CORRELATION FUNCTIONS OF THE QCD VACUUM AND INSTANTONS*

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QCD point-to-point correlation functions for various mesonic and baryonic channels are calculated using the so called Random Instanton Liquid Model. The results are compared to experimental data and the results of recent lattice calculations. We also briefly discuss the present status of the theory of interacting instantons.

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1. Introduction

The structure of the QCD vacuum is, in a way, the central problem of strong interaction physics. I have wrote some reviews about it [1, 2], so let me remind only one general point. "Elementary particles", or hadrons, are but collective excitations of this complicated matter. Respectively, like for phonons in condense matter, one cannot really understand them without first understanding the structure of matter itself.

Hadronic spectrum and excitation cross sections can be converted into more fundamental quantities, the correlation functions, which provide more direct information about short-range structure of matter itself. Say, neutron scattering on solids and liquids have produced enough examples of the kind. Recent review [3] compile available information on such correlator functions, being extracted directly from experimental data. With such phenomenological input, one can discuss models of the vacuum structure and results of lattice simulations with greater confidence.

It turns out that it is extremely important to study *point-to-point* correlation functions rather than (3d) plain-to-plain one, traditionally used by

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lattice people. The reason is short-range forces between quarks and anti-quarks are much more complicated than previously believed, and those can better be studied if the distance between the points is sufficiently small. In other words: studying *virtual* hadrons, the wave packets of small size, one can learn many things which it is difficult to get out of properties of lightest hadronic states alone[1].

During the last decade it was becoming more and more clear, that instanton-induced effects are in fact responsible for many features of light hadrons, possibly being even more important than the confinement effects. The present mini-review contains a report on numerical studies of the correlation functions data [4] in the framework of the simplest model of the kind, the so called Random Instanton Liquid Model (RILM). Its parameters were fixed about a decade ago by global vacuum properties, the gluon and quark condensates, so these calculations in fact had no free parameters at all. Somewhat unexpectedly, the model is surprisingly accurate in many cases, and can claim *quantitative* agreement with data. Another encouraging fact is that its predictions in some cases for which experimental data are absent (*e.g.* nucleon and delta correlations functions) happen to be in good agreement with recent lattice calculations [23].

2. Introductory instanton physics and approximations

Tunneling phenomena in gauge theories were discovered in 70's. This *one-instanton era* is marked by the discovery of the instanton solution [5], its physical interpretation as tunneling, semiclassical integration and relation with chiral anomalies [6]. Their first applications to QCD problems [7] based on *dilute gas approximation*, attracted a lot of attention in late 70's. However, as no explanation for diluteness of the instanton ensemble and to their semiclassical nature were suggested from first principles, pessimism has prevailed and most people has left the field.

However, phenomenological studies of possible instanton effects [9, 10] have shown, that instanton-induced effects definitely can explain many puzzles of the hadronic world, and should not be forgotten. The so called "instanton liquid" model [10] was suggested, with two free parameters, the main radius and distance between the instantons. In this work we essentially use it again, demonstrating that it does work, at new and more detailed level.

Attempts to describe *interacting* tunneling events were initiated by Dyakonov and Petrov [11], who have used the *variational approach* and have

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[1] One can use here analogy with nuclear physics. In 30's, when only general properties of nuclei were known, the NN interaction were believed to be a simple attractive central forces. Only much later, due to precise NN scattering experiments, its full complexity was revealed.

qualitatively reproduced the "instanton liquid" parameters. Further numerical studies of this problem [18] have allowed to get rid of many approximations and eventually included fermionic effects to all orders in *'t Hooft effective Lagrangian* [6]. We return to their discussion in Section 4.

The main reason why instantons are believed to be so important in physics of light fermions is related to the so called "zero modes", solutions of the Dirac equation

$$D_\mu \gamma_\mu \phi_0(x) = 0\,, \tag{1}$$

where D is a covariant derivative containing the instanton field. Formally, importance of small eigenvalues of D follows from the (Euclidean) definition of the quark propagator

$$S = -\frac{1}{iD_\mu \gamma_\mu + im} \tag{2}$$

at $m \to 0$. Their existence explains chiral anomaly: while tunneling, quarks with one chirality "dive into Dirac sea", and with another "emerge" from it. Also, quark condensate is known to be proportional to density of "nearly-zero" modes.

