SPACE-TIME STRUCTURE OF HADRON SOURCES AND INTERMITTENCY*

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Following the idea that the phenomenon of intermittency is related to HBT correlations between identical pions, we investigate the origin of the power law behaviour of the correlation functions. The measurements of the higher-order correlation functions indicate that the observed intermittency is an effect present in individual events rather than the result of incoherent superposition of many events. Also uncorrelated emission from a space-time volume of any shape does not seem to account for the data. These results suggest a genuine fractal space-time structure of particle production.

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1. Introduction

Recently, several experiments [1–3] found that the phenomenon of intermittency [4, 5] is dominated by very short range correlations between momenta of identical hadrons (HBT effect [6]). As is well known, the HBT correlations reflect the size and shape of the space-time region from which the observed identical particles are emitted [7]. Remembering that intermittency is equivalent to a power law dependence of the hadronic correlation functions, one may conclude that a power law dependence must also be present in the distribution of space-time shapes and/or sizes of the region of hadron emission [8]. It was suggested recently that this observation may indicate a critical behaviour of the system [9, 10].

In the present paper we investigate in somewhat more detail the relation between the space-time structure of the sources of particles created in

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high-energy collisions and the observed power law behaviour of the correlation functions in momentum space. We show that the measurements of the correlation functions of order higher than two is necessary to obtain a non-trivial information on the space-time structure of the source. Comparison with existing preliminary data indicates that this structure may indeed be fractal, as conjectured already in Ref. [8–10]. In the next section we discuss the general formulae describing the HBT effect and propose a set of measurements of higher order correlations which are most suitable for studying intermittency. In Sections 3 and 4 two specific examples of the physical systems are considered. In Section 3 uncorrelated emission from a source with space-time density described by a power law is described. In Section 4 we discuss a multicomponent model of intermittency, *i.e.*, an incoherent superposition of non-intermittent events (chosen in such a way that the resulting inclusive spectra are intermittent). In Section 5 the breaking of intermittency at very small intervals is analyzed. Our conclusions are listed in the last section.

2. Intermittency and multiparticle HBT correlations

To explain our argument, which is a generalization of the one presented in [8–10], let us consider a totally incoherent source of identical particles with space-time multiparticle densities $d_1(x)$, $d_2(x_1, x_2)$,..., where x denotes position in space and time.

Following the standard procedure [7] and neglecting final state interactions we find for multiparticle densities in momentum space

$$\rho_k(p_1,\ldots,p_k) = \frac{v^{-k}}{k!}\int dx_1\ldots dx_k \mid \sum_{\mathrm{perm}}\prod_{j=1}^{k}\exp(ix_jp_j)\mid^2 d_k(x_1,\ldots,x_k)\,, \tag{1}$$

where the summation extends over all permutations of the momenta $p_1\ldots p_k$. The constant V can be determined from the normalization condition

$$\int \rho_k(p_1,\ldots,p_k)dp_1\ldots dp_k = \int d_k(x_1,\ldots,x_k)dx_1\ldots dx_k = \langle n(n-1)\ldots(n-k+1)\rangle\,, \tag{2}$$

where n is the total multiplicity. For $k=1$ we obtain from (1)

$$V^{-1} = \frac{\rho_1(p)}{\langle n\rangle}\,. \tag{3}$$

A sketch of the proof of Eq. (1) is given in the Appendix.

It is seen from (1) that the particle density in momentum space is a linear combination of Fourier transforms of particle densities in space-time which we shall denote by D_k:

$$D_k(q_1,\ldots,q_k) \equiv V^{-k} \int dx_1 \ldots dx_k \prod_{j=1}^{k} \exp(ix_j q_j) d_k(x_1,\ldots,x_k). \quad (4)$$

D_k is a symmetric function with respect to exchange of any of its arguments (because d_k is symmetric). Furthermore, $q_1 + \ldots + q_k = 0$ for all D_k's on the r.h.s. of Eq. (1). Finally, only the symmetric combinations $D_k(q) + D_k(-q)$ are present.

