THERMODYNAMICS OF A QUARK PLASMA IN THE MEAN FIELD* †

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The thermodynamics of a quark plasma is studied within the two-flavour Nambu–Jona-Lasinio model. We review the results that have been obtained in the mean-field (Hartree) approximation, including chiral and quark number susceptibilities, in which we treat contributions to the thermodynamic potential which arise to order 1 in the number of colours N_c.

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1. Introduction

An understanding of the behavior of quantum chromodynamics at finite temperature and density is of fundamental importance. In particular, it is needed for a description of the development of the early universe and for the interpretation of results from ultra-relativistic heavy-ion collisions, where high temperatures and densities are reached, and a transition from a hadronic phase of matter to one containing quarks and gluons is thought to occur.

Numerical simulations of quantum chromodynamics have long since indicated a chiral symmetry restoration phase transition that should occur in the temperature range $T = 100 - 200\,\text{MeV}$ [1]. In addition, measurements of the Polyakov loop [2] indicate a deconfinement transition in which the composition of a system moves from one containing hadronic degrees of freedom to one having solely a quark and gluonic content. For systems with

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net baryon number zero, the numerical simulations seem to indicate that the two transition temperatures coincide [2].

As a step on the way to gaining an understanding of finite temperature and density QCD, we investigate here the thermodynamics of a system of interacting quarks within an effective model for QCD. As other authors have done, we use the SU(2) Nambu–Jona-Lasinio (NJL) model [3], in which the gluonic degrees of freedom are reduced to an effective local interaction between quarks, in a manner that respects chiral symmetry. This model has been successfully used to quantitatively describe the meson spectrum at low excitation energies and other static properties [4]. In this paper, we shall restrict ourselves to a description of the mean field approximation [5–10]. In particular, we will keep the Hartree term only, in anticipation of the fact that this is the leading term in an expansion in the inverse number of colors, $1/N_c$. As far as this goes, some of the results that we will derive are already known, while others are specific to this manuscript. We will in general develop the formalism in a broad fashion that will more easily permit us to formally incorporate fluctuations about the mean field, or terms that are of higher order in an expansion in $1/N_c$. This is important, since while mean field dynamics enable us to understand the mechanism of chiral symmetry breaking, it describes the thermodynamics of *quarks* in a temperature bath. Fluctuations however are necessary to include the essential hadronic components into the general formalism. For details of this, we refer the reader to the preprint [11].

Our procedure commences with a subdivision of the Lagrangian into mean field and fluctuating parts. Since we have a contact four-fermion interaction, we can simply construct the self-energy, and the thermodynamic potential for the mean field part of the Lagrangian. Then, in dealing with thermodynamical aspects of the quark system, the usual bulk thermodynamic quantities such as pressure, energy and entropy densities and the specific heat may be obtained directly from the thermodynamic potential. In addition, we introduce a measure for the deviation of the order parameter $\langle\bar{\psi}\psi\rangle$, or effectively the quark mass, from its value in the chiral limit. We call this the chiral susceptibility, χ_χ. In the same way that the presence of magnetic fields destroys the superconducting state in Ginzburg–Landau theory, so the current quark mass is responsible for destroying the chirally symmetric ordered phase in QCD. Thus χ_χ is an indicator for the stability of the phase transition with respect to variations in the current quark mass.

In regarding the meson spectrum, one may query whether a Goldstone mode should be expected at finite temperature. Within the NJL model at the Hartree or mean field level, this can be explicitly demonstrated to be the case.

This paper is organized as follows. In Section 2, we present a general theoretical framework for the thermodynamic approach and we discuss the mean field theory. In Section 3, we present detailed numerical calculations, and we summarize and conclude in Section 4.