Instead of going into discussion of all these complicated phenomena, we now concentrate on evaluation of a quark propagator in the multi-instanton field configuration. Trying to understand spectrum and eigenfunctions of the Dirac operator, one can use the following analogy. "Zero modes" can be viewed as quark bound states to instantons (a "potential wells"), and at finite density of such wells they are collectivized (as electrons in condensed matter) and form the so called "zero mode zone". In simple physical language, quarks can easily jump from one instanton to another, and thus travel very far. If they can go infinitely far in this way, modes with infinitely small eigenvalues appear, and chiral symmetry becomes spontaneously broken even for massless quarks.

Significant efforts were done in the past to work out some set of approximations providing quark propagators in the "instanton vacuum". Our expression looks as follows:

$$S(x,y) = \sum_{\mathrm{ZMZ}} \frac{\phi_\lambda(x)\phi_\lambda^+(y)}{\lambda - im} + iS_{\mathrm{NZM}}(x,y)\,. \tag{3}$$

Here the first term is the sum over all states belonging to "zero mode zone" (ZMZ), or those being linear combinations of zero modes of individual instantons. The non-zero modes (analogs of "scattering states") are taken into account by the last term. For *single* instanton and for *massless* quarks the analytic expression for fermion propagator is known from explicit solution of the Dirac equation. Generalizing it for *many* instantons, we use an approximation summing all deviations for instantons and anti-instantons:

$$S_{\rm NZM} = S_0 + \sum_{\rm I}(S_{\rm I} - S_0) \tag{4}$$

valid for sufficiently dilute system. The last step is generalization of this expression to the case of *non-zero quark mass*: this is needed for discussion of effects related to strange quarks. The resulting expression is lengthy, and we do not give it here.

A simplified instanton ensemble we are going to use as a simplest example (RILM) is based on the following assumptions: *(i)* all instantons have the same size ρ_0; *(ii)* they have *random* positions and orientations; *(iii)* instanton and anti-instanton densities are equal to $n_0/2$.

Thus, there is one "diluteness" parameter $f = (\pi^2/2)n_0\rho_0^4$ which describes the model, apart from the overall scale given *e.g.* by the distance R defined by $n_0 = 1/R^4$. We are going to show, that already this simple model leads to reasonable description of many correlation functions. For definiteness, we use below $R = 1$ fm (corresponding to right gluon condensate) and $\rho_0 = 1/3$ fm, without attempting to fit them to the correlators. For random model, results are not very sensitive to them, apart of obvious change in overall scale.

3. Correlators in random instanton liquid

We start with showing in Fig. 1 our measurements of the quark propagators[2]. At small x part (b) is normalized to a propagator of massless quarks, so it starts at 1, while the part (a) is normalized so that it should be equal to quark mass. Part (b) can be approximately represented by a "constituent quark model" (curves), although with a surprisingly large quark mass. The chirality-flipping part (a) does not agree with this model at all. One may therefore doubt whether it is possible in RILM context to explain hadronic properties based on "constituent quark" picture. In addition, one should not forget that actually quark propagator contains also many "hidden components", which vanishes in average but show up in the mesonic correlation functions, where they enter squared.

Let us now consider correlators, starting with few general remarks. One can classify correlation functions considering quark paths, recognizing two different types of diagrams: the *one-loop* ones, in which both quark and

[2] We remind that those quantities are gauge dependent and shown only for illustration.

