THERMAL PROPERTIES OF NUCLEAR AND NEUTRON MATTER WITH MYERS–SWIATECKI POTENTIAL

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The Thomas–Fermi model is used to calculate the equation of state (EOS) for nuclear as well as neutron matter. We apply the effective nucleon-nucleon interaction of Myers and Swiatecki which is velocity and density dependent. The calculations cover a wide density range of interest in heavy-ion collisions and astrophysics. We explore the thermodynamical behaviour of symmetric nuclear matter and neutron matter at finite temperature as it results from our EOS. As an illustration, the maximum stable mass of neutron star is calculated.

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1. Introduction

The equation of state (EOS) of nuclear matter is a subject of growing interest in the field of heavy-ion collisions at intermediate energy, where a regime of high density and high temperature seems to be reached. A comparable interest in EOS exists also in astrophysics, where the dynamics of supernovae explosions and the structure of neutron stars is very sensitive to the properties of nuclear matter far from the saturation point.

The Thomas–Fermi model with an adjustable effective nucleon-nucleon interaction is nearly an ideal macroscopic model [1–4]. It is efficient in its use of adjustable parameters.

In the present work, we formulate the Thomas–Fermi model with a flexible effective nucleon-nucleon interaction of Myers and Swiatecki (MS) [5] which is velocity and density dependent to study the properties of nuclear and neutron matter at non-zero temperature.

In the previous work, the density dependent term was not considered. In this case the Thomas–Fermi model predicts that the optical potential turns repulsive beyond about 70 MeV incident nucleon energy, which is in clear disagreement with measurements. This is a direct indication of the need for a density dependence of the effective interaction, which can then take over some of the burden of nuclear saturation from the velocity-dependent terms.

The paper is organized as follows. In Section 2, the theory and model applied are given. The results and discussion are presented in Section 3.

2. Theory and model

We used the (MS) [5] potential which is velocity and density dependent.

$$v = -\left[\tfrac{1}{2}(1 \mp \xi)C_\alpha - \tfrac{1}{2}(1 \mp \zeta)(C_\beta P_{12}^2 - C_\gamma P_{12}^{-1} + C_\sigma \bar{\rho}^{2/3})\right] \frac{1}{4\pi a^3} \frac{e^{-r/a}}{r/a} . \quad (1)$$

Here P_{12} stands for the magnitude of the relative momentum of the interacting particles and $\bar{\rho}$ is a mean density $\left(\bar{\rho}^{2/3} = \frac{1}{2}\left(\rho_1^{2/3} + \rho_2^{2/3}\right)\right)$, where ρ_1 and ρ_2 are the relevant neutron or proton densities at points 1 and 2. The $\pm$ sign refers to unlike (neutron-proton) interactions and to like (neutron-neutron or proton-proton) interactions.

Using the Thomas–Fermi model, the energy per particle for asymmetric nuclear matter up to x^2 [x is the neutron excess parameter defined by $x = (N - Z)/A$] is given by

$$E(T = 0, \rho) = E_v(T = 0, \rho) + x^2 E_x(T = 0, \rho) , \quad (2)$$

where

$$E_v(T = 0, \rho) = T_0\left[\frac{3}{5}\left(\frac{\rho}{\rho_0}\right)^{2/3}(1 - \gamma_L - \gamma_U) - \frac{1}{2}\left(\frac{\rho}{\rho_0}\right)(\alpha_L + \alpha_U) + \frac{3}{5}\left(\frac{\rho}{\rho_0}\right)^{5/3}(B_L + B_U)\right] \quad (3)$$

and

$$E_x(T = 0, \rho) = T_0\left[\frac{1}{3}\left(\frac{\rho}{\rho_0}\right)^{2/3}(1 - \gamma_L + 2\gamma_U) + \frac{1}{2}\left(\frac{\rho}{\rho_0}\right)(\alpha_U - \alpha_L) + \frac{2}{3}\left(\frac{\rho}{\rho_0}\right)^{5/3}(2B_L - B_U)\right. \quad (4)$$

where $\alpha_{L,U}$, $B_{L,U}$, $\gamma_{L,U}$ and $\sigma_{L,U}$ are dimensionless parameters related to C_α, C_β, C_γ and C_σ, respectively. T_0 is the nuclear matter Fermi energy, ρ

is the density of nuclear matter and ρ_0 is the equilibrium density of nuclear matter.

