DOMAIN WALLS IN THERMAL GAUGE FIELD THEORIES — MYTH OR REALITY?*

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We argue that different Z_N thermal vacua of hot pure Yang-Mills theory distinguished in the standard approach by different values of Polyakov loop average $\langle P \rangle_T$ corresponds actually to one and the same physical state. A critical discussion of the argument which are usually put forward in favor of the opposite conclusion (that, in pure continuum Yang-Mills theory, distinct Z_N -phases may coexist in the physical space being separated by the domain walls with finite surface energy) is given. In particular, we note that the same arguments can be applied with an equal ease to abelian theories and would lead to the existence of the walls in the high-T 4-dim QED and to appearance of the queer high-T solitons with the mass $\propto T^2/e$ in the Schwinger model. We emphasize that these configurations may be relevant for the Euclidean path integral but whether they correspond to real Minkowski space objects is unclear.

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1. Introduction

It was shown sometime ago that the pure Yang-Mills theory (without fermions) undergoes a phase transition at some temperature T_c [1, 2]. This phase transition exhibits itself in a radical change of behaviour of the correlator

$$C(\mathbf{x}) = \langle P(\mathbf{x})P^*(0)\rangle_T, \tag{1}$$

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where P(x) is the thermal Wilson line:

$$P(x) = \frac{1}{N_c} \text{Tr} \left\{ P \exp \left[ig \int_0^\beta \hat{A}_0(x, \tau) d\tau \right] \right\}$$
 (2)

 $(\hat{A}_0 = A_0^a t^a, \ \tau \text{ is the Euclidean time, and } \beta = 1/T).$

At small T, the correlator (1) falls down exponentially at large |x| whereas, at large T, it tends to a constant. Physically, this phase transition corresponds to deconfinement: at small T, the interaction part of free energy of a test heavy quark-antiquark pair at distance R grows linearly with R whereas, at large T, it tends to zero at large R.

It was argued later that, at high temperature, there are actually not one but N_c different phases distinguished by the expectation value $\langle P(0)\rangle_T = C\exp\{2\pi ik/N_c\}$, $k=0,\ldots,N_c-1$. This has been interpreted as a spontaneous breaking of the Z_N center subgroup of the $SU(N_c)$ gauge group [2-5]. Recently, the surface tension on the boundary separating two different phases has been calculated [6] (in the assumption that they exist, of course). Coexistence of different phases at high temperature could lead to nontrivial cosmological consequences [7].

However, the assertion that a symmetry which the system enjoys at low temperature can be broken spontaneously at high temperature is very radical and unusual. Much more natural and very common in physics is the opposite situation where a spontaneously broken symmetry is restored at high temperature.

Further, the statement of the spontaneous breaking of Z_N looks suspicious as the true symmetry of the pure Yang-Mills lagrangian in the continuum limit is $SU(N)/Z_N$ rather than SU(N) (gluon fields are not transformed under the action of the elements of the center). There seems to be nothing which can be broken.

The problem has many aspects and we are not able to discuss all of them in this talk. We shall concentrate on the question whether narrow domain walls separating the different Z_N phases in hot Yang-Mills theory exist and if so — in what sense?

2. Effective potential in constant A_0 -background

Let us remind here one of the ways of reasoning which has led to the conclusion of the existence of distinct Z_N -bubbles [3]. Consider the partition function of the SU(2) Yand-Mills system at high temperature presented as

an Euclidean path integral:

$$Z(\beta) = \mathcal{N} \int \prod_{\boldsymbol{x}} \{1 - \cos[g\beta\phi(\boldsymbol{x})]\} d\phi(\boldsymbol{x}) \prod_{\tau} d\boldsymbol{A}_{i}(\boldsymbol{x}, \tau)$$

$$\times \exp\left\{-\frac{1}{2} \int_{0}^{\beta} d\tau \int d\boldsymbol{x} [(\bar{\nabla}\phi)^{2} + (\partial_{0}\boldsymbol{A}_{i} + g\phi\boldsymbol{3} \times \boldsymbol{A}_{i})^{2} + \boldsymbol{B}_{i}\boldsymbol{B}_{i}]\right\}, (3)$$

where \mathcal{N} is a normalization factor, 3 is the unit vector along the third isotopic axis, the gauge $A_0(x,\tau) = \phi(x)3$ is chosen, and B_i is the colour magnetic field. The factor $1 - \cos[g\beta\phi(x)]$ is the invariant measure on the group.