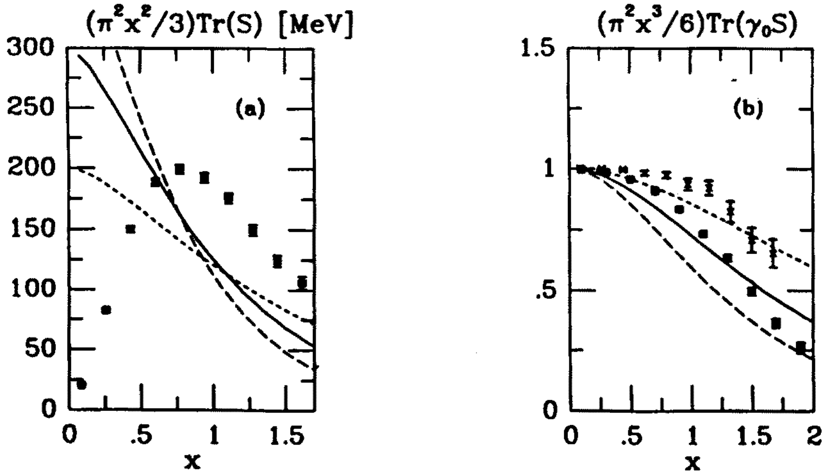


Fig. 1. Chirality-flipping (a) and chirality conserving (b) parts of the quark propagator, normalized as indicated in the figure. Three curves correspond to "constituent quark model" with masses 200,300,400 MeV (solid, dashed, long-dashed, respectively). Squares are full propagators, stars in (b) show the contribution of non-zero modes alone.

anti-quark travel from 0 to x, and the *two-loop* ones, in which they return back to where they started. Isospin $I=1$ correlators (*e.g.* of $\bar{u}d$ currents) get contribution only from one-loop diagrams, and therefore most of lattice work deal with this case. Although we also mainly deal with $I=1$ mesons, we have also performed measurements of some important two-loop diagrams as well, see below.

Our second comment emphasizes some observations related to chiral properties of fermionic zero modes. They have chirality, directly related to the topological charge of the gauge field: there is only a *left-handed* solution for the instantons and a *right-handed* one for the anti-instanton. Therefore, the first order corrections in 't Hooft effective interaction are: *(i)* present in the scalar and pseudoscalar correlators, but absent in the vector and axial ones; *(ii)* they have the opposite sign for the scalar and pseudoscalar channels; *(iii)* and, since it has $\bar{u}u\bar{d}d$ flavor structure, they have the opposite sign for the isospin 1 and 0 channels. All three points are phenomenologically welcomed, and this is by itself a very strong hint, suggesting that instanton-induced effects are crucial for short-range correlators.

The pion and kaon solid curves in Fig. 2 correspond to phenomenology [3]: notice the scale: the ratio to free quark propagator is very large, because pion is so light! Thus, it turns out that RILM predict properties of the pion nearly perfectly: both the mass and the coupling. In fact, the fit gave the pion mass extrapolated to physical quark mass the value 142±12 MeV.

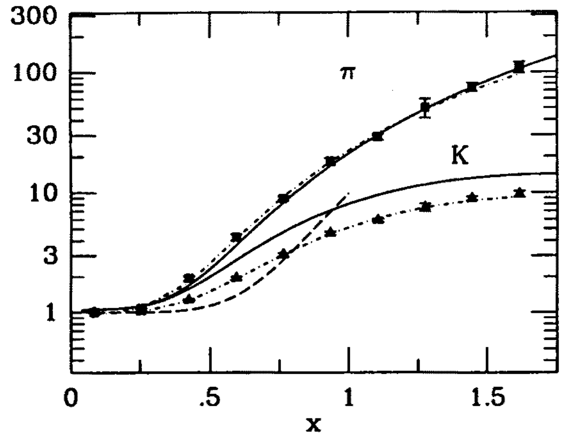


Fig. 2. Pseudoscalar π, K correlators, normalized to the free quark loop. The solid curves here and below correspond to phenomenology, as explained in the text. Squares and triangles correspond to RILM calculations, for $\bar{u}d$ and $\bar{u}s$ flavor structure of the currents. The dash-dotted line is a fit used to extract particle masses and coupling constants. The dashed line shows the correlator in a simplified version of "vector dominance" approximation.

(Such exact agreement is of course accidental.)

