

## PERTURBATION THEORY AND RELATIVE SPACE\*

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The validity of non-perturbative methods is questioned. The concept of relative space is introduced.

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## 1. Introduction

Both success and failure have been spectacular in present day field theory and particle physics. High energy experiments, so far, confirm the Standard Model to an astonishing degree. That same model has many arbitrary features; all attempts at understanding these features have failed abysmally. The origin of the particular symmetries of the Standard Model,  $SU(3) \times SU(2) \times U(1)$  is unknown. We have no idea why there are three generations of particles, nor do we have a clue as to the particular multiplets chosen by nature. Coupling constants and masses of the model are unexplained. And so on.

The only theoretical successes in explaining the Standard Model are certain consequences of the  $SU(5)$  grand unification scheme. That theory provided us with numerical values for the bottom quark mass and the weak mixing angle, with reasonable agreement with experiment. Furthermore, massless neutrino's are natural in that scheme. At some point the model fails, and it must also not be overlooked that it is at most a very partial solution to our problems: it leaves most of the basic questions unanswered.

The idea of renormalizable field theory, so successful in its application to the Standard Model, has resulted in further theoretical advances, notably supergravity and string theory. Unfortunately, not one single question of

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the type cited above has been answered, and the theories show unmistakably signs of that same old malady that we will lump under the name "epicycles".

The concept of naturalness is usually cited as the underlying motivation for supersymmetry. We will challenge that concept, and in any case need to point out that there is nothing natural about the development of the theory itself. Its main success is its agility in dodging the facts. The dubious explanation of the convergence of the three scaling coupling constants into a single point can not be taken seriously. It is just another fit, using some of the many free parameters.

The list of failures is as long as the list of attempts, and it is pointless to discuss the often extremely ingenious constructs. However, it must be noted that there are some real difficulties in the Standard Model, quite apart from questions concerning the origin of the particular features of that model. The most pronounced of these concern the cosmological constant and strong  $CP$  violation. Despite great theoretical and experimental efforts (axions) no reasonable solution to these problems has been offered. More in general, failure seems automatic whenever non-perturbative aspects of the Standard Model are considered.

In the background, as always, lurks non-renormalizable gravitation with its black and other holes. The Higgs system and the associated problem of the cosmological constant evokes the impression that there is a deep and fundamental connection that so far remains completely hidden to us. It certainly appears that the problem is too difficult for us, and very likely, only experiment can help us to gain insight.

A re-examination of some very basic concepts appears timely and expedient. We must clarify the presently achieved description of nature for the simple reason that there is more than one viewpoint. There are different formalisms, presumably describing physical reality, but it is not clear that the descriptions are actually equivalent.

## 2. Complementarity

This century has seen the introduction of two great theoretical creations, relativity and quantum mechanics. Almost from the start, conflict has surrounded the meeting of the two, and in fact persists to this day. It must be understood that the problem of the cosmological constant is precisely a consequence of the basic features of both theories. Einstein's theory of gravitation, by necessity, introduces the metric of the underlying space as a free parameter, the cosmological constant. Within the context of classical general relativity that constant may be chosen to be zero, even if this appears to us today as an arbitrary choice. Quantum mechanics however radically changes the situation, simply because it affects that constant. If

it is initially chosen to be zero radiative corrections will change that. The present day observation of a very small if not zero cosmological constant is in flagrant contradiction with the scale of the corrections suggested by quantum theory.

In first instance, in the twenties, the conflict manifested itself through the famous discussions between Bohr and Einstein. While on the face of it Bohr appeared to have the upper hand, it is nonetheless clear that there is more to the issue. Bell's inequalities and the associated literature testify to that. Einstein, to the very end, has refused to accept the quantum concepts, and it may well be that he has perceived the fundamental conflict more clearly than anyone else.

In the Copenhagen philosophy there is the concept of complementary, which we will interpret to mean the following. There are two alternative descriptions of particles, namely the particle description and the wave description. The particle description is by means of momentum and energy, the wave description concerns location in time and space. Basically, one is the Fourier transform of the other, and we could simply state that either description is complete and supposedly fully equivalent to the other. A particle may be specified by a superposition of momentum states or by a wave function in space-time (coordinate space). Either space, momentum space or coordinate space, may be used to describe the situation. We may use this or that representation in setting up Hilbert space.

