QUANTUM MECHANICS OF SPACETIME SIGNATURE*,**

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It is suggested that the Wick rotation angle is a genuine dynamical degree of freedom, rather than just a technical device needed to improve the convergence of functional integrals. The one-loop effective potential $V(\theta)$ of the Wick angle θ is calculated, using heat-kernel regularization. It is found that when the number of fermionic degrees of freedom exceeds the number of bosonic degrees of freedom, the real part of $V(\theta)$ is minimized, and the imaginary part is stationary, uniquely in $D = 4$ dimensions, at the value $\theta = \pi$, corresponding to Lorentzian signature.

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We usually take for granted the fact that spacetime is Lorentzian, *i.e.* locally Minkowski rather than, say, Euclidean, in a suitable reference frame. In this talk I will argue that there may be a dynamical reason why spacetime signature is Lorentzian in D=4 dimensions. My discussion will be based on work reported in Refs [1, 2]; the second reference written in collaboration with Alberto Carlini.

The word "signature" refers to real, symmetric matrices G, which can always be expressed in the form

$$G = EDE^T , \tag{1}$$

where

$$D = \text{diag}[-1, -1, \ldots, -1, +1, +1, \ldots, +1, 0, 0, \ldots 0] . \tag{2}$$

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The (unique) diagonal entries of the matrix D are known as the signature of the matrix G. In general relativity, the metric tensor $g_{\mu\nu}$ of a D-dimensional manifold is a real symmetric matrix, and can therefore be written

$$\begin{aligned} g_{\mu\nu} &= e^a_\mu \eta_{ab} e^b_\nu \,, \\ \eta &= \text{diag}[-1,-1,\ldots,-1,+1,+1,\ldots,+1]\,, \end{aligned} \tag{3}$$

with zeros in the signature excluded if $g_{\mu\nu}$ is non-degenerate. Two important special cases are

$$\eta = \text{diag}[+1,+1,\ldots,+1]\,, \tag{4}$$

known as Euclidean signature, and

$$\eta = \text{diag}[-1,+1,\ldots,+1]\,, \tag{5}$$

which is Lorentzian signature.

Classical relativity theory gives no clue as to why spacetime is Lorentzian, rather than having some other signature. The Einstein field equations

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}\,, \tag{6}$$

actually make no reference at all to the spacetime signature. There are Euclidean solutions to these equations and there are Lorentzian solutions; there are even solutions, found in Ref. [3], which are Euclidean on part of the manifold, and Lorentzian on the rest. The simplest of these "mixed" solutions is for pure gravity with a cosmological constant $\lambda = 3H^2/\kappa$. Then the Robertson-Walker metric

$$ds^2 = N(t)dt^2 + R^2(t)\left(\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)\,, \tag{7}$$

with

$$\begin{aligned} -\frac{\pi}{2H} < t < 0: &\quad N = +1 \quad R(t) = \frac{\cos(Ht)}{H} \quad \text{Euclidean}\,, \\ t > 0: &\quad N = -1 \quad R(t) = \frac{\cosh(Ht)}{H} \quad \text{Lorentzian}\,, \end{aligned} \tag{8}$$

is a solution of Einstein's equations, which can be visualized as joining a section of a Euclidean hypersphere to an expanding (Lorentzian) DeSitter space.

If classical physics doesn't provide an explanation for Lorentzian signature, then the explanation (if there is one) must be found in quantum phenomena. Let us consider, in the spirit of standard quantum field theory and statistical mechanics, the possibility that Lorentzian signature is somehow selected by the vacuum state of quantized fields. More precisely, suppose we consider η_{ab} as a dynamical variable. Then in principle, by integrating out all other degrees of freedom, we could calculate an effective potential for the signature $V_{\text{eff}}(\eta)$, and look for the minima/stationary points of this potential, which would dynamically determine the signature of the vacuum state. To carry out this program in practice, we must first decide on the range of the variables

$$\eta = \text{diag}[\sigma_1, \sigma_2, \ldots, \sigma_D]\,. \tag{9}$$

These might be taken as Ising-like variables, *i.e.* $\sigma_n = \pm 1$, but that choice restricts signature fluctuations, except in some special cases, to pathologically discontinuous metrics. Nevertheless, one expects $|\sigma_n| = 1$, since any variation in the modulus of σ_n can be absorbed in the vielbeins e^a_μ. The modulus of σ_n must therefore be fixed, for otherwise we are overcounting degrees of freedom. Thus, if we wish to consider tangent-space metrics η_{ab} which interpolate continuously between, *e.g.*, Euclidean and Lorentzian signature, it is necessary to consider complex entries of unit modulus $\sigma_n = \exp[i\theta_n]$. In fact, a special case of such a metric is a well established tool of quantum field theory, where it goes under the name of Wick rotation.

