MESON SCREENING MASSES IN THE NAMBU – JONA-LASINIO MODEL*, **

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The spatial dependence of the finite-temperature meson correlation function in the scalar and pseudoscalar channel is studied in the Nambu–Jona-Lasinio model. The screening masses, obtained from the asymptotic behaviour of the static correlation function, are found to differ substantially from the dynamic masses, defined by a pole of the meson propagator. In particular, at high temperatures, the meson screening masses are large, although there are no well defined meson modes. In the high-temperature limit, the screening masses approach $2\pi T$, which corresponds to a gas of non-interacting, massless quarks. However interaction effects remain substantial well beyond the chiral transition temperature. The overall temperature dependence of the screening masses is in agreement with lattice results.

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1. Introduction

Recently, much effort has been put into the investigations of the hadronic correlation functions. As it was pointed out by Shuryak [1], the importance of these functions relies on the fact that they can be calculated in several ways: *(i)* one can express them in terms of the fundamental QCD fields, quarks and gluons; *(ii)* one can evaluate a part of them using hadron phenomenology; or *(iii)* they can be calculated in the framework of effective models. Consequently, these correlation functions can be used for making comparisons between various approaches. In particular, the results of the lattice simulations of QCD can be compared with the results of approximate models. In this way we can observe how good the latter are in reproducing the features of the fundamental theory.

In this paper we study the temperature dependence of the meson correlation functions in the pseudoscalar (pion) and scalar (sigma) channel. Our calculations are based on the Nambu–Jona-Lasinio (NJL) model. This model was constructed already in the sixties [2] as the theory of nucleons interacting via an effective two-body interaction. Now it has been reinterpreted as a theory with quark degrees of freedom (for an overview see [3] or [4]).

An essential feature of this model is that it respects several important symmetries of QCD, the most prominent one being the chiral symmetry. The NJL model also exhibits spontaneous symmetry breaking. The relative simplicity of the model makes it useful in studying this phenomenon. Effects, like dynamical mass generation and the appearance of Goldstone bosons, are exhibited by the model. Another interesting feature is the phase transition; at high temperature the chiral symmetry, which is spontaneously broken in the vacuum, is restored. In this respect the model resembles QCD where we expect that an analogous phase transition takes place.

There are two basic shortcomings of the NJL model. The first one is connected with the pointlike character of the quark-antiquark interaction which makes the theory non-renormalizable. The second one is that in this model the quarks are not confined. Because of these two disadvantages the model has been strongly criticized. Nevertheless, the NJL model is, in spite of its deficiencies, an interesting tool for studying phenomena related to the chiral symmetry and the chiral phase transition.

The properties of the NJL model at finite temperature (and/or density) have been studied already by many authors [5–12]. One of the main aims of these investigations was to calculate the dynamic meson masses determined by the poles of the correlation functions. In the present paper we consider a different, in some respect complementary problem. We study the spatial dependence of the static correlation functions. In particular, we calculate the meson screening masses which characterize their asymptotic behaviour.

The difference between the dynamic and screening mass will be discussed in more detail below. At this point we note only that both masses are determined by the singularities of the correlation functions. Therefore, the analytic structure of the latter should be studied in detail.

We compare the qualitative behaviour of our results obtained in the NJL model with those produced in the lattice simulations of QCD. The lattice measurements of the hadronic correlation functions were initiated by DeTar and Kogut [13] and continued by other groups [14–17]. The results of these calculations show that at high temperatures, $T > T_c$, the hadronic screening masses of chiral partners are equal, but non-vanishing. This is consistent with the restoration of chiral symmetry. Moreover, Eletski and Ioffe [18] pointed out that the meson screening masses approached the value $2\pi T$, whereas those of baryons were close to $3\pi T$. They argued that such a behaviour is typical for a gas of non-interacting massless quarks. Nevertheless, in the (pseudo)scalar channel, in contrast to the (axial)vector one, the screening masses are still considerably below $2\pi T$. This result indicates that also well above the phase transition, the residual interaction between quarks and antiquarks in these channels is non-negligible.

Our attempt to interpret the lattice results in terms of a simple model is not the first one. So far, several models have been discussed in this context. Gocksch [19] argues that the hadronic screening masses as well as the quark number susceptibility can be qualitatively understood in the linear sigma model. Hansson and Zahed [20] show that the static correlation functions in high-temperature QCD can be calculated from an equivalent problem of non-relativistic quarks in a dimensionally reduced theory. Similarly, Koch *et al.* [21] find that the propagation of a light fermion in a spatial direction at high temperatures can be described effectively by a two-dimensional Schrödinger equation with a heavy effective mass $m_{\text{eff}} = \pi T$. They then introduce an effective potential for propagation of the quark-antiquark pair. The results reproduce the lattice data [16] in the ρ and π channels.

In the end of the introduction let us present the organization of our paper. In the next Section we define the model. In Section 3 we give the details of our calculations. The temperature dependence of the constituent quark mass and of the dynamic meson masses are discussed in Section 4. Our main results concerning the screening meson masses are presented in Section 5. Conclusions and the list of references close the paper.

2. Definition of the model

We consider the following lagrangian of the NJL type

$$\mathcal{L} = \overline{\psi}(i\gamma_\mu \partial^\mu - m)\psi + \sum_{a=0}^{3} \frac{G_S}{2}\left[(\overline{\psi}\sigma_a\psi)^2 + (\overline{\psi}i\gamma_5\sigma_a\psi)^2\right], \qquad (1)$$

where ψ is the Dirac field carrying additional flavour (N_f=2) and colour (N_c=3) degrees of freedom, σ_a are the flavour Pauli matrices (with σ_0=1), G_S is the coupling constant, and m is the current quark mass. We take into account the isospin symmetric case $m_u = m_d = m$.

The interaction term in (1) has the chiral symmetries $U_A(1)$ and $SU_A(2)$. If m is small then, in approximation, the whole lagrangian is chirally invariant. The symmetries $U_A(1)$ and $SU_A(2)$ are characteristic for QCD where, however, the first is broken due to the instanton effects [22] and the second is spontaneously broken. In the NJL model the $SU_A(2)$ symmetry is also spontaneously broken and this fact is one of the most attractive features of the model, showing its closeness to QCD. The breaking of the $U_A(1)$ symmetry can be achieved by adding an extra term to the lagrangian (1). For sake of simplicity we do not take this term into account. Consequently, besides the pseudoscalar isovector (I=1) almost massless mesons, *i.e.* pions, we have a pseudoscalar isoscalar (I=0) meson having the same mass.