From (1) one can see that the power law behaviour of the densities in momentum space requires a power law behaviour of the Fourier transforms D_k and thus a power law behaviour of the space-time densities. To obtain more precise information about the mechanism(s) responsible for such a power law we shall now discuss in more detail two different scenarios.

3. Uncorrelated emission in space-time

The sum over permutations in the r.h.s. of (1) implies that the momentum distributions will exhibit correlations even if the space-time distribution is uncorrelated, *i.e.*, if

$$d_n(x_1,\ldots,x_n) = d_1(x_1)\ldots d_1(x_n). \quad (5)$$

In this case, using Eqs. (1) and (4) and well-known relations between the multiparticle distributions and correlation functions [11], one finds the general formula

$$C_k(p_1,\ldots,p_k) = \sum D_1(p_1 - p_{a_1})\ldots D_1(p_k - p_{a_k}); \quad k \geq 2, \quad (6)$$

where C_k is the correlation function and the sum extends over all $(k-1)!$ combinations of different pairs $(1, a_1), \ldots (k, a_k)$ with $a_i \neq i$.

The power law behaviour of the correlation functions is obtained if the Fourier transform D_1 of the density d_1 exhibits a power law dependence and thus if the density d_1 itself follows a power law. Indeed, taking $d_1(x)$ in the form

$$d_1(x) = A\,|x|^{\gamma - D}, \quad (7)$$

where A is a constant and D is the topological dimension of the x-space, one obtains for the Fourier transform (4)

$$D_1(q) = A\,|q|^{-\gamma} \int \exp[iuG(Q)]u^{\gamma-1} du\, d\Omega. \quad (8)$$

Here Ω is the D-dimensional solid angle and $G(\Omega)$ is the scalar product of the two unit vectors in the x-space.

One sees from (8) and from (6) that the correlation function C_k follows the power law with "intermittency exponent"

$$f_k = k\gamma\,; \quad k \geq 2\,. \tag{9}$$

Thus in the case of uncorrelated production intermittency exponents are simply proportional to the rank of the correlation function.

The existing data show intermittency exponents increasing with increasing rank of the moments. However, there exist yet no published data on intermittency for more than two-particle spectra of identical pions. The preliminary results from NA22 experiment [12] gives $\frac{f_3}{f_2} \approx 2$ rather than expected $\frac{3}{2}$. If this result is confirmed it would mean that the hypothesis of independent production must be abandoned.

4. Multicomponent model of intermittency

Consider now the situation when the power law spectrum is not present at the level of individual events but is generated by superposition of events characterized by uncorrelated emission from space-time volumes of varying sizes [8]. In this case we have

$$d_k(x_1,\ldots,x_k) = \int dR\, F(R)\bar{d}_1(x_1;R)\ldots\bar{d}_1(x_k;R)\,, \tag{10}$$

where $\bar{d}_1$ is a non-singular function of x confined to the region of size R. We take the $\bar{d}_1$ in the generic form

$$\bar{d}_1(x;R) = h\left(\tfrac{x}{R}\right)\Phi(R) \tag{11}$$

with $h(u)$ confined to the region $u < 1$. The Fourier transform (4) reads

$$D_k(q_1,\ldots,q_k) = \int F(R)\, dR\, [\Phi(R)]^k\, R^{kD}\, \tilde{h}(Rq_1)\ldots\tilde{h}(Rq_k)\,, \tag{12}$$

where

$$\tilde{h}(z) = \int h(u) \exp\left[i\,|u|\, zG(\Omega)\right] |u|^{D-1} d\,|u|\, d\Omega \tag{13}$$

is a regular function of z. The non-integer power law at the l.h.s. of the Eq. (12) can thus be generated if the functions $F(R)$ and $\Phi(R)$ follow the power law. Taking

$$F(R) = \text{const}\; R^{-\gamma}\,; \quad \Phi(R) = \text{const}\; R^{\lambda-D} \tag{14}$$

one obtains

$$D_k(q_1,\ldots,q_k) = \text{const}\; q^{\gamma-1}q^{-k\lambda}\int dz\, z^{-\gamma+k\lambda}\tilde{h}\left(\tfrac{zq_1}{q}\right)\ldots\tilde{h}\left(\tfrac{zq_k}{q}\right), \quad (15)$$

