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ISOSPIN MASS SPLITTINGS AND THE m_s CORRECTIONS IN THE SEMIBOSONIZED SU(3)-NAMBU–JONA-LASINIO MODEL

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The mass splittings of hyperons including the isospin splittings are calculated with $O(m_s^2)$ and $O(m_s\Delta m)$ accuracy respectively within the semibosonized SU(3)-Nambu–Jona-Lasinio model. The pattern of the isospin splittings is not spoiled by the terms of the order $O(m_s\Delta m)$, and both splittings between the different isospin multiplets and within the same multiplet are well reproduced for acceptable values of m_s and Δm.

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1. Introduction

Although Quantum Chromodynamics (QCD), as the ultimate theory of strong interactions, should be able to give precise predictions of all physical quantities such as masses, mass splittings etc., its predictive power is obscured by technical difficulties. One way out is to employ effective models, which share some important features with QCD. The chiral quark model

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[1] (or semibosonized Nambu–Jona-Lasinio model) exhibits chiral symmetry breaking, which, together with confinement, is relevant for the low energy regime of QCD. Moreover it can be "derived" from QCD within the instanton liquid model of the QCD vacuum [2].

The NJL model [3] has been therefore thoroughly examined against the low energy hadronic data and it turned out to give surprisingly good predictions for mass splittings [4, 5], electromagnetic properties [6] and axial properties of hyperons [7, 8]. The *hadronic* part of the isospin breaking, which is due to $\Delta m = m_{\rm d} - m_{\rm u}$ mass difference is perhaps the most striking example of the accuracy reached by the model. In Ref. [9] the isospin mass differences have been calculated for the octet and decuplet of baryons. Although the error bars on these splittings are large (since the poorly known electromagnetic part of the splittings has to be subtracted) the model predictions fall nicely within the error bars for all spin 1/2 and 3/2 baryons for $\Delta m \approx 3.5$ MeV.

These results have been obtained within the perturbative approach, where one expands the collective hamiltonian describing baryonic states in terms of m_s and Δm up to terms linear in both parameters. It has been claimed in the literature [10, 11] that, at least within the Skyrme model approach, the next orders in m_s spoil the nice pattern of the isospin splittings obtained in the linear approximation. It is the purpose of this work to examine if this is also the case in the present model. As will be seen in the following, the inclusion of the terms of the order $\Delta m\ m_s$ does not spoil the pattern of the isospin breaking, however Δm which reproduces the experimental data is shifted towards the higher value of the order of 4.5 MeV. We consider this result as fully satisfactory. Other chiral models either underestimate the isospin splittings by factor of 2 or overestimate it by approximately the same amount[1].

The crucial point of our analysis is the fact that, though we are considering the isospin breaking effects caused by the non-strange current masses, we are using the SU(3) version of the NJL model. In this framework the symmetry breaking operator is proportional to λ_3. In the SU(2) model it reduces to τ_3 and most of the relevant matrix elements after the collective quantization vanish. In SU(3) the polarization of the strange Dirac sea provides a number of terms which are crucial for the splitting pattern.

2. Mesonic sector

The prominent feature of the NJL model consists in the fact that the same effective action describes meson physics and also, through the solitonic solutions, baryon physics. It is customary to fix the parameters of the

[1] See Introduction in Ref. [11].

model by fitting meson properties. This procedure leaves usually one free parameter, namely the constituent quark mass M. The regularization cut-off (or cut-off function) becomes then a function of M. Then M is fixed to fit *one* splitting in the soliton sector, *e.g.* N$-\Delta$, and then all other baryonic observables come out as predictions. In practice there is always some freedom in tuning other parameters like m_s or Δm, although they are always fairly constrained by the meson sector.

There are in fact different ways to treat the meson sector. One can either solve the gap equations and then evaluate meson masses at zero momentum $q = 0$ from the curvature of the effective potential. One can also evaluate the curvature at $q^2 = -m^2_{\text{meson}}$. Or one can solve Bethe–Salpether equations for the meson propagators. Another way is to fix the parameters in the gradient expansion, which results in an effective meson Lagrangians whose coupling constants are known from the meson scattering. Each of these methods produces slightly different results, which are not important for the gross features of the soliton sector, but may influence the numerics. This influence is not very significant if one is interested in the quantities which are not too small, however they might turn out important, if one considers such tiny effects as the isospin splittings.

It is not our purpose to make a complete calculation of the isospin splittings. That would require to include the electromagnetic effects, which is certainly beyond the scope of this paper. Our goal is by far more modest: we want to investigate the effect of the strange quark mass on the hadronic part of the isospin splittings, very much in the spirit of the calculations performed in the Skyrme model [12]. Electromagnetic contributions will be simply parametrized by means of some convenient Ansatz; in the meson sector we use the Dashen Ansatz:

$$m^2_{\text{meson}} = m^2_{\text{H}} + Q^2 c \,, \tag{1}$$

where Q is the meson charge, c is a constant which does not depend on the meson in question and subscript H stands for the hadronic (quark mass dependent) part of the meson mass.

