is as follows: The initial momentum of the quark is

$$\vec{p} = (p\cos\alpha)\vec{e}_z + (p\sin\alpha\sin\phi)\vec{e}_y + (p\sin\alpha\cos\phi)\vec{e}_x\,. \quad (2.1a)$$

The final momentum of the quark is

$$\vec{p}_{\rm f} = (p_{\rm f}\cos\theta)\vec{e}_z + (p_{\rm f}\sin\theta\sin\phi_{\rm f})\vec{e}_y + (p_{\rm f}\sin\theta\cos\phi_{\rm f})\vec{e}_x\,. \quad (2.1b)$$

The magnetic field is

$$\vec{H} = H\vec{e}_z\,. \quad (2.1c)$$

From (2.1) follows that

$$\Delta p_z = p_{\rm f}\cos\theta - p\cos\alpha,\; \Delta p_x = p_{\rm f}\sin\theta\cos\phi_{\rm f} - p\sin\alpha\cos\phi\,,$$

and

$$\frac{\vec{p}\cdot\vec{p}_{\rm f}}{pp_{\rm f}} = \cos\chi = \cos\theta\cos\alpha + \sin\theta\sin\alpha\cos(\phi - \phi_{\rm f})\,.$$

From (1.4) we obtain

$$\Gamma_{\rm fi} = 2\pi\langle|\mathcal{H}_{\rm fi}|^2\rangle\delta(\varepsilon_{\rm f}-\varepsilon_{\rm i}) = \tilde{g}^2\Big(1+v_{\rm f}v(2\cos\theta\cos\alpha-\cos\chi)-\frac{m^2}{p_{f0}p_0}\Big)$$
$$\times\frac{(2\pi)^3}{L^4}\frac{|\tilde{\Phi}_n(p_{\rm f}\sin\theta\cos\phi-p\sin\alpha\cos\phi)|^2}{2E(\Delta p_z,n,S_z)}\delta(F(p_{\rm f},E)), \qquad (2.2)$$

where

$$E(\Delta p_z,n,S_z)=\sqrt{(p\cos\alpha-p_{\rm f}\cos\theta)^2+2|gH|(n+\tfrac{1}{2})-2gHS_z},$$

and

$$F(p_{\rm f},E)=\sqrt{p^2+m^2}-\sqrt{p_{\rm f}^2+m^2}-E(\Delta p_z,n,S_z)=0,$$

because it is the argument of Dirac's δ. $v_{\rm f}$ and v are the velocities of the final and initial quark, respectively.

Expression (2.2) can be simplified through a gauge transformation which is equivalent to a rotation of the reference frame around z-axis. Let p, α and ϕ be fixed. Then (2.2) gives the rate of scattering to $p_{\rm f}$, θ and $\phi_{\rm f}$. The vector potential $\vec{A}$ which gives $\vec{H}$ along $\vec{e}_z$ is (compare [2])

$$\vec{A}=\vec{e}_yHx.$$

We do the following gauge transformation

$$\vec{A}'=\vec{A}+\vec{\nabla}f=\vec{e}_yHx+\vec{e}_x\Big(\frac{-p\sin\alpha\cos\phi}{g}\Big),$$

hence we choose the scalar function as

$$f=(-p\sin\alpha\cos\phi)x.$$

This implies a change of phase in $\Phi_n(x)$:

$$\Phi'_n(x)=\Phi_n(x)e^{igf}=\phi_n(x)e^{-ixp\sin\alpha\cos\phi}.$$

Thus

$$\tilde{\Phi}'_n(\Delta p_x)=\int\limits_{-\infty}^{+\infty}dx e^{-ix(\Delta p_x+p\sin\alpha\cos\phi)}\Phi_n(x)=\tilde{\Phi}_n(p_{\rm f}\sin\theta\cos\phi_{\rm f}), \quad (2.3)$$

which simplifies the argument of $\tilde{\Phi}_n$ in (2.2).

Note that we may equally well fix the final configuration and get rid of the dependence of the argument of $\tilde{\Phi}_n$ on the final configuration and have the transition rate to a fixed final configuration from any initial configuration. This symmetry between the initial and final configurations follows from the fact that the argument of $\tilde{\Phi}_n$ in (2.2) is a sum of two contributions with the variables of these two contributions separated.

