HEAVY HADRONS*,**

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I discuss the general aspects of heavy hadron spectroscopy. The interplay between heavy-light hadrons and light mesons is discussed using a double expansion in the number of colors and the heavy quark mass. For a large number of colors heavy hadrons emerge as solitons.

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1. Introduction

Hadrons with an infinitely heavy quark exhibit a new type of symmetry — invariance under spin-flip of the heavy quark [1]. This invariance can be used to understand the hadronic structure and properties of heavy-light systems. A number of relations have been recently derived showing that the excitation spectra and form factors of heavy hadrons are independent of the mass and spin of the heavy quark [2].

Heavy excited hadrons decay mainly by spitting out light pseudoscalars. The interplay between light and heavy hadrons can be elucidated in the heavy quark limit by combining chiral symmetry with heavy quark symmetry. The basic observation is to note that the hard part in the heavy quark field is kinematical and factorizable. The remaining part is soft and constrained by chiral dynamics to leading order in the heavy quark mass.

The purpose of these notes is to provide a simple discussion of the salient features of heavy hadron systems and discuss the role of the pion in heavy hadron dynamics. In Section 2, I summarize some essential facts on heavy hadrons. In Section 3, I discuss the general symmetry structure of the

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effective action formulation. In Section 4, I discuss the large N_c limit and suggest that heavy baryons may be understood as solitons. My conclusions are summarized in Section 5.

2. Generalities

In what follows a heavy hadron will be understood as a meson or baryon with at least one heavy quark. By heavy quark I mean ..., b(4.7 − 5.3 GeV), c (1.3 − 1.7 GeV) and to some extent s (100 − 300 MeV). Heavy mesons $Q\overline{q}$ in their ground state can be organised into multiplets with $I(J^P) = \frac{1}{2}(0^-, 1^-)$. The multiplets are invariant representation of the heavy quark symmetry group. Empirically $(K, K^*) = (493, 892)$ MeV, $(D, D^*) = (1869, 2010)$ MeV and $(B, B^*) = (5278, 5324)$ MeV. The D^* decay is mostly through $D\pi$ with 55% into neutral and 27% into charged pions.

The P-orbital excited states of heavy mesons can be organized by Clebsch–Gordaning : $(\vec{\frac{1}{2}} \times (\vec{\frac{1}{2}} + \vec{1}))^+ = (\vec{\frac{1}{2}} \times \vec{\frac{1}{2}} + \vec{\frac{1}{2}} \times \vec{\frac{3}{2}})^+$. The multiplet $(\overline{D}, \overline{D}^*)$ has $I(J^P)$ assignment $(\frac{1}{2}(0^-, 1^+))$ and corresponds to the even parity partners of (D, D^*). The multiplet $(D_1, D_2^*) = (2420, 2460)$ MeV corresponds to the assignment $(\frac{1}{2}(1^+, 2^+))$ and decay mostly into $D^*\pi$. Its parity partner $(\overline{D}_1, \overline{D}_2^*)$ is composed of D-orbital excited states with $(\frac{1}{2}(1^-, 2^-))$ assignment. This multiplet is not observed. Notice that as the mass of the heavy quark is increased the splitting within the multiplet decreases.

The heavy baryons we will be discussing carry one heavy quark. They are of the type Qqq. The conventions being $\Lambda_Q = 0\frac{1}{2}^+$, $\Sigma_Q = 1\frac{1}{2}^+$ and $\Sigma_Q^* = 1\frac{3}{2}^+$. Few heavy baryons have already been observed. For instance $\Lambda_s, (\Sigma_s, \Sigma_s^*) = 1115, (1189, 1382)$ Mev ; $\Lambda_c, (\Sigma_c, \Sigma_c^*) = 2284,\ (2455, \ldots)$ MeV; $\Lambda_b, (\Sigma_b, \Sigma_b^*) = 5641, (\ldots,\ \ldots)$. The dots indicate that the corresponding heavy hadrons have not been measured, and the brackets refer to the heavy hadron multiplets invariant under heavy quark symmetry. Heavy hadrons with two heavy quarks QQq such as Ξ_{ss} are the analogue of heavy mesons $\overline{Q}q$. They will not be discussed here.