Here and below the dashed curves stand for "vacuum dominance approximation", by which we mean the that quark propagator is taken as free one plus the quark condensate. One can see, that this approximation always predict the right sign of the deviations, but not the correct magnitude.

The next pair of channels is shown in Fig. 3: those are vector and axial. Three ρ, K^*, ϕ solid curves in (a) and region between the two A_1, π curves in (b) correspond strictly to phenomenology [3]. Actually, those correlation functions have the smallest ambiguities because we have rather good data for electromagnetic annihilation into hadrons and weak τ lepton decays. The most interesting feature of these data is "superduality" in vectors: all three flavor channels are very close to 1 till rather large distances, about 1 fm. It is reproduced by the model, although deviation is about 20-30 percent[3]. Axial-vector splitting is also reproduced reasonably well.

In Fig.4 we show two examples of channels, involving two-loop (or annihilation) diagrams. Splitting of η, η' channels is the famous Weinberg U(1) problem. To reproduce their splitting, especially at small distances like 0.3–0.4 fm, still remains a challenge to lattice calculations. One can see from Fig. 4(a) that RILM strongly "overshoot" at this point, making so strong

[3] Note that the first perturbative correction is $(1+\alpha_s/\pi)$, which explains about 10 percent of it.

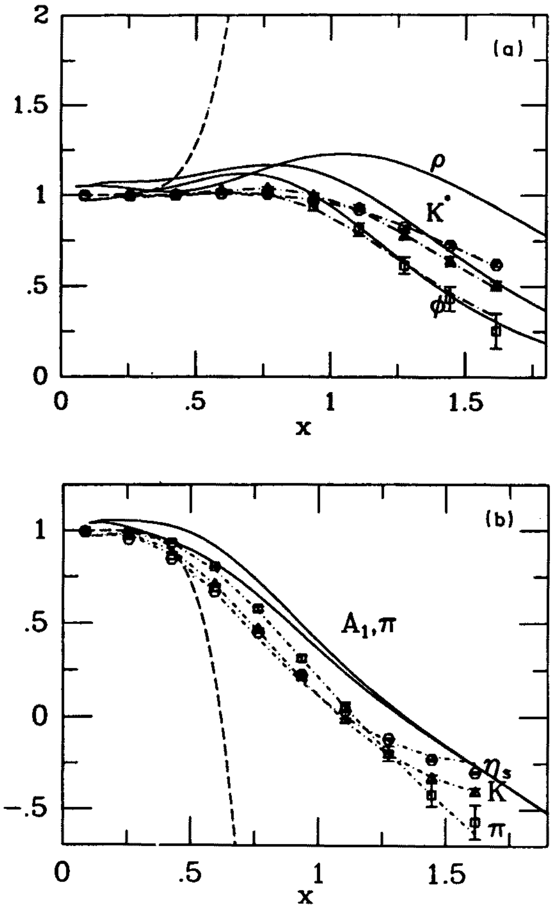


Fig. 3. Vector (a) and axial (b) correlation functions. Three sets of points correspond to ūd, ūs and s̄s (without annihilation) flavor structure of the currents. All notations are as in Fig. 1.

splitting, that the η' correlator becomes negative. This cannot be true, and demonstrate limitations of the model.

In Fig. 5 we compare our results with those obtained on the lattice [23]. The agreement is generally good. The only exception is another "repulsive" channel, this time the I=1 scalar channel (denoted by δ), where again the RILM "overshoots" the repulsion.

Finally let us show some results for baryonic systems (Fig. 6). In this case we do not have phenomenological input, thus we use predictions based on QCD sum rules, [12] and [19], respectively.

