TOPOLOGICAL CONFINEMENT IN QCD_2* ,**

A. FERRANDO[a]

Institute for Theoretical Physics, University of Bern
Sidlerstrasse 5, CH-3012 Bern, Switzerland

AND

V. VENTO

Departament de Física Teòrica and I.F.I.C.
Centre Mixt Universitat de València – C.S.I.C.
E-46100 Burjassot (València), Spain
Vento@evalvx.ific.uv.es

In two dimensional SU(N) theories confinement can be understood as a topological property of the vacuum. In the bosonized version of two dimensional theories non trivial boundary conditions (topology) play a crucial role. They are inevitable if one wants to describe non singlet states. In abelian bosonization, color is the charge of a topological current in terms of a non-linear meson field. We show that confinement appears as the dynamical collapse of the topology associated with its non trivial boundary conditions.

PACS numbers: 12.38. Aw

1. Brief overview of confinement in QCD_2

The study of confinement in two dimensional Quantum Chromodynamics has been a much discussed problem in the literature since the pioneering work of 't Hooft [1]. Many controversies about the realization of confinement in QCD_2 have arisen over the years. Contrary to 't Hooft's results, other approaches show several phases in parameter space (M (quark mass),

---

* Presented at the XXXIII Cracow School of Theoretical Physics, Zakopane, Poland, June 1–11, 1993.

** Supported in part by CICYT grant # AEN93-0234 and DGICYT grant # PB91-0119-C02-01.

[a] Post Doctoral Fellow of CICYT- Plan Nacional de Altas Energías.

e (coupling contant) and N (number of colors)) [2–4]. The large N approximation appears responsible for the exceptionality of the confinement mechanism in the model of 't Hooft.

In the large N limit strong suppresions occur in the set of diagrams of the theory. Only planar diagrams contribute, there are no vertex corrections and quark loops dissapear. Under these severe restrictions the quark–antiquark interaction is controled by the One Gluon Exchange Potential (OGEP) exclusively. Unlike in the four dimensional case the 2D OGEP is confining. The 2D Poisson equation in the presence of a static charge tells us that the zero component of the gauge field must raise linearly with the distance. The gluon propagator is then,

$$\partial_1^{-2}(x,y) = \tfrac{1}{2}|x_1 - y_1|\delta(x_0 - y_0)\,, \tag{1}$$

which to first order in $1/N$ is proportional to the $q\bar{q}$ interaction. Thus confinement here is a peculiarity of the dimensionality of space-time [5]. The resolution of 't Hooft's equation confirms this mechanism leading to a discrete, stable and infinite spectrum [1]. However the formalism is much more powerful eliminating those amplitudes which would violate confinement explicitly [6, 7]. Let us study for example the process *meson* $\to q\bar{q}$. It can be shown that the amplitude for it is given by

$$F_n^{a\bar{b}}(t, r_-) = \frac{e^2}{r_-\sqrt{\pi N}} \mathrm{P}\int_0^1 dt' \frac{\varphi_n^{a\bar{b}}(t')}{(t-t')^2}\,, \tag{2}$$

if $t \in [0,1]$. The notation follows that of Ref. [1]. In order to obtain the physical amplitude one has to impose the on mass shell restrictions, *i.e.*,

$$p^2 = M_a^2\,, \quad (p-r)^2 = M_{\bar{b}}^2\,, \quad r^2 = r_n^2\,, \tag{3}$$

where M_i are the renormalized quark masses and r_n the corresponding meson mass. The on mass shell condition leads to the following relation for the adimensional momentum t

$$\mu^2 = \frac{\alpha_a}{t} + \frac{\alpha_{\bar{b}}}{1-t}\,, \tag{4}$$

and therefore 't Hooft's equation becomes

$$\mathrm{P}\int_0^1 \frac{\varphi^{a\bar{b}_n}}{(t-t')^2} = 0\,, \tag{5}$$

which implies the vanishing of the amplitude for the process. Therefore an expected consequence of the confinement mechanism is that no quarks can be liberated from a bound state.

Another limit in which the confinement mechanism can be understood is that of very heavy quarks. In the leading order, that is when quarks are infinitely heavy $M \to \infty$, QCD_2 becomes a pure gauge theory with static external color sources. Confinement for such a theory is trivial. As in the large N limit the quark propagator, as well as the $q\bar{q}$ potential, becomes linearly raising with the distance. Analogously to the $N \to \infty$ limit, in a pure gauge theory with static external sources there are no sea quark contributions, nor non-planarity effects, nor vertex corrections. Therefore no special assumptions are needed to prove confinement when $e \ll M \to \infty$. In the next to leading order in $1/M$, the eigenvalue equation for the mesonic amplitudes can be reduced to a one dimensional Schrödinger equation [8]. The $q\bar{q}$ potential appearing in this equation is again linear. In this case only the zero component of the gauge field survives the Non Relativistic limit and it is proportional to the gluon propagator. The spectrum exhibits the same features as t' Hooft's one. It is discrete, infinite and stable. No quarks are allowed.

Nevertheless once we move away from these two limits in the space of parameters, QCD_2 becomes extremely complex. The aforementioned sea quark excitations, vertex corrections and non-planar contributions are now of great relevance. Consequently the confinement mechanism becomes much more complicated than in the above cases.

In this paper we give an alternative description of the confinement mechanism for two dimensional Quantum Chromodynamics, which has the advantage of being universal in the space of parameters. We develop it in an SU(2) gauge theory, without any restriction associated with large N or large mass. Therefore our results hold in any regime of the theory including those where *naive* confinement cannot take place.

2. Realization of color symmetry in the bosonized *free* massless theory

The bosonized action for a massless quark (1 flavor, $N = 2$ colors) using abelian bosonization is [10]

$$\mathcal{S} = \int d^2x \left\{ \tfrac{1}{2}(\partial_\mu \varphi)^2 + \tfrac{1}{2}(\partial_\mu \eta)^2 \right\}. \tag{6}$$

The bosonized Lagrangian (6) shows an explicit splitting of flavor and color degrees of freedom when expressed in terms of the bosonic fields φ and η.

There is nothing special about the realization of the U(1) flavor symmetry of the original fermion model. The φ field takes care of the $U(1)_V \otimes U(1)_A$ flavor symmetry in a complete manner. The vector symmetry is conserved in a topological way (that is without resorting to the φ equation of motion) thanks to the presence of the fully antisymmetric tensor in the bosonized form of the flavor current ($J^\mu = (1/\sqrt{2\pi})\epsilon^{\mu\nu}\partial_\nu\varphi$). On the contrary the axial flavor current is a real Noether current. Its conservation is guaranteed by the massless character of the φ field.

The φ field realizes the U(1) flavor symmetry in a *linear and local* way[1]. Nevertheless it holds Non Trivial Boundary Conditions (NTBCs) solutions as it is required by the vector current bosonization rule, which relates the baryon charge to the φ boundary conditions ($B = (1/\sqrt{2\pi})[\varphi(+\infty) - \varphi(-\infty)]$). The full equivalence of the fermion and boson theories forces its bosonized version to have operators generating states carrying fermion quantum numbers [12]. Because baryon number (as well as the third component of color-isospin) is a topological charge, the only way to achieve baryonic charged operators in the $U(1)_F$ sector of the bosonized model is through NTBCs ($\Delta_\infty\varphi = \sqrt{\pi/2}B \neq 0$).