The problem in relativistic quantum field theory is to calculate Feynman path integrals in Minkowski space of the form

$$Z_F = \int d\mu(e, \phi, \psi, \overline{\psi})\ \exp\left(-i \int d^D x \sqrt{-g}\mathcal{L}\right)\,, \tag{10}$$

where $d\mu(e, \phi, \psi, \overline{\psi})$ is the integration measure for the tetrads, and other bosonic (ϕ) and fermionic ($\psi, \overline{\psi}$) fields. The restriction to Lorentzian spacetime is enforced by working with a fixed signature

$$\begin{aligned} g_{\mu\nu} &= e^a_\mu \eta_{ab} e^b_\nu\,, \\ \eta_{ab} &= \text{diag}[-1, 1, \ldots, 1]\,, \end{aligned} \tag{11}$$

and in the case of a flat background, one simply sets $g_{\mu\nu} = \eta_{\mu\nu}$. To improve the convergence of the functional integration, it is useful to make the rotation $t \to it$ to get the Euclidean path integral

$$Z_{\text{E}} = \int d\mu(e, \phi, \psi, \overline{\psi}) \exp\left(- \int d^D x \sqrt{g}\mathcal{L}\right)\,, \tag{12}$$

where

$$\eta_{ab} = \text{diag}[1, 1, \ldots, 1]\,. \tag{13}$$

A path integral interpolating between the Minkowski and Euclidean forms is simply

$$Z = \int d\mu(e, \phi, \psi, \overline{\psi}) \exp\left(-\int d^D x \sqrt{g}\mathcal{L}\right) , \tag{14}$$

where

$$\eta_{ab} = \mathrm{diag}[e^{i\theta}, 1, \ldots, 1] . \tag{15}$$

In the interpolating path-integral, the limit $\theta \to \pi$ from below gives the correct Feynman prescription for propagators in Minkowski space, which are ill-defined at $\theta = \pi$ exactly. The value $\theta = 0$ gives the Euclidean theory, and is a particularly useful choice for Monte Carlo enthusiasts. Finally, the range $\theta > \pi$ (or $\theta < -\pi$) must be excluded, since the real part of the kinetic term in scalar field theories goes like

$$\begin{aligned} \mathrm{Re}[\sqrt{g}\mathcal{L}] &= \mathrm{Re}\left(e^{-i\theta/2}(\partial_0\phi)^2 + e^{i\theta/2}(\nabla\phi)^2\right) \\ &= \cos\left(\frac{\theta}{2}\right)[(\partial_0\phi)^2 + (\nabla\phi)^2] , \end{aligned} \tag{16}$$

which is unbounded from below for $|\theta| > \pi$.

Wick rotation ($\theta = \pi \to \theta = 0$) is usually thought of as just a technical trick to improve convergence of the Feynman path-integral, and recover the correct propagator prescription. Instead, we propose to treat θ in the range $[-\pi, \pi]$ as a genuine dynamical variable, and will compute the one-loop contribution to the effective potential $V(\theta)$ due to massless bosons and fermions.

Allowing θ to fluctuate calls for some generalization of ordinary quantum mechanics. Begin from the usual path-integral definition of transition amplitudes

$$G[\phi_f, t_f|\phi_i, t_i] \equiv \int_{\phi_i}^{\phi_f} d\mu(\phi) \exp\left(-\int_{t_i}^{t_f} dt \int d^{D-1}x\sqrt{g}\mathcal{L}\right) , \tag{17}$$

which leads to the generalized Schrödinger equation

$$\partial_t \Psi[\phi] = -e^{i\theta/2} H \Psi[\phi] , \tag{18}$$

where H is the standard (and hermitian) Hamiltonian. For any $\theta \neq \pm\pi$ the norm of Ψ can change. Therefore, to conserve probability, we must define

$$\langle Q \rangle \equiv \frac{\langle \Psi | Q | \Psi \rangle}{\langle \Psi | \Psi \rangle} , \tag{19}$$

so that

$$\partial_t\langle Q\rangle = \sin\frac{\theta}{2}\langle i[H,Q]\rangle \\ - \cos\frac{\theta}{2}\{\langle HQ + QH\rangle - 2\langle Q\rangle\langle H\rangle\}. \quad (20)$$