Again, agreement between RILM and lattice results is surprisingly good. The most interesting aspect of these curves is a qualitative difference be-

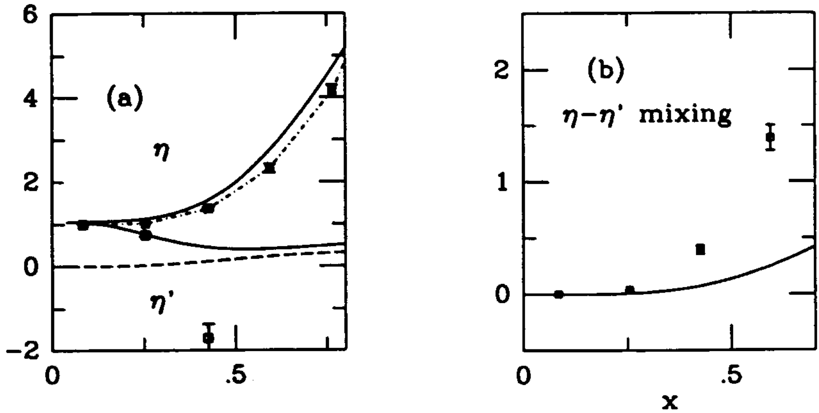


Fig. 4. The SU(3) octet and singlet pseudoscalar correlators (a) and their mixing (b). All notations are as in Fig. 1. The η, η' solid curves correspond to phenomenology.

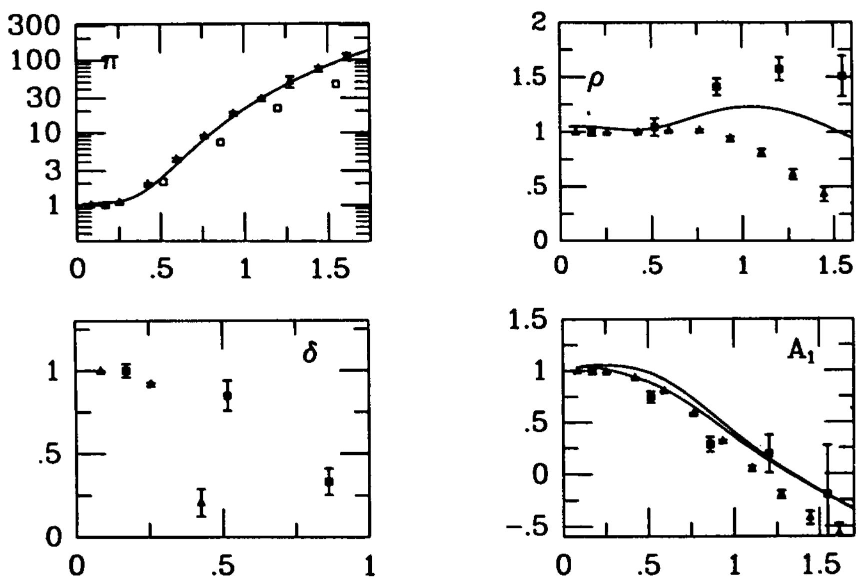
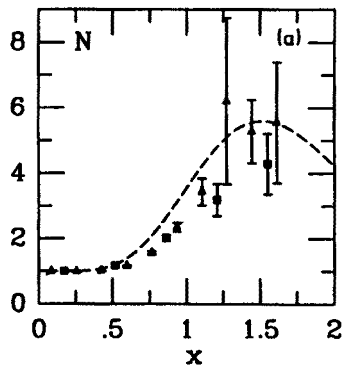


Fig. 5. Comparison between RILM (closed triangles) and lattice calculation (open squares) for pseudoscalar, vector, scalar and axial vector mesonic correlators.

tween the nucleon and delta case: this is due to different role of instanton-induces forces in these two systems, see discussion of it in [14–16]. Fit to RILM points have reproduced a surprisingly accurate value for the nucleon mass, while delta is somewhat too heavy.

Let me also emphasize, that more detailed recent studies (including



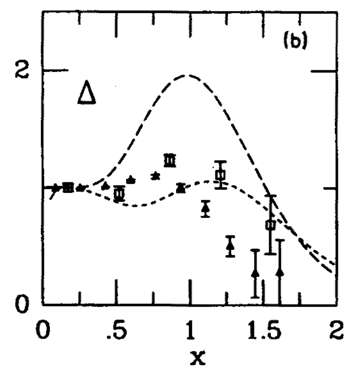


Fig. 6. The nucleon (a) and delta (b) correlation functions normalized to the free quark ones. As in Fig. 4, the closed triangles correspond to RILM, while the open squares to lattice calculation. The long-dashed and short-dashed curves correspond to QCD sum rule predictions [12] and [19], respectively.

measurements of the so called wave functions) show that in RILM there exist sufficiently strong attraction between "constituent quarks", binding them together to, say, nucleon. It is very non-trivial statement, taking into account that both the perturbative (Coulomb-like) and confining forces between quarks are not included in this calculation.