On the contrary the realization of color symmetry is much more subtle. It can seem surprising that such a simple action as (6) can realize the full global SU(2) color symmetry. A mere counting of color degrees of freedom seems to imply that some color field might be missing[2]. The answer to this apparent contradiction lies in the *topology* of the η field. The non-triviality of its BCs forces η to transform under a *non-linear non-local* representation of SU(2). Nevertheless η can be related to linearly transforming objects through non-linear non-local expressions. For example, it can be related to:

i) Objects in the fundamental representation:

$$S^\alpha(x,t) =: \exp i(-)^{\alpha+1/2}\sqrt{\frac{\pi}{2}}\left\{\int\limits_{-\infty}^{x} d\zeta\,\dot{\eta}(\zeta,t) + \eta(x,t)\right\}:, \quad (7)$$

where $\alpha = -{}^1\!/_2, +{}^1\!/_2$.

---

[1] This can be easily seen if we rewrite the U(1) part of the bosonized lagrangian in its 'group form', $\partial_\mu\varphi\partial^\mu\varphi = (1/2\pi)\partial_\mu(\exp(i\sqrt{2\pi}\varphi)\partial^\mu \exp(-i\sqrt{2\pi}\varphi)$.

[2] Non-abelian bosonization would lead to an action in terms of the SU(2) fields $g = \exp(i\tau^a\pi^a/2), a = 1,2,3$, with three color degrees of freedom, where the π^a fields transform according to the adjoint representation.

ii) Objects in the adjoint representation[3]:

$$J_{\pm}(x,t) =: \exp \pm i\sqrt{2\pi} \left\{ \int_{-\infty}^{x} d\zeta \dot{\eta}(\zeta,t) + \eta(x,t) \right\} :,$$

$$\bar{J}_{\pm}(x,t) =: \exp \pm i\sqrt{2\pi} \left\{ \int_{-\infty}^{x} d\zeta \dot{\eta}(\zeta,t) - \eta(x,t) \right\} :,$$

$$J_3^0(x,t) = \frac{1}{\sqrt{2\pi}} \partial_x \eta(x,t), \tag{8}$$

as can be checked explicitly.

But, as we already mentioned, in order for these relations to occur the η field must possess non-trivial boundary conditions, *i.e.*,

$$T^3 = \frac{1}{\sqrt{2\pi}} [\eta(+\infty) - \eta(-\infty)]. \tag{9}$$

Thus the action in Eq. (6) can only realize the full SU(2) group if one incorporates besides the conventional solutions $\eta_0(\pm\infty) = 0$, those associated with non-trivial boundary conditions ($\eta(+\infty) = \pm\sqrt{\pi/2}$, $\eta(-\infty) = 0$). That they exist for the free theory has been proven by construction [10]. Moreover they are dynamically allowed since the finiteness of the energy for physical states,

$$E_\eta = \int dx H(\eta) = \int dx \tfrac{1}{2} (\partial_x \eta)^2 < \infty, \tag{10}$$

only requires the asymptotic vanishing of the derivative. Thus the η field can tend to different constants at $\pm\infty$, and generate solitonic solutions. These states are stable because they are protected from desintegrating into conventional $T^3 = 0$ states by topological conservation laws.

By means of the canonical commutation relations of the η field one can prove that the operators in (8) close a SU(2)⊗SU(2) current algebra,

$$[J_+(x), J_+(y)] = 4J^3(x)\delta(x-y) + i\frac{2}{\pi}\partial_x \delta(x-y), \tag{11}$$

(identically for $\bar{J}$) [10]. Their conservation is guaranteed by topology in the case of J_3^μ and by the equations of motion in the case of $J_\pm$ and $\bar{J}_\pm$. The

[3] In our notation $J = J_0 + J_1$ and $\bar{J} = J_0 - J_1$. The subindex $\pm$ stands for the lowering and raising color-isospin currents.

spatial integrals of the zero components of the $(J^\mu_\pm, J^\mu_3)$ currents define the conserved SU(2) color charges $(T_\pm, T_3)$. Analogously, by using the same techniques leading to the current algebra (11) one finds that the soliton operator (7) is *really* in the fundamental representation of SU(2) because

$$[T^a, S^\alpha(x,t)] = \left(\frac{\tau^a}{2}\right)_{\alpha\beta} S^\beta(x,t)\,. \tag{12}$$

In this way η transforms under the action of the lowering and raising operators of SU(2) as follows [10],

$$[T_\pm, \eta(x,t)] = \pm\sqrt{2\pi}\int\limits_{-\infty}^{x} d\zeta J^0_\pm(\zeta,t)\,. \tag{13}$$

The fact that the S^α and $J_\pm$ operators lie in irreducible representations of the color group prevents the η field to transform *linearly and locally* under the group. The complexity of the relation between $T_3 \neq 0$ tensorial objects and the field supporting the symmetry (Eqs (7) and (8)) causes this particular realization of the non-abelian structure. Observe that is *only* the value of η at $+\infty$ -not η itself- which transforms properly under the adjoint representation of SU(2). This is because $\eta(+\infty) = \sqrt{2\pi}T_3$, Eq. (9)[4]. Thus the BCs contain crucial information associated with the realization of color symmetry.

Now it is easy to understand how the bosonization procedure operates. We have just learned that it is possible to build non-trivial color operators based upon the η field. For this reason the original Fermi field can be expressed in terms of soliton operators carrying ($B = T = {}^1\!/_2$, $T_3 = \pm{}^1\!/_2$) quantum numbers. That is to say[5],

$$q^\alpha_+(x,t) = k\mu^{1/2} S(x,t) S^\alpha(x,t)\,, \tag{14}$$

where k is a numerical constant and μ a renormal ordering mass [13]. The S operator creates the $B = {}^1\!/_2$ 'flavor' soliton in terms of the φ field, whereas S^α does the same for the color soliton. Thus any fermion operator is capable of being expressed in terms of the φ and η boson fields by means of the previous relation. Diagonal operators in color ($T_3 = 0$) will be local in η. In the same way $B = 0$ operators will be local in φ. Any other operator will contain non-local pieces, as those occuring in the charge operators of Eqs (7) and (8).

---

[4] We have chosen $\eta(-\infty) = 0$.

[5] In 2D q is a bi-spinor, $q = (1/2^{1/4})(q_+, q_-)$.

In the more modern language of Conformal Field Theories (CFT), the above construction expresses the possibility of writing the same CFT — defined by its central charge c and its level k- in different free field representations. For a theory realizing an affine U(1)⊗SU(N) symmetry at level $k = 1$, the central charge is $c = N$. This is fulfilled both for a theory of N free complex fermions and for a theory of N free bosons. Because c and k define completely the current algebra of the theory (Kac–Moody algebra), both representations have the same current algebra (Eq. (11)) and therefore preserve the same symmetries. Our diagonal current J_3^μ $(= (1/\sqrt{2\pi})\epsilon^{\mu\nu}\partial_\nu\eta)$ represents the Cartan subalgebra of color SU(2), whereas the charged currents $J_\pm^\mu$, are 'vertex operators' with conformal weights (1,0) and (0,1) representing the remaining SU(2) currents. The η field itself is not a conformal field because of its 2D infrared behavior. It is not a primary field and it has not well defined conformal dimensions. Only its derivative (J_3^μ) or its exponential ($J_\pm^\mu$) have good conformal properties. Due to the deep link beetwen conformal symmetry (Virasoro algebra) and internal symmetry (Kac–Moody algebra) in 2D, the strong IR behavior of the η field also spoils its SU(2) properties. Because η is not a primary field it cannot provide a linear representation of the color group either [14][6]. This peculiar regeneration of the whole SU(N) structure out of its Cartan subalgebra in 2D is called the Frenkel–Kac–Segal or 'Vertex Operator' construction [11][7].