But is it really true that momentum space and coordinate space are equivalent? Here is the basic conflict: in Einstein's concept of gravitation they are not possibly equivalent. In general relativity space-time plays a very particular role, it is intimately connected with gravitation. Gravitation is interpreted as nothing else but the structure of space-time. In turn, that structure is determined by the distributed matter.

Why then, if the description in momentum space is equivalent, do we not introduce a metric in momentum space? Is that space flat by definition? Putting the issue this way the basic conflict between gravitation and quantum mechanics becomes obvious. Gravitation is particular to space-time. By its very nature general relativity assigns properties to space-time. But it is completely unclear whether a definition of physics in momentum space would adhere to these assumed properties of space time. It may conflict.

In order to investigate this question we must clarify our description of physical reality. We can not, on the one hand, do gauge theories and renormalizable field theory in momentum space, and on the other hand solve classical equations of motion and play with black holes in coordinate space. In other words, we must realize that these descriptions may not be equivalent, and that we may have to make a choice depending on the agreement with observed physics.

### 3. Theoretical framework

For the moment we will leave gravitation and concentrate on quantum field theory. Here, today, we have three seemingly equivalent descriptions. They are:

- The canonical formalism, involving Lagrangian, coordinates and their conjugate momenta, and an  $S$ -matrix defined in terms of time-ordered products of operators in Hilbert space.
- The path integral formalism where the  $S$ -matrix is defined as a sum over paths in coordinate space.
- The purely pragmatic description of the  $S$ -matrix in terms of Feynman rules with in addition the prescriptions of dimensional regularization. We will call this the dimensional formulation.

The third prescription is totally perturbative, but otherwise complete in itself. Also, it may assure us with respect to anomalies. Both the canonical and path integral formalism are at least in principle not restricted to the perturbative domain, but on the other hand they need in addition a regulator method. Usually somewhere along the line dimensional regularization is used, but it must be stated that that method can not be formulated within either canonical or path integral formalism. The conventional wisdom is that one can use any regulator method desired and that with the appropriate counter terms the results are independent of the particular regulator method used. Ward identities are the crucial instruments. Then one may use any method that can be formulated in either scheme, be it Pauli-Villars regularization, or a finite lattice spacing, or whatever.

The crucial question is whether the conventional wisdom is correct. The answer is that it appears correct, but only within the context of renormalizable perturbation theory, that is within the domain of the dimensional formulation.

The question of regularization scheme is an old one, and we may perhaps briefly reflect on that. Discovering the basic difficulty of infinities in field theory, Lorentz speculated that perhaps the electron has an extended structure in space-time. The idea of renormalization is that many physical properties do not depend on the details of that structure, and that the only consequence is a redefinition of the mass of the electron. However, it must be realized that this idea has become totally untenable in modern gauge field theory. The problem is that any attempt at regularization through a finite extension in space-time hopelessly conflicts with gauge invariance. To make it explicit, suppose the electron coupling to vector bosons involves a form factor (the Fourier transform of its spatial structure). That, through Ward identities for the case of vector boson electron scattering, has its implications for the three vector boson vertex, but it is not obvious that these

implications can be put in terms of a form factor for the three vector boson vertex (it can not). And Ward identities for vector boson scattering, that involve also the four point vector boson vertex, are conflicting with the assumption of form factors for the vertices involved. In other words, the intuitive idea that a finite space-time structure of particles would ultimately solve the problems of infinities in field theory is removed farther than ever from realization.

Another possible point of view is that all particles are basically massless and acquire mass through an essentially low energy mechanism. The divergence structure of the theory is then the divergence structure of a massless theory. No one has achieved to regulate that in a physically appealing way. It is interesting to note in this context that in the Standard Model indeed all particles, fermions as well as vector bosons, acquire their mass through the Higgs mechanism. The way the Higgs boson itself acquires mass is less clear.

It should be added that the renormalization prescription has become much more decoupled from the physical situation. While in Lorentz's view all difficulties could be concentrated in a simple physical picture, that of an extended electron, present day renormalization is much less directly related to any physical image. The renormalization prescription is now simply a matter of fixing parameters, accidentally involving infinities, and no one associates that with any particular physical visualization. Again, a view based on a particular perspective in coordinate space fails, in fact is contradictory to the theory.