In this approach, the dynamic fields $A_i(x,\tau)$ satisfy the periodic boundary conditions:

$$A_i(x,\beta) = A_i(x,0). \tag{4}$$

The quantity of a particular interest for us is the constrained effective potential defined as

$$Z^{-1}\exp\{-\beta V^{(3)}V^{\text{eff}}(\phi^c)\} = \left\langle \delta \left[\phi^c - \frac{1}{V^{(3)}} \int d\boldsymbol{x}\phi(\boldsymbol{x})\right] \right\rangle, \quad (5)$$

where $V^{(3)}$ is the 3-dimensional volume of the system, and the averaging in Eq. (5) is performed with the weight specified in Eq. (3). $V^{\text{eff}}(\phi^c)$ can be interpreted as the free energy with a *constant* background $A_0 = \phi^c 3$.

A small β and small g (the effective coupling decreases with temperature), the integral can be done, and one gets

$$V_{\text{eff}}(\phi^c) = \frac{\pi^2 T^4}{12} \left\{ 1 - \left[\left(\frac{g\phi^c}{\pi T} \right)_{\text{mod.2}} - 1 \right]^2 \right\}^2, \tag{6}$$

The potential (6) has minima at $\phi^c = 2\pi nT/g$ with integer n. It is easy to see that half of them (with even n) correspond to $\langle P \rangle_T = 1$ while the other half (with odd n) to $\langle P \rangle_T = -1$.

 A_0 is the nondynamical variable canonically conjugated to the Gauss law constraint. The latter is essentially the generator of the gauge transformations, and the matrix

$$\Omega = \exp\{i\beta g A_0^a t^a\} \tag{7}$$

can be interpreted as a matrix of a finite gauge transformation of the dynamic variables $A_i(x)$. Even and odd minima of the potential (6) correspond to $\Omega = 1$ and $\Omega = -1$, respectively. Note, however, that both $\Omega = 1$

and $\Omega = -1$ correspond to one and the same trivial gauge transformation of $A_i^a(x)$ and are physically undistinguishable.

3. Wall-like configurations

However, this argument alone does not yet exclude the existence of domain walls separating different phases in field theory. Really, one can imagine a planar field configuration satisfying boundary conditions $\phi(-\infty,y,z) = 0$, $\phi(\infty,y,z) = 2\pi T/g$. It corresponds to the classical vacuum (one and the same in the quantum mechanical sense) at $x = \pm \infty$ but is nontrivial in between presenting a noncontractable loop in the $SU(N)/Z_N$ group (the real symmetry group of the continuum Yang-Mills theory). Such loops exist due to nontrivial $\pi_1(SU(N)/Z_N) = Z_N$. It is even possible to determine the profile of this wall-like configurations and to find their surface energy [6]. T. Bhattacharaya et al. have written down the effective hamiltonian depending on inhomogeneous $\phi(x)$ as the sum of the tree-level kinetic term and the periodic potential term (6) with x-dependent argument:

$$\mathcal{H}^{\text{eff}} = \int d\boldsymbol{x} \left[\frac{1}{2} \left(\frac{\partial \phi(\boldsymbol{x})}{\partial \boldsymbol{x}} \right)^2 + V^{\text{eff}} [\phi(\boldsymbol{x})] \right]. \tag{8}$$

This effective lagrangian resembles the Sine-Gordon hamiltonian and supports the wall-like classical solutions. These walls have the width $\sim (gT)^{-1}$ and the surface energy

$$\sigma = \frac{4\pi^2}{3\sqrt{6}} \, \frac{T^3}{g} + O(gT^3) \tag{9}$$

(for SU(2), or better to say for SU(2)/ $Z_2 \equiv SO(3)$ gauge group).