It is important to stress that it is the existence of non-trivial boundary conditions which generate the full $SU(2)_C$ algebra. The raising and lowering operators connect different topological sectors, *i.e.*,

$$T_-|\eta(\infty) = \sqrt{2\pi}t\rangle \propto |\eta(\infty) = \sqrt{2\pi}(t-1)\rangle\,. \tag{15}$$

If NTBCs did not exist the non-local pieces of the non-diagonal operators would dissapear leading us to a *trivial abelian current algebra* instead of (11). The 'Vertex Operator' construction would not be possible anymore. NTBCs are a *necessary* condition for the non-linear non-local realization of the whole non-abelian symmetry. This is precisely the crux of the matter which we will exploit systematically in next sections.

3. Vacuum structure of the bosonized *free* theory

The introduction of a mass term in the fermion Lagrangian (6) does not alter the previous construction [10, 12]. The bosonized form of the mass op-

[6] Note the difference between η and the non-abelian bosonization field g^α_β. The latter is *really* a primary field of the WZWN model and thus it belongs to the $(1/N, 1/N)$ representation of SU(N)⊗SU(N).

[7] We are grateful to E. Alvarez for this reference.

erator is easily obtainable from the soliton-fermion operator correspondence (14) leading to

$$S_M = \int d^2x\{k^2\mu M(\cos\sqrt{2\pi}\varphi)_\mu(\cos\sqrt{2\pi}\eta)_\mu\}, \tag{16}$$

M being the fermion mass. The classical potential has a minimum at

$$V_M(\tilde{\varphi},\tilde{\eta}) = -k^2\mu M, \tag{17}$$

which implies a degenerate minima structure formed by the infinite set of points [10]

$$\emptyset = \emptyset_I \cup \emptyset_{II}, \tag{18}$$

where

$$\emptyset \equiv \begin{cases} (\sqrt{2\pi}n, \sqrt{2\pi}m) & \in I \\ (\sqrt{2\pi}(n+\frac{1}{2}), \sqrt{2\pi}(m+\frac{1}{2})) & \in II \end{cases} \\ (n,m) \in Z. \tag{19}$$

Although the mass term has no effect on the 'Vertex Operator' construction, it possesses a very appealing property, namely, it allows us to relate $\emptyset$ to the SU(2) content of the theory. The minima give us precise information about the solutions of the bosonized theory, since they are related to the possible boundary conditions [5, 15], *i.e.*,

$$\lim_{x\to\pm\infty}(\varphi(x),\eta(x)) = (\varphi_{n_\pm},\eta_{m_\pm}) \in \emptyset. \tag{20}$$

Therefore there exists also a close link between the minima structure and the non-linear non-local realization of color symmetry. Charge operators ($B \neq 0, T_3 \neq 0$) connect different minima (different BCs) in the infinite lattice defined by $\emptyset$. Thus any charge operator of the fermionic theory can be represented by means of solitonic operators linking different lattice points (see Fig. 1). In this way the realization of the whole SU(2) color symmetry is guaranteed. Every state generated by any of these operators will transform linearly under the color group.

We can use the (φ,η) minima structure plane $\emptyset$ as a diagram for physical states, just recalling the relation between (B,T_3) charges and the asymptotic conditions of the fields. Fig. 1 shows that all states described by arrows of the same length and direction are equivalent, *i.e.*, they have the same (B,T_3) charges. Thus we can define an equivalence relation and choose just one representative per class. For example, we proceed by attaching the arrows to the same point ($\varphi(-\infty) = \eta(-\infty) = 0$), since we have the freedom to select one of the boundary conditions. With this restriction all

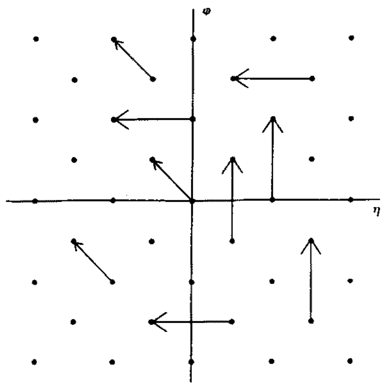

Fig. 1. Examples of allowed states: a) Diagonal arrows: up-color quark state ($B = 1/2,\ T^3 = 1/2$); b) Horizontal arrows : up-color vector state ($B = 0,\ T^3 = 1$); c) Vertical arrows: color singlet baryon state ($B = 1,\ T^3 = 0$).

the physical states will be represented by the points of the vacuum structure lattice, which becomes in this way a (B, T_3) plane (see Fig. 2).

Global SU(2) transformations leave the potential invariant. This is easily seen in the non-abelian bosonization scheme where the potential is proportional to $tr(g) = \cos\sqrt{2\pi}\eta$, an SU(2) invariant, and $\cos\sqrt{2\pi}\varphi$, a function of φ, invariant by construction [16]. Therefore the set of physical states, the lattice, is invariant under these transformations. Moreover it is also invariant under the discrete 'orthogonal' translations (horizontal and vertical shifts) [10]

$$\begin{aligned} \varphi &\to \varphi + \sqrt{2\pi}n \\ \eta &\to \eta + \sqrt{2\pi}m \\ &\quad (n, m) \in Z \end{aligned}$$

and the 'diagonal' ones

$$\begin{aligned} \varphi &\to \varphi + \sqrt{\frac{\pi}{2}}n \\ \eta &\to \eta + \sqrt{\frac{\pi}{2}}m \\ &\quad n + m = \text{even}\,. \end{aligned} \tag{21}$$

Before closing this section it is important to emphasize that the topological structure just studied coincides with that of the chiral limit of the theory. The topological charges of the physical states are mass independent. They survive in the chiral limit together with the asymptotic conditions that generate them. The minima of the potential become the NTBCs, which are allowed by energy considerations.

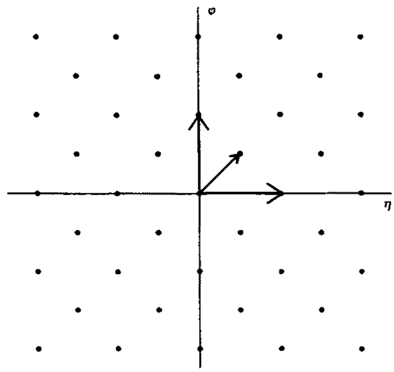

Fig. 2. Representation of the physical states in the (B, T^3) plane.