The Dirac δ-function in (2.2) takes care of the energy conservation and gives the following expression for the scattering angle θ as a function of the final momentum p_f (with α, p and n fixed):

$$\cos\theta = c(p_f, \alpha, p, n) = \frac{p\cos\alpha}{p_f} - \frac{1}{p_f}\sqrt{(\Delta p_0)^2 - H(n)}\,, \tag{2.4}$$

where

$$\Delta p_0 = \sqrt{p^2+m^2} - \sqrt{p_f^2+m^2}, \quad H(n) = 2|gH|(n+\tfrac{1}{2}) - 2gHS_z\,.$$

With α, p, n fixed we get from (2.2) either of the two differential rates $\frac{d\Gamma_{fi}}{d(\cos\theta)}$ or $\frac{d\Gamma_{fi}}{dp_f}$. Since $F(p_f, E) = 0$ along the (2.4) curve, we have

$$\frac{\partial F}{\partial p_f}dp_f + \frac{\partial F}{\partial(\cos\theta)}d(\cos\theta) = 0\,,$$

and

$$-\frac{d(\cos\theta)}{dp_f} = \frac{\partial F}{\partial p_f}\frac{E}{p_f(p\cos\alpha - p_f\cos\theta)}\,,$$

thus

$$\frac{d\Gamma}{dp_f} = \frac{d\Gamma}{d(\cos\theta)}\frac{\partial F}{\partial p_f}\frac{E}{p_f(p\cos\alpha - p_f\cos\theta)}\,. \tag{2.5}$$

And we get (compare also [5])

$$\begin{aligned}\frac{d\Gamma_{fi}}{d\theta} =& L\sqrt{\frac{2gH}{(n+\frac{1}{2})}}\frac{L^3}{(2\pi)^3}\sin\theta\int d\phi_f p_f^2\, dp_f\, \Gamma_{fi}\\ =& \int_0^{2\pi} d\phi_f \tilde{g}^2\sqrt{\frac{gH}{(n+\frac{1}{2})}}\sin\theta\Big[1 + 2v_f v(\cos\theta - \tfrac{1}{2}\cos\chi) - \frac{m^2}{p_{f0}p_0}\Big]\\ &\times \frac{p_f^2}{|\frac{\partial F}{\partial p_f}|}\frac{|\tilde{\Phi}_n(p_f\sin\theta\cos\phi_f)|^2}{E(\Delta p_z, n, S_z)}\,,\end{aligned} \tag{2.6}$$

or

$$\frac{d\Gamma_{\rm fi}}{dp_{\rm f}} = \int_0^{2\pi} d\phi_{\rm f}\bar{g}^2 \sqrt{\frac{gH}{(n+\frac{1}{2})}} \left[1 + 2v_{\rm f}v(\cos\theta - \tfrac{1}{2}\cos\chi) - \frac{m^2}{p_{f0}p_0}\right] \times \frac{p_{\rm f}|\tilde{\Phi}_n(p_{\rm f}\sin\theta\cos\phi_{\rm f})|^2}{2(p\cos\alpha - p_{\rm f}\cos\theta)}. \tag{2.7}$$

Note that, from (2.4),

$$p\cos\alpha - p_{\rm f}\cos\theta = \sqrt{(\Delta p_0)^2 - H(n)}.$$

Note also that when $\alpha = 0, \cos\alpha = 1$ (the initial quark moves along the magnetic field $\vec{H} = \vec{e}_z H$) we obtain the case considered in Ref. [5].

One can perform approximately the integrals over $\phi_{\rm f}$ remembering that we can write $\cos\chi = \cos\theta\cos\alpha + \sin\theta\sin\alpha\cos\phi_{\rm f}$ but our main conclusions do not depend on detailed form of these integrals.

3. Discussion and conclusions

Both differential rates (2.6) and (2.7) have singularities.

(a) The angular distribution (2.6) has a singularity at

$$\frac{\partial F}{\partial p_{\rm f}} = 0, \text{ or } v_{\rm f} = \left(\sqrt{1 - \frac{H(n)}{(\Delta p_0)^2}}\right)\cos\theta. \tag{3.1}$$

We shall call this singularity the "Cherenkov" singularity, because it defines a distinctive cone although it does not lead to an infinite rate (compare also [5]).