The energy per particle for neutron matter is

$$E_{\mathrm{n.m}}(T=0,\rho) = T_0\left[\frac{3}{5}\left(\frac{2\rho_n}{\rho_0}\right)^{2/3}(1-\gamma_L) - \frac{1}{2}\left(\frac{2\rho_n}{\rho_0}\right)\alpha_L + \frac{3}{5}\left(\frac{2\rho_n}{\rho_0}\right)^{5/3} B_L\right]. \tag{5}$$

The pressure of nuclear matter can be obtained from the relation

$$P(T,\rho) = \rho^2\left(\frac{\partial E}{\partial \rho}\right)_T. \tag{6}$$

Therefore, the pressure of the system can be written as

$$P(T,\rho) = P_v(T,\rho) + x^2 P_x(T,\rho), \tag{7}$$

where $P_v(T,\rho)$ and $P_x(T,\rho)$ represent the volume and symmetry pressures, respectively. The sound velocity in nuclear matter is given by

$$V_s(T,\rho) = \left(\frac{\partial P(T,\rho)}{\partial \rho}\right)^{1/2} = \left[\frac{\partial}{\partial \rho}\Big(P_v(T,\rho) + x^2 P_x(T,\rho)\Big)\right]^{1/2}, \tag{8}$$

The free energy per particle for homogeneous nuclear matter is

$$F(T,\rho) = E(T,\rho) - TS, \tag{9}$$

where S is the entropy per nucleon in the Thomas–Fermi model, the entropy can be written as [6]

$$S = \frac{\pi^2}{2}\rho^{-1}\sum_J \rho_J \frac{2m_J^*}{\hbar^2 k_f^2} T + O(T^3), \tag{10}$$

where ρ_J is the density of nucleons with isospin J and k_f is the Fermi momentum.

The thermodynamic functions can be represented by a power series in the temperature. Then the free energy per particle up to T^2 approximation is

$$F(T,\rho) = F_v(T,\rho) + x^2 F_x(T,\rho), \tag{11}$$

where

$$F_v(T,\rho) = E_v(T=0,\rho) - \frac{T^2}{6}\frac{2m^*}{\hbar^2}\left(\frac{3}{2}\pi^2\right)^{1/3}\rho^{-2/3} \tag{12}$$

and

$$F_x(T,\rho) = E_x(T=0,\rho) + \frac{T^2}{54}\frac{2m^*}{\hbar^2}\left(\frac{3}{2}\pi^2\right)^{1/3}\rho^{-2/3} \tag{13}$$

m^* is the nucleon effective mass, related to the parameters of potential and the nuclear mass m by the relation

$$\frac{m}{m^*} = 1 + \left(\beta + \tfrac{1}{2}\gamma\right). \tag{14}$$

The pressure of neutron matter at zero temperature ($P = \rho^2 \partial F/\partial\rho$) is

$$P_{\rm n.m}(T=0,\rho) = T_0\left[\frac{2}{5}\left(\frac{2}{\rho_0}\right)^{2/3}(1-\gamma_L)\rho_n^{5/3} - \left(\frac{\rho_n^2}{\rho_0}\right)\alpha_L + \left(\frac{2}{\rho_0}\right)^{5/3} B_L \rho_n^{8/3}\right]. \tag{15}$$

The sound velocity in neutron matter is calculated using the relation $V_{\rm sn.m.}(T=0,\rho) = (\partial P_{\rm n.m.}(T=0,\rho)/\partial\rho)^{1/2}$. Since the temperature inside neutron star is less than 1 MeV. This gives the energy as

$$E_{\rm n.m.}(T,\rho) = E_{\rm n.m.}(T=0,\rho) + \frac{T^2}{12}\left(\frac{2m*}{h^2}\right)(3\pi^2)^{1/3}\rho_n^{-2/3}, \tag{16}$$

The pressure is

$$P_{\rm n.m.}(T,\rho_n) = P_{\rm n.m.}(T=0,\rho_n) + \frac{T^2}{18}\left(\frac{2m*}{h^2}\right)(3\pi^2\rho_n)^{1/3}. \tag{17}$$

The hydrostatic equilibrium of a neutron star is established by the Tolman–Oppenheimer–Volkof (TOV) equation

$$\frac{dP(r)}{dr} = -G\frac{[\rho(r) + P(r)/C^2]\,[m(r) + 4\pi r^3 P(r)/C^2]}{r(r - 2m(r)G/C^2)}, \tag{18}$$

which is derived from the fact that the inward gravitational force must be balanced by the outward pressure given by the EOS.