4. The vacuum structure of QCD$_2$

The introduction of the color gauge interaction is carried out in the usual way, that is, by minimally coupling gluons to matter in the free fermionic Lagrangian (6) and by adding a gauge invariant gluon kinetic term. The new lagrangian is local gauge invariant and thus it is, in particular, global gauge invariant. Therefore there must exist conserved charges associated to this global symmetry. Certainly these charges are nothing but the color charges of the SU(N) symmetry and they are conserved[8]. But now we have two different sources generating this internal degree of freedom: the fermions (quarks) and the colored gauge particles (gluons). The conserved color current is the sum of both contributions,

$$J_a^\nu = j_a^\nu + G_a^\nu \quad a = 1, \ldots, N^2 - 1, \tag{22}$$

where j_a^ν is the quark current and

$$G_a^\nu = i[A_\mu, F^{\mu\nu}]_a . \tag{23}$$

The J_a^ν current is conserved through the equations of motion of the gluon field $D_\mu F^{\mu\nu} = ej^\nu$,

$$\partial_\mu F_a^{\mu\nu} = ej_a^\nu + eG_a^\nu = eJ_a^\nu \tag{24}$$

and thus $\partial_\mu J_a^\mu \equiv 0$. The quark current is a covariant object under local gauge transformations. This is not the case of the gluon current which does not transform as a tensor, as one can see from its definition (23) ($\delta_U G^\nu =$

[8] Because the SU(N) *vector* symmetry is anomaly free this holds also at the quantum level.

$e^{-1}[(\partial_\mu U)U^{-1}, F^{\mu\nu}])$. However, global color symmetry is preserved in any gauge (24). Therefore, also in any gauge, we have at our disposal a set of $N^2 - 1$ different color charges commuting with the hamiltonian. These charges will be related in different gauges by local gauge transformations.

In order to perform our calculation we proceed to fix the gauge. The techniques developed in the previous section require gauges in which the topological character of the color current shows up explicitly. The so-called 'hybrid gauges', involving restrictions on both the gauge field and the field strength, can be used for our purpose. In particular to generate, for $N = 2$, a topological J_3^ν current it is enough to demand $G_3^\nu = 0$ in the desired gauge. In this way, $J_3^\nu = j_3^\nu$.

Once we integrate out the gluons, we are left with a theory with only fermionic degrees of freedom. By means of the bosonization techniques we will obtain a theory for the η field possessing the topological color conservation law, $\partial_\mu J_3^\mu \equiv 0$, since

$$J_3^\nu = j_3^\nu = \frac{1}{\sqrt{2\pi}}\epsilon^{\mu\nu}\partial_\nu\eta\,. \tag{25}$$

A 'Vertex Operator' construction provides the conditions to understand the effects of the η dynamics on the $\mathrm{U}(1)_C$ topology, *i.e.*, the color structure of the theory.

If we write the $F^{\mu\nu} = \epsilon^{\mu\nu}F$ and A^μ adjoint fields in the spherical basis[9], then the condition $G_3^\nu = 0$ is equivalent to require

$$F_+A_-^\mu = F_-A_+^\mu\,, \quad \mu = 0,1\,. \tag{26}$$

There are many gauges which fulfill (26) and thus the topological condition (25). We take the gauge of Baluni, in which a complete bosonization of the theory has been already worked out [3]. This gauge generates a very convenient framework to study the topological realization of color symmetry in QCD_2. Baluni's gauge clearly satisfies the two above conditions, since it requires that the non-diagonal terms of the field strength (F_+, F_-) be zero. To fix the gauge completely another independent condition is needed, which in Baluni's gauge is $A_3^1 = 0$.

The bosonized form of the *3rd* component of the field strength is the same in any 'topological' gauge (26). From the equation of motion (24) and the bosonization rule (25) it is clear that

$$\partial_\mu F_3 = \frac{e}{\sqrt{2\pi}}\partial_\mu\eta \tag{27}$$

and then

---

[9] $F = F_-T_- + F_+T_+ + F_3T_3$ (the same for A^μ).

$$F_3(x) = \frac{e}{\sqrt{2\pi}}\eta(x) + E\,, \tag{28}$$

where E is a constant color background field which can be attributed to the existence of classical charges at spatial infinity. In the abelian case such a background field has physical relevance. It generates the θ angle of the massive Schwinger model. θ is a mass and coupling constant independent parameter possessing important physical properties [17]. However, in the non-abelian case, the corresponding angle associated to E has no physical consequences. Thus we consider that there is no color background field by setting $E = 0$ [5]. This bosonizes unambiguosly the gluon kinetic term producing a mass term for the η field.

The bosonization of the other terms is more involved but straightforward. Nevertheless we would like to draw attention to an important detail of the bosonization procedure. After the integration of the gluons in any 'topological' gauge, one is able to bosonize completely the effective fermionic action by means of the fermion-boson correspondence (14). The gauge dependent effective fermion interaction can be extremely complicated, but it has to verify a very nice property when bosonized, namely, that it must be *local* in the color field η. This has to be so because the hamiltonian is a *diagonal* operator in color ($T_3 = 0$). An analogous argument holds for the 'flavor' field φ. Therefore in any 'topological' gauge the final outcome of the bosonization procedure is a local action in the bosonic fields φ and η. Moreover this action must include a mass term for the color field η, corresponding to the bosonized form of the pure gauge action.

Baluni's bosonized action is an explicit example of a 'topological' gauge and therefore satisfies the features just described. In this case the potential arising from the quark-gluon interaction is

$$V_i(\eta) = \frac{e^2}{32\pi}\eta^2 + \sqrt{\pi}k^4\mu^2\left\{1 - \frac{\sin\sqrt{2\pi}\eta}{\sqrt{2\pi}\eta}\right\}\,, \tag{29}$$

where k and μ are the parameters appearing in the bosonization formula (14). What are the implications of the above potential in the $U(1)_C$ color topology of the gauge theory? We proceed to provide an answer.

Due to the relation between the minima structure and NTBCs (20), our first task is to find the constraints induced by $V_i(\eta)$ into the full action,

$$S = \int d^2x\,\left\{\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta + k^2\mu M(\cos\sqrt{2\pi}\varphi)_\mu(\cos\sqrt{2\pi}\eta)_\mu - V_i(\eta)\right\}\,. \tag{30}$$

The potential V_i is positive definite and it has only one absolute minimum $V_i(\eta) = 0$ which occurs for $\eta = 0$

$$V_i(\eta) = 0 \leftrightarrow \eta = 0\,. \tag{31}$$

Therefore the full potential has a lower bound

$$V(\varphi, \eta) = V_M(\varphi, \eta) + V_i(\eta) \geq -k^2 \mu M, \;\; \forall(\varphi, \eta)\,. \tag{32}$$

The equality is only saturated if the following conditions are met

$$\cos\sqrt{2\pi}\varphi \cos\sqrt{2\pi}\eta = 1 \quad \text{and} \quad V_i(\eta) = 0\,. \tag{33}$$

Thus the minima form the set

$$\emptyset_{\mathrm{QCD}_2} = \{(\varphi_n,\, \eta_0 = 0)\;\; n \in Z\}\,, \tag{34}$$

which correspond to the shaded lattice points shown in Fig. 3.

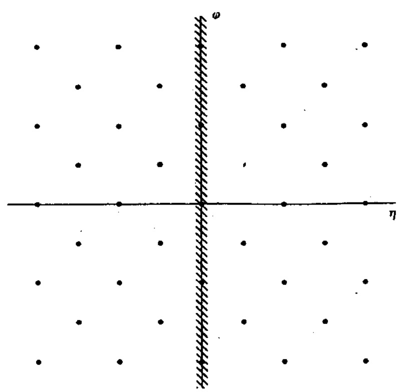

Fig. 3. The color singlet sector of QCD_2.