One of our main observations is that the same arguments which have led people to the conclusion on existence of the walls in pure YM theory can be transferred without essential change also to high-temperature QED: if the walls appear in the former, they appear also in the latter. The topological reason for their existence in QED is nontrivial π_1 of the U(1) gauge group.

To understand better what happens, let us write down the effective potential in constant A_0 -background in high-T QED. The one-loop result is [3, 8]

$$V_F^{\text{eff}}(A_0) = -\frac{\pi^2 T^4}{12} \left\{ 1 - \left[\left(\frac{\beta e A_0}{\pi} + 1 \right)_{\text{mod.2}} - 1 \right]^2 \right\}^2$$
 (10a)

for spinor QED and

$$V_B^{\text{eff}}(A_0) = \frac{\pi^2 T^4}{12} \left\{ 1 - \left[\left(\frac{\beta e A_0}{\pi} \right)_{\text{mod}, 2} - 1 \right]^2 \right\}^2$$
 (10b)

for scalar QED. The potentials (10a) and (10b) are periodic in A_0 with the period $2\pi T/e$. As far as the quantum mechanical aspect of the problem is concerned, the fields A_0 and $A_0 + 2\pi T/e$ are equivalent — the gauge transformations $\Omega = \exp\{ie\beta A_0\}$ acting on dynamic field variables are exactly the same. But it does not resolve the question yet as, in the full analogy with the Yang-Mills theory, one can consider a wall-like configuration of the kind depicted in Fig. 1 (the field is assumed to be uniform in y and z). This configuration carries topological charge (it presents a non-contractible loop in U(1) group space) and cannot be trivialized.

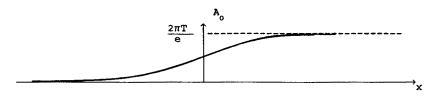


Fig. 1. A wall-like configuration.

It is obvious that taking Eqs (10) at face value also for space-dependent fields $A_0(x)$, adding the kinetic term $\propto (\partial_x A_0(x))^2$, and minimizing the energy functional

$$\mathcal{H}^{\text{eff}}[A_0(\boldsymbol{x})] = \int d^3\boldsymbol{x} \left[\frac{1}{2} \left(\frac{\partial A_0(\boldsymbol{x})}{\partial \boldsymbol{x}} \right)^2 + V^{\text{eff}}(A_0(\boldsymbol{x})) \right]$$
(11a)

thus obtained, with boundary conditions

$$A_0(-\infty, y, z) = 0, \qquad A_0(\infty, y, z) = 2\pi T/e,$$
 (11b)

we would arrive at a wall-like configuration with the width of order of Debye screening length $R_D \sim (eT)^{-1}$ and the surface energy $\sigma_{\rm wall} \sim T^4 R_D \sim T^3/e$. It is not difficult to work out the exact coefficients:

$$\sigma_B = \frac{4\pi^2}{3\sqrt{6}} \frac{T^3}{e} \tag{12a}$$

for scalar QED (the calculation is just identical to that in pure YM theory [6]) and

$$\sigma_F = \frac{2\pi^2(2\sqrt{2} - 1)}{3\sqrt{6}} \frac{T^3}{e}$$
 (12b)

for spinor QED. Thus, the same line of reasoning as that in Ref. [6] for nonabelian gauge theories leads to the conclusion that the walls separating different phases exist also in high temperature QED?!