The Schwinger–Dyson equation for the self-energy of quarks in the Hartree-Fock approximation has the following form

$$\Sigma = G_S\, i \sum_{a=0}^{3} \int \frac{d^4p}{(2\pi)^4} \sigma_a \mathrm{Tr}\left[\sigma_a S(p)\right], \qquad (2)$$

where Tr is the trace over flavour, colour and spinor indices. The quantity $S(p)$ is the quark propagator $S(p)^{-1} = \not{p} - \Sigma - m + i\varepsilon$. The quark condensate is defined by equation

$$\langle \overline{q}q \rangle = -\frac{i}{2}\mathrm{Tr}\; S(x=0) = -4N_c\, i \int \frac{d^4p}{(2\pi)^4} \frac{\Sigma + m}{p^2 - (\Sigma+m)^2 + i\varepsilon}. \qquad (3)$$

The effective mass $\Sigma + m$ can be regarded as the constituent quark mass M, whose origin is the quark-antiquark interaction. Eqs (2) and (3) lead to the following relation connecting M directly to the condensate

$$M = m - 2G_S\langle \overline{q}q \rangle. \qquad (4)$$

This is so-called gap or self-consistency equation.

The zeroth-order correlation function (called also a generator) is represented by the following expression

$$\chi^{(0)}_{AB}(Q) = 2iN_c \,\mathrm{Sp} \int \frac{d^4p}{(2\pi)^4}[\Gamma_A S(p+Q)\Gamma_B S(p)]\,.$$

Here $Q^\mu = (\omega, \vec{q})$ is the external momentum. The indices A and B take on the values P or S and Γ_P, Γ_S are the Dirac tensors: $\Gamma_P = i\gamma_5$ and $\Gamma_S = 1$. Calculation of the trace over the spinor indices, denoted here by Sp, shows that the crossing terms ($A \neq B$) vanish. Therefore, we are left with two non-vanishing generators: $\chi^{(0)}_{PP}$ and $\chi^{(0)}_{SS}$. Using the random phase approximation, we find that the full correlation function in the pseudoscalar channel ($A = P$) and in the scalar channel ($A = S$) has the form

$$\chi_{AA}(Q) = \frac{\chi^{(0)}_{AA}(Q)}{1 - G_S\chi^{(0)}_{AA}(Q)}\,. \tag{6}$$

In the medium, the correlation function $\chi_{AA}(Q)$ depends on Q through the variables ω^2 and q^2. Therefore, in the following we shall use the notation $\chi_{AA}(\omega^2, q^2)$ rather than $\chi_{AA}(Q)$. The dynamic mass is defined by the position of the pole of the function $\chi_{AA}(\omega^2, 0)$ closest to the origin. In the NJL model it can be easily found by solving the equation

$$1 - G_S\chi^{(0)}_{AA}(m^2_{\mathrm{dyn}}, 0) = 0\,. \tag{7}$$

This follows directly from Eq. (6). On the other hand, the screening mass is defined by the asymptotic behavior of the correlation function in space, namely

$$m_{\mathrm{scr}} = -\lim_{r\to\infty} \frac{d\ln\chi_{AA}(r)}{dr}\,, \tag{8}$$

where

$$\chi_{AA}(r) = \int \frac{d^3q}{(2\pi)^3}\,\chi_{AA}(0, q^2)\,e^{i\vec{q}\vec{r}} = \frac{1}{4\pi^2 ir}\int\limits_{-\infty}^{+\infty} dq\,q\,\chi_{AA}(0, q^2)\,e^{iqr}\,. \tag{9}$$

Let us now consider an arbitrary correlation function $\chi_{AA}(\omega^2, q^2)$, not necessarily obtained from the NJL model. One can ask the question when the dynamic mass is equal to the screening one. At $T = 0$ the system is explicitly Lorentz invariant and $\chi_{AA}(\omega^2, 0) = \chi_{AA}(0, -\omega^2)$. Therefore, if the function $\chi_{AA}(\omega^2, 0)$ has a pole for $\omega = m_{\mathrm{dyn}}$ then the function $\chi_{AA}(0, q^2)$ has a pole for $q = im_{\mathrm{dyn}}$. The latter gives the contribution to the integral

(9), which has the form $\sim \exp(-m_{\rm dyn} r)$. For very large r this contribution is the dominant one because $im_{\rm dyn}$ is the nearest pole. Consequently, we find that $m_{\rm dyn} = m_{\rm scr}$. Of course one has to be careful because the analytic structure of the correlation function can be complicated and there may be other important contributions to the integral (9). Nevertheless, at $T = 0$ we expect that $m_{\rm dyn} = m_{\rm scr}$. A different situation takes place when $T > 0$. In this case the Lorentz invariance is implicitly broken by the existence of the preferable reference frame defined by the heat bath. The fact that $\chi_{AA}(\omega^2, 0)$ has a pole for $\omega = m_{\rm dyn}$ does not imply that $\chi_{AA}(0, q^2)$ has a pole for $q = im_{\rm dyn}$. It is also well known that at $T > 0$ the contribution to Eq. (9) from the cuts of the correlation function is important and it can substantially change the space asymptotics of the function $\chi_{AA}(r)$. Consequently, at $T > 0$, the dynamic and screening masses are different.

3. Calculation of the physical quantities at zero and finite temperature

(i) Imaginary time formalism

The calculation of all quantities that interest us, like the condensate or the correlation functions, can be done in the imaginary time formalism [23, 24]. Schematically, this is achieved by the following substitution

$$\int \frac{d^4p}{(2\pi)^4} \to iT \sum_n \int \frac{d^3p}{(2\pi)^3}. \tag{10}$$

Here T is the temperature and the integration over the energy is replaced by the sum over discrete complex energies $p^0 = i\omega_n$. In the case of fermions $\omega_n = (2n+1)\pi T$ and in the case of bosons $\omega_n = 2n\pi T$. Such sums can be converted to the contour integral in the complex energy plane. Moreover, a deformation of this contour allows us to write the final result as a sum of two terms [24]. In the case of interest, *i.e.*, for fermions, one finds

$$\begin{aligned} iT \sum_n f(i\omega_n) = & \frac{1}{2\pi} \int\limits_{-i\infty}^{+i\infty} dp^0 f(p^0) \\ & - \frac{1}{2\pi} \int\limits_{-i\infty+\epsilon}^{i\infty+\epsilon} dp^0 \left[f(p^0) + f(-p^0)\right] \frac{1}{e^{p^0/T}+1}. \end{aligned} \tag{11}$$

This decomposition is useful, since it separates the explicitly temperature dependent part (*"matter part"*) from the *"vacuum part"*. The former contains the Fermi–Dirac distribution and vanishes in the limit $T \to 0$. Using, *e.g.*, Eqs (3), (10) and (11) we can decompose the condensate into two parts

$$\langle \bar{q}q \rangle = \langle \bar{q}q \rangle_{\rm vac} + \langle \bar{q}q \rangle_{\rm mat} . \tag{12}$$

