HOT QCD*,**

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I discuss a comprehensive approach to the space-like physics in high temperature QCD in three dimensions. The approach makes use of dimensional reduction. I suggest that this approach is useful for high temperature QCD in four dimensions.

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1. Introduction

The physics of hot and dense hadronic matter has received a lot of attention recently. Due to asymptotic freedom, one expects that at sufficiently high temperatures and/or densities, hadronic matter in thermal and/or chemical equilibrium will behave as a weakly interacting system of quarks and gluons (quark-gluon plasma). As a result, the bulk thermodynamical quantities such as the energy, the pressure, the entropy, ... should display black-body behaviour. The plasma should screen for space-like momenta and display collective behaviour (plasma waves, hydrodynamical waves, ...) for time-like momenta.

The low temperature phase of QCD is dominated by the physics of pions and the general role of the spontaneous breaking of chiral symmetry. The high temperature phase is not. To what extent one expects a phase transition because of the qualitative change in the ground state properties is not clear. Lattice simulation results are not yet definitive on this issue. In so far, they seem to suggest a first order phase transition for four flavours, and a second order phase transition for two flavours. The real world with

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three flavours seems to lie somewhere in between. For pure Yang–Mills the transition is believed to be first order.

In these notes I will not attempt to address the issue of the phase transition in QCD. Rather, I will focus on the some of the aspects of the lattice results at high temperature and try to put forward a general framework for their global understanding. In Section 1, I review some of the lattice results at high temperature. In Section 2, I outline a general strategy for understanding these results based on dimensional reduction. In Section 3, this strategy is applied to three dimensional QCD at high temperature. My conclusions are summarized in Section 4.

2. Lattice results

In the vacuum, chiral symmetry is believed to be spontaneously broken. What this means is that vacuum fluctuations allow for mixing between left and right handed fermions in the massless case. In the chiral limit, the order parameter is $\langle\overline{q}q\rangle$, its behaviour versus temperature is shown in Fig. 1 for four flavours [1]. For temperatures of the order of 150 MeV a substantial decrease in $\langle\overline{q}q\rangle$ is noted.

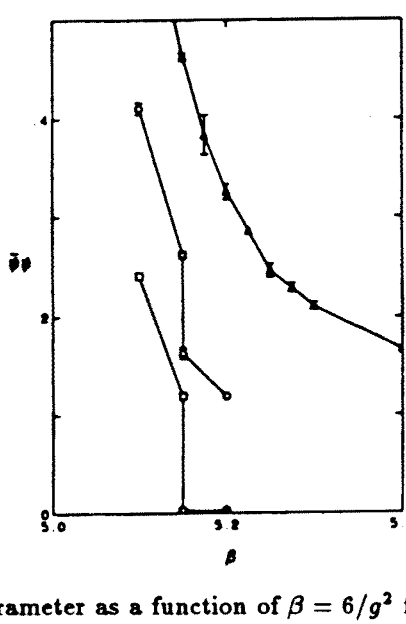

Fig. 1. Chiral order parameter as a function of $\beta = 6/g^2$ for 4 flavours and quark masses $m_q a = .025$ (triangles), .01 (circles) and 0 (squares) [1].

In the vacuum, color is expected to be confined at zero temperature. The order parameter for confinement is the Polyakov loop

$$\langle P \rangle = \langle \mathrm{Tr} \ \exp P \int igA_4 dx_4 \rangle ,$$

where the average is a thermal average, A_4 is the imaginary part of the gauge field and the trace is over one period $\beta = 1/T$ of the imaginary time. $\ln |\langle P \rangle|$ is usually assumed to be the free energy of an infinitely heavy quark. A sharp rise in the expectation value of the Polyakov loop is usually seen following the drop in the chiral condensate [2].

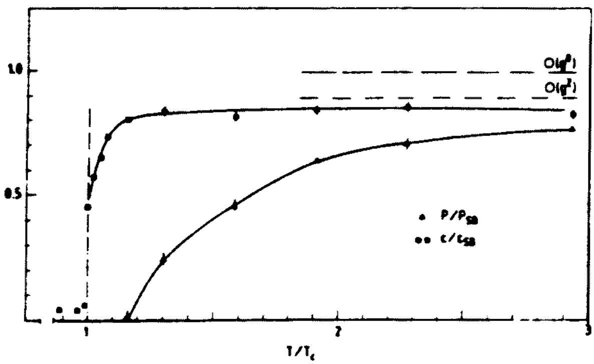

Fig. 2. Energy density and pressure normalized to the black body limit on a finite lattice *versus* T/T_c [3]. The dashed horizontal lines refer to the perturbative corrections to the black body limit.