One could even give to these "phases" the same interpretation in terms of Polyakov loop as the one usually given to Z_N -phases of nonabelian theories. Though, the standard Polyakov loop

$$\langle P_1 \rangle_T = \langle \exp\{ie\beta A_0(x)\} \rangle_T$$
 (13)

is equal to 1 at all classical minima of the potentials (10), one could consider as well

$$\langle P_{1/2} \rangle_T = \left\langle \exp \left\{ \frac{ie}{2} \beta A_0(x) \right\} \right\rangle_T.$$
 (13a)

The loop (13a) describes the interaction of a static heavy source with the charge e/2 with the electron and photon heat bath. $P_{1/2}=\pm 1$ at classical potential minima $A_0^{(n)}=2\pi nT/e$ depending on whether n is even or odd.

Likewise, one can consider

$$P_{1/q} = \left\langle \exp\left\{\frac{ie}{q}\beta A_0(x)\right\} \right\rangle_T, \tag{13b}$$

with any integer q, which describes the interaction of a heavy source with the charge 1/q, with the heat bath, and, classically, can acquire q different values. Introducing heavy sources with fractional charges to probe the system involving only dynamic fields with unit charge is completely equivalent to considering fundamental heavy color sources probing the pure Yang-Mills system involving only adjoined dynamic fields.

Thus, if one takes seriously the existence of such walls in hot QED, the global Z-symmetry of electrodynamics [where $Z = \pi_1[\mathrm{U}(1)]$ is the factor in the bundle $R_1 \xrightarrow{Z} \mathrm{U}(1)$ and plays the same role as the factor $Z_2 = \pi_1[\mathrm{SO}(3)]$ in the bundle $\mathrm{SU}(2) \xrightarrow{Z_2} \mathrm{SO}(3)$ and the factor Z_N for $\mathrm{SU}(N)$] breaks spontaneously at high temperature exactly in the same sense (if any) as the global Z_N -symmetry breaks spontaneously in high temperature Yang-Mills theory.

For sure, that looks very strange and suspicious and the natural desire is to look around in search for arguments which could disprove this conclusion. To this end, let us simplify the problem still more and go to 2 dimensions.

4. Schwinger model

Consider Schwinger model — the 2-dimensional QED with one massless charged fermion [9]. The action of the model reads

$$S = \int \left[-\frac{1}{4} F_{\mu\nu}^2 - i \bar{\psi} \mathbb{D} \psi \right] d^2 x , \qquad (14)$$

with $\mathbb{D} = \gamma_{\mu}(\partial_{\mu} - ieA_{\mu})$. The charge e has the dimension of mass. The model is exactly soluble, and any reasonable physical question can, in principle, be given the exact answer.

To begin with, let us find the effective potential on the constant background A_0 at high temperature Schwinger model in the same way as it has been done in [3, 8] for 4-dimensional theories. Thus, let us assume $A_0(x,\tau)$ to be constant in space and time and impose the periodic (antiperiodic) b.c. on the dynamic boson (fermion) fields:

$$A_1(x,\beta) = A_1(x.0), \qquad (15a)$$

$$\psi(x,\beta) = -\psi(x,0). \tag{15b}$$

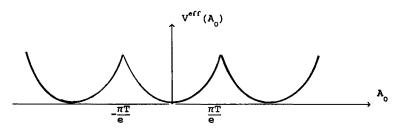


Fig. 2. Effective potential on constant A_0 background in the Schwinger model.

On the one-loop level (and in the Schwinger model there is actually nothing beyond), the background A_0 is coupled only to the charged fermion fields, and one has only to calculate the determinant of the Euclidean Dirac operator $\mathbb{D}(A_0) = [\gamma^0(\partial_0 - ieA_0) + \gamma^1\partial_1]$ in the constant A_0 background. Not dwelling on details, we give the result

$$V^{\text{eff}}(A_0) = \frac{e^2}{2\pi} \left[\left((A_0 + \frac{\pi T}{e})_{\text{mod.} \frac{2\pi T}{e}} - \frac{\pi T}{e} \right)^2 .$$
 (16)

The profile of the potential is shown in Fig. 2. We see that, at small A_0 , the effective potential is just

$$V^{\text{eff}}(A_0) = \frac{\mu^2}{2} (A_0)^2, \quad \text{(for small } A_0)$$
 (17)

where $\mu = e/\sqrt{\pi}$ is the photon mass.