The structure of a neutron star is determined by three basic equations: the EOS, the TOV and the mass equation. The mass equation is

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r). \tag{19}$$

These three basic equations must be written in a consistent form. This is done through two steps:

1. Writing the TOV equation in the density differential form

$$\frac{d\rho(r)}{dr} = \frac{dP(r)}{dr}\,\frac{d\rho(r)}{dP(r)} \tag{20}$$

and

2. normalizing all the variables in the equations. The normalization method reported by Colpi, Shapiro and Teukolsky [7] is used. For a given central density of the neutron star, the EOS is used to integrate the TOV and the mass equations simultaneously from the centre of the star to its surface. This is done using a four-point Runge–Kutta method with a variable step (Arnett and Bowers [8]). In this way the mass of the star and its radius are obtained as a function of the input central density. The mass has a maximum value which corresponds to each particular EOS.

3. Results and discussion

The neutron matter equation of state is adjusted to fit Friedman and Pandharipande (FP) [9] equation of state up to neutron density $\rho \approx 5\rho_0$ (at temperature $T = 0$) (see Fig. 1). This gives

$$\alpha_L = 0.9538\,, \quad B_L = 0.2112\,, \quad \gamma_L = 0.0547\,.$$

The remaining parameters are adjusted to fit the following parameters of nuclear matter (at $T = 0$).

$$\rho_0 = 0.16545\,\mathrm{fm}^{-3}\,, \quad E_v = -15.8\,\mathrm{MeV}\,, \quad E_x = 29.7\,\mathrm{MeV}\,,$$

$$K = 220\,\mathrm{MeV}\,, \quad m*/m = 0.519\,.$$

K is the compressibility of nuclear matter. The output potential parameters are

$$a = 0.5954\,\mathrm{fm}\,, \quad \alpha = (\alpha_L + \alpha_U) = 1.6455\,, \quad B = (B_L + B_U) = 0.9230\,,$$

$$\gamma = (\gamma_L + \gamma_U) = 1.2507\,, \; \beta = (\beta_L + \beta_U) = 0.3004\,, \; \sigma = (\sigma_L + \sigma_U) = 0.7471\,,$$

$$\xi = 1 - \frac{2\alpha_L}{\alpha} = -0.1593\,, \quad \zeta = 1 - \frac{2\gamma_L}{\gamma} = 0.9126\,.$$

We notice that our results are nearer to the calculation of Friedman and Pandharipande (FP) [9] than that of (MS) [5].

The binding energy of nuclear matter *vs.* density is represented in Fig. 2. In this figure, the $T = 0 (F = E)$ isotherm is the usual saturation curve for symmetric nuclear matter. Comparing our results with the variational calculations of Friedman and Pandharipande (FP) [9] and (MS) [5], we notice that our results at $\rho < 2.20\,\rho_0$ reasonably agree with (FP) and nuclear matter is globally unstable when $\rho \geq 2.20\,\rho_0$.

As an application to these parameters, we calculated the square of the speed of sound for both (nuclear and neutron matter) as a function of density

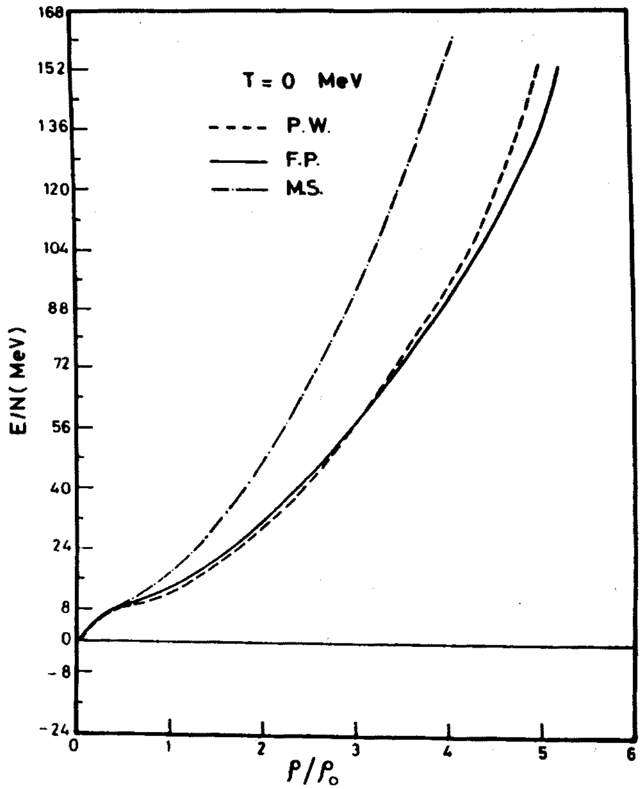

Fig. 1. The energy per neutron of neutron matter in the present work (PW) together with (FP) and that of (MS).