Lattice measurements of the thermodynamical quantities in QCD show a rapid cross over in these quantities at about 150 MeV. Fig. 2 shows the behaviour of the energy density and pressure versus T/T_c, normalized to the black-body result [3]. In the regime $T_c < T < (2-3)T_c$, there are substantial deviations from the free massless gas limit, $\Theta^{00} = \mathcal{E} - 3\mathcal{P} \neq 0$. The fermionic susceptibilities (singlet and isotriplet) show also a rapid variation in the same temperature range as shown in Fig. 3 [4]. At low temperature the susceptibilities are exponentially suppressed by the mass of the hadronic modes.

The Polyakov–anti–Polyakov (connected) correlation function

$$\langle P(x)\, P^{+}(0) \rangle$$

has been measured both at low and high temperature in the pure Yang–Mills theory. At low temperature, the correlation function displays an area

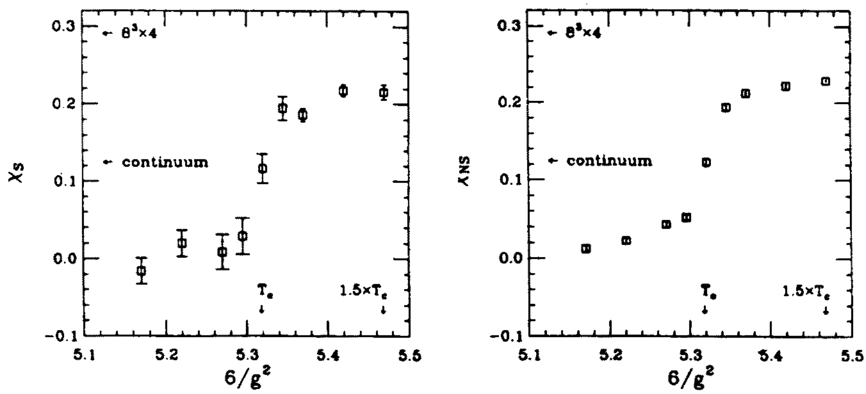

Fig. 3. Singlet (χ_S) and triplet (χ_{NS}) as a function of $\beta = 6/g^2$ for two flavours [4].

law behaviour. Recent simulations at high temperature indicate a screening behaviour [5]. The screening range is estimated to be of the order of the inverse screening mass $1/gT$. The time-like Wilson loop shows an area law behaviour at low temperature, and a perimeter law at high temperature. This is to be contrasted with the space-like Wilson loop which displays an area law behaviour both at low and high temperature. The space-like string tension at high temperature has been found to scale with T^2 [5].

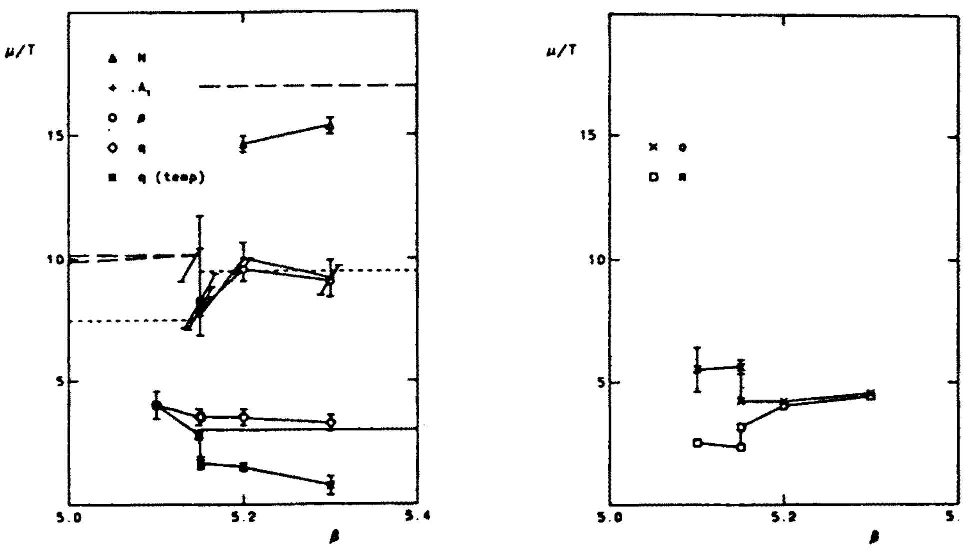

Fig. 4. (a) Screening masses $\mu/T = m_H/T$ as a function of β for four flavours and $m_q a = .01$ [6]. $\beta = 5.15$ is the transition point. (b) the same as (a) but for the π and σ propagators in the spatial directions.