4. Studies of interacting instantons

Our discussion above have ignored correlation between tunneling events (instantons). However, at least the very phenomenon studied above, namely quark "jumping" from one instanton to another, produce strong correlation between them. Another obvious source of interaction is non-linear gluonic Lagrangian: a superposition of instanton field always have action different from a sum of two independent ones. Interacting instantons can be described by a statistical system with a partition function

$$Z = \int \Pi_i[d\Omega_i \exp(-S_i)] \exp(S_{int}) \Pi_{f=1,N_f}[\det(i\hat{D} + im_f)] \,, \qquad (5)$$

where by $d\Omega_i$ we have denoted the measure in space of *collective coordinates* of the i-th instanton[4]. The action of an individual instanton is $S_i = 8\pi^2/g^2(\rho_i)$. The next term $S_{\rm int}$ describes the classical (gluonic) interaction and the *fermionic determinant* describes quark-induced interactions.

[4] There are 12 of them in QCD, including the size ρ, 4-d coordinates of the center and 7 color rotation angles (one of the SU(3) rotations does not change the instanton configuration).

This formulation leads to a problem similar to those traditionally studied in statistical mechanics [5]. At fixed value of the collective coordinates, evaluation of the fermionic determinant resembles a problem of quantum chemistry: in both cases one has to determine spectrum of fermionic operator in some multi-centered background field.

Treating instantons as atoms (and quarks as electrons) one is lead to the following natural questions: *Is the instanton ensemble in solid, liquid or gas phase? Is the chiral symmetry broken or not?* The answer, obtained in [11, 20, 17], reads as follows: for densities roughly n=0.01–64. Λ^4_{PV} it is liquid, which is "freezing" only at the upper end of this interval. The instanton density in the real QCD vacuum is not known accurately, but in any case it is in the middle of this interval $n \sim 1\Lambda^4_{PV}$. Chiral symmetry is broken in this phase, and the obtained $\langle\bar{\psi}\psi\rangle$ has reasonable magnitude.

The main new element which has emerged during the last few years is that (rather arbitrary) trial functions for gauge field configurations used previously, are substituted by the so called *streamline configurations*, corresponding to the bottom of the $I\bar{I}$ valley. Those configurations can be found numerically, "descending" down the valley [21], which produce the solution to the "streamline" equation [13]. For gauge theories that was recently found by Verbaarschot [22], using conformal symmetry of the classical Yang-Mills theory. It was also found, that the specific ansatz proposed by Yung [13] give very accurate description of the action of these configurations.

However, straightforward application of the "streamline-based" gluonic and fermionic interaction has not improved the results obtained previously. In fact, it was found that a significant fraction of the instantons and anti-instantons form pairs with small spatial separation and, what is most important, with small total action. Such "over-correlated" liquid leads to very small quark condensate and wrong correlators, unless the total instanton density is strongly increased. In principle it can be motivated by the fact, that total density of instantons was fixed without such strongly correlated pairs.

However, *a problem of close instanton–anti-instanton pairs* need much closer attention. Configurations with small separation and particular relative orientation have small action, and therefore they are not semiclassical. As it was emphasized by Yung [13], those fluctuations were already included in the usual perturbation theory (which, by definition, includes *all* small fluctuations of the fields) and they should not be included twice.

While this challenging problem still waits for its theoretical solution, one may introduce an artificial repulsive core. If it is done, one can reproduce

[5] It is somewhat more complicated than traditional atomic systems because of fermionic determinant: however, it is still orders of magnitude simpler than full lattice gauge theory

the correlators. Moreover, the results obtained on this way are even better than those shown above: in particular, large fluctuations of the topological charge become screened and negative values η' correlators disappear.