In the similar way, using Eqs (5), (10) and (11) we find (for $A = B$)

$$\chi^{(0)}_{AA}(\omega^2, q^2) = \chi^{(0)}_{AA,\rm vac}(Q^2) + \chi^{(0)}_{AA,\rm mat}(\omega^2, q^2) . \tag{13}$$

(ii) Zero temperature expressions and their regularizations

Let us now concentrate on the vacuum parts of the physical quantities defined in the end of the last subsection. Introducing the variables p_4 and q_4, such that $p^0 = ip_4$ and $\omega = iq_4$, we obtain

$$\langle \bar{q}q \rangle_{\rm vac} = -\frac{M}{2} I_{1,\rm vac}(M^2) , \tag{14}$$

$$\chi^{(0)}_{PP,\rm vac}(-q_E^2) = I_{1,\rm vac}(M^2) + q_E^2 I_{2,\rm vac}(M^2, q_E^2) \tag{15}$$

and

$$\chi^{(0)}_{SS,\rm vac}(-q_E^2) = I_{1,\rm vac}(M^2) + (q_E^2 + 4M^2) I_{2,\rm vac}(M^2, q_E^2) . \tag{16}$$

Here $q_E^2 = q^2 + q_4^2$ and the functions $I_{1,\rm vac}(M^2)$ and $I_{2,\rm vac}(M^2, q_E^2)$ are defined as follows

$$I_{1,\rm vac}(M^2) = 8N_c \int \frac{d^4p_E}{(2\pi)^4} \frac{1}{p_E^2 + M^2} , \tag{17}$$

$$I_{2,\rm vac}(M^2, q_E^2) = -4N_c \int \frac{d^4p_E}{(2\pi)^4} \frac{1}{\left[(p_E + q_E/2)^2 + M^2\right]\left[(p_E - q_E/2)^2 + M^2\right]} . \tag{18}$$

The integration measure in these integrals is $d^4p_E = d^3p\,dp_4$.

Two points have to be emphasized here. Firstly, all the scalar products of vectors have now the Euclidean metric, for example $(p_E + q_E/2)^2 = (\vec{p} + \vec{q}/2)^2 + (p_4 + q_4/2)^2$. Secondly, the integrals (17) and (18) are diverging, so that the quantities (14), (15) and (16) are not well defined. In order to get finite results one has to apply a regularization procedure. In our case we adopt a subtraction scheme which is close to the Pauli–Villars method, namely, we replace functions $I_{1,\rm vac}$ and $I_{2,\rm vac}$ by the series

$$I_{1,\rm vac}(M^2) \to I^R_{1,\rm vac}(M^2) = \sum_{i=0}^{N} A_i I_{1,\rm vac}(\Lambda_i^2) \tag{19}$$

and

$$I_{2,\mathrm{vac}}(M^2,q_E^2) \to I_{2,\mathrm{vac}}^R(M^2,q_E^2) = \sum_{i=0}^{N} A_i I_{2,\mathrm{vac}}(\Lambda_i^2,q_E^2)\,. \tag{20}$$

It is not the proper Pauli–Villars regularization scheme because we do not subtract the expressions corresponding to the appropriate diagrams. We only make subtractions at the level of the functions $I_{1,\mathrm{vac}}$ and $I_{2,\mathrm{vac}}$. This allows us, however, to take into account the chiral symmetry of the system in the right way.

In Eqs (19) and (20) N is the number of subtractions, $A_0 = 1$ and $\Lambda_0 = M$. If $N = 0$ then the series in Eqs (19) and (20) simplify to the expressions (17) and (18). The coefficients A_i, for $i > 0$, have to be chosen in such a way as to provide the finite result for $I_{1,\mathrm{vac}}^R$ and $I_{2,\mathrm{vac}}^R$. At the same time the correlation functions should have a good asymptotics, such that the Fourier transform used to calculate the screening masses exists. These requirements lead to the set of the equations of the form

$$\sum_{i=0}^{N} A_i = 0,\quad \sum_{i=0}^{N} A_i\Lambda_i^2 = 0,\quad \ldots\quad ,\sum_{i=0}^{N} A_i\Lambda_i^{2(N-1)} = 0\,. \tag{21}$$

A straightforward calculation, using Eqs (21), yields

$$I_{1,\mathrm{vac}}^R(M^2) = \frac{N_c}{2\pi^2}\sum_{i=0}^{N} A_i\,\Lambda_i^2\,\ln\Lambda_i^2 \tag{22}$$

and

$$I_{2,\mathrm{vac}}^R(M^2,q_E^2) = \frac{N_c}{2\pi^2}\sum_{i=0}^{N} A_i\left[\frac{2\Lambda_i}{q_E}\sqrt{1+\left(\frac{q_E}{2\Lambda_i}\right)^2}\ln\left(\sqrt{1+\left(\frac{q_E}{2\Lambda_i}\right)^2}+\frac{q_E}{2\Lambda_i}\right)+\ln\Lambda_i\right]. \tag{23}$$

Substituting Eqs (22) and (23) into (14), (15) and (16) gives us finite expressions for the condensate and for the generators. We can use them directly for the calculation of the screening masses. Nevertheless, if we want to calculate the dynamic masses we have to know the function $I_{2,\mathrm{vac}}^R(M^2,-\omega^2)$. This can be obtained by performing the analytic continuation of the function defined on the right hand side of Eq. (23). The substitution $q_E \to i\omega \pm \varepsilon$ leads to the following formula

$$I_{2,\mathrm{vac}}^R(M^2,-\omega^2 \pm i\varepsilon) =$$

$$\frac{N_c}{2\pi^2}\sum_{i=0}^{N} A_i \left\{ \Theta(2\Lambda_i-\omega)\left[\frac{2\Lambda_i}{\omega}\sqrt{1-\left(\frac{\omega}{2\Lambda_i}\right)^2}\arcsin\left(\frac{\omega}{2\Lambda_i}\right)\right.\right.$$
$$\left.\left. +\ \ln\Lambda_i\right] + \Theta(\omega-2\Lambda_i)\left[\frac{2\Lambda_i}{\omega}\sqrt{\left(\frac{\omega}{2\Lambda_i}\right)^2-1}\left(\text{Arch}\left(\frac{\omega}{2\Lambda_i}\right)\pm\frac{i\pi}{2}\right)+\ln\Lambda_i\right]\right\}. \tag{24}$$

From Eq. (24) we can see that the function $I^R_{2,\text{vac}}(M^2,-\omega^2)$ has a cut starting at $\omega = 2M$. In the NJL model this cut corresponds to the (unphysical) possibility of the decay of a meson into a quark-antiquark pair.