Lattice measurements of static hadronic correlation functions show that the screening lengths asymptote $2\pi T$ (mesons) and $3\pi T$ (baryons) as shown in Fig. 4 [6]. The exception being the pion and the sigma (scalar-isoscalar). The pattern displayed by the measured screening lengths suggests that the thermal state preserves chiral symmetry. The pion-sigma channel, however, suggests the presence of still large fluctuations possibly related with a chiral phase transition. The lattice measurements of the transverse correlations in the hadronic channels are shown in Fig. 5 , for the pion and the rho [7]. Almost no change is detected from the low to the high temperature regime. The correlations are expected to be absent in the free gas limit.

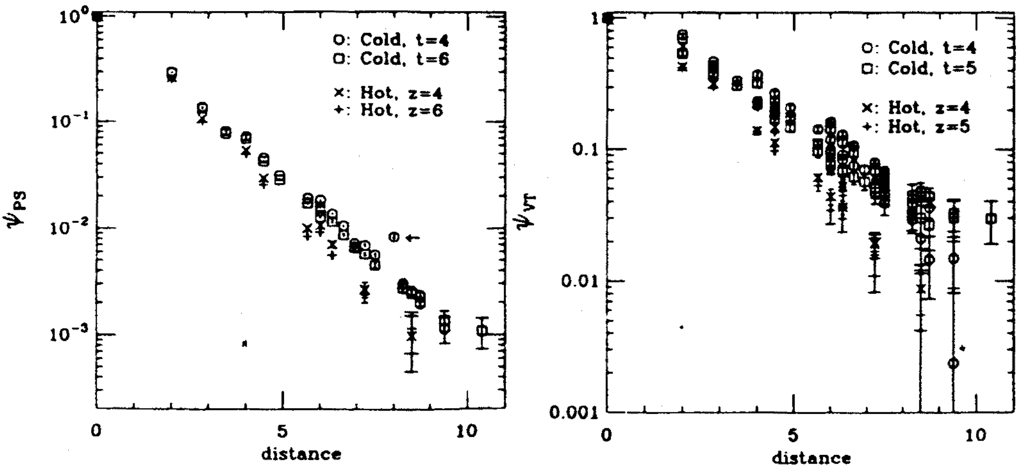

Fig. 5. The pion (PS) and ρ (VT) "wavefunctions" at $T = 0$ and $T = 1.5$ [7].

Recent lattice measurements of the fermionic distribution around the Polyakov line (heavy quark) are shown in Fig. 6 [8]. At low temperature the fermionic distribution is localized around the heavy source in a range of the order of $1/\Lambda$ (QCD scale), and sums up to -1. At high temperature, the distribution is considerably smeared.

The lattice measurements discussed so far are somewhat contradictory. On one hand, they suggest that the bulk thermodynamical quantities such as the energy density, the pressure, the entropy, the susceptibilities, *etc.* when measured at high temperature are consistent with the black body limit. On the other hand, the space-like correlators whether gluonic or hadronic, show strong evidence of correlations and thus an a priori non-black body limit. How come ?

Before answering this question, let me emphasize the following : all the

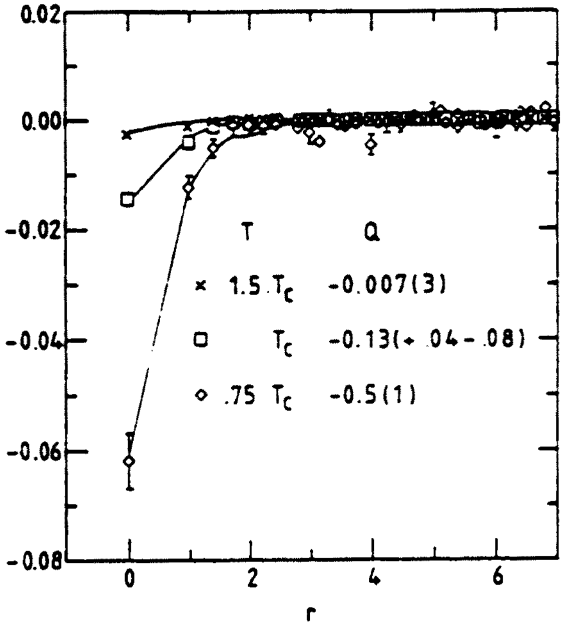

Fig. 6. Quark number density induced by a fixed quark at the origin at three temperatures [8].

lattice calculations performed up to date reflect on space-like physics. They have no bearing on time-like physics. The latter is what matters for rate calculations at high temperature.