Because of these considerations one may feel inclined to break with all these vague hopes and insights. As a possible alternative to Hamiltonian formalism or path integrals we therefore bluntly take as starting point perturbative field theory, defined in terms of Feynman diagrams, using dimensional regularization, i.e. the dimensional formulation. Thus loop momenta are in  $n$  dimensional space, and at the end the limit  $n = 4$  must be taken. Fitting the free parameters to the experiment provides excellent agreement with experiment, from Coulomb law via Lamb shift to LEP observations.

At this point it must be mentioned that the dimensional formulation is not free of problems either. There are situations where perturbation theory is non-convergent, and it is not clear how to deal with that. A point in case is the non-convergence of quantum chromodynamics in the infrared. This is usually referred to as the confinement problem, a very specific coordinate space type qualification. Indeed, in momentum space one has a problem. But let us not forget that no solution of the confinement problem has been offered so far. Perhaps a non-coordinate point of view is more productive.

#### 4. Space-time

In the dimensional formulation no assumptions concerning space and time occur, they do not occur altogether. This is the central point. Space and time do not occur in any way in this definition of physical reality. We repeat and emphasize: space and time do not occur in the dimensional definition of physical reality.

It follows that behaviour in space-time is solely defined by Fourier transformation. It must be understood that the Fourier transform of  $n$  dimensional momentum space (with continuous  $n$ ) is not simply  $n$  dimensional coordinate space. It is unclear how to define the Fourier transform. Obviously then the dimensional formulation is in contradiction with the assumption of four dimensional coordinate space. Whether in the limit  $n = 4$  the conflict resolves is another matter. However, this formulation of physical reality is not equivalent to canonical or path integral formulations. As argued above, these latter formalisms need dimensional regularization to show their consistency, obviously restricted to the perturbative regime. In other words, the other formalisms are well defined insofar they are perturbative. We do not know to what extent non-perturbative results are true.

In the dimensional formulation, therefore, space and time have no a priori existence. They exist exclusively as Fourier transforms of a momentum space description. Space and time are defined solely relative to momentum space. To what extent absolute properties can be ascribed to momentum space is not our concern here, nor is there any need. But we must not assign absolute properties to space-time as they may simply conflict with the starting point. For example, a cosmological constant assigns a metric to space-time as a boundary condition, in the absence of matter. However, how can we assign a metric to a space defined through Fourier transforms? What is a boundary condition in a Fourier transformed space? To make it very explicit, consider the idea that one would impose a boundary condition in momentum space, or define a metric in that space. It just makes no sense.

Within the dimensional scheme we must abandon the idea of the absolute existence of a space-time continuum. There are arguments leading up to the idea of an absolute time continuum, but this is not the moment to discuss that. The statement that if a particle moves from one point to another, it must necessarily pass through some series of points in between is meaningless. Of course, some kind of continuity property can certainly be derived, but it is not true a priori. It is not an intrinsic property of space-time, but at best a derived property.

The idea of a vacuum expectation value never really occurs in the dimensional formulation. We simply start with Feynman rules that correspond to a Lagrangian with the Higgs field shifted by a constant. The issue surfaces only when discussing gravitation and the cosmological constant.

The battered concept of causality loses much of its meaning in the dimensional viewpoint described here. Unlike unitarity it becomes a derived property, no fundamental assumption. The mathematical properties that guarantee unitarity actually largely imply locality as well. But then, who cares about causality in some mathematical space? Indeed, quantum mechanics has always been very ambiguous on this point, and the situation becomes intolerable when gravitation and its black holes enter the discussion. Recent arguments in the literature point to drastic difficulties.

It is curious to note that Einstein was well aware of the concept of absolute space-time in relation to his theory. In certain editions of his book [1] "The Meaning of Relativity" he makes some general remarks on the issue. The discussion is very interesting, but any partial quote would do injustice to the whole and we leave it at this. It appears quite possible that Einstein suspected that his philosophy was at odds with the ideas of quantum mechanics. That could explain his reluctance towards accepting quantum theory.