2. General framework for the mean field approximation.

2.1. The thermodynamic potential Ω

The standard approach for dealing with the thermodynamics of a variable number of particles is via the grand canonical ensemble. The grand thermodynamic potential is related to the Hamiltonian of the system through the defining equation,

$$e^{-\beta\Omega} = \mathrm{Tr}\, e^{-\beta(H-\mu N)}. \tag{2.1}$$

where $\beta = 1/T$ is the inverse temperature and μ the chemical potential. Here N is the number operator. In Eq. (2.1), the trace Tr is to be taken over all states. In general, one ignores the colour fluctuations that are implicit in such a sum. These are thought not to play a major role because of their large excitation energy, and technically, they can be removed by projecting onto color neutral states [12]. Once Ω is known, the thermodynamic functions that measure the bulk properties of matter can be obtained. For an infinite system, these are the pressure density p, the entropy density s, the quark number density n, the energy density ϵ and the specific heat c, that are defined as

$$\begin{aligned} p &= -\Omega\,, \\ s &= -\frac{\partial\Omega}{\partial T}\,, \\ \epsilon &= -p + Ts + \mu n\,, \qquad n = -\frac{\partial\Omega}{\partial\mu}\,, \\ c &= \frac{\partial\epsilon}{\partial T}\,. \end{aligned} \tag{2.2}$$

If the system in question undergoes a phase transition and has associated with it an order parameter m say, then it is expected that Ω be minimal with respect to this parameter, *i.e.*

$$\frac{\partial\Omega}{\partial m}(T,\mu,m) = 0, \qquad \frac{\partial^2\Omega}{\partial m^2}(T,\mu,m) \geq 0. \tag{2.3}$$

This equation is the so-called "gap equation" of the system. In effect, (2.3) is nothing other than the generalization of the analogous zero temperature

BCS approach to finite temperatures, in which a variational ansatz for the ground state wave function is made, and the total ground state energy is minimized with respect to the variational parameter to give the gap equation.

In what follows, we make a particular choice of the Lagrange density, and evaluate Ω in the mean field approximation.

2.2. *The Lagrange density.*

The two flavor version of the NJL model is defined through the Lagrangian [4]

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i \not\partial - m_0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \\ &= \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} . \end{aligned} \tag{2.4}$$

Here G is a constant of dimension $[\text{GeV}]^{-2}$, and ψ, $\bar{\psi}$ are Dirac spinors with flavor, color and spinor indices suppressed. The model is non-renormalizable, and is only fully specified with the inclusion of a regulator Λ for divergent quantities. One can separate the scalar $\bar{\psi}\psi$ into its mean field and fluctuating parts,

$$\bar{\psi}\psi = \langle\bar{\psi}\psi\rangle + \delta_s \tag{2.5}$$

with $\delta_s = \bar{\psi}\psi - \langle\bar{\psi}\psi\rangle$. The thermal average $\langle\bar{\psi}\psi\rangle$ is assumed real but is otherwise as yet unspecified. Then the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{mf} + \mathcal{L}_{fl} , \tag{2.6}$$

where

$$\mathcal{L}_{mf} = \bar{\psi}[i \not\partial - (m_0 - 2G\langle\bar{\psi}\psi\rangle)]\psi - G\langle\bar{\psi}\psi\rangle^2 , \tag{2.7}$$

$$\mathcal{L}_{fl} = G(\delta_s^2 + \delta_{ps}^2) . \tag{2.8}$$

Here $\delta_{ps} = (\bar{\psi}i\gamma_5\vec{\tau}\psi)$ represents the pseudoscalar fluctuations.

Keeping only the mean field part of the Lagrangian, one identifies the mass term as

$$m = m_0 - 2G\langle\bar{\psi}\psi\rangle \tag{2.9}$$

which is the usual result obtained in the Hartree approximation when $\langle\bar{\psi}\psi\rangle$ satisfies the gap equation. Here we will go another way, namely by determining $\langle\bar{\psi}\psi\rangle$ from thermodynamic considerations [8], using the equations of Section 2.1. The thermodynamic potential associated with the NJL Lagrangian (2.6) may also be subdivided into a mean field and a fluctuating contribution,

$$\Omega(T,\mu,m) = \left(\frac{(m-m_0)^2}{4G} + \Omega_q(T,\mu,m)\right) + \Omega_{fl}(T,\mu,m) , \tag{2.10}$$