3. Hot O(N)

I will now proceed to outline the strategy that I will follow to address the issues raised by the lattice results. First, I will focus on space-like physics, second I will work at $T = \infty$ and then expand in $1/T$. The motto for this approach is dimensional reduction (DR). To illustrate the approach, I will first discuss it for the $O(N)$ sigma model in two dimensions.

Consider the $O(N)$ model at finite temperature. The field $\vec{\phi}$ consists of N bosonic components constrained by $\phi\cdot\phi = 1/g^2$. The Lagrangian density is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\vec{\phi})\cdot(\partial_\mu\vec{\phi})\,. \tag{1}$$

The free energy of the system reduces to the free energy of $N-1$ free bosons, to order g^0, $\mathcal{F}/V_1 \sim (N-1)\,\pi T^2/6$. The correlation function can be worked out in powers of g. To leading order and large distances

$$< \vec{\phi}(x)\cdot\vec{\phi}(0) >= \frac{1}{g^2}\left(1-\frac{1}{2}g^2(N-1)T|x|+\ldots\right)\sim\frac{1}{g^2}e^{-\frac{1}{2}g^2(N-1)T|x|}\,. \tag{2}$$

These results illustrate rather well the points to be emphasized below. Indeed, while the free energy reflects on a free bosonic system to leading order, the correlation function has an exponential fall off with a correlation length of the order of $1/(N-1)g^2T$. This correlation length is due to the long wavelength modes in the $O(N)$ model which are sensitive to the curvature of the S^N manifold. Indeed, the Euclidean partition function reads

$$\begin{aligned} Z(T) &= \int d\vec{\phi}\,\delta(\vec{\phi}^2-\frac{1}{g^2})\,e^{-\int_0^{1/T}d\tau\int dx\,\frac{1}{2}(\partial_\mu\vec{\phi})^2} \\ &= \int d\lambda(\det(-\nabla^2+\lambda))^{N/2}e^{-\frac{1}{g^2}\int\lambda\,dx}\,. \end{aligned} \tag{3}$$

The second equality follows from the integration over the $\vec{\phi}$ field after introducing a Lagrange multiplier λ. In the large N limit, the saddle point approximation to (3) gives

$$N\frac{T}{2\pi}\sum_{-\infty}^{+\infty}\int_0^\Lambda\frac{dk}{k^2+(2\pi nT)^2+\lambda}=\frac{1}{g^2}\,, \tag{4}$$

which can be rearranged to give

$$\frac{NT}{2\sqrt{\lambda}}=\frac{1}{g^2(1+\frac{N}{2\pi}g^2\ln(\frac{T}{\Lambda}))}=\frac{1}{g^2(T)} \tag{5}$$

the right hand side of (5) is the $n=0$ contribution in (4). The nonzero modes in (4) renormalize the coupling constant $g^2\to g^2(T)$.

To enhance the physics of the nonzero modes it is more efficient to take the $T=\infty$ limit first, in which case the high temperature problem for the $O(N)$ model reduces to a quantum mechanics problem (field theory in $0+1$ dimensions). Indeed, on the strip $[0,1/T]\times R$, the bosonic fields are periodic,

$$\vec{\phi}(\tau,x)=\vec{\phi}_0(x)+\sum_{n\neq 0}e^{-i2\pi nT\tau}\vec{\phi}_n(x)\,. \tag{6}$$

If we choose to redefine $\vec{\phi}_0(x)\to\vec{x}(t)T$, then at high temperature only the zero modes contribute to $Z(T)$,

$$Z(T\to\infty)=\int d\vec{x}\,\delta(\vec{x}^2-\frac{1}{g^2T^2})\,e^{-\int_0^{1/T}dt\,\frac{T}{2}\dot{x}^2}\,. \tag{7}$$

This is the partition function of a quantum mechanical particle of mass T on a sphere of radius $R = 1/gT$. The Hamiltonian is $H = \vec{L}^2/2TR^2$ where $\vec{L}$ is the angular momentum in N-dimensions. The eigenstates of the Hamiltonian are hyperspherical harmonics and the spectrum is given by $E_n = n(n + N - 2)/2TR^2$. There is a gap in the spectrum. The constant modes contribute zero to the free energy to leading order, but dominate the correlation function

$$T^2 < 0|\vec{x}(t) \cdot \vec{x}(0)|0 > \sim e^{-E_1 t} \sim e^{-\frac{1}{2}(N-1)g^2 T t}. \tag{8}$$

4. Hot QCD$_3$

I now proceed to discuss the high temperature limit of three dimensional QCD. The dimensional reduction scheme in QCD$_3$ is exact, as opposed to QCD$_4$ which has shortcomings beyond one-loop. Many of the results derived in three dimensions are amenable to four dimensions. Also the three dimensional theory has the advantage of being easier to track down numerically.