So far only mathematics have been invoked. Now we have to put the previous properties into physical words. This will require a careful analysis of both the global SU(2) properties of the vacuum and the non-linear non-local realization of color symmetry.

We start by paying attention to the first of these two important issues. The full potential V is SU(2) invariant just by construction. In order to obtain it, we have rewritten the invariant interaction hamiltonian of the fermion model in terms of the η field by means of the fermion-boson equivalence (14). Because bosonization is exact, when writing V in terms of η dependent *tensorial* objects we must obtain the same SU(2) fermionic invariant interaction, no matter how complicated the transformation law of the η field is.

Consequently, in any 'topological' gauge the shape of the full potential must be preserved by global color transformations (this is exactly what happened in the free case too). Because the V_i potential is exactly the same before and after a SU(2) transformation its minimum in $\eta_0 = 0$ must stay. In other words, a global SU(2) transformation cannot move the V_i minimum from $\eta_0 = 0$. Certainly a color transformation will not shift a 'flavor' minimum φ_n into another for the simple reason it does not act on the φ field. Thus any of the $\emptyset_{\text{QCD}_2}$ minima will remain unaltered by a global color rotation.

The special realization of color symmetry has provided us with a potential depending on *one single real scalar* field. This means, that the set of minima $\emptyset_{\text{QCD}_2}$ can only be a discrete set (no typical 2D 'Mexican hat' potential is allowed because we have a function depending only on *one* real variable). Therefore only *discrete* transformations can shift one of this minima to another. A global continuous transformation will leave these minima untouched. If we choose one of them as the real vacuum, it will be necesseraly invariant under *continuous* global color tranformations. This is nothing more than an explicit example of the more general result, that a continuous symmetry cannot be spontanously broken in 2D [9][10]. This is a general statement valid for any value of the quark mass, the coupling constant and the number of colors.

The second issue is to unveil the consequences of the new interaction and the new vacuum structure on the realization of color symmetry. This cannot be done in a standard group theory fashion because η does not realize the color symmetry in a standard way.

To see this, in a more transparent way, let us proceed *ad absurdum.* Assume that η can be *naively* interpreted as a conventional color field, which does not carry third component of color charge (recall $[T_3, \eta] = 0$ and (13)). Furthermore, let it belong to an irreducible representation of SU(2), *e.g.*, η behaves like π_0 in the pion triplet. Then the hamiltonian can not be a color singlet. To see this, one realizes that the hamiltonian may be expanded in powers of $\eta^2(\sim \pi_0^2)$, but $\eta^2(\sim \pi_0^2)$ is not a SU(2) invariant object (only $\pi_a \pi_a$ is). Thus we arrive to the absurd result that the bosonization procedure is inconsistent[11].

The η field is not adequate to study the color symmetry, but its BCs are. The η field NTBCs determine not only the topology of the $U(1)_C$ subgroup of color SU(2) but its whole structure. We have therefore to establish

---

[10] In Coleman's paper Noether currents are used. Nevertheless it is easy to see that a topological current like (25) verifies also the theorem.

[11] Note that we could use the same argument for the free massive action (see Eq. (16)).

the new $U(1)_C$ topology induced by the interaction. This is easy using the relation (20) between BCs and the minima structure of the potential. We can understand what happens just by comparing the set of minima of the free theory $\emptyset$ to that of QCD_2. Because of the interaction the $U(1)_C$ topology experiments a complete collapse. Only *trivial* BCs solutions, *i.e.*, $\eta(+\infty) = \eta(-\infty) = \eta_0 = 0$, are allowed by the minima structure of QCD_2.

This result can be expressed in a more dynamical way. If we wish to calculate the mass of a $T_3 = n/2$ ($n \in \mathcal{Z}$) charged particle, we need to evaluate the expectation value $\langle\psi_n|\hat{V}|\psi_n\rangle$, $\hat{V}$ being the interaction potential operator and $|\psi_n\rangle$ a generic charged state. In bosonized language such a state will be built upon the vacuum by means of 'vertex operators' of the kind S^α or $J_\pm$. Because the vacuum is not charged — it is color invariant — the T_3-charge of this state will coincide with that of the 'vertex operator'. These states are certainly responsible of NTBCs solutions because they produce the soliton 'jump' of the η operator [12],

$$\langle\psi_n|\hat{\eta}(x)|\psi_n\rangle \rightarrow \begin{cases} 0 & x \rightarrow -\infty \\ \sqrt{\frac{\pi}{2}}n, \; n \in \mathcal{Z} & x \rightarrow +\infty \, . \end{cases} \tag{35}$$

If we expand this expectation value in powers of the Planck's constant, the leading term will be just the classical soliton solution,

$$\langle\psi_n|\hat{\eta}(x)|\psi_n\rangle = \eta_n^{\text{cl}}(x) + O(h) \, . \tag{36}$$

Analogously we can expand the total energy of the charged particle (at rest) in powers of h,

$$\langle\psi_n|\hat{V}|\psi_n\rangle = \mathcal{E}(\eta_n^{\text{cl}}) + O(h) \, , \tag{37}$$

where the static classical energy $\mathcal{E}$ can be calculated just by means of the classical potential [15]

$$\mathcal{E}(\eta_n^{\text{cl}}) \sim \int_{-\infty}^{\infty} dx \; V(\eta_n^{\text{cl}}) \, . \tag{38}$$

But this integral is necesseraly divergent unless $\eta_n^{\text{cl}}(+\infty)$ is an absolute minimum η_n of the classical potential V. Then it becomes clear from the $\emptyset_{QCD_2}$ structure that only $n = 0$ solutions can remain with finite energy. The interaction has eliminated all the possible $(\eta_n, n \neq 0)$ minima thus giving an infinite mass to any $(T_3 = (n/2) \neq 0)$ state[12].

---

[12] In a recent paper by Ellis *et al.* [4] they find color solitons in bosonized QCD_2 with infinite energy. We claim that their *constituent quarks* correspond to solitons which do not connect absolute minima.

Apparently the previous mechanism only allows to eliminate the T_3 charged particles from the physical spectrum. In principle, finite energy color states are still possible. Those states belonging to non-singlet multiplets but with $T_3 = 0$ charge have still a chance of remaining in the spectrum. We will prove next that these states have also infinite mass. We only need to remember two basic properties of the QCD_2 interaction which we have extensively discussed:

(i) The interaction is SU(2) global color invariant,

$$[\hat{V}, T^a] = 0\,.$$

(ii) It gives rise to a neutral color vacuum,

$$T_a|0\rangle = 0\,. \tag{39}$$

These properties force the physical states to accomodate in representations of the SU(2) color group. Let us now consider a representation having total color charge $T = m$ ($m = 1, 2, 3, \ldots$). It certainly contains a state with $T_3 = 0$. According to our previous arguments, states described by a non-trivial boundary condition, get through the dynamics infinite energy. Consequently, all the states of this representation with $T_3 \neq 0$ acquire an infinite mass. But so does the $T_3 = 0$ state because it is *degenerate* with its multiplet partners since the color symmetry is not broken. However there is one representation which evades this mechanism, namely the singlet one ($T = 0$). It only contains a *single* state arising from trivial boundary conditions ($T_3 = 0$) and therefore has finite energy.

The previous result is not a surprise. As we showed in the last section, NTBCs are a *necessary* condition to recover the whole SU(2) structure in the 'Vertex Operator' framework. Now the dynamics does not allow us to keep NTBCs and therefore the non-abelian character of the charge operators disappears in the realization of the spectrum.