where the two terms in the bracket relate to the mean field. The first expression in the bracket originates from the last term in Eq. (2.7) and reflects the contribution to the total energy that is required in the Hartree approximation over and above that which arises from summing single particle states (see, for example [5]). Physically, this term delivers a contribution to $\Omega(T, \mu, m)$ from the vacuum. $\Omega_q(T, \mu, m)$ is the thermodynamic potential that is associated with free constituent quarks and antiquarks of mass m given by (2.9). $\Omega_{fl}(T, \mu, m)$ represents the contribution due to the fluctuations from the mean field and has both pseudoscalar-isovector and scalar-isoscalar components. In what follows, we briefly review and then expand on the thermodynamics associated with the mean field approximation, and refer the reader to Ref. [11] for the calculation that includes the fluctuating components.

2.3. Mean field thermodynamics.

2.3.1. Thermal properties of particle masses

The thermodynamic potential associated with the mean field approximation [5] is

$$\Omega_{mf}(T, \mu, m) = \frac{(m - m_0)^2}{4G} + \Omega_q(T, \mu, m), \tag{2.11}$$

with

$$\Omega_q(T, \mu, m) =$$
$$-2N_cN_f \int \frac{d^3p}{(2\pi)^3} E_p - \frac{2N_cN_f}{\beta} \int \frac{d^3p}{(2\pi)^3} \ln \left[1 + e^{-\beta(E_p+\mu)}\right] \left[1 + e^{-\beta(E_p-\mu)}\right]. \tag{2.12}$$

Here $E_p^2 = p^2 + m^2$, β is the inverse temperature, and N_c, N_f are the number of colors and flavors respectively. Equation (2.11) holds for any value of the scalar quark condensate $\langle\bar{\psi}\psi\rangle$, which is the order parameter of the chiral phase transition. Since this quantity and the effective quark mass m are related in a simple equation, Eq. (2.9), we may also call m the order parameter. In the spirit of thermodynamics, the physical system is described by that value of m for which $\Omega_{mf}(T, \mu, m)$ is a minimum,

$$\frac{\partial \Omega_{mf}(T, \mu, m)}{\partial m} = 0. \tag{2.13}$$

Since $\Omega_q(T, \mu, m)$ depends only on m^2, we have

$$m\left(\frac{1}{4G} + \frac{\partial \Omega_q(T, \mu, m)}{\partial m^2}\right) = \frac{m_0}{4G}. \tag{2.14}$$

This equation is identical to the usual thermal gap equation in the Hartree limit, which we write as

$$m = 8mGN_cN_fI_1 + m_0 \tag{2.15}$$

defining

$$I_1 = \int \frac{d^3p}{(2\pi)^3}\frac{1}{4E_p}\left(\tanh\tfrac{\beta}{2}(E_p+\mu) + \tanh\tfrac{\beta}{2}(E_p-\mu)\right) . \tag{2.16}$$

Equation (2.15) gives the quark mass m as a function of the temperature T and the chemical potential μ. For a vanishing current quark mass m_0, Ω_{mf} exhibits a chiral phase transition. The phase transition line in the $T-\mu$ plane separates the regions with $m \neq 0$ (chiral symmetry broken) from the one with $m = 0$ (chiral symmetry restored).

In the self-consistent Hartree approximation, it is well-known that the masses of the π and σ mesons satisfy the dispersion relations [4]

$$1 - 2G\Pi_M(k=0, i\nu_n = m_M) = 0\,, \tag{2.17}$$

where m_M is the mass of the respective meson, and Π_M the associated retarded polarization function, analytically continued to real frequencies $i\nu_n \to m_M + i\eta$. For completeness, we list these functions. One has in the pseudoscalar and scalar channels respectively,

$$\Pi_\pi(0,m_\pi) = N_cN_f\int\frac{d^3p}{(2\pi)^3}\frac{1}{E_p}\frac{E_p^2}{E_p^2-\frac{m_\pi^2}{4}}[\tanh\tfrac{\beta}{2}(E_p+\mu)+\tanh\tfrac{\beta}{2}(E_p-\mu)], \tag{2.18}$$

and

$$\Pi_\sigma(0,m_\sigma) = N_cN_f\int\frac{d^3p}{(2\pi)^3}\frac{1}{E_p}\frac{E_p^2-m^2}{E_p^2-\frac{m_\sigma^2}{4}}[\tanh\tfrac{\beta}{2}(E_p+\mu)+\tanh\tfrac{\beta}{2}(E_p-\mu)]. \tag{2.19}$$

The divergent integrals here and in what follows are understood to be regulated by a cutoff Λ on the three momentum.