The basic physics in a heavy quark system can be understood from a simple bag model description. The energy budget inside the bag can be organised in powers of the heavy quark mass. Generically : $E = m_Q + E_0 m_Q^0 + E_{-1} m_Q^{-1} + \ldots$. The order m_Q^0 part corresponds to

$$E_0 = \frac{2.04}{R} + \frac{4}{3}\pi R^3 B - \frac{4}{3}\alpha_s \frac{1}{R} - Z_0 \tag{1}$$

with the kinetic, volume, magnetic and zero point energy respectively. The order m_Q^{-1} corresponds to

$$\frac{E_{-1}}{m_Q} = \frac{(\pi/R)^2}{2m_Q} + \frac{\vec{\mu}^a \cdot \vec{B}^a}{2m_Q} \tag{2}$$

with the recoil and spin interaction, respectively. For charmed mesons (recoil, spin) $\sim$ (400, 20) MeV, while for bottom mesons (recoil, spin) $\sim$ (100, 10) MeV.

3. Heavy mesons

In the limit where the mass of the heavy quark is taken to infinity and the masses of the light quarks are taken to zero, the QCD action is invariant under a combined set of symmetries : $SU(2)_Q \times (SU(2)_L \times SU(2)_R)$. Heavy quark symmetry $SU(2)_Q$ means invariance under rotation of the heavy quark spin : $Q \to e^{i\vec{S}\cdot\vec{\beta}}Q$. Chiral symmetry $SU(2)_L \times SU(2)_R$ means invariance under : $q_{L,R} \to e^{i\alpha_{L,R}} q_{L,R}$.

An efficient way to implement these symmetries is in the form of an effective action. To illustrate the procedure, we will consider a toy model. Let $\psi = (Q, q)$ be a doublet of a heavy and light field, whose dynamics in the infrared is given by

$$\mathcal{L} = \overline{\psi}(i\not{\partial} - m)\psi - \theta(|x| - \frac{1}{\Lambda})\ \frac{g^2}{\Lambda^2}\ \overline{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi\ \overline{\psi}\gamma^\mu \frac{\lambda^a}{2}\psi\,, \tag{3}$$

where Λ is some infrared scale. The fermions in (3) carry a constituent mass say $\Sigma \sim \Lambda$ and the gluons a mass $\Lambda/\sqrt{2}$.

The effective action for the composite mesons $q\overline{q}$, $Q\overline{Q}$, ... follows by integrating out the constituent fermions in the long wavelength approximation. In the heavy quark sector, this is achieved by decomposing the heavy quark field into [3]

$$Q = \frac{1+\not{v}}{2} e^{-im_Q v\cdot x} Q_v^+ + \frac{1-\not{v}}{2} e^{-im_Q v\cdot x} Q_v^-\,, \tag{4}$$

the fields $Q_v^\pm$ are soft. A Fierz rearrangement of the four fermion interaction in (3) in the heavy quark limit gives

$$\begin{aligned}\overline{Q}_v q\overline{q} Q_v - \overline{Q}_v\gamma_5 q\overline{q}\gamma_5 Q_v - \overline{Q}_v\gamma_\mu \frac{1-\not{v}}{2} q\overline{q}\gamma^\mu \frac{1-\not{v}}{2} Q_v \\ + \overline{Q}_v\gamma_\mu \frac{1-\not{v}}{2}\gamma_5 q\overline{q}\gamma^\mu \frac{1-\not{v}}{2}\gamma_5 Q_v\end{aligned}$$

The bolded combinations carry 0^+, 0^-, 1^- and 1^+ spin-parity assignment. They can be bosonized in the standard way using the multiplets $H = (0^-, 1^-)$ and $G = (0^+, 1^+)$ which are $\mathrm{SU}(2)_Q$ invariant. The procedure is standard. The result is a nonlocal effective action for the H and G fields. In the long wavelength approximation, the nonlocal effective action can be organised using a double expansion in powers of $\partial/\sqrt{N_c}\Lambda$ and $1/m_Q$.