Let us show how this mechanism works in an explicit way. In QCD_2 dynamics forces any state to verify ordinary BCs ($n = 0$). According to (35) this implies that

$$\langle\psi|\hat{\eta}(x)|\psi\rangle \xrightarrow{x\to\pm\infty} 0\,, \tag{40}$$

for every physical state $|\psi\rangle$. The question is, what kind of η-dependent operators are allowed now under the restriction (40)?

The $|\psi\rangle$ state will be generated out of the vacuum by some generic operator $\hat{S}$ depending on η. The color quantum numbers of the state will be the same as those of the operator since the vacuum is a singlet. The topological current (25) is conserved independently of the dynamics and does not generate charged states when applied to the vacuum (it preserves

(40)). Thus the T_3 charge is a good quantum number. What happens to T_3 charged operators when the new BCs (40) are dinamically imposed?

A $T_3 = n/2$ charged operator has the following commutation relation with the T_3 current

$$[\hat{S}^n(x), J_3^0(y)] = \frac{n}{2}\delta(x-y)\hat{S}^n(x)\,. \tag{41}$$

This implies that the η and S^n operators will verify the non-trivial commutation relation

$$[\hat{S}^n(x), \hat{\eta}(y)] = \sqrt{\frac{\pi}{2}}n\theta(x-y)\hat{S}^n(x)\,. \tag{42}$$

But if this relation holds, then

$$\langle\psi_n|\hat{\eta}(x)|\psi_n\rangle \stackrel{x\to+\infty}{\longrightarrow} \sqrt{\frac{\pi}{2}}n \neq 0, \quad |\psi_n\rangle = \hat{S}^n|0\rangle\,. \tag{43}$$

in contradiction with the dynamical constraint (40).

From the preceeding argument we conclude that charge operators are no longer allowed. In particular, we cannot construct the non-diagonal currents $J_\pm^\mu$ to enlarge the symmetry from $U(1)_C$ to $SU(2)_C$, as we did in the free case. Therefore the only symmetry that remains is the topological $U(1)_C$ *restricted* to zero charge particles.

The η field does not transform under color transformations, since if it did, the existence of the non-diagonal currents $J_\pm^\mu$ would be required Eq. (13), and those cannot be constructed due to the dynamical restrictions. Thus the color symmetry has dissapeared from the bosonized QCD_2 action!

It is clear why we do not run into any inconsistency with the color invariance of the hamiltonian. The η is not a $T_3 = 0$ particle in the adjoint representation. The special realization of the color symmetry on η along QCD_2 boundary conditions tells us, that the η is really a *color singlet field*. Consequently the potential $V(\eta)$ is trivially invariant, as it must be according to the bosonization procedure.

An important remark becomes necessary before concluding. We have learned that the cornerstone of the confinement mechanism in QCD_2 is the minima structure $\emptyset_{QCD_2}$ induced by the interaction. However this set of minima is obtained by looking at the shape of a *classical* potential V. This fact can be worrisome because we know examples in which radiative corrections can shift away the classical minimum from its original position [18]. We have to take into account that Coleman's Theorem does not prevent the spontaneous breaking of the *discrete* symmetry $\eta \to -\eta$. Thus we cannot exclude *a priori* the possibility of a radiatively induced spontaneous symmetry breakdown.

Let us assume that this possibility really occurs. For the sake of simplicity we consider the case that radiative corrections induce only two new symmetric minima. That is, we deal with the case of a 1D 'Mexican hat' effective potential. We call this couple of minima $(v, -v)$, v being some dimensionless function of the action parameters M and e. Due to this new topology we are now energetically allowed to construct two soliton operators connecting this two minima. A state generated by this operator will have finite energy and $T_3 = \sqrt{2/\pi}v$. For $v = {}^1\!/_2\sqrt{\pi/2}$ we would get a couple of finite energy 'quark-like' solutions, $T_3 = \pm {}^1\!/_2$. Once we choose one of the two minima as the real vacuum we break the $\eta \to -\eta$ (Z_2) symmetry of the action. We define a new η' field having zero *vev* just by shifting η (we chose $-v$ as the real vacuum),

$$\eta' = \eta + v\,. \tag{44}$$

If we look now at the bosonization rule of the field strength (Eq. (28)), we realize that the previous operation is equivalent to a shift in F_3 ,

$$F_3' = F_3 + \frac{e}{\sqrt{2\pi}}v\,. \tag{45}$$

The *vev* of the η field gives rise to a constant color background field $E = (e/\sqrt{2\pi})v$. We already argued that such a constant background field had not physical sense in a non-abelian theory, which motivated our choice, $E = 0$.

The reason to set this background field equal to zero in the non-abelian case arises when we compute the above shift for the pure gauge action. Since the pure gluon term is quadratic in the field strenght, we pick up after the shift one term of the form

$$\int d^2x \left(-\tfrac{1}{2}E_a\tilde{F}_a(x)\right), \tag{46}$$

where $\tilde{F}_a$ is the dual field strength and $E_a = (0, 0, E)$. E_a is an *external* constant background field which does *not* change under local gauge transformations. A term like (46) *violates* local gauge invariance[13]. Local gauge invariance prevents the interaction to produce the violating term (46) *to all orders* in h. The classical potential cannot have such a term and quantum corrections are not allowed to produce it. Thus we are forced to take $E = 0$ and consequently $v = 0$ as well.

Local gauge invariance teaches us that the original Z_2 symmetry of the bosonized lagrangian cannot be spontanously broken. The original $\emptyset_{\mathrm{QCD}_2}$

[13] Notice the peculiar fact that in the abelian case this term is perfectly gauge invariant and therefore allowed.

structure is thus preserved and the mechanism of confinement explained above is valid in its full extent.

5. Confinement and topology

We have learned that the vacuum structure of the *free* field theory is the consequence of the existence of non-trivial boundary conditions. The stability of the soliton numbers (B, T^3) is guaranteed by the existence of *topological* conservation laws for the $\mathrm{U}(1)_F$ and $\mathrm{U}(1)_C$ currents

$$j^\mu = \frac{1}{\sqrt{2\pi}}\varepsilon^{\mu\nu}\partial_\nu\varphi\,, \quad J_3^\mu = \frac{1}{\sqrt{2\pi}}\varepsilon^{\mu\nu}\partial_\nu\eta\,. \tag{47}$$

Because the (B, T^3) charges depend on the values of the (φ, η) fields on the border of the one-dimensional space, they can be related to topological properties of the groups associated with them. The (B, T^3) charges generate the $\mathrm{U}(1)_F\otimes\mathrm{U}(1)_C$ explicit symmetry of the *free* Lagrangian. Its action can be written in terms of the $\mathrm{U}(1)_F\otimes\mathrm{U}(1)_C$ fields $\big(\exp(i\sqrt{2\pi}\varphi), \exp(i\sqrt{2\pi}\eta)\big)$ using

$$\partial_\mu\phi\partial^\mu\phi = \frac{1}{2\pi}\partial_\mu(\mathrm{e}^{i\sqrt{2\pi}\phi})\partial^\mu\mathrm{e}^{-i\sqrt{2\pi}\phi}\,, \tag{48}$$

where $\phi = \varphi,\ \eta$. This means, that when calculating any functional integral of this theory in a finite volume (finite length L), we have to consider all different sectors induced by the topological non-trivial mapping from the S^1 sphere (the compactified one-dimensional space) into the $\mathrm{U}(1)_F\otimes\mathrm{U}(1)_C$ group space $(S^1 \otimes S^1)$. Each of the different homotopic solutions can be characterized by two integers (ν_F, ν_C). These topological charges are obtained by means of the integral formulas in terms of group elements [5]

$$\nu_i = \frac{i}{2\pi}\int_0^{2\pi} d\theta g_i\frac{d}{d\theta}g_i^{-1}\,, \tag{49}$$

where $g_i \in U_i(1), i = F, C$ and therefore

$$\nu_F = \sqrt{\frac{2}{\pi}}[\varphi(2\pi) - \varphi(0)] = 2B\,,$$
$$\nu_C = \sqrt{\frac{2}{\pi}}[\eta(2\pi) - \eta(0)] = 2T^3\,. \tag{50}$$