The potential (16) undergoes the change of regime at

$$A_0^{\text{sing.}} = \frac{\pi T(2n+1)}{e},$$
 (18)

and repeats itself going to zero at $A_0 = 2\pi nT/e^1$.

Now, we can play the same game as in four dimensions. After noting that all minima of the effective potential (16) are physically equivalent (they correspond to one and the same gauge transformation), we consider the field configuration $A_0(x)$ which interpolates between the equivalent vacua $A_0=0$ and $A_0=2\pi T/e$ as shown in Fig. 1. It presents a topologically nontrivial configuration (corresponding to a noncontractable loop in U(1) group) and cannot be smoothly transformed to the trivial configuration $A_0(x)=0$. One may try to find the profile and the energy of this configuration by solving the equations of motion with the effective hamiltonian

$$\mathcal{H}^{\text{eff}} = \int \left\{ \frac{1}{2} (\partial_x A_0)^2 + V^{\text{eff}} [A_0(x)] \right\} dx, \qquad (19)$$

with boundary conditions $A_0(-\infty) = 0$, $A_0(\infty) = 2\pi T/e$ (cf. Eqs (8), (11)). The result is

$$A_0(x) = \begin{cases} \frac{\pi T}{\epsilon} \exp\{\mu(x - x_0)\}, & x \leq x_0 \\ \frac{\pi T}{\epsilon} \left[2 - \exp\{-\mu(x - x_0)\}\right], & x \geq x_0 \end{cases}$$
 (20)

for the profile and

$$M = \frac{\pi^{3/2}T^2}{e} \tag{21}$$

for the mass. The width of the solution (20) is of order of correlation length $\mu^{-1} \sim e^{-1}$.

An exact analysis shows that the assumption (19) was actually too naive. Infrared singularities (which are much more malicious in 2 than in 4 dimensions) destroy the local form of the effective hamiltonian completely. The true effective hamiltonian (which as everything can be found in Schwinger model exactly) is highly nonlocal and rather intricate. It is remarkable, however, that the form and the energy of the solution to this correct effective hamiltonian turn out to be exactly the same as in Eqs (20), (21).

Thus, we are led to the conclusion that wall-like quasiclassical field configurations appear universally in hot pure Yang-Mills theory, hot 4-dim QED (both spinor and scalar), and hot Schwinger model. But what is their physical meaning?

¹ The reason for nonanaliticity of the effective potential at the points (18) are severe infrared singularities occurring at these points. We refer the reader to our paper [10] for the detailed discussion.

5. Discussion

Consider first the simplest theory where this phenomenon occurs, *i.e.* Schwinger model. Note that the configuration (20) carries nontrivial 2-dim Euclidean topological charge

$$\nu = \frac{e}{2\pi} \int_{0}^{\beta} d\tau \int_{0}^{L} dx F_{01}(x,\tau).$$
 (22)

 $(L\gg 1/e)$ is the size of the spatial box introduced for infrared regularization). Another name for it is the instanton number and the classical solution (20) in the sector $\nu=-1$ should be called the thermal instanton.

Note that the high-T solution (20) has much nicer properties than its counterpart at T=0. Zero-temperature instanton is a configuration describing the electric field which is constant in space and Euclidean time and very small $E=F_{01}=\pm 2\pi/e\beta L$, so that the net flux (22) is equal to ± 1 . Thus, it is a highly delocalized configuration [11, 12]. Also, the quasiclassical picture does not really work here — characteristic fields providing main contribution to the path integral involve strong fluctuations of the field density [13] and are similar to the homogeneous classical solution².