where we have introduced a new variable

$$q = \left[(q_1)^2+\ldots+(q_k)^2\right]^{\frac{1}{2}}. \quad (16)$$

It is seen from (15) that the obtained singularity in q depends linearly on k: the intermittency exponent is given by

$$f_k = 1-\gamma+k\lambda. \quad (17)$$

At this point it is worth to note that the parameter λ is related to the multiplicity of the different components of the model by the formula

$$\langle n(R)\rangle = \text{const}\; R^{\lambda}, \quad (18)$$

and thus for positive λ (which is necessary for (17) to account for the data) the range of the correlations in momentum space is predicted to decrease with increasing multiplicity. This does not seem to be followed by the data of UA1 Collaboration [13] and, therefore, we are forced to conclude that the multicomponent models are unlikely to be realized in nature.

5. Breaking of intermittency

The exact power law implies that particle densities and correlation functions must be singular (*i.e.* infinite) in the limit of vanishing momentum differences. This is unphysical and thus at some point the power law must be broken. The examples shown in Sections 3 and 4 allow to discuss this phenomenon a little more precisely.

To this end let us observe that the formula (7) cannot be exact because, taken literally, it implies the infinite multiplicity. It must, therefore, be cut at some (large) $|x| = L$. If we take the simplest possibility *i.e.* $d_1 = 0$ for $|x| > L$, we obtain for $D_1(q)$ the formula identical to (8), except that the limits of integration in $|u|$ are from 0 to qL rather than from 0 to ∞. Consequently, the integral behaves as $(qL)^{1-\gamma}$ for small $q(q \ll L)$ and the singularity at $q = 0$ is cancelled. The deviation from the power law behaviour should be seen already for $qL \approx 1$ and thus observation of such a deviation gives information on the range of the extension of the source in space time.

Similar remarks apply also for the model discussed in Section 4. In this case the probability density $F(R)$ must be cut above some value of R, because otherwise the probability distribution would not be normalizable. Again, cut-off at large R implies that the singularity at $q = 0$ in Eq. (15) disappears. Also here the point at which the data deviate from the power law gives information about the maximal range of R. In Ref. [10] a simple example of this effect is studied numerically.

This argument explains the physical interpretation of the minimal interval in momentum space, where the power law still holds. This minimal interval is determined by largest accessible space-time volume of the system. The recent data of the UA1-MB and NA22 collaborations [1, 2] are consistent with power law up to momentum difference 30–40 MeV. This indicates presence of indeed very large volumes (6–7 fm), which are not easy to explain by standard arguments.

6. Conclusions

We have investigated consequences of the experimentally observed relation between intermittency and HBT correlations. Our conclusions can be summarized as follows.

(a) Measurements of two-particle correlations do not provide sufficient information for analysis of the space-time structure of the sources of particles created in high-energy collisions.

(b) Information about the statistical properties of this space-time structure is accessible only through measurements of correlations between more than two particles.

(c) The existing data seems incompatible with the possibility that the effect of intermittency is a consequence of a "multicomponent" character of the production process.

(d) It seems also unlikely that the existing data can be explained in terms of uncorrelated production from a source whose space-time distribution follows a power law.

(e) The points (c) and (d) make it plausible that the space-time structure of the source of particles is actually fractal. However, the data on higher order correlations are needed to confirm this conjecture.

(f) Confirmation of the power law behaviour of two-particle correlation function up to very small momentum intervals [1, 2] indicates the presence of unusually large space-time structures in particle production.

As a final comment we would like to point out that we have considered only the simplest case of the totally incoherent source of particles. We think that it illustrates correctly the essential points of our argument. In the real analysis of the data, however, more complicated situations are known to occur and the assumption of total incoherence cannot be maintained. Although we do not expect that the main qualitative features will change, we fully appreciate the need of such a refined analysis for a detailed comparison with the data.