In the heavy light sector the effective action to order $m_Q^0 N_c^0$ reads [4, 5]

$$\begin{aligned}\mathcal{L}_v = &-\frac{i}{2}\mathrm{Tr}\left(\overline{H}v\cdot\partial H - Hv\cdot\partial\overline{H}\right) + m_H\mathrm{Tr}(\overline{H}H)\\ &+\mathrm{Tr}(\mathcal{V}\cdot v\overline{H}H) - g_H\mathrm{Tr}(\mathcal{A}\cdot\gamma\gamma_5\overline{H}H)\\ &+\sqrt{\frac{Z_H}{Z_G}}\mathrm{Tr}(\gamma_5\overline{G}Hv\cdot\mathcal{A}) + (H\leftrightarrow G).\end{aligned} \tag{5}$$

The light currents $\mathcal{V}$ and $\mathcal{A}$ are the vector and axial currents carried by the light particles. If only pions are included these currents are entirely expressed in terms of the pion field π (see below). When more light mesons are included such as the rho (ρ), the axial (a_1), the omega (ω), ... and vector dominance assumed, they read

$$\begin{aligned}\mathcal{V} &= \omega + \rho + \dots\\ \mathcal{A} &= a_1 - \frac{\alpha}{f_\pi}\partial\pi + \dots,\end{aligned} \tag{6}$$

where $\alpha = (m_\rho/m_A)^2 \sim 1/3$ and f_π is the pion decay constant. The mass m_H, axial coupling g_H and the GH-coupling $\sqrt{Z_H/Z_G}$ depends on the infrared scale Λ through the induced constituent mass $\Sigma \sim \Lambda$. The effective action (5) is invariant under the induced chiral transformation on the coset $\mathrm{SU}(2)_v$ ($H \to Hh^+$, *etc.*) and the heavy quark transformation $\mathrm{SU}(2)_Q$ ($H \to SH$).

The behaviour of the axial couplings for the H, G multiplets and their excited version T, S, are shown in Fig. 1 *versus* Σ/Λ, in the case where only pions are retained. The axial coupling of the D multiplet g_H is about 1/3 in this case . The decay width $D^* \to D\pi$ is empirically bounded from above

$$\Gamma(D^* \to D\pi) = \frac{g_A^2}{12\pi f_\pi^2}|p_\pi|^3 \leq 50\ \mathrm{KeV} \tag{7}$$

at a measured pion momentum of about 40 MeV. This suggests that $g_A \leq 0.56$ which is to be compared with 0.33 when only pions are retained, and 0.13 when all the light mesons are retained. The mass splitting between the even and odd parity multiplets is $m(\overline{D}^*) - m(D^*) = m(\overline{D}) - m(D) \sim \Sigma$, of the order of the constituent quark mass. In the bag description of Section 2, this splitting is centrifugal and of order $m_Q^0 N_c^0$.

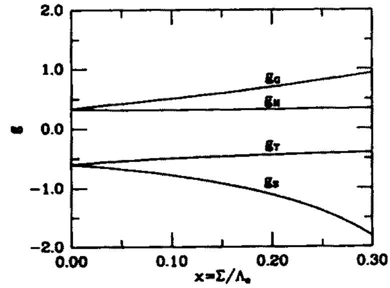

Fig. 1. Behaviour of the axial couplings for the H, G, T and S multiplets *versus* Σ/Λ.

4. Heavy baryons

The effective action (5) shows that the heavy quark mass m_Q through the redefinition (4) drops. This implies that the $1/N_c$ expansion is compatible with the $1/m_Q$ expansion. Upsetting terms of the form $m_Q/N_C\Lambda, \ldots$ do not arise. We note that the πHH coupling $g_H \sim 1/\sqrt{N_c}$ as expected.