(iii) Finite temperature contributions

Let us now consider the matter parts of the physical quantities of interest. In the case of the condensate, using Eqs (3), (10) and (11), we find

$$\langle\bar{q}q\rangle_{\text{mat}} = -\frac{M}{2} I_{1,\text{mat}}(M^2), \tag{25}$$

where

$$I_{1,\text{mat}}(M^2) = -16iN_c \int\limits_{-i\infty+\epsilon}^{i\infty+\epsilon} \frac{dp^0}{2\pi}\int\frac{d^3p}{(2\pi)^3}\frac{1}{p_0^2-\vec{p}^{\,2}-M^2}\frac{1}{e^{p^0/T}+1}. \tag{26}$$

To calculate the function $I_{1,\text{mat}}(M^2)$ we deform the contour of the integration in the complex p^0 plane. Picking up the pole in the right half plane, which is placed at $p^0 = \omega_p \equiv \sqrt{M^2+\vec{p}^{\,2}}$, we find that

$$I_{1,\text{mat}}(M^2) = -\frac{4N_c}{\pi^2}\int\limits_0^\infty \frac{dp\,p^2}{\omega_p}\frac{1}{e^{\omega_p/T}+1}. \tag{27}$$

From now on we use the notation $p = |\vec{p}|$.

The matter parts of the generators depend separately on ω^2 and q^2. Nevertheless, the following decompositions, analogous to those obtained in vacuum, are possible

$$\chi^{(0)}_{PP,\text{mat}}(\omega^2,q^2) = I_{1,\text{mat}}(M^2) - Q^2 I_{2,\text{mat}}(M^2,\omega^2,q^2) \tag{28}$$

and

$$\chi^{(0)}_{SS,\text{mat}}(\omega^2,q^2) = I_{1,\text{mat}}(M^2) - (Q^2-4M^2) I_{2,\text{mat}}(M^2,\omega^2,q^2), \tag{29}$$

where

$$I_{2,\mathrm{mat}}(M^2,\omega^2,q^2) = -8iN_c \int\limits_{-i\infty+\varepsilon}^{i\infty+\varepsilon} \frac{dp^0}{2\pi} \int \frac{d^3p}{(2\pi)^3}$$

$$\times \frac{1}{\left[(p+Q/2)^2-M^2\right]\left[(p-Q/2)^2-M^2\right]} \frac{1}{\exp(p^0-\omega/2)/T+1}. \tag{30}$$

The calculation of the function $I_{2,\mathrm{mat}}(M^2,\omega^2,q^2)$ proceeds in the same way as the calculation of the function $I_{1,\mathrm{mat}}(M^2)$. To calculate it we deform the contour of the integration in the complex p^0 plane. The modified contour surrounds the poles of the integrand, lying on the right hand side of the imaginary axis. The position of these poles is now given by the equation $p^0 = \mp\omega/2 + \sqrt{M^2 + (\vec{p} \pm \vec{q}/2)^2}$. Calculating the residues we find

$$I_{2,\mathrm{mat}}(M^2,\omega^2,0) = \frac{N_c}{\pi^2} \int\limits_0^\infty \frac{dp\,p^2}{\omega_p} \frac{1}{e^{\omega_p/T}+1} \frac{1}{\omega_p^2-\omega^2/4} \tag{31}$$

and

$$I_{2,\mathrm{mat}}(M^2,0,q^2) = -\frac{N_c}{q\pi^2} \int\limits_0^\infty \frac{dp\,p}{\omega_p} \frac{1}{e^{\omega_p/T}+1} \ln\left|\frac{2p-q}{2p+q}\right|. \tag{32}$$

In order to calculate the screening mass, we need to know the analytic structure of the function $I_{2,\mathrm{mat}}(M^2,0,q^2)$ in the whole complex q plane. To find it we represent $I_{2,\mathrm{mat}}(M^2,0,q^2)$ as a sum of two functions, namely

$$I_{2,\mathrm{mat}}(M^2,0,q^2) = I^{(+)}_{2,\mathrm{mat}}(q) + I^{(-)}_{2,\mathrm{mat}}(q) \tag{33}$$

where

$$I^{(\pm)}_{2,\mathrm{mat}}(q) = -\frac{N_c}{2q\pi^2} \int\limits_0^\infty \frac{dp\,p}{\omega_p} \frac{1}{e^{\omega_p/T}+1} \ln\frac{2p-q\pm i\varepsilon}{-2p-q\pm i\varepsilon}. \tag{34}$$

The calculation should be done for an infinitesimal value of ε and then the limit $\varepsilon \to 0$ has to be taken. The function $I^{(\pm)}_{2,\mathrm{mat}}(q)$ has a cut at $\mathrm{Im}\, q = \pm i\varepsilon$, and stretching from minus to plus infinity parallel to the real axis. The cut is due to the discontinuity of the imaginary part of the function $I^{(\pm)}_{2,\mathrm{mat}}(q)$. In the regions just above and below the cut the imaginary part has opposite sign. However, for q on the real axis, the imaginary part of $I^{(+)}_{2,\mathrm{mat}}(q)$ cancels exactly the imaginary part of $I^{(-)}_{2,\mathrm{mat}}(q)$. Thus, the final result for $I_{2,\text{mat}}(M^2,0,q^2)$ is real as it should be. These properties can be summarized by the following equations

$$I^{(+)}_{2,\text{mat}}(q = q_R + i\varepsilon \pm i\delta) = I^{(-)}_{2,\text{mat}}(q = q_R - i\varepsilon \pm i\delta) \tag{35}$$

and

$$\text{Im}\, I^{(+)}_{2,\text{mat}}(q = q_R + i\varepsilon \pm i\delta) = \mp \frac{N_c T}{2q_R\pi} \ln \left[1 + \exp\left(-\frac{\sqrt{M^2 + q_R^2/4}}{T}\right)\right] . \tag{36}$$

Here q_R denotes the real part of q. The numbers $q_R \pm i\varepsilon$ determine the position of the cut of the function $I^{(\pm)}_{2,\text{mat}}(q)$ and $\pm i\delta$ is the small shift up or down from the cut. Using Eqs (33) and (34) we can find the analytic continuation of $I_{2,\text{mat}}(M^2,0,q^2)$ to purely imaginary values of q. Substituting $q = ik$ we obtain the following expression

$$I_{2,\text{mat}}(M^2,0,-k^2) = -\frac{N_c}{k\pi^2} \int\limits_0^\infty \frac{dp\, p}{\omega_p} \frac{1}{e^{\omega_p/T}+1} \left[\pi - 2\text{arctg}\left(\frac{k}{2p}\right)\right] . \tag{37}$$

The functions $I_{1,\text{mat}}$ and $I_{2,\text{mat}}$ defined by Eqs (26) and (30) are finite and do not have to be regularized. We note, however, that in the limit $T \to \infty$ they are diverging. Moreover, in this limit, we recover expressions (17) and (18) but with the minus sign. This means that in the limit $T \to \infty$ the non-regularized expressions for the condensate and the generators formally vanish. This fact follows also directly from Eq. (11). This property is destroyed in a procedure, where only the vacuum part is regularized. Can we recover this property by modifying the regularization scheme? The answer is yes; if we regularize also the matter parts analogously to the vacuum part the regularized condensate as well as the generators vanish as $T \to \infty$. The proper regularization of the matter parts guarantees that for arbitrary temperature $M > m$. It also leads to the cancellations which ensure the expected from QCD asymptotics of the correlation functions.