In the phase in which chiral symmetry is broken, the quark mass is a solution of the equation

$$1 - 8GN_cN_fI_1(T,\mu,m) = 0. \tag{2.20}$$

One observes from Eq. (2.18) that the pion polarization function for zero pion mass and all temperatures and chemical potentials, $\Pi_\pi(0,0)$ exactly equals $I_1/8N_cN_f$, which implies that the mass of the pion is zero in the chirally broken phase. Thus the pion is the Goldstone mode of this transition. At the same time, one sees by inspection that $m_\sigma = 2m$ whenever $m_\pi = 0$. The explicit expressions for the gap equation and the pion polarization functions show that the Hartree approximation to the gap equation plus the RPA approximation to the polarization functions conserves the properties associated with chiral symmetry. This fact makes the mean field approximation to the self-energy attractive for a study of the chiral phase transition in the NJL model.

One can also determine the critical exponents for the chiral phase transition. For instance, if we denote the transition temperature by T_χ, we find for the quark and meson masses $m(T)$ and $m_M(T)$,

$$\begin{aligned} m(T) &\sim |T - T_\chi|^{1/2}\,, \\ m_M(T) &\sim |T - T_\chi|^{1/2}\,, \end{aligned} \tag{2.21}$$

in the neighbourhood of T_χ. The critical exponents $^1/_2$ are simply a consequence of the fact that $I_1(T,\mu,m)$ in Eq. (2.16) and the polarization functions $\Pi_M(T,\mu,m_M,m)$ depend quadratically on the masses m and m_M. One has a simple zero for $m^2(m_M^2)$ as a function of T and hence Eqs (2.21) follow.

2.3.2. Thermodynamic functions

The first two terms that occur in $\Omega_{mf}(T,\mu)$ in Eqs (2.11) and (2.12), *i.e.*

$$\Omega_{\rm vac} = \frac{(m - m_0)^2}{4G} - 2N_cN_f \int \frac{d^3p}{(2\pi)^3} E_p\,, \tag{2.22}$$

represent energy contributions from the vacuum, since they do not vanish with $T \to 0$ and $\mu \to 0$. Physically, the quantity $\Omega(T,\mu,m)$ corresponds to the pressure density (except for a sign) and only the pressure of a system relative to the vacuum can be measured. We therefore introduce the rescaled thermodynamic potential

$$\bar{\Omega}(T,\mu,m) = \Omega(T,\mu,m) - \Omega(0,0,m)\,, \tag{2.23}$$

which from now on, we denote by $\Omega(T,\mu,m)$ for simplicity. From $\Omega(T,\mu,m)$, we can obtain the thermodynamical functions that measure the bulk properties of matter, as given by Eq. (2.3).

2.3.3. Chiral and quark number susceptibilities

In order to discuss the stability of the chiral phase transition with respect to the current quark mass m_0, we introduce the concept of chiral susceptibility,

$$\chi_\chi = \left.\frac{\partial m}{\partial m_0}\right|_{m_0=0}\,, \tag{2.24}$$

where m_0 is the current mass. This quantity indicates how the order parameter m changes, when we introduce a small current quark mass m_0. We can derive an explicit expression for this from the gap equation (2.15). Letting $m \to m + \delta m$, we find that δm satisfies the equation

$$\delta m \left[1 - 8GN_cN_fI_1(m^2) - 16GN_cN_fm^2 \frac{\partial I_1(m^2)}{\partial m^2}\right] = \delta m_0 \,, \qquad (2.25)$$

so that

$$\chi_\chi(T,\mu) = \left[1 - 8GN_cN_fI_1(m^2) - 16GN_cN_fm^2 \frac{\partial I_1(m^2)}{\partial m^2}\right]^{-1} . \qquad (2.26)$$