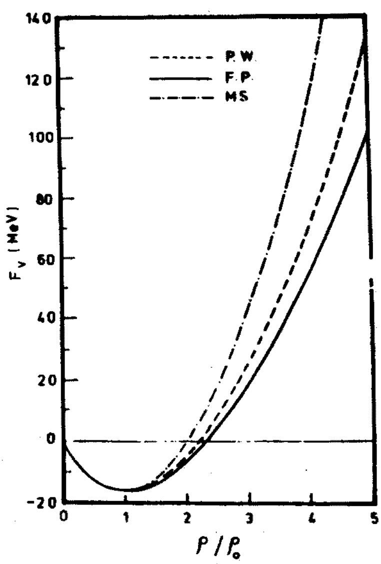

Fig. 2. The free energy of nuclear matter at zero temperature in the present work (PW) together with (FP) and that of (MS).

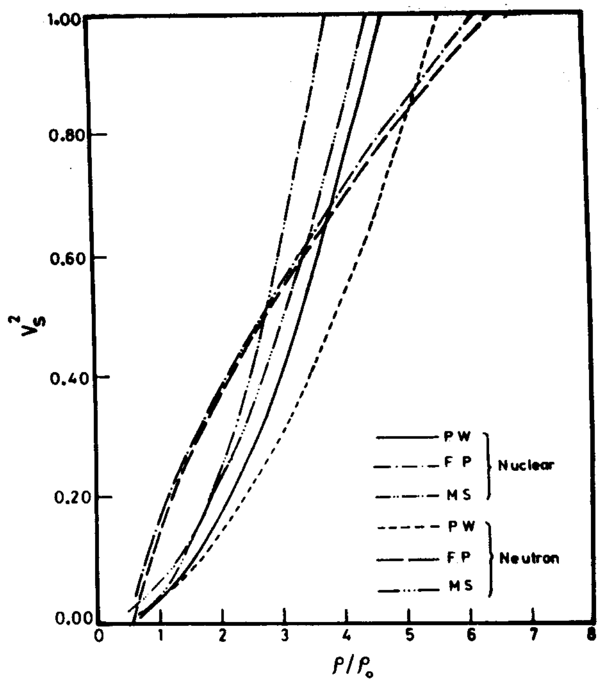

Fig. 3. The square of the speed of sound for both (nuclear ($x = 0$) and neutron matter) as a function of density together with (FP) and that of (MS).

and the dependence of the neutron star mass (in units of the solar mass $M_\odot$) on the star central density as shown in Figs 3 and 4 together with (FP) [9] calculations.

The mean value of the nucleon velocity in the medium is about $0.3c$ (c is the velocity of light) at normal nuclear density ($\rho_0 = 0.16545\ \mathrm{fm}^{-3}$) and at all densities V_s is smaller than the velocity of light ($v/c < 1$) which is consistent with causality. Notice that the speed of sound exceeds the speed of light when $\rho > 4.75\,\rho_0$ in case of nuclear matter whereas it happens when $\rho > 5.7\,\rho_0$ in the case of neutron matter. The corresponding values for (MS) [5] are $\rho > 3.85\,\rho_0$ and $\rho > 4.5\,\rho_0$, respectively. The calculations of (FP), show that causality violation occurs at $\rho \approx 6.25\,\rho_0$.

The mass of neutron star is a monotonic increasing function of central density, until a maximum mass is attained. The maximum mass (M_{max}), known as the limiting mass is interesting because it must exceed that of the most massive neutron star observed.

In the present work, we have $M_{\mathrm{max}} = 2.05\,M_\odot$ at $\rho_c \approx 7\,\rho_0$ corresponding to radius ($R = 10.23$ km). In Fig. 4, we can see that our results are very close to those of (FP) [9] ($M_{\mathrm{max}} \approx 1.97\,M_\odot$). Therefore, the parameters of (MS) [5] are not suitable to calculate neutron star properties (in this case,