Thus for Q hermitian, $\langle Q\rangle$ is real, and probability is conserved. On the other hand

$$\partial_t\langle H\rangle = -2\cos\frac{\theta}{2}\langle(H - \langle H\rangle)^2\rangle, \quad (21)$$

which means that conservation of energy (along with Lorentz invariance), is violated for any non-stationary state, at $\theta \neq \pm\pi$. In general, for non-Lorentzian spacetime, an arbitrary initial state will relax to the lowest energy eigenstate that has a non-vanishing overlap with the initial state. This means that any theory of dynamical signature faces a very strong challenge: not only must $\theta = \pm\pi$ come out as the most probable signature, but also it must be shown that fluctuations away from Lorentzian signature are enormously suppressed.

In order to compute the most probable θ, we need to find the effective potential $V(\theta)$ obtained after integrating out all other fields. The integration measure, as well as the Lagrangian, may be θ-dependent. I will fix the θ-dependence of the measure by assuming:

1. For free fields of mass m, the contributions to Z in Eq. (14) from each (propagating) bosonic degree of freedom are equal, and inverse to the contributions from each fermionic degree of freedom. Thus, *e.g.*, $Z = 1$ at any θ for a supersymmetric combination of free fields.
2. The integration measure for scalar fields is given by the real-valued, invariant volume measure (DeWitt measure) in superspace $d\mu(\phi) = D\phi\sqrt{|G|}$, where G is the determinant of the scalar field supermetric $G(x,y) = \sqrt{g}\delta(x-y)$.

Under these assumptions, the one-loop contribution $V_S(\theta)$ to the effective potential $V_{\text{eff}}(\theta)$, due to a massless scalar field propagating in a flat ($g_{\mu\nu} = \eta_{\mu\nu}$) background is

$$\exp\left(-\int d^Dx V_S(\theta)\right) = \int D\phi \exp\left(-\int d^Dx\sqrt{\eta}\eta^{ab}\partial_a\phi\partial_b\phi\right) \\ = \det{}^{-1/2}[-\sqrt{\eta}\eta^{ab}\partial_a\partial_b]. \quad (22)$$

Heat kernel regulation of an operator M is given by

$$\det{}^{-1}(M) = \exp\left(\int_\epsilon^\infty \frac{ds}{s}\,\mathrm{Tr}\,e^{-sM}\right), \quad (23)$$

so that

$$\begin{aligned}
&\det{}^{-1/2}[-\sqrt{\eta}\eta^{ab}\partial_a\partial_b] \\
&= \exp\left(\frac{1}{2}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\int\frac{d^Dp}{(2\pi)^D}\exp\left[-s(e^{-i\theta/2}p_0^2+e^{i\theta/2}\vec{p}^{\,2})\right]\right) \\
&= \exp\left(\int d^Dx\frac{\Lambda^D}{D(4\pi)^{D/2}}\exp\left[-i(D-2)\frac{\theta}{4}\right]\right) \\
&= \exp\left(-\int d^Dx V_S(\theta)\right) ,
\end{aligned} \tag{24}$$

or

$$V_S(\theta) = -\frac{\Lambda^D}{D(4\pi)^{D/2}}\exp\left(-i(D-2)\frac{\theta}{4}\right) , \tag{25}$$

where Λ is a high-momentum cutoff which, given the non-renormalizability of gravity, is taken to exist at the Planck scale.

Then, for a set of massless fields with n_B bosonic and n_F fermionic physical degrees of freedom, the one-loop contribution is

$$V(\theta) = (n_F - n_B)\frac{\Lambda^D}{D(4\pi)^{D/2}}\exp\left(-i(D-2)\frac{\theta}{4}\right) . \tag{26}$$

This potential is complex. We therefore look for a value of θ in the range $\theta \in [-\pi,\pi]$ such that Im$[V]$ is stationary, and Re$[V]$ is a minimum. This requires simultaneously satisfying

$$\left.\begin{array}{r}\cos\left((D-2)\frac{\theta}{4}\right) = 0 \\ \min\left[\mathrm{Re}[V(\theta)]\right] = 0\end{array}\right\} \qquad \theta \in [-\pi,\pi] . \tag{27}$$

It is easy to show that these conditions can only be solved, for massless fields, in D=4 dimensions, at $\theta = \pm\pi$. There are five cases to consider:

(i) $n_F < n_B$. Then $\min\mathrm{Re}[V] < 0 \to$ no solution.