5. Generalization to finite temperatures and chiral symmetry restoration

In principle, generalization of the theory under consideration to finite temperatures is straightforward: the periodic instanton solution, the "caloron" is known, and their (much more complicated) interactions were studied in [26]. Simulations of the interacting system is under way, but no quantitative results are obtained. Let me therefore make only qualitative remarks.

First of all, instantons are known to be suppressed at high T by the one loop quantum effects, essentially by the Debye screening [27]. The Pisarski-Yaffe [28] suppression factor describes it, but only in the limit of high temperatures, at which the heat bath can be treated perturbatively. The instanton density as temperatures below T_c remains unknown.

Moreover, it can be that the instanton suppression is not even the main phenomenon: another one is a strong "pairing" of instantons, leading to formation of $\bar{I}I$ molecules. There are several reasons for that. First, as the box size become comparable to the molecule diameter, attraction goes both way on the Matsubara torus, so it is about doubled. Also, quarks have problems propagating in space direction: their zero modes and propagators obtain the famous behaviour $\exp(-\pi T r)$, where πT is the lowest possible Matsubara frequency. In time direction such factor is absent.

Now, if the whole "instanton liquid" is split into molecules, the chiral symmetry is restored. The first attempt to look at chiral restoration phase transition at this angle was made in [30]. However, that attempt was in many respect oversimplified. In particular, the "random liquid" and "molecular gas" were considered as two phases, and the question addressed was where their thermodynamical potentials may become equal. However, some pairing exist already at $T = 0$, and this tendency grows gradually till $T = T_c$. One can adopt another simple picture of instanton ensemble at these temperatures, as a *two-component mixture.* The "liquid" component should disappear at T_c, together with the condensate $\langle\bar{q}q\rangle$, while the "molecular" one is there for any temperature.

First exploratory studies of the correlation functions have been made in this simple model [29]. It is easy to generate such ensemble, for various values of the "molecule fraction" $f_m = \frac{2N_{\text{molecules}}}{N_{\text{all}}}$, the main parameter of the model. In order not to introduce new parameters, we have kept total instanton density fixed, and ignored the non-sphericity of the instantons at finite T. In short, the only difference between the ensemble considered above and the one to be discussed now is in *correlation* between positions of some fraction of the instantons.

The main order parameter, $\langle\bar{q}q\rangle$, depends on f_m in a way which is very similar to its T-dependence, measured on the lattice: it changes little first, and then rapidly vanishes at $f_m = 1$. It was further found, that different correlation functions depend on it quite differently, and some examples are shown in Fig. 7 . One can see, that some correlators are rather insensitive to it. For example, the vector one does not change by more than about 10 % for all compositions, from $f_m = 0 - 1$. Even the pion correlator shows remarkable stability for $f_m = 0 - 0.8$, with subsequent rapid drop if its coupling constant toward $f_m = 1$, at which point it coincides with its scalar "relatives" (see below). At the same time, some correlators show dramatic sensitivity to f_m, especially the scalar ones.

Let me now point out the main physics of the phenomenon. Its one side was repeatedly emphasized before (see *e.g.* [31]): as the quark condensate "melts down", so does a "constituent quark mass" (whatever it means) and hadrons in average should become lighter. However, there is another side of the story: the instanton-induced interaction between quarks does "melt" as well. In the channels where this interaction is *attractive*, such as ρ, π, these two effects work in the opposite directions and can somewhat cancel each other. However, in the channels where it is *repulsive*, such as the axial vector A_1 or the $I = 1$ scalar δ, they work in the same direction, and may lead to much stronger T-dependence of the correlator.

Another aspect of this model is that it assumes rapid restoration of not only SU(N_f) chiral symmetry, but approximate restoration of the U(1) chiral symmetry at T_c as well. Whether it is the case or not, remains so far unknown.

New generation of instanton-based and lattice-based calculation is needed, in order to answer this and other qualitative questions, and to provide the detailed knowledge of correlation functions at finite temperatures.