Note that the pure Yang-Mills version of QCD$_3$ is also Z_N symmetric. If the deconfinement phase transition is indeed related to the spontaneous breaking of Z_N at high temperature, then the three dimensional theory should display also a phase transition that could be mapped onto the Z_N Potts model. In this case for both $N = 2, 3$ the transition could be second order. In three dimensions parity could be spontaneously broken. If we were to assume that a condensate forms at low temperature and disappears at high temperature, then there is a possibility of a Z_2 transition related to parity restoration. If confirmed, this transition is of the Ising type in two dimensions.

At high temperature QCD$_3$ dimensionally reduces to QCD$_2$ plus a massive Higgs. To see this consider three dimensional QCD on the cylinder $[0, 1/T] \times R^2$

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}^2 + \psi^+(-i\partial\!\!\!/ + g_3\, A\!\!\!/)\psi. \tag{9}$$

In three dimensions QCD is superrenormalizable. g_3 is a dimensionful coupling constant. The gauge fields obey periodic boundary conditions modulo periodic gauge transformations, and the fermions antiperiodic boundary conditions. Explicitly

$$\begin{aligned} A_\mu(\tau, x) &= A_{\mu,0}(x) + \sum_{n\neq 0} e^{i2\pi nT\tau} A_{\mu,m}(x) \\ \psi(\tau, x) &= \sum_{\pm} e^{\pm i\pi T\tau}\psi_{\pm} + \sum_{n\neq 0,-1} e^{i(2n+1)\pi T\tau}. \end{aligned} \tag{10}$$

At high temperature the theory truncates to R^2 with massless magnetic gluons A_i, massive electric gluons A_0 and "heavy fermions" with mass $m = (2n+1)\pi T$ [9],

$$\begin{aligned}\mathcal{L} =& \frac{1}{4}F_{ij}^2 + \frac{1}{2}|(\partial_i - g_3\sqrt{T}A_i)\phi|^2 + \frac{1}{2}m_H^2\phi^2 + V_H(\phi) \\ &+ \psi^+(-i\not{\partial} + g_3\sqrt{T}A + \gamma^3 m + \gamma^3 g_3\sqrt{T}\phi)\psi\,. \end{aligned} \quad (11)$$

We have rescaled the fields to their canonical dimensions in two dimensions, $A_3 \rightarrow \sqrt{T}\phi$, $A_i \rightarrow \sqrt{T}A_i$ and $\psi \rightarrow \sqrt{T}\psi$. The screening mass m_H is infrared sensitive in perturbation theory. Its self consistent determination will be discussed below.

Fig. 7. Induced Higgs potential V_H by dimensional reduction. The wavy lines refer to the magnetic gluons.

The effective potential V_H for the Higgs is induced by integrating over the magnetic gluons as shown in Fig. 7. Dimensional arguments yield [9]

$$V_H(\phi) = \sum_{n\leq 3} C_n \frac{(g_3\sqrt{T})^n}{T^{n-2}}\phi^n\,. \quad (12)$$

At high temperature and for a fixed screening mass (to be specified below), the Higgs potential is subleading in $1/T$. Thus to leading order the Higgs field is massive and interacts solely with the magnetic gluons and the "heavy" fermions. Note that the fermions carry an energy m. However through a pertinent γ_3 rotation of the spinors, this energy can be turned to a mass [9]. This will be assumed throughout.

Fig. 8. Leading contribution to the Polyakov–anti–Polyakov correlator (connected) in the dimensionally reduced theory.

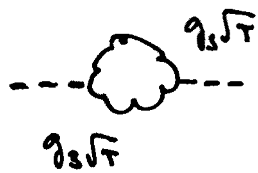

Fig. 9. One-loop contribution to the screening mass.