(ii) $n_F = n_B$ or $D = 2$. Then $V(\theta)$ is independent of θ, and no θ is preferred.

(iii) $n_F > n_B$ and $\frac{D-2}{4}\pi < \frac{\pi}{2}$. Then $\min\mathrm{Re}[V] > 0 \to$ no solution.

(iv) $n_F > n_B$ and $\frac{D-2}{4}\pi > \frac{\pi}{2}$. Then $\min\mathrm{Re}[V] < 0 \to$ no solution.

(v) $n_F > n_B$ and $\frac{D-2}{4}\pi = \frac{\pi}{2}$. In this case, both conditions are satisfied at $\theta = \pm\pi$, which corresponds to Lorentzian signature. The equality $\frac{D-2}{4}\pi = \frac{\pi}{2}$ can, of course, only be achieved for a spacetime dimensionality $D = 4$.

Case (v) is the unique solution; we have therefore found an interesting connection between the Lorenztian signature of spacetime, and the $D = 4$ dimensionality of spacetime. The next question is how far θ can fluctuate away from $|\theta| = \pi$. Also, we would like to know if a more generalized form of the signature

$$\eta_{ab} = \mathrm{diag}[e^{i\theta_1}, e^{i\theta_2}, \ldots, e^{i\theta_D}], \tag{28}$$

should be considered. To answer this, we must first consider whether there are any restrictions which should be imposed, for the sake of consistency, on $\theta(x)$ in curved spacetime.

There are three requirements which I believe are reasonable:

1. For the metric $g_{\mu\nu} = \eta_{\mu\nu}$, spacetime is flat in the sense that the Riemann tensor vanishes;
2. The number of gravitational degrees of freedom (= inequivalent vielbein degrees of freedom) is independent of the Wick angle;
3. Covariant derivatives of spinors have appropriate properties; *e.g.* Dirac gamma matrices are covariantly constant.

It will be shown that all three of these requirements are satisfied by the following conditions:

$$\begin{aligned} \theta &= \theta(T(x)), \\ e^0_\mu &= \partial_\mu T(x). \end{aligned} \tag{29}$$

Let us begin with the first requirement. In cartesian coordinates, the Riemann tensor vanishes for $g_{\mu\nu} = \eta_{\mu\nu}$ if $\theta = \theta(t)$; *i.e.* θ depends only on a (preferred) time coordinate. A preferred time direction is, of course, quite a violation of Lorentz invariance, but remember that this symmetry is violated anyway, for any $\theta \neq 0, \pm\pi$ (the goal is to show that the symmetry arises dynamically, through a preferred value of θ). The obvious generalization of $\theta = \theta(t)$ to curved space is to say that the gradient of θ is parallel to e^0_μ, *i.e.* $\partial_\mu\theta = f(x)e^0_\mu$, but then consistency requires that

$$0 = (\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\theta = \partial_\mu(fe^0_\nu) - \partial_\nu(fe^0_\mu), \tag{30}$$

which is easily seen to be fulfilled by the conditions (29).

Turning next to the second requirement, we note that in ordinary Lorentzian or Euclidean spacetimes, $g_{\mu\nu}$ is real and symmetric, which means that the metric has $D(D+1)/2$ independent components (of course we can subtract $2D$ degrees of freedom, due to diffeomorphism invariance, to get the actual number of physical degrees of freedom). On the other hand $g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu$, and the vielbeins e^a_μ have D^2 independent components. This mismatch in independent components between the metric and the vielbeins is resolved by the fact that there is a local Lorentz invariance which is $O(D-1,1)$ in Lorentzian spacetimes, or $O(D)$ in Euclidean spacetimes, and therefore we should subtract the dimension of the Lorentz group $(D(D-1)/2)$ from the D^2 vielbein components to get the actual number of degrees of freedom, up to diffeomorphisms. This gives $D(D+1)/2$ components, the same as for the metric.

However, for $\theta \neq 0, \pm\pi$, the local Lorentz invariance is only $O(D-1)$ and therefore, without further restrictions on the vielbeins, there would be more independent vielbein components in non-Lorentzian spacetimes than in Lorentzian spacetime. It is easily seen that the conditions (29) are just what is needed so that the number of independent components is θ-independent. The counting is: one component $(T(x))$ for e^0_μ, $D(D-1)$ components for the other vielbein components, minus the dimension of $O(D-1)$ which is $(D-1)(D-2)/2$. The number of independent components (before taking account of diffeomorphism invariance) again adds up to $D(D+1)/2$, which is our second reason for imposing (29).