6. Conclusions and discussion

Summarizing this work, let me say that about 40 different correlation functions were calculated, in the framework of the simplest "random instanton liquid model". It was found that these functions are very similar to what is observed in nature. For example, the model happen to predict correct values of pion and nucleon masses, inside the errors. What is also important, the overall agreement in many cases is reached. The only two channels where the model does not work well are the "repulsive" channels, δ and η', in which the repulsion seems to be too strong.

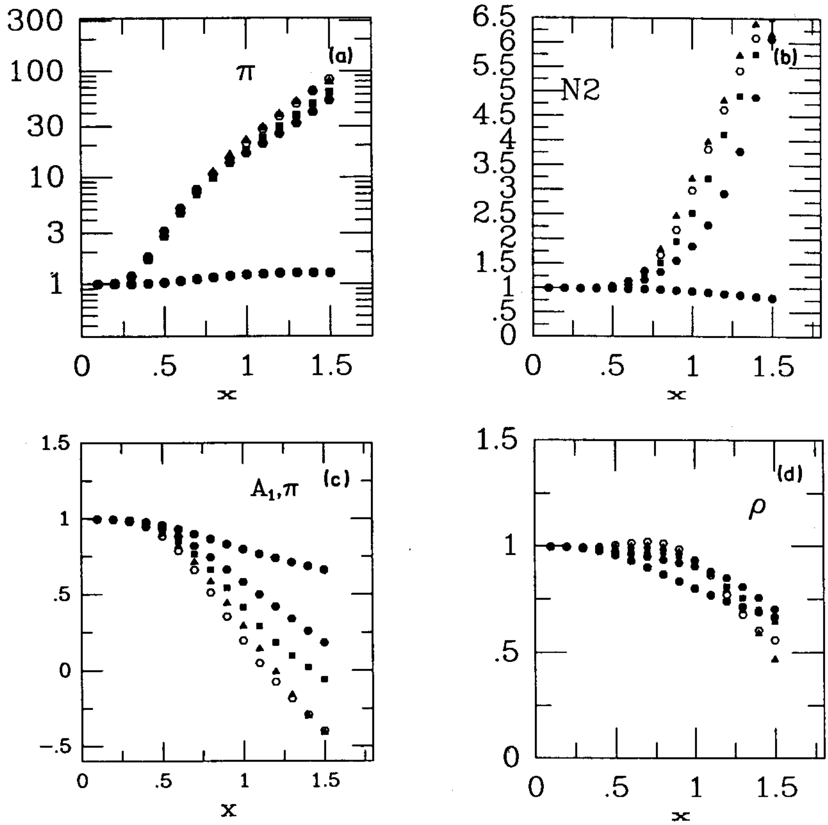


Fig. 7. Correlation functions in the "two-component" model [29] for pion (a), nucleon (b), axial vector (c) and vector (d) channels. Open points correspond to pure "random" instanton ensemble, while four closed set of points correspond to admixter of 0.25, 0.5, 0.75 and 1. fraction of instantons united into the instanton-antiinstanton molecules.

A crucial test of the whole instanton-based approach to the QCD vacuum can be provided by lattice calculations. First of all, one can compare the correlation functions themselves, and we have already shown above that such comparison looks encouraging by itself. However, lattice people can do much more, comparing the configurations. In fact, the so called "cooling" of lattice configuration is the well known method to remove all components of the QCD vacuum, except instantons, the local minima of the action. Indications on the general importance of instantons were made in [24, 25]. Calculation of the correlation functions for "cooled" configurations are in progress (J. Negele, private communications), and the first qualitative conclusion is that they do not differ much from those in full quantum vacuum.

Another qualitative idea which was emphasized in this work deals with chiral symmetry restoration at finite temperatures. The main phenomenon driving this transition is suggested to be "pairing" tendency between instantons and anti-instantons. Although present at $T = 0$ as well, it becomes much stronger around the critical temperature. Simple two-component model was suggested and studied, and remarkable correlation between the results of this model and lattice simulations at finite T are observed. The main lesson from those (still preliminary) studies is that different correlation functions are changed in this transition in a quite different way.

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