In the high temperature limit, the Polyakov line simplifies to

$$P(x) = \mathrm{Tr}\, \boldsymbol{P}\exp(ig_3 \int_0^{1/T} A_0(\tau, x)d\tau) \sim \mathrm{Tr}\, \exp(i\frac{g_3}{\sqrt{T}}\phi(x)) . \qquad (13)$$

To leading order in $1/T$ the connected part of the Polyakov–anti–Polyakov correlator is given by

$$\langle P(x)\, P^+(0)\rangle_c \sim \frac{(g_3\sqrt{T})^4}{16T^4}\langle \phi^2(x)\, \phi^2(0)\rangle_c . \qquad (14)$$

This is the correlation function for two electric gluons. The correlator is dominated by the diagrams shown in Fig. 8. The asymptotic form of the correlator in Euclidean space is just the tail of the closest singularity to zero in Minkowski space. This singularity corresponds to a bound state in $1+1$ dimension. The bound state equation is given by

$$-\frac{1}{m_H}\psi''(x) + \frac{1}{2}g_3^2 NT|x|\psi(x) = E\psi(x) \qquad (15)$$

in the regime where $m_H/T \ll 1$ (nonrelativistic limit). The linear potential reflects on the Coulomb potential in $1+1$ dimension. The solutions to (15) are Airy functions. The spectrum follows from the zeros of the derivative of the Airy function $Ai'(-E_n) = 0$ (after proper rescaling). The lowest state

$$E_0 \sim \left(\frac{g_3^2 T}{\sqrt{m_H}}\right)^{2/3} \qquad (16)$$

determines the slope of the correlation function (14). Indeed, at high temperature we expect

$$\langle P(x)\, P^+(0)\rangle \sim K_0((2m_H + E_0)|x|) \sim \frac{\exp(-(2m_H + E_0)|x|)}{((2m_H + E_0)|x|)^{1/2}} , \qquad (17)$$

which is to be contrasted with the free screening correlator (free bubble of Fig. 8)

$$\langle P(x)\, P^+(0)\rangle \sim K_0^2(m_H|x|) \sim \frac{\exp(-2m_H|x|)}{m_H|x|} . \qquad (18)$$

Note that at high temperature the difference between (17) and (18) is in the preexponent and not the exponent, since $E_0/m_H \to 0$. For QCD in four dimensions $K_0 \to K_1$.

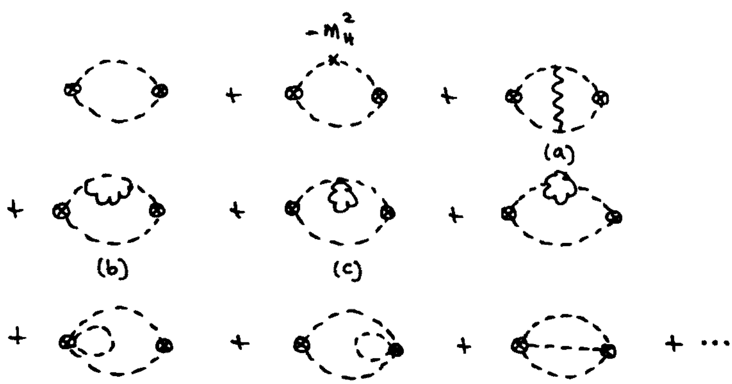

Fig. 10. Two-loop contributions to the Polyakov–anti–Polyakov correlator (connected).

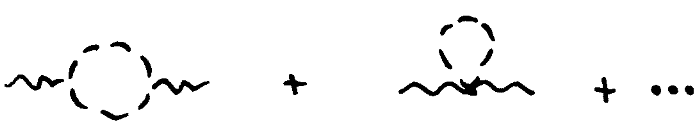

Fig. 11. Magnetic polarisation in the dimensionally reduced theory, to leading order.

In QCD_3, the screening mass $m_H \sim g_3^2 T$ is infrared sensitive and gauge dependent in perturbation theory as indicated by the graph of Fig. 9. A self-consistent and gauge independent analysis of the screening mass can be performed from the loop-expansion of the Polyakov–Polyakov correlator at high temperature. The two-loop contribution to (14) is shown in Fig. 10. The cross refers to the $-m_H^2$ insertion in the self-consistent (Hartree) definition of the electric mass, where the electric propagators carry a mass m_H^2. All the infrared and ultraviolet divergences cancel to two-loop. The finite contributions from diagram 10a and 10b are scale (μ) dependent : $g_3^2 NT(a + b\ln(m_H/\mu))$. The finite contribution of diagram 10c is also scale dependent $g_3^2 NT(\overline{a} + \frac{1}{2\pi}\ln(m_H/\mu))$. If we choose $\mu = m_H$ then the contribution from 10c can be made logarithmically large compared to the con-