The third argument for conditions (29) concerns fermionic actions in curved spacetime. For bosonic fields, the action in the generalized metric at arbitrary θ is obtained by using $g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu$ in the standard action. The corresponding construction, for Dirac fields, is to use

$$\{\gamma^a, \gamma^b\} = -2\eta^{ab}, \tag{31}$$

in the standard Dirac action

$$S_D = \int d^D x \sqrt{g}\,\overline{\psi}(-i\gamma^\mu D_\mu + m)\psi, \tag{32}$$

with

$$\begin{aligned}
\gamma^\mu &= e^\mu_a \gamma^a, \\
D_\mu &= \partial_\mu + \tfrac{1}{2}\sigma^{ab}\omega_{\mu ab}, \\
\sigma^{ab} &= \tfrac{1}{4}[\gamma^a, \gamma^b], \\
\omega_{\mu ab} &= e^\rho_a e_{b\rho;\mu},
\end{aligned} \tag{33}$$

where the γ^a satisfy (31). The local Lorentz invariance of S_D is only $O(D-1)$ at general θ, rather than $O(D)$ $(\theta = 0)$ or $O(D-1,1)$ $(|\theta| = \pi)$, but we can still ask whether the derivative D_μ defined in (33) has the usual property of commuting with the γ^μ, or whether the metric is covariantly constant w.r.t. D_μ. Introducing the "ict" notation

$$\begin{aligned}
g_{\mu\nu} &= \tilde{e}^a_\mu \tilde{e}^a_\nu, \\
\tilde{e}^a_\mu &= \begin{cases} e^{i\theta/2} e^0_\mu & (a = 0) \\[2ex] e^a_\mu & (a \neq 0) \end{cases}, \\
\{\gamma^a_E, \gamma^b_E\} &= -2\delta^{ab},
\end{aligned} \tag{34}$$

it is clear that the covariant derivative should have the property

$$\begin{aligned} 0 = D_\mu g_{\alpha\beta} &= D_\mu(\tilde{e}^a_\alpha \tilde{e}^a_\beta)\,, \\ &= (D_\mu \tilde{e}^a_\alpha)\tilde{e}^a_\beta + \tilde{e}^a_\alpha(D_\mu \tilde{e}^a_\beta)\,, \end{aligned} \tag{35}$$

which implies

$$D_\mu \tilde{e}^a_\nu = \tilde{e}^a_{\nu;\mu} + \tilde{\omega}_\mu{}^a{}_b \tilde{e}^b_\nu = 0\,. \tag{36}$$

So the spin-connection in "ict" notation must be

$$\tilde{\omega}_{\mu ab} = \tilde{e}^\rho_a \tilde{e}_{\rho b;\mu}\,, \tag{37}$$

with the corresponding covariant derivative for spinor fields

$$\begin{aligned} D^{\text{"ict"}}_\mu &= \partial_\mu + \tfrac{1}{2}\sigma^{ab}_E \tilde{\omega}_{\mu ab}\,, \\ \sigma^{ab}_E &\equiv \tfrac{1}{4}[\gamma^a_E, \gamma^b_E]\,. \end{aligned} \tag{38}$$

The question is whether D_μ defined in (33) and $D^{\text{"}ict\text{"}}_\mu$ are the same. These two derivatives are identical if

$$\sigma^{ab}\omega_{\mu ab} = \sigma^{ab}_E \tilde{\omega}_{\mu ab}\,, \tag{39}$$

which is easily shown to be a consequence of (29). The requirement that the derivative D_μ have the properties expected of a covariant derivative on spinor fields is thus the third reason for imposing the conditions (29).

We may now ask whether there can be more than one Wick angle in the generalized signature, *i.e.*

$$\eta_{ab} = \mathrm{diag}[\mathrm{e}^{i\theta_1}, \mathrm{e}^{i\theta_2}, \ldots, \mathrm{e}^{i\theta_D}]\,. \tag{40}$$

The requirements that spacetime is flat for $g_{\mu\nu} = \eta_{\mu\nu}$, and $D_\mu = D^{\text{"ict"}}_\mu$ for spinors, would lead to the condition that $\partial_\mu \theta_a \propto e^a_\mu$. But then the number of independent components of the vielbeins (modulo diffeomorphisms) would be less than $D(D+1)/2$, in violation of the requirement that the number of gravitational degrees of freedom is independent of Wick angle. We conclude that only one Wick angle is allowable.