From the gap equation, this may be simplified to read

$$\chi_\chi(T,\mu) = \begin{cases} -\left[16GN_cN_fm^2 \frac{\partial I_1(m^2)}{\partial m^2}\right]^{-1} & \text{CSB} \\ +[1 - 8GN_cN_fI_1(0)]^{-1}, & \text{CSR} \end{cases} \qquad (2.27)$$

where CSB and CSR refer to the phases in which chiral symmetry is broken or restored, respectively.

For completeness, we also examine the quark number susceptibility χ_q [13, 14]. This quantity measures the susceptibility of the quark number density with respect to changes in the chemical potential, and is defined as

$$\chi_q(T) = \left.\frac{\partial n}{\partial \mu}\right|_{\mu=0} , \qquad (2.28)$$

Lattice simulations of QCD [1, 15, 16] have indicated that a sharp rise occurs in χ_q at $T = T_\chi$, and for $T > T_\chi$, $\chi_q \to 2T^2$. There is some speculation that at such temperatures, QCD may be treated approximately as an ideal gas of weakly interacting quarks.

It is interesting to note that the isothermal compressibility $\kappa_T = 1/n(\partial n/\partial p)_T$ is closely connected to χ_q, since κ_T can be rewritten as

$$\kappa_T = \frac{1}{n^2}\frac{\partial n}{\partial \mu} . \qquad (2.29)$$

The isothermal compressibility is a measure of the density fluctuations, which can be directly seen through the relationship

$$\kappa_T = \frac{\beta}{n}\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle} , \qquad (2.30)$$

that can be derived from the connection of n with the partition function Z and the relation $P = -\Omega$, see Ref. [17].

3. Numerical results.

In this section, we present the numerical calculations of the quark and meson masses, and the bulk quantities specific to the thermodynamics of the problem.

As was pointed out in Section 2, the quark mass m can be calculated as a function of the temperature T and chemical potential μ via the gap equation, Eq. (2.15). For a vanishing quark mass m_0, Ω_{mf} exhibits a chiral phase transition. The phase transition line is determined from the equation

$$\left[\frac{1}{4G} + \frac{\partial \Omega_q}{\partial m^2}\right]\Bigg|_{m=0} = 0 \tag{3.1}$$

in the chiral limit, and separates the regions with $m \neq 0$ (chiral symmetry broken) from the one with $m = 0$ (chiral symmetry restored). This is shown in Fig. 1(a) in the $T - \mu$ plane, and in Fig. 1(b) in the density-T plane. The baryon density used in this figure is denoted as n_b. The behavior of the quark and meson masses with temperature ($\mu = 0$) is sketched in Fig. 2, for the parameter set $\Lambda = 0.65$ GeV and $G = 5.01(\mathrm{GeV})^{-2}$, for which the quark condensate density $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = (-0.250\mathrm{GeV})^3$ and the pion decay constant $f_\pi = 0.093$GeV. For $\mu = 0$, the critical temperature T_χ for the phase transition is found to be 190 MeV, and the standard results $m_\sigma(T) = 2m(T)$ and $m_\pi(T) = 0$ hold for $T < T_\chi$. The π and σ mesons meet after the critical temperature for chiral symmetry restoration is reached, and thereafter their thermal energy dominates. Above the critical temperature, the situation is more complicated, since the quark mass is zero, and the π as