It is perhaps necessary at this point to state explicitly that most consequences of Einstein's theory remain true, also in the dimensional formulation. The philosophy changes, and furthermore cosmology needs re-examination. Black holes are probably nothing else but commercially viable figments of the imagination. Unfortunately, Einstein's beautiful dream, to formulate forces as properties of space-time, has already been next to untenable for quite some time, given the multitude of forces that we have to deal with. Few would reject the dimensional formulation because it is at odds with a geometric interpretation.

The point of view arising from the dimensional formulation is utterly alien to our usual intuition concerning space-time. Yet there is no logical reason that can be put against it; moreover the usual troubles of quantum mechanics, such as wave function collapse, Bell's inequalities etc. rather obviously point in the direction described. And let us not forget the old ugly aspect of curved space: general coordinate transformations have no half integer spin representations. That beauty defect disappears. Momentum space is perfectly flat. Gauge invariance, *i.e.*, Ward identities, dictate the graviton-fermion interactions. The equivalence principle is a consequence of gauge invariance and remains as true as ever. Because of gauge invariance, the definition of length and time measurement must involve the gravitational field, ultimately to the same effect as if operating in curved space-time [2, 3].

## 5. Non-perturbative solutions

The question is now to what extent non-perturbative results remain true in the dimensional approach. Let us be clear about the extent to

which non-perturbative results in Hamiltonian or path integral formalism have been derived and used. By necessity, since radiative corrections need regularization, most (but not all) applications are based on the tree level approximation. That produces classical electromagnetism and the general theory of relativity. There may or may not be differences with the dimensional formulation, depending on whether some extension of perturbation theory may be made plausible. Furthermore, path integrals have been used as tools for numerical calculations, but also for certain theoretical considerations. It is hard to say whether this proves anything one way or the other. It would be quite interesting if the gap between perturbation theory and the strong coupling limit within the path integral approach could be bridged. In short, while many blindly accept the validity of the Hamiltonian or path integral formalism, there is really no objective basis on which to accept the validity in the non-perturbative region. This in addition to all kinds of problems in the case of path integrals with respect to chiral fermions.

Various different situations must be envisaged. We first turn to ordinary bound states such as the atom. Do such solutions survive in a perturbative approach?

As is well known the Schrödinger, or rather the Lippmann-Schwinger equation, can be derived from diagram theory by means of a partial sum of diagrams. In this case the diagrams to be summed are the ladder diagrams. In the approximation of low momentum transfer the Lippmann-Schwinger equation for situations such as electron proton scattering can be derived. Bound states can be understood from the analytic continuation of the amplitude to negative energies, and these states manifest themselves as poles in the complex energy plane along the negative real axis. This type of analytic continuation is analogous to that encountered in connection with unstable particles. Also there one has a perturbation series that diverges in a particular momentum region; there the Dyson summation provides us with a solution whose perturbation expansion coincides with perturbation theory wherever that expansion converges, and which is otherwise an analytic continuation to the region where perturbation theory fails. We will accept such solutions as quasi-perturbative solutions. In that sense then the more obvious non-perturbative situations such as atoms and planetary systems can still be understood from the perturbative point of view. And nuclei would also fall in this class.

The situation becomes more difficult with respect to other solutions of the field theoretical equations of motion. Usually one writes formally equations of motion in coordinate space, and in the dimensional approach we cannot accept those solutions unless they somehow can be understood from the point of view of momentum space. The above discussion exemplifies that. There is now one type of argument that is completely beyond any



momentum space formulation, and that is the treatment of a  $\theta$  term as commonly introduced to derive  $CP$  violating effects in quantum chromodynamics [4]. Such a term is a total derivative and as such has no counterpart in momentum space. The instanton vacuum with its winding number requires the concept of absolute space and boundary conditions to that space. From a perturbative point of view, with space defined mathematically through Fourier transformation, such solutions are incomprehensible. In general, assigning absolute properties to the vacuum in coordinate space is meaningless in momentum space. In the perturbative approach we must reject such constructs as based essentially on the prejudice of an absolute space. Thus in the dimensional formulation there is no strong  $CP$  problem as there is no such thing as the instanton vacuum.