The non-trivial boundary conditions produce the *winding numbers* associated with the homotopy classes of these mappings. The lattice of physical states gives just the first homotopy class of the group

$$\emptyset \approx \Pi_1[\mathrm{U}(1)_F \otimes \mathrm{U}(1)_C] \approx \Pi_1[\mathrm{U}(1)_F] \times \Pi_1[\mathrm{U}(1)_C] \approx Z_F \otimes Z_C\,. \tag{51}$$

Let us now add a confining interaction. The free vacuum structure collapses into $\emptyset_{QCD_2}$ (see Fig. 3). In topological language the non-trivial topology induced by the mapping

$$\Pi_1[\mathrm{U}(1)_C] \approx Z_C\,, \tag{52}$$

disappears. The interaction forces the physical solutions to have *trivial* BCs ($T^3 = \nu_C = 0$). Because NTBCs are a *necessary* condition to recover the whole SU(2) structure in the 'Vertex Operator' construction, triviality in $\mathrm{U}(1)_C$ implies triviality in $\mathrm{SU}(2)_C$ as well. The trivial topology induced by the interaction in the maximal abelian subgroup of $\mathrm{SU}(2)_C$ ($\mathrm{U}(1)_C$) determines that all physical states have to be color singlets.

Confinement is recognized in the effective action (30) by the fact that the η field is a color singlet field. Since η is a color singlet it must have simple relations with $\mathrm{SU}(2)_C$ invariant quantities. This is easily established when we compare simple operators in their abelian and non-abelian forms. For example, we can express the non-abelian bosonized field g in its abelian form

$$tr g = g_1^1 + g_2^2 = 2\cos\sqrt{2\pi}\eta\,. \tag{53}$$

But because $g \in \mathrm{SU}(2)_C$ it also has a standard representation in terms of the adjoint fields π^a, $g^\alpha_\beta = \exp\left(i\pi^a\,(\tau^a/2)_{\alpha\beta}\right)$ and therefore $\eta(x) = \left(\mathrm{Sign}(x)/2\sqrt{2\pi}\right)\sqrt{\pi_a\pi_a}$[14]. We may write the full potential $V(\eta)$ in terms of the scalar singlet $|\pi|$ thus generating an *explicit* color invariant interaction $V(|\pi|)$. We could proceed to calculate straight off the spectrum of the theory.

In the chiral limit the φ and $|\pi|$ actions decouple. The first gives rise to a massless pseudoscalar particle, the second is the responsible for the resonant meson masses [16]. Resonant states appear as *massive*[15] bound states of the color singlet field $|\pi|$. Obviously, every eigenstate of this hamiltonian will be a singlet. The spectrum of $\hat{V}(|\pi|)$ contains *no trace* of colored particles.

As we have just seen, color dissapears of the bosonized action completely. However the $\mathrm{U}(1)_F$ non-trivial topology remains since we did not gauge the 'flavor' degree of freedom. Nevertheless, it is still affected by the way $\mathrm{U}(1)_C$ collapses. If we compare to the free case, due to the minima structure of the free theory $\emptyset$ (Eq. (19)), the allowed 'flavor' states have $B = (m/2)$ $(m \in Z)$ charges. Once we turn on the interaction $\emptyset$ becomes

---

[14] The Sign function is necessary because η is a pseudoscalar Eq. (10), whereas $|\pi| \equiv \sqrt{\pi_a\pi_a}$ is a scalar. Notice that the cosine in (53) is invariant under $\eta \longrightarrow -\eta$.

[15] Recall that $|\pi|$ is massive.

$\emptyset_{\mathrm{QCD}_2}$ (Eq. (34)) and only $B = m$ states can survive. Thus the singlet spectrum is formed by particles of *integer* baryon number $B = m$, *i.e.*, by mesons ($m = 0$) and by baryons ($m = 1, 2, ...$) or antibaryons ($m = -1, -2, \ldots$). Particles with half-integer baryon number have dissapeared from the spectrum as they should. This result agrees with what is expected from the fermionic theory. Color singlet states are operators which in color space are of the form

$$q_\alpha^+ q_\alpha\,,\quad \varepsilon^{\alpha\beta} q_\alpha q_\beta\,,\quad q_\alpha^+ q_\alpha \varepsilon^{\beta\gamma} q_\beta q_\gamma\,,\quad \ldots\,. \tag{54}$$

It must be clear that no restriction on the range of the QCD_2 parameters (M and e) has been made. The minima structure $\emptyset_{\mathrm{QCD}_2}$ of the theory is the same *no matter* what the values of M and e are. Even in the massless limit topology stays unaltered. Moreover it is easy to generalize our results to any N. The main features of the confinement mechanism remain: nonlocal non-linear realization of color symmetry in terms of N boson fields, vacuum invariance under global $\mathrm{SU}(N)_C$ and presence of quadratic terms in the bosonized action.

Finally let us point out that the formalism used to explore confinement in QCD_2 can also be extended to any 2D $\mathrm{SU}(N)$ theory. If we add to the *free* action a non-confining interaction, then the quarks must be the only asymptotic in/out states. Thus asymptotically the action is that of a free (φ, η) fields with non-trivial boundary conditions and therefore we obtain a free vacuum structure. The topological conservation law of the $\mathrm{U}(1)_F \otimes \mathrm{U}(1)_C$ charges ensures that the minima structure of the asymptotic theory is *identical* to that of the interaction theory. Thus the addition of a non-confining interaction leaves the free vacuum structure unaltered and therefore one can charaterize these interactions topologically by Eq. (51). For $N = 2$ the bosonized potential of a non-confining $\mathrm{SU}(2)$ theory has to be local in η and support the free minima structure $\emptyset$. That is to say, it must be periodic in η and invariant under 'diagonal' translations $\eta \to \eta + \sqrt{\pi/2}m$ (see Eq. (21)).

From this exhaustive survey on the topology and color structure of QCG_2 only the following conclusion can be drawn:

The confinement mechanism in QCD₂ is purely topological. Confinement occurs for any value of the quark mass, the gauge coupling constant and for any number of colors. There is only one phase of permanent confinement.

6. Conclusions

The problem of confinement in the fermionic formulation of QCD_2 becomes extremely complex beyond the large N and weak coupling limits. In these two limits the spectrum, as well as the vanishing of quark creation amplitudes, corroborate the non existence of *free* color non singlet states. The quark self-energy shows a confining behavior.