As an effective action for heavy-light meson dynamics in the large N_c limit, could it be that (5) allow for solitons ? To find out about this, consider the H field in the heavy quark rest frame

$$H = \frac{1+\gamma^0}{2}(-\gamma_i P_i^* + i\eta\gamma_5 P) \qquad \overline{H} = \gamma^0 H^+ \gamma^0 , \tag{8}$$

where the P, P^* fields refer to the axial and vector D, D^* fields respectively. Note that under an infinitesimal spin flip of the heavy quark,

$$\delta\vec{P}^* = \eta\vec{\alpha}P + (\vec{\alpha}\times\vec{P}^*) \qquad \delta P = -\eta^*\vec{\alpha}\cdot\vec{P} . \tag{9}$$

Here η is an arbitrary relative phase between P and P^*. Note that the P and P^* fields mix. In terms of (8) the effective action (5) reads [6, 7]

$$\begin{aligned}\mathcal{L}_{\pi H} =& - i\mathrm{Tr}(\dot{H}\overline{H}) + m_H \mathrm{Tr}(H\overline{H}) \\ &+ \mathrm{Tr}(H\mathcal{V}^0\overline{H}) - g_H \mathrm{Tr}(H\vec{\mathcal{A}}\cdot\vec{\sigma}\overline{H}) ,\end{aligned} \tag{10}$$

where we have dropped G the chiral partner of the H field. We will be seeking only even parity solitons. The vector and axial currents are totally expressible in terms of the chiral field $\xi = e^{i\pi/2f_\pi}$

$$\begin{aligned} \mathcal{V}^0 &= \frac{i}{2}(\xi\dot{\xi}^+ + \xi^+\dot{\xi}) \\ \vec{\mathcal{A}} &= \frac{i}{2}(\xi\vec{\nabla}\xi^+ - \xi^+\vec{\nabla}\xi)\,. \end{aligned} \tag{11}$$

Now, consider the hedgehog background $\xi_H = e^{i\tau\cdot\hat{x}F/2}$ with $F(0) = \pi$ and $F(\infty) = 0$. This configuration is invariant under rigid rotations with $\vec{K} = \vec{I} + \vec{J} = \vec{K}_L + \vec{S}_Q$ and carry $K^\pi = 0^+$. πH solitons arise from the *possible* binding of an H multiplet to the hedgehog pion field much like in the scheme advocated by Callan and Klebanov for the strange hyperons [8]. To see this, consider the following $K = \frac{1}{2}$ partial wave in the H field

$$\begin{aligned} P_\pm &= \frac{1}{4\sqrt{2\pi}}\phi_\pm^T\vec{\tau}\cdot\hat{x}P(r) \\ \vec{P}_\pm^* &= \frac{\mp}{4\sqrt{\pi}}\left(\sqrt{2}\vec{\epsilon}_\pm^{\,+}\phi_\mp^T - \vec{\epsilon}_0^{\,+}\phi_\pm^T\right)\vec{\tau}\cdot\hat{x}\;P(r)\,. \end{aligned} \tag{12}$$

Here $\phi_\pm$ refers to an isodoublet, $\vec{\epsilon}$ are the polarisations (longitudinal and circular) of the vector field $\vec{P}$, and $P(r)$ the profile for the heavy particles. In terms of (12) the H particle cary $K^\pi = \frac{1}{2}^+$. In the heavy quark limit, the range of $P(r)$ is of order $1/N_c m_Q$. With this in mind, the energy of the hedgehog in the presence of the H configuration (12) is given by [7] [1]

$$E = M_s + m_Q + m_H + \frac{3}{4}g_H F'(0)(1 - \frac{\eta - \eta^*}{2i})\,. \tag{13}$$

Since $g_H > 0$ and $F'(0) < 0$ the H particle can bind to the soliton to order $m_Q^0 N_c^0$. The details of the binding are of course model dependent. The classical configuration has a range of order $1/\Lambda$ with $K^\pi = \frac{1}{2}^+$. This is the precursor of the the Q-hyperon.