In view of the arguments above we regularize the matter parts of the condensate and the generators, in the same way as in the case of the vacuum parts:

$$I_{1,\text{mat}}(M^2) \to I^R_{1,\text{mat}}(M^2) = \sum_{i=0}^{N} A_i I_{1,\text{mat}}(\Lambda_i^2) \tag{38}$$

and

$$I_{2,\text{mat}}(M^2,\omega^2,q^2) \to I^R_{2,\text{mat}}(M^2,\omega^2,q^2) = \sum_{i=0}^{N} A_i I_{2,\text{mat}}(\Lambda_i^2,\omega^2,q^2) . \tag{39}$$

We note that this regularization procedure does not change the analytic structure of the function $I_{2,\mathrm{mat}}$.

The structure of Eqs (14)–(16), (25), (28) and (29) suggests that it is convenient to use the functions

$$I_1(M^2) = I^R_{1,\mathrm{vac}}(M^2) + I^R_{1,\mathrm{mat}}(M^2) \tag{40}$$

and

$$I_2(M^2,\omega^2,q^2) = I^R_{2,\mathrm{vac}}(M^2,q^2-\omega^2) + I^R_{2,\mathrm{mat}}(M^2,\omega^2,q^2). \tag{41}$$

4. Gap equation and dynamic masses

Using Eqs (4), (12), (14), (25) and (40) one finds the following form of the gap equation

$$M = m + MG_S I_1(M^2). \tag{42}$$

If the current quark mass is zero Eq. (42) has always a trivial solution ($M = 0$). The existence of the non-trivial solution ($M > 0$) depends on the choice of the parameters G_S and Λ_i ($i = 1,..,N$), and also on the temperature. At $T = 0$ one usually fits G_S and Λ_i in such a way as to ensure the existence of a solution with $M > 0$, which minimizes the free energy. Nevertheless, when the temperature exceeds some critical value T_c the trivial solution to the gap equation is stable. This means that the system passes the phase transition as the temperature is increased above T_c. If $m > 0$ and $T > 0$ we can always find the solution of the gap equation satisfying the condition $m < M < \Lambda_1$. However with increasing temperature $M \to m$. This property of the solution can be seen from equation (42) because for $T \to \infty$ we have $I_1(M^2) \to 0$.

The dynamical masses of the pion and the sigma are obtained from Eq. (7). Assuming that the gap equation has a non-trivial solution, we find

$$\frac{m}{M} + m^2_{\mathrm{dyn},\pi} I_2(M^2, m^2_{\mathrm{dyn},\pi}, 0) = 0 \tag{43}$$

and

$$\frac{m}{M} + (m^2_{\mathrm{dyn},\sigma} - 4M^2) I_2(M^2, m^2_{\mathrm{dyn},\sigma}, 0) = 0. \tag{44}$$

For vanishing current quark mass, the pion is massless (it is the true Goldstone boson) and the mass of sigma is simply $2M$. Eqs (43) and (44) are correct not only in vacuum but also at finite temperature, as long as a non-trivial solution to the gap equation exists.

If the current quark mass does not vanish, the pion becomes a massive particle. Nevertheless, for small current masses it is still a light particle. In the case of the sigma meson we expect that a non-zero current quark mass will cause an increase of its mass to the value larger than $2M$. However, as it was discussed in the previous Section, the correlation functions have cuts for arguments larger than $2M$. In consequence there is no isolated pole which can be identified with the mass of the sigma. One way of avoiding this difficulty is to define the mass by taking the real part of Eq. (7), *i.e.*,

$$1 - G_S \,\mathrm{Re}\, \chi^{(0)}_{AA}(m^2_{\mathrm{dyn}}, 0) = 0 \,. \tag{45}$$

At finite temperature, we have to use Eq. (45) also in the pseudoscalar channel. With the increasing temperature the constituent quark mass drops down and the pion mass increases. Consequently, at some stage the well isolated pion pole is shifted into the cut.

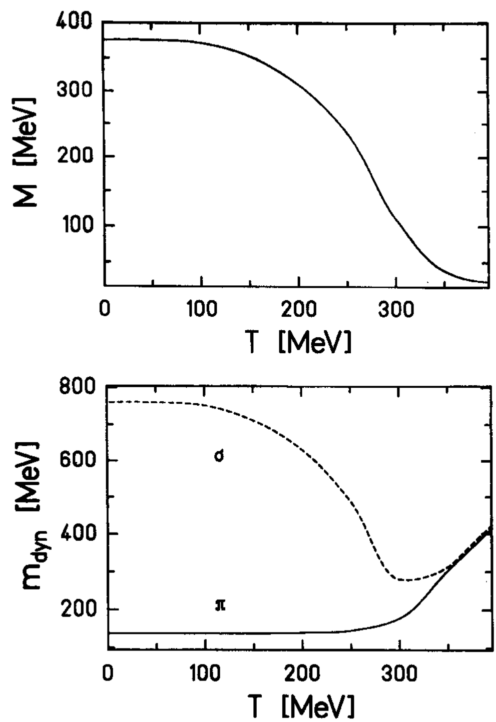

Fig. 1. The temperature dependence of (a) the constituent quark mass M and (b) of the dynamic meson masses: $m_{\mathrm{dyn},\pi}$ (solid line) and $m_{\mathrm{dyn},\sigma}$ (dashed line). The calculation is done with $N = 3$ subtractions. The regulating masses are: $\Lambda_1 = 680$ MeV, $\Lambda_2 = 2.1\Lambda_1$ and $\Lambda_3 = 2.1\Lambda_2$. The coupling constant $G_S = 0.75$ fm^2 and the current quark mass $m = 8.56$ MeV.

Our results concerning the temperature dependence of the constituent quark mass and the dynamic masses of mesons are shown in Figs 1a, 1b.