However sea quark excitations, vertex corrections and non-planar contributions turn out to be of great relevance in the most general case, complicating our understanding of the confinement mechanism. Nevertheless the fermionic description seems to indicate in a qualitative manner that QCD_2 is a confining theory beyond leading order in the $1/N$ expansion [6].

In this paper we have given and alternative general method to solve the confinement problem beyond the large N and large mass restrictions. According to our investigation the crucial ingredients to understand confinement in QCD_2 are:

i) Boundary Conditions. Non Trivial Boundary Conditions (NTBC) allow the existence of a non-linear non-local realization of the SU(2) color symmetry in 2D. This realization is built upon one single colored field transforming under a *non-linear non-local representation* of the group. The color content of the theory *dramatically* depends on the structure of the BCs of this field.

ii) Vacuum Invariance. The invariance of the QCD_2 vacuum under SU(2) *global* transformations is an *exact* statement (Coleman's Theorem [9]). Its validity is universal for any value of the parameters (M and e and easily generalizable to any N). These two statements lead to a unique conclusion:

There is only one phase of permanent confinement in QCD_2 for every value of M, e and N. No colored states are allowed for any value of the parameters.

In the bosonized version of fermionic two dimensional theories topology plays a crucial role. States with baryon number and color charge are described by solitons. The properties of the vacuum, which give rise to non-trivial boundary conditions, determine the quantum number structure of the Fock space. We have analyzed initially the rich spectrum of non confining theories by discussing the role of boundary conditions. The existence of non-trivial boundary conditions is a consequence of the non-trivial topology of the SU(N) maximal abelian subgroup $U(1)^N$ on the S^1 compactified 1D space. This non-trivial topology allows the enlargement of the explicit $U(1)^N$ symmetry into a complete color SU(N) symmetry by means of the so-called 'Vertex Operator' construction. The discussion of color symmetry can be reduced to the study of the minima structure of the bosonized potential when written in terms of the $U(1)^N$ color abelian fields. It is appealing that in the bosonized version of the theory , this discussion can be carried out purely at the classical level.

When the interaction is switched on the $U(1)^N$ color topology breaks down. The $U(1)^N$ color abelian charges are dynamically screened and the

$SU(N)_C$ symmetry cannot be reconstructed anymore. The invariance of the vacuum under $SU(N)_C$ transformations ensures that all $SU(N)_C$ charges are likesome screened. Only singlet states can remain in the spectrum. This topological mechanism is independent of the relation between the coupling constant and the quark mass and it is valid to all orders in h. QCD_2 shows only one single phase of permanent confinement. The mechanism can be cast in a more mathematical language by invoking homotopy groups, but one should not avoid the very naive dynamical statement, namely that color solitons are given infinite energy [4].

The symplicity of the topological description and its generality can be used in a wider spectrum of 2D $SU(N)$ theories. They can be classified according to its $U(1)^N$ minima structure. Then very strong statements can be made about their confining properties.

Unluckily at this moment, four dimensional calculations seem to be pretty unrelated to two dimensional theories. Recent investigations (string theory [19], dimensional reduction techniques [20], observable effects of string-like confining mechanisms [21],...) suggest that it might be possible to make a connection with realistic problems in a near future. The deep knowledge of QCD_2 and related theories could be of big help.

Moreover the abstraction associated with the mathematical language might guide one into the four dimensional case. Is it not possible to write an approximate bosonized theory in four dimensions? Skyrme type models have been extremely succesful in describing the low energy flavor properties of the theory , but they avoid confinement simply by assuming its existence [22]. They have been already attempts of writing an approximate bosonized lagrangian in terms of the color degrees of freedom. Quark-like solutions appear as color Skyrmions of this effective action [23]. Could we find here an analogous mechanism that produced the collapse of the spectrum ? The work of 't Hooft [24] has been pioneering in this respect, but again the dimensionality of space-time makes difficult the connection. No relation between our topological scheme and his has been as of yet found, but the endeavor seems sufficiently appealing to embark.

We have received useful comments and criticism from M. Asorey, A. González-Arroyo, D. Espriu, J. Ros and A. Santamaría. One of us (A.F.) is grateful to H. Leutwyler, P. Minkowski, E. Alvarez and J. Soto for illuminating discussions. We would like to thank J. Segura for technical assistance.

REFERENCES

[1] G. 't Hooft, *Nucl. Phys.* **B75**, 461 (1974).
[2] A. Patrasciou, *Phys. Rev.* **D15**, 3592 (1977); P. Mitra, P. Roy, *Phys. Lett.* **B79**, 469 (1978); M. Partovi, *Phys. Lett.* **B80**, 377 (1979); G. Bhattacharya, *Nucl. Phys.* **B205**, 461 (1982).
[3] V. Baluni, *Phys. Lett.* **B90**, 407 (1980).
[4] J. Ellis, Y. Frishmann, M. Karliner, *Nucl. Phys.* **B382**, 189 (1992).
[5] S. Coleman in *Aspects of Symmetry*, Cambridge University Press, 1985.
[6] C.G. Callan, N. Coote, D.J. Gross, *Phys. Rev.* **13**, 1649 (1976); R.C. Brower, J. Ellis, M.G. Schmidt, J.H. Weis, *Nucl. Phys.* **B128**, 131 (1977); *Nucl. Phys.* **B128**, 537 (1977).
[7] M.B. Einhorn, *Phys. Rev.* **D12**, 3451 (1976).
[8] S. Huang, J.W. Negele, J. Polonyi, *Nucl. Phys.* **B307**, 669 (1988).
[9] S. Coleman, *Comm. Math. Phys.* **31**, 259 (1973).
[10] M.B. Halpern, *Phys. Rev.* **D12**, 1684 (1975); *Phys. Rev.* **D13**, 337 (1976).
[11] I.B. Frenkel, V.G. Kac, *Inv. Math.* **62**, 23 (1980); G. Segal, *Comm. Math. Phys.* **80**, 301 (1981).
[12] S. Mandelstam, *Phys. Rev.* **D11**, 3026 (1975).
[13] S. Coleman, *Phys. Rev.* **D11**, 2088 (1975).
[14] P. Ginsparg, *Applied Conformal Field Theory. Les Houches, Session XLIX,* 1988, *Fields, Strings and Critical Phenomena*, ed. by E. Brézin and J. Zinn-Justin. Elsevier Science Publishers B.V. 1989.
[15] T.P. Cheng, L.F. Li in *Gauge Theory of Elementary Particle Physics*, Oxford University Press, 1984.
[16] A. Ferrando, V. Vento, *Phys. Lett.* **B256**, 503 (1991); *Phys. Lett.* **B265**, 153 (1991); Ph. D. Thesis, Universitat de València (1991).
[17] S. Coleman, *Ann. Phys.* **101**, 239 (1976).
[18] S. Coleman, E. Weinberg, *Phys. Rev.* **D7**, 1888 (1973).
[19] D.J. Gross, Preprint LBL 33415, PUPT 1356, December 1992.
[20] A. Ferrando, U.J. Wiese, work in progress.
[21] A. Ferrando, V. Vento, Preprint UTF-304/93.
[22] I. Zahed, G.E. Brown, *Phys. Rep.* **142**, 1 (1986).
[23] D.B. Kaplan, *Nucl. Phys.* **B351**, 137 (1991).
[24] G. 't Hooft, *Acta Physica Austriaca, Suppl.* **XXII**, 531 (1980).