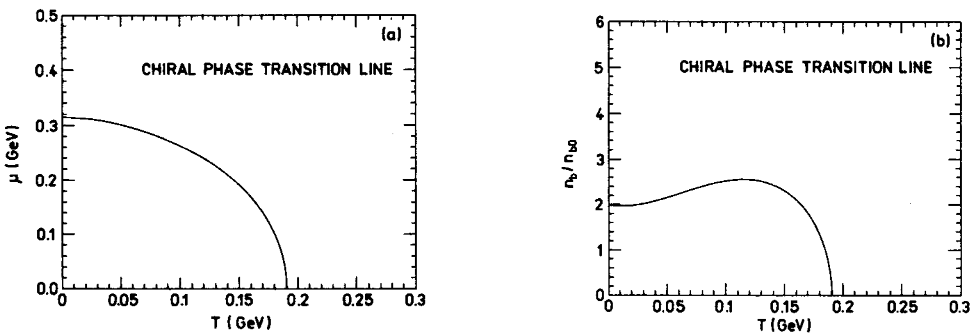

Fig. 1. The chiral phase transition line is given (a) in the $\mu - T$ plane, and (b) in the density-T plane. Here the baryon density has been normalized to the value for normal nuclear matter, $n_{b0} = 0.17\mathrm{fm}^{-3}$.

well as the σ can decay into $q\bar{q}$ pairs. They are thus not stable bound states, but rather resonances. One is therefore required to find *complex* solutions of Eq. (2.17) of the form $m_M \to m_M - i\Gamma/2$. In Fig. 2, the dashed lines indicate the spreading $m_M \pm \Gamma/2$ for the quark mass due to the presence of this width.

In Fig. 2(b), the same calculation is indicated for the case of a non-vanishing current quark mass of $m_0 = 5.0$ MeV. For all values of T, the σ-meson is a resonance state, which can be seen from the dashed lines indicating the magnitude of the width.

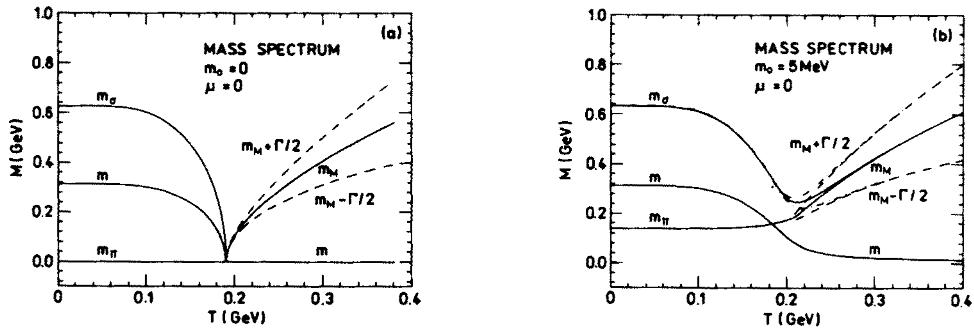

Fig. 2. The mass spectrum is indicated as a function of temperature for (a) the chiral limit, $m_0 = 0$, and (b) the case $m_0 = 5$ MeV. Shown are the dynamically generated quark mass m, and the meson masses m_π and m_σ. The dashed lines indicate $m_M \pm \Gamma/2$, where m_M, $M = \sigma, \pi$ is the mass of the state and Γ its width.

The thermodynamics functions for $m_0 = 0$ given in Eq. (2.2) are shown in Figs 3(a)–3(d). One observes that the curve for the pressure is continuously differentiable at $T = T_\chi$, while the entropy and energy densities are continuous, but display a kink. The specific heat, on the other hand, is discontinuous. These findings are in agreement with a second order phase transition. Above T_χ, we expect the thermodynamic functions to correspond with those of an ideal quark gas with mass zero. In that case, the ratios p/T^4, and s/T^3 should be independent of temperature, and are determined by the number of degrees of freedom. The expected behavior is indicated in all four graphs by the dot-dashed line. The calculated curves undershoot this value significantly, and furthermore decrease with temperature for $T > T_\chi$. This last feature is a consequence of the use of a finite cut-off in the expression (2.12) for Ω_q.