## 6. Dimensional regularization

At this time we do not want to go into technical details concerning dimensional regularization, but feel nonetheless compelled to state that there are some non-trivial problems there that have not yet been settled satisfactorily, at least in our opinion. There is of course the usual problem of treating  $\gamma^5$ ; it is our understanding that this problem is essentially understood and that there is no difficulty there [5]. Here we will not enter into any discussion on that, and neither will we consider related issues such as chiral invariance. There are however further problems in case of fermion lines that end in external lines, and that may also be part of closed loops. Then it is not clear how  $\gamma$ 's are to be treated, since one is dealing with a string terminated by spinors that have no trivial generalization to continuous  $n$ . We refer to the literature for a more extensive discussion of this problem [6].

Another problem that arises concerns the  $CPT$  theorem. In spinor space the transformation matrix corresponding to the  $CPT$  transformation is  $\gamma^5$ , and we may have a problem. Stated otherwise, in  $n$  dimensions the  $PT$  transformation involves only the first four components of vector quantities and not those beyond the fourth dimension. Thus components of loop momenta beyond the fourth behave anomalously under  $CPT$ . However, the deviations arising from that will go to zero in the limit  $n = 4$ , and appear not to result in observable consequences if all singularities at  $n = 4$  have been subtracted properly<sup>1</sup>.

Let us close this section with a remark. One might think that  $CPT$  in  $n$  dimensions can be defined as reversal of all coordinates. But that poses

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<sup>1</sup> In case of anomalies there is seepage out of four dimensional space due to an unsubtracted pole.

a problem: going from odd to even dimension the determinant of the  $PT$  transformation changes sign, and in odd dimensions the  $PT$  transformation is not continuously related to the identity. In any case, in the usual formulation, in four dimensions, the  $CPT$  transformation is represented in spinor space by  $\gamma^5$ , and the problem of generalizing  $\gamma^5$  is known to all. Now perhaps this can be remedied by limiting oneself to transformations that involve only one of the spatial-like coordinates and in addition the time-like coordinate, but we will not discuss that here.

## 7. Naturalness, supersymmetry and the mass equation

In a previous publication [7] we have argued for a mass equation resulting from the requirement that there be no quadratic divergences. The meaning of a quadratic divergence is completely unclear within the method of dimensional regularization, and the treatment offered was based on a vague analogy between the dimensional method and a momentum cut-off scheme.

At this point we would like to distance ourselves from such an approach. Quadratic divergencies do not exist within the dimensional formulation. The concept of naturalness with respect to scalar particle masses needs revision. There are no large corrections related to quadratic divergencies as these divergencies do not exist in the dimensional method. Of course, corrections to scalar particle masses involving masses of heavier particles could still occur, but that is a quite different subject. Only within a well defined model can conclusions be drawn.

Supersymmetry has evolved on the premise that this solves the naturalness problem with respect to the Higgs mass. That is really not a very strong argument if we realize that this requires the idea that somehow quadratic divergencies become finite through some physical cut-off mechanism, and that there is a scale associated with that. The singular non-success of supersymmetry so far supports, and in fact to some extent produced our negative view. Within the dimensional approach it is simply not clear what purpose would be served by a supersymmetric theory. That is no proof against a possible existence, but it certainly weakens the case.

The question is if there is any equation left relating the top and Higgs mass. Very speculatively we would like to argue that tadpole type diagrams should add up to zero in view of difficulties with the cosmological constant. In lowest order that results in the same equation as before. If both top mass  $m_t$  and Higgs mass  $m_H$  are large with respect to all other masses the relation is roughly  $m_H = 2m_t$ . We emphasize the highly speculative nature of this relation.

## 8. Gravitation

The theory of gravitation can be formulated as a gauge theory, and that is of course precisely as it is always done in quantum field theory. The trouble is that the theory is non-renormalizable, and the dimensional scheme offers no new insight here. Thus we are facing an unsatisfactory theory from the start, no matter what starting point. Nonetheless we may perhaps spend some words on the question of black holes and the problem of the cosmological constant. The arguments may well be naive and incomplete.

In the traditional formulation of gravitation, black holes are an unavoidable consequence of Einstein's equations. In the dimensional formulation, lacking coordinate space, we may ask to what extent such solutions remain valid. Can they somehow be seen as analytical continuations of solutions valid in a perturbative domain, like the bound states discussed above?