The spectrum of the bound H-soliton can be obtained using the standard quantization of the zero modes. This can be achieved through a slow rotation of the fields using the isospin valued transformation A as a collective variable

$$\xi \to A\xi A^+ \qquad\qquad H \to HA^+\,. \tag{14}$$

In terms of (14) the induced action reads ($H = G\vec{\tau}\cdot\hat{x}$)

$$\mathcal{S}_A = -\Omega\mathrm{Tr}(A^+\dot{A})^2 + i\mathrm{Tr}(GA^+\dot{A}\overline{G})\,. \tag{15}$$

[1] In Ref. [7] the factor $\frac{3}{4}$ was quoted as $\frac{1}{4}$ following an unnecessary approximation.

The first term is the usual rotor contribution from the hedghehog background, with Ω the moment of inertia. The second term is a Berry-type phase induced by the $\dot{H}$ particle. In the heavy quark limit, the Berry-type phase vanishes identically [7]

$$\vec{\Phi}(\infty) = -\mathrm{Tr}(G\frac{\vec{\tau}}{2}\overline{G}) = \tfrac{1}{2}\mathrm{Tr}(P\frac{\vec{\tau}}{2}P^{+} + \vec{P}^{*}\frac{\vec{\tau}}{2}\vec{P}^{*}) = 0\,. \tag{16}$$

Because of the hedgehog structure and the heavy quark symmetry, the induced phase by the D meson is equal and opposite to the induced phase by the D^* meson. As a result, the net action (15) is rotor-like.

The corresponding Hamiltonian is $H_{1/2} = \boldsymbol{I}^2/2\Omega$, with $\boldsymbol{I} = \boldsymbol{J}_R = \boldsymbol{J} - \boldsymbol{S}_H = \boldsymbol{J}_L$, where the latter is the angular momentum carried by the light degrees of freedom. The Hilbert space factorises into $|H> \otimes |A>$. The eigenstates carry good isospin, angular momentum and light angular momentum $|I, J^{\pi}, J_L>$. In our case, the spectrum consists of

$$\begin{aligned} |\Lambda_Q> &= |0, \tfrac{1}{2}^{+}, 0> \\ |\Sigma_Q> &= |1, \tfrac{1}{2}^{+}, 1> \\ |\Sigma_Q> &= |1, \tfrac{3}{2}^{+}, 1>\ . \end{aligned} \tag{17}$$

The states $|\Sigma_Q>$ and $\Sigma_Q^*>$ are degenerate in agreement with heavy quark symmetry. The splitting between $\Lambda_Q>$ and $|\Sigma_Q>$ is rotational and about the $N - \Delta$ splitting [7]

$$m_{\Sigma_Q} - m_{\Lambda_Q} = \tfrac{2}{3}(m_\Delta - m_N)\,. \tag{18}$$

This relation is to be compared with the one originally derived by Callan and Klebanov [8]

$$\tfrac{1}{3}(2m_{\Sigma^*} + m_\Sigma) - m_\Lambda = \tfrac{2}{3}(m_\Delta - m_N)\,. \tag{19}$$

In the heavy quark limit (19) reduces to (18). If we were to relax the $m_Q = \infty$ limit, the Berry phase (16) no longer vanishes. This means that the Hamiltonian is no longer a simple rotor, involving explicitly the spin of the heavy quark.

5. Conclusions

I have shown how the concepts of chiral and heavy quark symmetry could be combined to discuss the interplay between heavy and light degrees of freedom. In the long wavelength limit, the induced action allows for a useful description of the interactions between the pion, the rho, ... on one hand and the D, D^*, ... on the other, using the rationale of large N_c and large m_Q. I have shown that concepts such as vector dominance may imply specific constraints on the heavy light dynamics.

The effective action approach leads naturally to the concept of solitons. In the heavy quark limit, heavy mesons may bind to ordinary solitons. In the process, the heavy mesons quantum numbers are transmuted to the soliton and give rise to heavy hyperons with good heavy quark symmetry.

The present construction may be extended in different directions. Photons, kaons could be added to the picture. Subleading effects in $1/m_Q$ could be analysed. Also, a more quantitative analysis of the effects discussed here will be of interest.

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