(b) The momentum distribution (2.7) has a pole at

$$p\cos\alpha - p_{\rm f}\cos\theta = \sqrt{(\Delta p_0)^2 - H(n)} = 0. \tag{3.2}$$

We shall call this singularity the "bremsstrahlung" singularity, because for $\alpha = 0$ it becomes the familiar bremsstrahlung singularity as $p_{\rm f} \to p$. (3.2) leads to an infinite rate and this fact is of primary importance, as we shall presently see.

Indeed, let us continue — in spite of a somewhat academic quality of such an approach — to treat the magnetic vacuum as a piece of solid matter with a light object (quark) traversing it. The process has two distinct regimes:

Regime I, $\cos\alpha < 1$. The quark is set in motion at an angle to $\vec{H}$. Then, the configuration (3.2) dominates the process. Since $p > p_f$, we must have $\cos\theta > \cos\alpha$, hence the final momentum of the quark is being pushed towards $\vec{H}$. Then, immediately, the next scattering follows because the rate is enormous, and then the next scattering, and the next ... , until the momentum is aligned with $\vec{H}$. Note that from this process the quark emerges with momentum which is, virtually, parallel to $\vec{H}$ and its length is given by the initial momentum and angle:

$$p\cos\alpha\,,$$

and thus the second regime of the process starts.

Regime II, $\cos\alpha' = 1$. This is the case we discussed in detail in Ref. [5]. Now, the rates are finite and the scattering angles very small. Also, only $n = 0$ state of the vacuum contributes, and $H(n) < 0$. Should a scattering in this regime took us to a large enough angle and shift the quark back to the first regime — it will be immediately pushed back into the second regime by the singularity (3.2).

What we have is indeed a process of **channeling of the quark**. Similar to the channeling of charged particles traversing some crystals. The channel is here defined by $\vec{H}$.

One could go on and on with a detailed description based on the formulae (2.4), (2.6) and (2.7) but the value of such details would be very doubtful. These would be wild speculations indeed: detailed extrapolations of the above formulae to a quark set into motion inside a nucleon by *e.g.* a high energy lepton.

Yet, it is tempting to identify the inside of a nucleon with a piece of magnetic vacuum which does lead to the asymptotic freedom of quarks [2] and, moreover, its very existence is supported by calculations on a lattice [3]. Also, a possible influence of the chromomagnetic vacuum fields on high energy hadronic reactions has already been discussed (see *e.g.* [6]). What then our calculations do suggest?

A qualitative consequence appears reasonable. It refers to the structure of the nucleon. When a quark is moved, the most probable regime to be realized is the Regime I. Then, as we have seen, a large fraction of its momentum is immediately shared with the gluons. In other words, we are dealing, on the one hand, with asymptotically free quarks which, on the other hand, are strongly coupled to the gluons. So, the nonperturbative character of the magnetic vacuum **qualitatively** explains sharing of momenta between quarks and gluons observed in experiment [7]. This effect is commonly interpreted as a nonperturbative one and, as we see, can be accommodated by filling the interior of the nucleon with the magnetic vacuum.

We close with reiteration of our conclusions: Let us consider a deep inelastic lepton–nucleon scattering. If it were not for the quark coupling to the vacuum gluon field, the structure function would have reflected sharing of the nucleon momentum only between quarks. In our case, when a quark is struck by the photon and starts moving at an angle α relative to $\vec{H}$, it imparts a fraction of the received energy to the vacuum gluon field. In the end the quark will leave the chromomagnetic vacuum moving along the direction of $\vec{H}$. However, the energy it had lost in the process is transferred to the nucleon remnants and is a function of α. Assuming a random distribution of α, we get the following relation between the initial and the average final momenta of the quark in **Regime I**

$$\langle (p_{\rm f}^{\rm I})^2 \rangle = p^2 \langle \cos^2 \alpha \rangle = p^2 0.5\,,$$

$$\langle p_{\rm f}^{\rm I} \rangle \cong 0.7p\,.$$

But then **Regime II** takes over and reduces the above input momentum, $0.7p$, by another factor ~ 0.7 (compare [5]). So, finally, we obtain the average sharing of the original quark momentum with the gluons

$$\langle p_{\rm f} \rangle \cong 0.7 \langle p_{\rm f}^{\rm I} \rangle \cong 0.5p\,, \tag{3.3}$$

in complete agreement with experiment (compare [7]). In principle, assuming such a random distribution of α, we could calculate the quark energy loss distribution and the corresponding structure function. The reason we do not pursue this exercise is the feeling that our approach can represent at best the very general and qualitative outline of such a theory.

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