First, the approximation in terms of ladder diagrams, so successful in understanding the hydrogen atom. Here there is a peculiar difficulty with gauge invariance: ladder diagrams do not form a gauge invariant set. In the case of electromagnetism one can include crossed ladders, and then the result is gauge invariant. However, in the case of gravitation the approximation is never gauge invariant because of gravitational self-coupling. So, even to explain planetary systems, one must somehow approximate further, and we leave it to the reader to realize precisely the approximations involved in the standard classical approach.

However, as a consequence we have no description of the bound state problem that can be extended or analytically continued to the case that the gravitational self-coupling becomes important.

In terms of the gravitational field itself we may consider the solutions of the classical equations of general relativity. For a Schwarzschild black hole with radius  $R$  the spatial components of the gravitational tensor in cartesian coordinates are  $h_{jk} = \frac{x_j x_k}{r^2} \frac{R}{r-R}$ . The Fourier transform of this is non-existent, and also cannot be defined as a function of  $R$  in some region and then continued to the region of positive real  $R$ .

The arguments presented here are certainly not complete. The fields  $h_{\mu\nu}$  are not gauge invariant, and perhaps there is a choice of gauge (choice of coordinates) in which the Fourier transform exists or can be defined in some way. Nonetheless it is tempting to deny the existence of black holes, and in any case, it must be realized what the underlying assumptions are in the traditional approach. It might be added that the remarkable absence of black holes outside the domain of astrophysical speculation tends to support the idea of relative space and the perturbative approach.

Concerning the cosmological constant problem, at first sight one might think that the problem is non-existent. Given that there is no such thing as

the coordinate space vacuum there is no way of assigning properties such as curvature to that. Unfortunately the problem surfaces in a different form.

The gravitational Lagrangian is of the form:

$$\mathcal{L}_{\text{grav}} = -\sqrt{g} (R + \lambda) ,$$

where  $g$  is the determinant of the metric tensor  $g_{\mu\nu}$  and  $R$  is the Riemann scalar. The quantity  $\lambda$  is the cosmological constant, and radiative corrections affect it. When expanding  $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$ , where  $h_{\mu\nu}$  is the gravitational field, a tadpole type term  $\lambda h_{\mu}^{\mu}$  arises. The conventional field theoretical treatment of terms linear in the field is to perform a shift,  $h_{\mu\nu} \rightarrow h_{\mu\nu} + a_{\mu\nu}$ , choosing  $a_{\mu\nu}$  such that the linear term disappears. Conventionally, as for example in the  $\sigma$ -model, the quantity  $a$  is a constant, but in the case of gravity the solution is space-time dependent, that is  $a_{\mu\nu} = a_{\mu\nu}(x)$ . The equation arising from the condition that no linear term arises is precisely the Einstein equation for the case of no matter; in more conventional terms of space-time it is the solution corresponding to a curved universe. One might say that there is a very strong background gravitational field (the field  $a_{\mu\nu}(x)$ ) present in the vacuum, and physical processes evolve against this background.

In the perturbative perspective this is all but transparent. Our main problem is with unitarity. The  $S$ -matrix arising when considering the theory with a momentum dependent shift of the gravitational field is not evidently unitary. There are then vertices coupling particles to a classical source (the  $a_{\mu\nu}$  field). It would mean spontaneous creation or absorption of particles. We are at a loss to make any sense out of this. In other words, a non-zero cosmological constant appears inconsistent within the perturbative approach, but there is no way to guarantee its vanishing.

Assigning absolute meaning to time, we could discuss questions concerning the beginning of the universe. At this point, given the unsatisfactory situation concerning gravitation we believe such a discussion to be premature.

## 9. Conclusions

The basic assumptions that may be used as a starting point for the description of physical reality are not equivalent. Space and time are not part of the perturbative dimensional formulation and are thus defined only through Fourier transformation. Many non-perturbative results of contemporary field theory may be questioned, and we have attempted to differentiate between solutions that can be obtained by some analytic continuation and those that have no connection whatsoever to perturbation theory.

The question may be raised if the issues discussed here can ever be resolved. The non-success of non-perturbative solutions may be considered circumstantial evidence against coordinate space formulations, but is no proof. If we do find however any phenomenon that is particular to the dimensional formulation then that could decide the issue. But then it may well be that that is not the ultimate formulation either. There certainly is need for improvement.

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