In our calculations we made three subtractions with the regulating masses: $\Lambda_1 = 680$ MeV, $\Lambda_2 = 2.1\Lambda_1$ and $\Lambda_3 = 2.1\Lambda_2$. Using three subtractions improves the convergence of the correlation functions in infinity. This is, from the numerical point of view, convenient in the calculations of the screening masses. The coupling constant $G_S = 0.75\,\mathrm{fm}^2$ and the current quark mass $m = 8.56$ MeV. Using these values of the parameters we find that at zero temperature the constituent quark mass $M = 376$ MeV, the pion mass $m_{\mathrm{dyn},\pi} = 138$ MeV and the sigma mass $m_{\mathrm{dyn},\sigma} = 760$ MeV. We note that the pion and sigma masses are connected by the relation

$$m^2_{\mathrm{dyn},\sigma} \approx 4M^2 + m^2_{\mathrm{dyn},\pi}\,. \tag{46}$$

With the increasing temperature the constituent quark mass remains almost constant, for $0 < T < 100$ MeV, and later drops smoothly down. The pion mass remains constant as long as the temperature is smaller than 250 MeV. Afterwards it increases suddenly. The temperature dependence of the sigma mass is rather complicated. In the interval $0 < T < 250$ MeV it behaves in the similar way as the constituent quark mass. Moreover, in this region the relation (46) is still fulfilled. When the temperature reaches 300 MeV then the sigma mass stops decreasing, remains for a while constant and later increases. At very high temperature the sigma mass and the pion mass are with a good approximation the same. This fact signals the restoration of the chiral symmetry.

One of the attractive features of the NJL model is that it yields the Gell-Mann–Oakes–Renner relation

$$f_\pi^2 m^2_{\mathrm{dyn},\pi} = -\frac{1}{2}(m_u + m_d)\langle \overline{u}u + \overline{d}d\rangle = -2m\langle\overline{q}q\rangle\,, \tag{47}$$

where f_π is the pion decay constant. This relation is practically independent of the regularization scheme. It can be used to find the value of f_π provided the values of m, m_π and $\langle\overline{q}q\rangle$ are known. In our case we find that $f_\pi = 94$ MeV.

5. Screening masses

In this Section we shall discuss our main results concerning the temperature dependence of the screening masses. The Fourier transform (9) used to calculate the screening mass can be represented as a sum of different contributions related to the singularities of the correlation function. Let us therefore summarize our knowledge of the analytic structure of this function.

In Section 3 we found that $\chi^{(0)}_{\mathrm{vac}}(-q^2)$ has cuts (in the following we shall call them *vacuum cuts*) along the imaginary axis starting at $q = \pm 2iM$ up to

$\pm i\infty$. On the other hand the function $\chi^{(0)}_{\rm mat}(0, q^2)$ has cuts (in the following *thermal cuts*) which run parallel to the real axis. Consequently, the full correlation function defined by Eq. (6) has all these cuts and, moreover, it can have poles for imaginary arguments in between the cuts. The pole exists only in the pseudoscalar channel. In the scalar channel it is shifted into the vacuum cut. This situation is shown in Fig. 2. Deforming the contour of integration in the upper half plane we can see that there will be, in general, three contributions to the Fourier integral: from the vacuum cut, the thermal cut and from the pole.

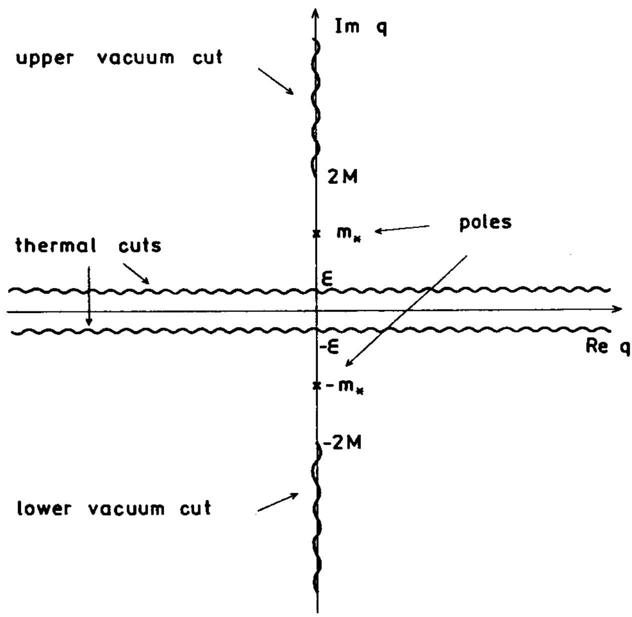

Fig. 2. A general analytic structure of the correlation function in the complex three-momentum plane $q = |\vec{q}\,|$.

The position of the pole is given by an equation of a form analogous to that of Eq. (7), namely

$$1 - G_S \chi^{(0)}_{AA}(0, -m_*^2) = 0\,. \tag{48}$$

One should consider only the interval $0 < m_* < 2M$ because for $m_* = 2M$ the cut starts. At zero temperature we have $m_* = m_{\rm dyn}$ but when $T > 0$ then $m_* \neq m_{\rm dyn}$ because the functions $I_{2,\,\rm mat}(M^2,\ k^2,\ 0)$ and $I_{2,\,\rm mat}(M^2,\ 0,\ -k^2)$ are different; see Eqs (31) and (32). The contribution to the Fourier transform from such a pole is, of course, of the form $\sim \exp(-m_* r)$. The integral around the vacuum cut can be written as follows

$$\chi_{AA}^{VC}(r) = -\frac{1}{4\pi^2 ir}\int_{2M}^{\infty} dk\, k\left[\chi_{AA}(0,-k^2+i\varepsilon) - \chi_{AA}(0,-k^2-i\varepsilon)\right] e^{-kr}. \tag{49}$$

It is easily seen [27] that for large distances this function behaves like $\sim \exp(-2Mr)$. This means that the contribution of the vacuum cut contribution is similar in character as that of the pole. We shall now discuss separately three different cases. The first two concern analytic results for $T = 0$ and $T \to \infty$. The third one concerns intermediate temperatures for which we have numerical calculations.

(i) The case $T = 0$

At zero temperature the thermal cut vanishes so we are left only with one or two contributions. In the case of the pseudoscalar channel we have an isolated pole and a cut. This pole, because of the Lorentz invariance, coincides with that determining the dynamic mass. For large values of r the contribution from the pole, $\sim \exp(-m_{\mathrm{dyn},\pi} r)$, is larger than that from the vacuum cut, $\sim \exp(-2Mr)$, since $m_{\mathrm{dyn},\pi} < 2M$. Therefore the pion screening mass at $T = 0$ is equal to the dynamic one: $m_{\mathrm{scr},\pi} = m_{\mathrm{dyn},\pi} = 138$ MeV. In the case of the scalar channel we do not have an isolated pole and the only contribution to the Fourier transform is due to the vacuum cut. Consequently, the sigma screening mass equals $2M$ and is smaller than the dynamic one; $m_{\mathrm{scr},\sigma} = 752$ MeV and $m_{\mathrm{dyn},\sigma} = 760$ MeV.