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APPENDIX

A sketch of the proof of Eq. (1)

Consider a system of n identical bosons emitted at point $(x_1, \ldots, x_n)$ and described by the wave function $\phi(x_1, \ldots, x_n; z)$, where z is a set of additional quantum numbers characterizing this system. The corresponding wave function in momentum space is

$$f_n(p_1, \ldots, p_n; z) = (2\pi)^{-n/2}(n!)^{-1} \int dx_1 \ldots dx_n \psi_n(p_1, \ldots, p_n; x_1, \ldots, x_n)\phi_n(x_1, \ldots, x_n; z), \tag{A.1}$$

where

$$\psi_n = \sum \exp\left(ip_1 x_{\alpha_1} + \ldots + ip_n x_{\alpha_n}\right), \tag{A.2}$$

and the sum extends over all $n!$ permutations of the set $a_1, \ldots, a_n$. The normalization in (A.2) is chosen such that, as usual,

$$\int dp_1 \ldots dp_n |f_n(p_1, \ldots, p_n; z)|^2 dz = \int dx_1 \ldots dx_n |\phi_n(x_1, \ldots, x_n; z)|^2 dz. \tag{A.3}$$

Let us now introduce the condition of incoherent emission in the form

$$\int \phi_n(x_1, \ldots, x_n; z)\phi_n^*(x_1', \ldots, x_n'; z) dz = \delta(x_1 - x_1') \ldots \delta(x_n - x_n') K_n \int |\phi_n(x_1, \ldots, x_n; z)|^2 dz, \tag{A.4}$$

where K_n is a constant, to be determined from the normalization condition.

Using (A.4) and (A.1) we obtain for the probability density of finding n particles with momenta $(p_1, \ldots, p_n)$

$$P_n(p_1, \ldots, p_n) \approx \int |f_n(p_1, \ldots, p_n; z)|^2 dz$$
$$= K_n \int dx_1 \ldots dx_n |\psi_n(p_1, \ldots, p_n; x_1, \ldots, x_n)|^2 W_n(x_1, \ldots, x_n)\,, \quad \text{(A.5)}$$

where $W_n(x_1, \ldots, x_n)$ is the probability of emitting n-particles at positions $(x_1, \ldots, x_n)$:

$$W_n(x_1, \ldots, x_n) = \int |\phi_n(x_1, \ldots, x_n; z)|^2 dz\,. \quad \text{(A.6)}$$

The normalization condition (A.3) substituted into (A.5) implies

$$K_n = \frac{V^{-n}}{n!}\,, \quad \text{(A.7)}$$

where V is the available volume in momentum space (the formula is valid in the limit $V \to \infty$) given by

$$V^{-1} = \frac{P_1(p)}{\int dx W_1(x)} \quad \text{(A.8)}$$

as is seen from (A.5) for $n = 1$ (it is seen from (A.5) that $P_1(p)$ is independent of p).

Eqs. (A.5)–(A.7) give the probability distribution of n identical particles in momentum space. To derive Eq. (1), it remains to translate this into standard multiparticle inclusive densities.

To this end, let us remind the definition of the k-particle inclusive density:

$$\rho_k(p_1, \ldots, p_k) = \sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1) \int P_n(p_1, \ldots, p_n) dp_{k+1} \ldots dp_n\,. \quad \text{(A.9)}$$

Using the identity

$$\int |\psi_n(p_1, \ldots, p_n; x_1, \ldots, x_n)^2 dp_{k+1} \ldots dp_n$$
$$= V_p^{n-k}(n-k)! \sum |\psi_k(p_1, \ldots, p_k; x_{\alpha_1}, \ldots, x_{\alpha_k}|^2\,, \quad \text{(A.10)}$$

where the sum extends over all $\binom{n}{k}$ sets $(a_1, \ldots, a_k)$ which can be selected from the set $(1, \ldots, n)$, we obtain from (A.5) and (A.7):

$$\rho(p_1, \ldots, p_k) = K_k \int dx_1 \ldots dx_k |\psi_k(p_1, \ldots, p_k; x_1, \ldots x_k)|^2$$

$$\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1) \int dx_{k+1} \ldots dx_n W_n(x_1, \ldots x_n)\,, \quad \text{(A.11)}$$

which is precisely formula (1).

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