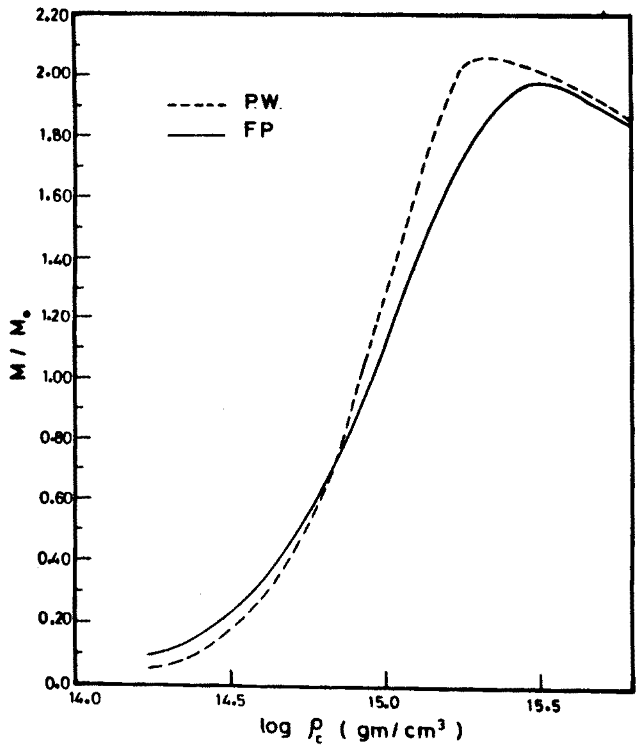

Fig. 4. Neutron star mass (in unit of solar mass ($M_\odot$) as a function of the star central density ρ_c (PW) together with (FP).

the sound becomes superluminar at $\rho \approx 3.85\,\rho_0$). Of the properties of nuclear and neutron matter at non-zero temperature, we concentrated on the calculation of the pressure for both nuclear and neutron matter as shown in Figs 5 and 6.

The critical temperature T_c and density ρ_c for the liquid-gas phase transition are determined by finding the turning point where $(\partial P/\partial\rho) = (\partial^2 P/\partial\rho^2) = 0$. This gives $T_c = 17.4$ MeV, $\rho_c = 0.064$ fm^{-3} (for MS [5] calculations, $T_c = 20.381$ MeV, $\rho_c = 0.0669$ fm^{-3}).

In Fig. 6 the pressure of neutron matter using our EOS (present work) is compared with the results of (FP) and (MS) at $T = 0, 16$ MeV. Since the pressure is related to the slope of the energy per neutron, it reflects the same behaviour as the EOS.

The effective nucleon–nucleon interaction of Myers and Swiatecki (MS) [5] has been very successful in describing a large number of zero temperature nuclear and neutron properties.

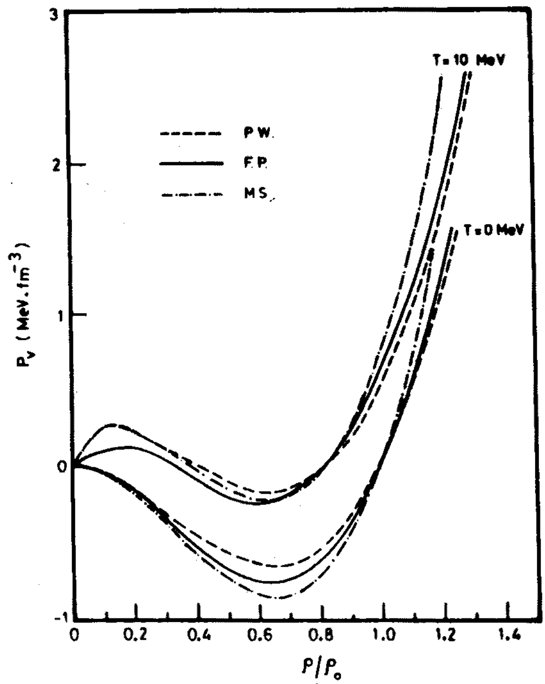

Fig. 5. The pressure-density isotherms for symmetric nuclear matter at $T = 0$, 10 MeV (PW) together with Friedman and Pandharipande (FP) and that of Myers and Swiatecki (MS).

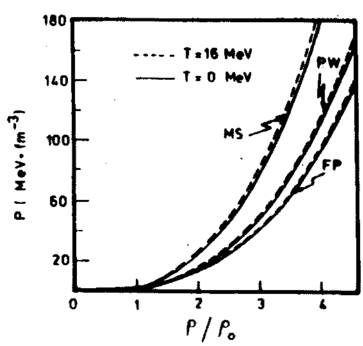

Fig. 6. The pressure of neutron matter at $T = 0$, 16 MeV together with (FP) and that of (MS).

The change of parameters done in these calculations is important to study the thermal properties of neutron star. The effect of using the present set of parameters on the other physical quantities calculated by (MS) will be the topic of future work.

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