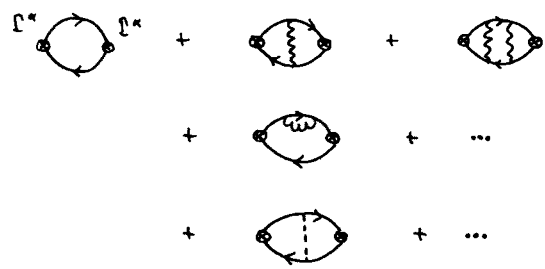

Fig. 12. Hadronic correlators in the dimensionally reduced theory.

tributions from 10a and 10b in the large temperature limit. This large contribution can be reabsorbed in a self-consistent definition of the electric mass in a Hartree-type approximation,

$$-m_H^2 + g_3^2 NT \frac{1}{2\pi} \ln\left(\frac{T}{m_H}\right) = 0 \, . \tag{19}$$

This result was first obtained by D'Hoker [10].

The fact that magnetic loops still obey an area law has a simple explanation in high temperature QCD in three dimensions. Indeed, to leading order the static part of the magnetic gluon correlator in static-axial ($A_2 = 0$) gauge follows from the diagrams of Fig. 11 ,

$$\Delta_{ij} \sim \delta_{i1}\delta_{j1}\delta(x_1)\left(a|x_2| + \frac{c}{\sqrt{Mx_2}}\mathrm{e}^{-M|x|}\right) , \tag{20}$$

where $a = g_3^2 NT/24\pi m_H^2$, $b = g_3^2 NT/240\pi m_H^4$, c an arbitrary coefficient and $M^2 = (1-a)/b$. In (20) only the leading temperature dependent part has been retained. The contribution of (20) to the space-like Wilson loop is due to the unscreened part of the magnetic gluon propagator. The result is $W_{xy} \sim \mathrm{e}^{-\sigma_{xy} A}$, where A is the area in the xy-plane and $\sigma \sim a$ the *temperature* dependent string tension to leading order. This result is gauge invariant. On the other hand, note that the magnetic-magnetic correlator is exponentially screened. At large distances, it is dominated by the second term of (20) in the static-axial gauge

$$\langle B(x)\cdot B(0)\rangle \sim \frac{e^{-M|x|}}{\sqrt{Mx_2}}\,. \tag{21}$$

The linear term drops by taking derivatives. This result is gauge dependent. Note that an area law behavior of the space-like Wilson loops is still consistent with an exponential fall-off in the correlation function of magnetic fields. I expect this to extend to four dimensions as well.

The static hadronic correlators can be analysed in the dimensionally reduced theory, by treating fermions as infinitely heavy (non-relativistically). To leading order in $1/T$

$$\begin{aligned} C_{\alpha,\alpha}(x) =&\frac{1}{\beta^2}\left\langle\int_0^\beta d\tau\psi^+\Gamma^\alpha\psi(\tau,x)\int_0^\beta d\tau'\psi^+\Gamma^\alpha\psi(\tau',0)\right\rangle\\ &\sim\left\langle\psi^+\Gamma^\alpha\psi(x)\psi^+\Gamma^\alpha\psi(0)\right\rangle\,. \end{aligned} \tag{22}$$

The fermions of the second part of (22) are restricted to the lowest Matsubara modes and carry a mass $m=\pi T$ after a γ_3-rotation. The dominant contribution to the hadronic correlator (22) follows from the diagram of Fig. 12. The Higgs and fermion insertions are subleading in the temperature. The large x-asymptotics of (22) is again given by the closest singularity to 0 in Minkowski space. In Minkowski space the quark and antiquark are massive (nonrelativistic) and interact via a Coulomb potential in $1+1$ dimensions. The bound state equation is

$$-\frac{1}{m}\psi''+\frac{e^2}{2}|x|\psi=E_\alpha\psi\,, \tag{23}$$

where $e^2=C_Fg_3^2T$ and C_F the value of the Casimir in the fundamental representation. The screening lengths are $m_\alpha=2m+E_\alpha$, and characterize $C_{\alpha\alpha}(x\to\infty)=e^{-m_\alpha|x|}$. For QCD_3 [9],

$$m_\alpha=2\pi T+\frac{3}{2}\left(\frac{C_F^2g_3^4T}{4\pi}\right)^{1/3}\,. \tag{24}$$

The wavefunctions are Airy functions with a size of the order of $1/(g_3T)^{2/3}$. These wavefunctions are directly related to the transverse correlations. For four dimensional QCD, the screening lengths can also be estimated [9]. The reduced wavefunctions directly relate to the correlations observed on the lattice as described above. The present discussion provides a simple explanation to the fact that while the screening lengths asymptote "free" quark values, they are in fact reflecting on strong correlations space-like. The correlations are subleading in the screening masses.