But at high temperatures the situation is different. One can show that at $T\gg e$, characteristic field fluctuations are $A_0^{\rm fluct}\sim \sqrt{T/e}$ which is much less than the amplitude of the classical solution (20) $A_0^{\rm cl}\sim T/e$. Thus, the higher is the temperature, the better the quasiclassical picture works: as temperature grows, the instantons cool down! Also, in contrast to the zero-temperature case, they become localized — the solution (20) involves nonvanishing flux density only in the region $|x-x_0|^{\rm char}\sim \mu^{-1}\sim 1/e^3$.

While interpreting the configuration (20) as an instanton, we think of it as an Euclidean field configuration contributing to the path integral. It

Quasiclassical picture is not adequate also in zero-temperature QCD. What is specific for the Schwinger model, however, is that, though quasiclassical picture does not work, quasiclassical calculations are still possible. The matter is that the path integrals are exactly Gaussian here and can be done irrespectively of whether characteristic field configurations are far away in the Hilbert space from the classical solutions or close to them.

³ The "cooling down" of instantons at high temperature occurs also in QCD — the effective coupling constant $\alpha_s(T)$ becomes small, and the instanton action $S^{\text{inst}}(T) = 2\pi/\alpha_s(T)$ (alias, the quasiclassical parameter) becomes large. That allows one to perform some exact instanton calculations in high-temperature QCD [8, 14, 15].

is seen, however, that it does not depend on Euclidean time τ , but only on x.

The question we are not able to answer definitely by now is whether this configuration can be interpreted also as a *soliton* configuration in the Minkowski space, in other words — whether it can be treated as a real physical object which can move, scatter on something, *etc*. We tend to think that these objects do *not* exist in *such* sense. But in case they do, they present a direct analog of the domain walls in 4-dimensional theories.

The latter appears due to nontrivial π_1 (gauge group) and can be called planar instantons (cf. the usual YM instantons which are localized and appear due to nontrivial π_3 (gauge group)). They exist (universally both in nonabelian and abelian theories) as Euclidean field configurations but probably can not be interpreted as real walls in Minkowski space, though, to repeat it once more, a satisfactory answer to this question is not yet obtained.

Finally, we want to emphasize that the question of whether or not domain walls really exist as physical object is equivalent to the question of whether spontaneous breaking of Z_N -symmetry really takes place. To illustrate this, we want to recall the discussion which had occurred some time ago about whether U(1) singlet axial symmetry in QCD is broken explicitly or spontaneously. The standard reasoning due to 't Hooft is that the singlet axial current is not conserved due to anomaly

$$\partial_{\mu}j_{\mu}^{5} = \frac{g^{2}}{16\pi^{2}} \text{Tr} \left\{ G_{\mu\nu} \tilde{G}_{\mu\nu} \right\},$$
 (23)

and hence $U_A(1)$ -symmetry is broken explicitly. However, Crewther argued that r.h.s. of Eq. (23) can be presented as a total derivative $\partial_{\mu}K_{\mu}$ and then the current

$$\tilde{j}_{\mu}^{5} = j_{\mu}^{5} - K_{\mu} \tag{24}$$

is conserved. The current (24) is not gauge invariant but the charge $\tilde{Q}^5 = \int d^3x \tilde{j}_0^5$ is. \tilde{Q}^5 generates shift of the vacuum angle θ , and the latter can be thought of as an order parameter corresponding to *spontaneous* breaking of $U_A(1)$ symmetry [16].

This is formally true but physically misleading as the "order parameter" θ is fixed by superselection rule and cannot fluctuate — the states with different θ cannot coexist in one and the same physical space. Thus, there is no massless Goldstone boson which always appear when a continuous symmetry is broken spontaneously in the physical meaning of this word.

Likewise, spontaneous breaking of a discrete symmetry should be physically associated with appearance of domain walls. The breaking does not really occur if the walls do not really appear.

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