In analogy to the bag constant in bag models, in the chiral limit, a chiral bag constant at $T = 0$ and $\mu = 0$ may be defined as the pressure (or energy) difference between the physical vacuum and the chiral perturbative vacuum,

$$B_0 = \epsilon_v(0,0)|_{m=0} - \epsilon_v(0,0)|_{m\neq 0}, \tag{3.2}$$

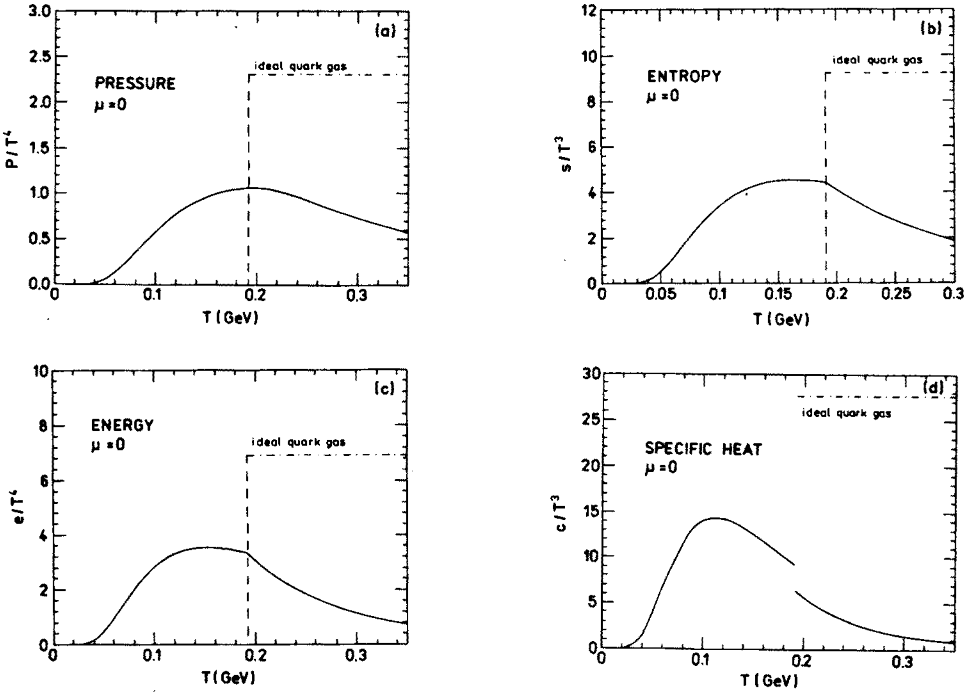

Fig. 3. The thermodynamic functions (a) pressure, (b) energy and (c) entropy densities, and (d) the specific heat, are plotted as a function of temperature for the case $m_0 = 0$ and $\mu = 0$. In each case, the ideal quark gas limit has been indicated by the horizontal dot-dashed line. The vertical line indicates the position of the critical temeprature T_χ.

where $\epsilon_v = \Omega_{\rm vac}$. In the NJL model, $\epsilon_v(0,0)|_{m=0} = -N_c N_f \Lambda^4/4\pi^2$. At finite temperature and density, we may *define* the chiral bag constant to be

$$B(T,\mu) = \epsilon_v(T,\mu)|_{m=0} - \epsilon_v(T,\mu)|_{m\neq 0}\,. \tag{3.3}$$

One sees that

$$B(T,\mu) = 0\,, \tag{3.4}$$

in the region of the phase diagram (Fig. 1) where chiral symmetry is restored. We indicate, for example in Fig. 4 the variation of the bag constant $B(T,0)$ with temperature for $\mu = 0$. The value at $T = 0$, $B^{1/4} = 157$ MeV is comparable with the MIT bag model value ($B^{1/4}_{MIT} = 145$ MeV). The bag constant goes to zero at $T = T_\chi$. This is not a result, but is an immediate consequence of the definition, Eq. (3.3) for B.

We now discuss the calculations of the chiral and quark number susceptibilities, which are shown in Figs 5(a) and 5(b) respectively. The chiral

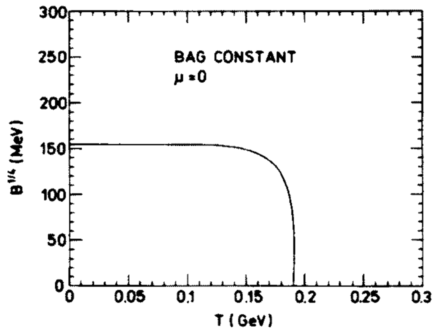

Fig. 4. The bag constant is shown as a function of temperature for the case $m_0 = 0$ and $\mu = 0$.

susceptibility probes the relationship between the current quark mass and the order parameter, and it is seen to display the expected singularity at the phase transition point. One sees that small variations in the current quark mass (the symmetry breaking term) lead to large effects on the order parameter in the phase transition region. Outside of this region, the value of χ_χ is of order one.