The size of the fermionic distribution around a heavy source has a simple understanding in the dimensionally reduced theory. Indeed, at high temperature

$$\langle P(0)\overline{\psi}\psi(x)\rangle \sim -\frac{g_3^2}{T}\langle \phi^2(0)\overline{\psi}\psi(x)\rangle \,. \tag{25}$$

At large temperature the Polyakov line is a source for the Higgs field. The correlation function (25) measures the fermionic distribution in the Higgs cloud. The range of the correlation is of the order of the inverse of twice the electric length $1/2m_H$.

While the above considerations have provided a large body of evidence for why the space-like physics at high temperature reflects on correlations, it remains puzzling why the bulk properties of the high temperature QCD phase are consistent to some extent with the black-body description. I will show now that this is in fact a consequence of the fact that the bulk quantities reflect on *all* distance scales. For that consider the isoscalar fermionic susceptibility

$$\begin{aligned}\chi(T) &= \int_0^{1/T} d\tau \int d^2x \langle \psi^+\gamma^3\psi(\tau,x)\psi^+\gamma^3\psi(0,0)\rangle \\ &\sim T\int d^2x\langle \psi^+\sigma^3\psi(x)\psi^+\sigma^3\psi(0)\rangle \,. \end{aligned} \tag{26}$$

All fermionic modes contribute to $\chi(T)$ to leading order. Indeed [11],

$$\chi(T) = \frac{N_cT}{\pi}\sum_{H=-\infty}^{+\infty}\sum_{n=0}^{+\infty}\sin^2(\frac{n\pi}{2})\frac{G_{nH}^2}{\mu_{nH}^2}\,. \tag{27}$$

The μ's are the invariant masses for bound quark-antiquark in $1+1$ dimensions with masses $m_F = (2F+1)\pi T$, and G_{nH} the form factors following from the 't Hooft equation. Asymptotically $\mu_{nH}^2 \to \pi\sqrt{2\pi n\sigma}/2\sigma$, $\sigma = g_3^2NT$ and $G_{nH} \to \pi/\sqrt{2}$. The density of states per unit energy in (27) grows linearly with the energy $dn/dE \sim \sqrt{E^2 - 4m_F^2}/\pi\sigma$. With this in mind, the sums in (27) can be done exactly, leading to [11]

$$\chi(T) - \chi(0) \sim \frac{N}{\pi}\int \frac{dz}{e^{z/T}+1} = N_c\left(\frac{\ln 2}{\pi}\right)T\,, \tag{28}$$

which is the free gas result to leading order. I believe this result to extend to the bulk energy density, pressure and entropy.

5. Conclusions

A global understanding of the lattice calculations can be achieved within the context of dimensional reduction at very high temperature. Even though the analytical calculations were performed for high temperature QCD_3, they can be generalized in a straightforward way to QCD_4. However, the arguments are not rigorous in the latter.

We have seen that the high temperature QCD phase is characterized by strong correlations space-like. These correlations are dominant in all correlators. Bulk quantities, however, are sensitive to all correlated modes. The latters sum up to the expected black body limits, a result familiar from asymptotic freedom when hadronic wavefunctions are probed in deep Euclidean region.

What does this imply for the high temperature phase time-like ? I really do not know. However, since the correlators are not free space-like why should they be time-like ?

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REFERENCES

[1] B. Petersson, *Nucl. Phys.* (Supp) **30B**, 66 (1993).
[2] J.B. Kogut, *Phys. Rev. Lett.* **56**, 2557 (1986).
[3] F. Karsch, *Nucl. Phys.* (Supp) **9B**, 357 (1989).
[4] S. Gottlieb, *et al.*, *Phys. Rev. Lett.* **59**, 2247 (1987).
[5] B. Peterson, private communication.
[6] R.V. Gavai *et al.*, *Phys. Lett.* **241B**, 567 (1990).
[7] C. Bernard *et al. Phys. Rev. Lett.* **68**, 2125 (1992).
[8] C. DeTar, *Nucl. Phys.* (Supp) **30B**, 66 (1993).
[9] T.H. Hansson, I. Zahed, *Nucl. Phys.* **374B**, 177 (1992).
[10] E. DeHoker, *Nucl. Phys.* **201B**, 401 (1982).
[11] M. Prakash, I. Zahed, *Phys. Rev. Lett.* **69**, 3282 (1992).