(ii) The case of extremely high temperature

In the limit $T \to \infty$ the generators of the correlation functions $\chi_{AA}^{(0)}$ vanish. It means that at sufficiently high temperature the generators are very small and, at the same time, they are a good approximation for the full correlation functions. In this case, the second term in the denominator of Eq. (6), $G_S\,\chi_{AA}^{(0)}(0,q^2)$ can be neglected compared to unity.

Using this fact we define the high temperature correlation function in space by equation

$$\chi_{AA}^{(0)}(r) = \frac{1}{4\pi^2 ir}\int_{-\infty}^{\infty} dq\, q\, \chi_{AA}^{(0)}(0,q^2) e^{iqr}. \tag{50}$$

(We hope that the notation used here is not very much misleading and that the reader will connect the index (0) with the "free theory" rather than with the case $T = 0$.) In Eq. (50) integration of the vacuum part $\chi^{(0)}_{AA,\mathrm{vac}}(-q^2)$ and the matter part $\chi^{(0)}_{AA,\mathrm{mat}}(0, q^2)$ defines the functions $\chi^{(0)}_{AA,\mathrm{vac}}(r)$ and $\chi^{(0)}_{AA,\mathrm{mat}}(r)$, respectively. Of course the simple relation holds

$$\chi^{(0)}_{AA}(r) = \chi^{(0)}_{AA,\mathrm{vac}}(r) + \chi^{(0)}_{AA,\mathrm{mat}}(r) \,. \tag{51}$$

The Fourier transform (50) can be calculated analytically. Using the same method as that developed by us in [26], we find that

$$\chi^{(0)}_{PP,\mathrm{vac}}(r) = \frac{N_c}{2\pi^3 r^3} \sum_{i=0}^{N} A_i \Lambda_i^2 \left[3K_2(2\Lambda_i r) + 2\Lambda_i r K_1(2\Lambda_i r)\right] . \tag{52}$$

where K_1 and K_2 are the modified Bessel functions [28]. On the other hand the matter piece has the form

$$\chi^{(0)}_{PP,\mathrm{mat}}(r) = \frac{N_c}{4\pi^3 r} \left(\frac{2}{r^3} - \frac{2}{r^2}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial r^2} \right) \left[G_1(r) + G_2(r)\right] , \tag{53}$$

where

$$G_1(r) = -\sum_{i=0}^{N} A_i \, \Lambda_i K_1(2\Lambda_i r) \tag{54}$$

and

$$G_2(r) = \pi T \sum_{i=0}^{N} A_i \sum_{l=-\infty}^{+\infty} \exp(-2r\sqrt{(2l+1)^2\pi^2T^2 + \Lambda_i^2}) \,. \tag{55}$$

Using the properties of the modified Bessel functions [29] we can check that in expression (51) the vacuum part (52) is exactly canceled by the first term of the matter part (53), *i.e.* by the term coming from differentiation of the function $G_1(r)$. We want to emphasize that this cancellation is independent of the number of subtractions N and of the values of the regulating masses Λ_i.

The function $G_2(r)$ has a simple asymptotic behaviour. For $r \to \infty$ it is enough to consider only the leading terms in the series, *i.e.* these corresponding to $l = 0, -1$ and $i = 0$. The terms for $i > 0$ are suppressed because, in this case, $\Lambda_i > \Lambda_0 = M$. Additionally we can neglect M with respect to T. Consequently, we obtain the following asymptotic expressions for the correlation function

$$\chi_{PP}(r) \sim \frac{2T^3 N_c}{r^2} e^{-2\pi T r} \,. \tag{56}$$

From Eq. (56) we can easily read off the screening mass in the pseudoscalar channel to be $2\pi T$. The calculation for the scalar channel is completely analogous, with the same result for the screening mass. At very high temperature the constituent quark mass drops down and is equal to the current one. The latter, however, is very small, so that the difference between the pseudoscalar and scalar channel, practically, disappears.

These considerations show how important the regularization of the matter part is. It leads to cancellations between the vacuum and matter parts and, finally, to the asymptotic behaviour of the form $\exp(-2\pi T r)$. If the matter part were not regularized then the asymptotic behaviour of the correlation function would be governed by the lowest regulating mass, *i.e.* by Λ_1. Such a case could not be accepted from the physical point of view.

(iii) Intermediate temperatures

For the temperature range $0 \leq T \leq 400$ MeV we did numerical calculations and read off the values of the screening masses from the logarithmic plots representing the correlation functions in space. We did the calculations in two ways checking whether the results were the same. The first method was to calculate the Fourier transform directly from Eq. (9). The second one was to calculate the contributions from the singularities separately and later to sum them up. Both methods encounter numerical difficulties. The direct calculation of the Fourier transform requires an integration of a slowly converging and oscillating function. On the other hand, the contributions from the cuts show large cancellations. Therefore, each contribution should be evaluated with very high accuracy. By the way, the appearance of these cancellations are expected, since from our analytic considerations we have learnt that such cancellations should really take place. Let us also note here that the parameters used to calculate the screening masses were the same as those used in the case of the dynamic ones.

We had to restrict ourselves to rather small values of r because the correlation functions decrease very rapidly and for large r their numerical evaluation becomes more and more difficult. The fact that they decrease exponentially suggests that the screening masses can be read off already for small r's. The exponential behaviour is shown in Figs. 3a and 3b, where we plotted the functions $r\chi^{(0)}_{PP}(r)$ and $r\chi^{(0)}_{SS}(r)$.

At zero temperature our numerical procedure gives the results which are in rather good agreement with what one expects from the analytic considerations; the numerically calculated pion screening mass agrees exactly with the analytic result whereas in the sigma channel we find a small discrepancy. This may be caused by the fact that we consider only small distances.