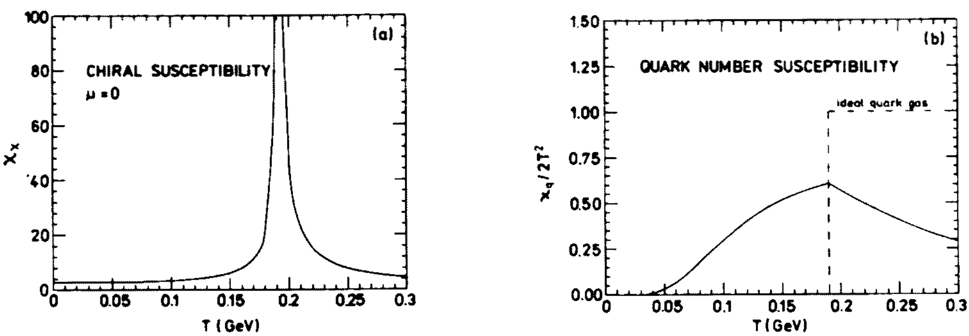

Fig. 5. (a) The chiral susceptibility χ_χ and (b) the quark number susceptibility are shown as a function of temperature for the case $m_0 = 0$ and $\mu_0 = 0$. In (b), the horizontal dot-dashed line indicates the expected ideal quark gas limit, while the vertical dashed line gives the critical temperature T_χ.

Since the quark number susceptibility (Fig. 5(b)) is not related to an order parameter, it is a continuous function. Once again, one would expect that this function should attain its ideal quark gas limit, that is also indicated on the figure. One sees that it undershoots this value significantly, and once again, the decrease in its value for $T > T_\chi$ may be attributed to the finite cut-off used to evaluate this quantity. The values of χ_q below T_χ also requires some explanation. The phase transition should separate a region containing only hadrons ($T < T_\chi$) from the region containing quarks and gluons ($T > T_\chi$). Therefore, one would expect that $\chi_q/2T$ should be small below the critical temperature, but this does not appear to be the case. In this model, however, free quarks can exist below the phase transition temperature, and it is this artifact that produces a non-zero contribution in this region. We refer the reader also to the results from lattice gauge theory [1, 15, 16]. These have not been incorporated into our figure, since they are scaled results, corresponding to fixed values of m_0/T. Finally, we comment that a similar calculation of the chiral susceptibility has been made in Ref. [14].

4. Conclusions

In this manuscript, we have discussed the thermodynamics of a quark plasma, using the NJL model to lowest order. This corresponds to the mean field, or Hartree approximation, and represents the lowest order term in an expansion in the inverse number of colours. The theoretical derivation starts from the thermodynamic potential, since all quantities of interest may be derived form it. Further, a formal extension of this calculation to include fluctuations can be made by noting the the thermodynamic potential for a four-fermion interaction can be expressed in general in terms of the self-energy and the associated Green function of a system via the method of coupling constant integration. This, in turn, leads to a Beth–Uhlenbeck like expression for the fluctuations, in which their contribution to the thermodynamic potential is related to the phase shifts of the scattering states. The details of this calculation can be found in Ref. [11].

To conclude, we comment on the limitations of the mean field calculation. While it has been useful in enabling one to understand the mechanism of chiral symmetry breaking, it is obviously lacking at finite temperature, since the physics that it describes contains no hadronic degrees of freedom. This can be seen directly from Eq. (2.12). However, this is not the physical scenario that we would expect. Fluctuations, on the other hand, introduce the mesonic degrees of freedom in a natural way, and thus indicates the importance of examining their inclusion and effects on the thermodynamic properties of the system.

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