Given the conditions (29), it is now possible to estimate the magnitude of quantum fluctuations $\delta\theta$ away from Lorentzian signature. Assuming a high-frequency cutoff at the Planck scale, the action for one Planck time is roughly

$$\Delta S \sim \Lambda^4 V l_P \delta\theta \sim \frac{V}{l_P^3}\delta\theta\,, \tag{41}$$

where l_P is the Planck-length, and V is the three-volume of the T = const. hypersurface. Therefore

$$\langle \delta\theta \rangle \sim \frac{l_P^3}{V} . \tag{42}$$

For a closed Universe of length scale on the order of 10^{10} light years, this gives an estimate

$$\delta\theta \sim 10^{-184} \text{ radians} , \tag{43}$$

which is surely unobservable. Of course, in the very early Universe, signature fluctuations could have been substantial.

Finally, we note that the calculation of $V(\theta)$ was carried out in a flat background $e^a_\mu = \delta^a_\mu$. However, $V(\pi)$ has the interpretation of an induced cosmological constant, whose magnitude is Planck-scale. This raises the question of how an expansion around flat-space can be justified, which is essentially the cosmological constant question. Possibly the cosmological constant is screened somehow at large distances [4], but here let us just consider the conventional approach of simply adding a counterterm

$$S_c = \int d^D x \sqrt{g} \lambda_c , \tag{44}$$

to remove the induced term. Then, writing

$$\lambda = (n_F - n_B) \frac{\Lambda^D}{D(4\pi)^{D/2}} , \tag{45}$$

the total effective potential to one-loop is

$$V_T(\theta) = \lambda_c e^{i\theta/2} + \lambda e^{-i(D-2)\theta/4} , \tag{46}$$

and it is impossible to set $V_T(\theta) = 0$ for *all* θ. Instead, for dynamical signature, the cancellation requirement is to choose λ_c such that $V_T = 0$ at the minimum/stationary point.

Denoting

$$\bar{\theta} \equiv \frac{D-2}{2} \theta , \tag{47}$$

the cancellation conditions at the minimum/stationary point are:

$$\text{Re}[V_T] = 0 : \qquad \lambda_c \cos\frac{\theta}{2} + \lambda \cos\frac{\bar{\theta}}{2} = 0 , \tag{48}$$

$$\text{Im}[V_T] = 0 : \qquad \lambda_c \sin\frac{\theta}{2} - \lambda \sin\frac{\bar{\theta}}{2} = 0 , \tag{49}$$

$$\frac{\partial \,\text{Im}[V_T]}{\partial\theta} = 0 : \qquad \lambda_c \cos\frac{\theta}{2} - \frac{D-2}{2} \lambda \cos\frac{\bar{\theta}}{2} = 0 , \tag{50}$$

$$\min \text{Re}[V_T] = 0 . \tag{51}$$

Conditions (48) and (50) give

$$\overline{\theta} = (2n+1)\pi\,, \qquad \theta = \pi\,, \tag{52}$$

while (49) implies

$$|\lambda_c| = \lambda\,. \tag{53}$$

Then condition (51) gives

$$\overline{\theta} = \pi\,. \tag{54}$$

However, $\theta = \overline{\theta} = \pi$ means that $D = 4$. Cancellation of the induced cosmological constant at the minimum/stationary point is only possible in $D = 4$ dimensions, and only for Lorentzian signature. Once again, the combination $D = 4$ and $\theta = \pi$ has been singled out.

The one-loop calculation I have discussed here was carried out for massless fields. Mass effects are discussed at length in Ref. [2]. They do not change the qualitative conclusions, except in the special case of $n_F = n_B$ and $D = 6$. It was noted above that, for $n_F = n_B$ or $D = 2$, the $V(\theta)$ computed for massless fields was θ-independent. Mass effects lift this degeneracy, and we find (providing the bosonic and fermionic masses satisfy a certain inequality) that there is another Lorentzian solution at $n_F = n_B$ and $D = 6$, which might correspond to some broken supersymmetric theory in six dimensions, in addition to the previous Lorentzian solution found for $n_F > n_B$ at $D = 4$.

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