The temperature dependence of the screening and dynamic masses is shown in Fig. 4. For $0 \leq T \leq 200$ MeV we observe that the screening

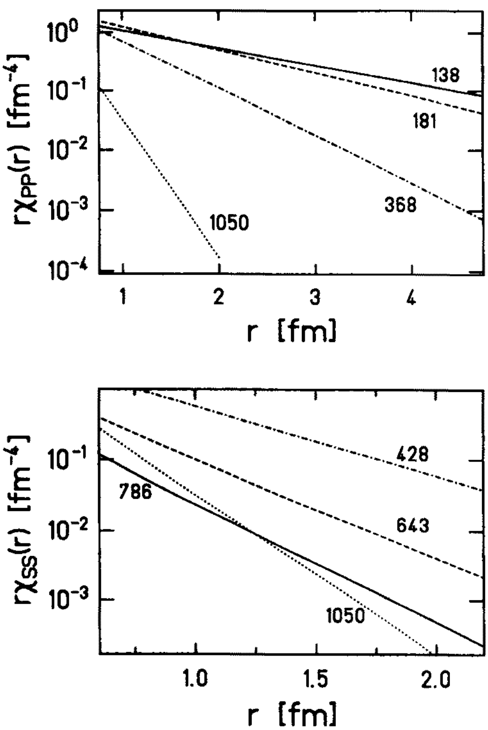

Fig. 3. The spatial dependence of the static correlation functions in the (a) pion and (b) sigma channels. The solid, dashed, pointed-dashed, and pointed lines correspond to the temperatures T =0, 200, 300 and 400 MeV, respectively. The numbers at the lines are the values (in MeV) of the screening masses.

masses are close to the dynamic ones. This is what one could expect because for small temperatures the Lorentz invariance is only slightly broken. Interesting things can be observed for larger temperatures. Both screening and dynamic masses exhibit the restoration of chiral symmetry although they substantially differ from each other. In the full interval, $0 \leq T \leq 400$ MeV, the qualitative behaviour of the screening masses and of the dynamic ones is similar and resembles the results of the lattice simulations of QCD.

Our observation that the dynamic masses are different from the screening ones contradicts the results of Ref. [10], where it was argued that they are the same even at finite T. In Ref. [10] the chiral limit, $m = 0$, is considered. Afterwards, the gap equation is used to find that the two masses are equal. The point is, however, that in Ref. [10] the non-trivial solution to the gap equation is used. It exists only if $T < T_c$ (in the chiral limit T_c is well defined) and for $T > T_c$ we have to take into account just the trivial solution $M = m = 0$. The trivial solution does not allow us to simplify the

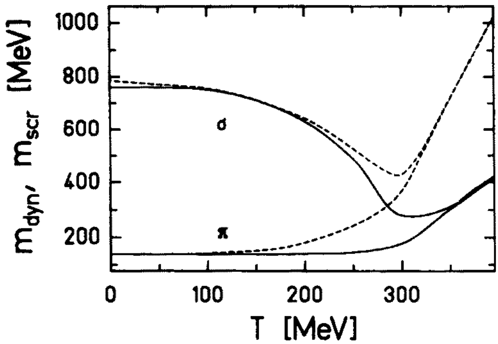

Fig. 4. The temperature dependence of the screening (dashed lines) and dynamic (solid lines) masses. The parameters as in Fig. 1.

denominator of the correlation function, what is essential for the proof of the equality of two masses in [10]. Consequently, the arguments of Ref. [10] are for $T > T_c$ not convincing and we cannot expect that $m_{scr} = m_{dyn}$. The problem remains what happens at $T < T_c$. The structure of the poles suggests that $m_{scr} = m_{dyn}$, but there exists a thermal cut whose contribution might be not negligible. We are, therefore, of the opinion that this situation deserves a separate study.

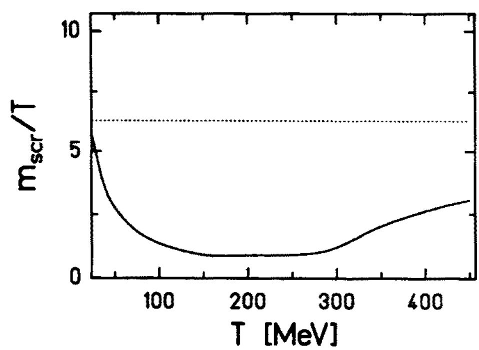

Fig. 5. The ratio $m_{scr,\pi}/T$ plotted as a function of the temperature. The pointed line corresponds to the Eletskii–Ioffe limit: $m_{scr} = 2\pi T$.

Another interesting point is to check whether the screening masses approach the Eletskii–Ioffe limit, *i.e.* if they are equal to $2\pi T$ for large T. Our numerical results for the pseudoscalar channel are shown in Fig. 5. One can see that the ratio $m_{scr,\pi}/T$ grows with the temperature for $T > T_c$ but even at $T = 400$ MeV it is still smaller than 2π. (In the scalar channel the high temperature behaviour is the same as in the pseudoscalar one.) As we have already mentioned this type of behaviour is observed in the lattice simulations. In the (pseudo)scalar channel the screening masses are smaller than $2\pi T$ what is in contrast to the (axial)vector channel where they are equal to $2\pi T$ already for the temperatures slightly exceeding T_c.

Our result suggests that the NJL model can explain the existence of the non-negligible (residual) interaction in the (pseudo)scalar channel. Nevertheless, before drawing any definite conclusions the calculations in the (axial)vector channel should be done. If the difference between the behaviour of the screening masses in the (pseudo)scalar channel and the (axial)vector one were found then it would act much more in favour of the model. Work in this direction is in progress.

6. Summary and conclusions

In this paper we studied static meson correlation functions in the NJL model. We did numerical calculations and, in the special cases, found approximate analytic results. We were mainly concentrated on the calculation of the meson screening masses, *i.e.* quantities which characterize the range of the correlation functions. Our general conclusion is that the results obtained in this model are in overall good agreement with those coming from lattice simulations of QCD.

We introduced a regularization scheme which is a modification of the Pauli–Villars method. It protects chiral symmetry, guaranties the appropriate convergence of the correlation functions and allows us to do analytic calculations. The possibility of doing analytic calculations is very important because the asymptotics of the correlation function is connected directly with its analytic structure.

In the upper half-plane of the complex three-momentum the static correlation function has two cuts (scalar channel) or two cuts and a pole (pseudoscalar channel). The Fourier transform defining the spatial static correlation function can be represented as a sum of different contributions related to these cuts and the pole.

At zero temperature one of the discussed cuts is absent. The pion screening mass is equal to the dynamic one. They are both determined by the position of the pole because the contribution from the second cut is, at large distances, negligible. On the other hand, the sigma screening mass is slightly different from the dynamic one. This difference is caused by the unphysical possibility of the decay of the sigma meson into a quark-antiquark pair. Mathematically, this fact is reflected by the absence of the isolated pole in this channel. The sigma screening mass is determined by the position of the cut whereas the dynamic mass is defined by the zero of the real part of the denominator of the correlation function.

At sufficiently high temperature the pion pole disappears and we have to deal, in both channels, with two cuts. The contributions from these cuts show large cancellations which lead to the exponential decay of the correlation functions. In the limit $T \to \infty$ we find that the screening masses are equal to $2\pi T$ independently of the channel. This result is characteristic of a gas of non-interacting quarks. At large but finite temperatures we observe, however, differences between the actual values of the screening masses and the limiting case $2\pi T$. These facts are in general agreement with the lattice simulations of QCD.

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