We fix the model parameters in the meson sector in the first order in the quark mass matrix. In this way we avoid unnecessary complications due to the $\eta^0 - \eta^8 - \pi^3$ mixing. We also refrain from such problems as boson loops and the validity of the Dashen Ansatz. All these complications result in the uncertainty in the value of Δm extracted from the mesonic data.

The first order meson mass formulae with the Dashen Ansatz have been analyzed in Ref. [9] They imply:

$$\mathcal{R} = \frac{\Delta m}{m_{\text{u}} + m_{\text{d}}} = 0.28 \,. \tag{2}$$

For $m_u + m_d = 12.2$ MeV, which is required by the regularization prescription we use and which falls within the commonly accepted range of values for $m_u + m_d$, we get $\Delta m = 3.4$ MeV. Ignoring completely electromagnetic contributions would yield $\mathcal{R} = 0.21$ and $\Delta m = 2.6$ MeV.

It should be however stressed that full next-to-leading order analysis of mesonic masses within the chiral perturbation theory does not constrain Δm at all [13]. This is due to the lack of knowledge of the electromagnetic contribution to meson masses at next order in quark masses. It is, however, not excluded that at this order the Dashen Ansatz is violated. Recent analysis of the decay $\eta \rightarrow 3\pi$ suggests that

$$(m^2_{K^\pm} - m^2_{K^0} - m^2_{\pi^\pm} + m^2_{\pi^0})_{EM} = 1300 \pm 400\text{MeV}^2 \,, \tag{3}$$

in contrast to 0 for the Dashen Ansatz. If this were true $\mathcal{R} = 0.34$ and $\Delta m = 4.2$ MeV.

Therefore, we fix cutoff *vs.* M dependence as in Ref. [8], where the details can found. The experimental numbers which are used to fix the model parameters are $f_\pi = 93$ MeV, m_π and m_K. Then it comes out that $m_u + m_d = 12.2$ MeV, $m_s \simeq 150$ MeV. The kaon decay constant is then constrained to $f_K = 105$ MeV about 10% below the experimental value. The allowed range for Δm is then, as discussed above, 2.6 – 4.2 MeV, however the lower values are rather unlikely, since they come out by completely ignoring the electromagnetic contributions.

3. Solitons and the quantization of zero modes

The effective Euclidean action for the semibosonized SU(3) NJL model reads (after integrating out the quark fields) [4, 5]:

$$S_{\text{eff}} = -\text{Sp}\,\ln(-i\partial\!\!\!/ + m + MU^{\gamma_5})\,. \tag{4}$$

Here M is the constituent quark mass, m the current quark mass matrix and U^{γ_5} describes the 8 Goldstone modes of the SU(3) chiral symmetry. The real part of Eq. (4) is assumed to be regularized with the proper-time regularization function of Ref. [4] and Sp denotes the functional trace.

First we make use of the trivial embedding [14] of the SU(2) chiral field $U_0(x) = (\sigma_{(2)} + i\gamma_5\vec{\tau}\vec{\pi})/f_\pi$ into the isospin subgroup of SU(3) according to

$$U(x) = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{5}$$

The soliton solutions of SU(2) are also solutions for SU(3) and the embedding (5) gives the correct constraint on the Hilbert space of the baryonic states [14].

The soliton solutions are found by employing the *hedgehog* Ansatz for the field U_0. The details are widely described in the literature. Stability of the soliton solution is achieved by an interplay of the energy of the polarized Dirac continuum and the discrete valence level.

Following the treatment of Ref. [15], we quantize the soliton by introducing time dependent rotations of the *hedgehog* field: $U(x,t) = A(t)\ U(x)\ A(t)^\dagger$. This rotation can be undone by rotating the quark fields: $\tilde{q} = A(t)q$ and $\bar{\tilde{q}} = \bar{q}A(t)^\dagger$. Defining "angular velocities" Ω:

$$A^\dagger \dot{A} = i\Omega_E = \tfrac{i}{2}\lambda_a \Omega_E^a \tag{6}$$

one can rewrite S_{eff} as:

$$S_{\text{eff}} = -\text{Sp}\ \ln(\partial_\tau + H + i\Omega_E - i\gamma_4 A^\dagger m A)\,. \tag{7}$$

The following relation between Euclidean and Minkowski velocities holds: $i\Omega_E = \Omega_M$ and $\Omega_E^\dagger = \Omega_E$.

Expanding (7) up to the quadratic order in Ω (in Minkowski metric and in the chiral limit) one gets Ref. [4]:

$$L_0 = -M_{\text{cl}} + \tfrac{1}{2}\Omega_a I_{ab} \Omega_b\,, \tag{8}$$

where tensor of inertia $I_{ab} = \text{diag}(I_1, I_1, I_1, I_2, I_2, I_2, I_2, 0)$ can be found in Ref. [4].

To calculate the mass splittings one has to expand the effective action in powers of the current quark mass $m = \mu_0\,\lambda_0 - \mu_8\,\lambda_8 - \mu_3\,\lambda_3$ ($\lambda_0 = \sqrt{2/3}\,1$) with:

$$\mu_0 = \tfrac{1}{\sqrt{6}}(m_{\text{u}} + m_{\text{d}} + m_s), \quad \mu_8 = \tfrac{1}{\sqrt{12}}(2\,m_s - m_{\text{u}} - m_{\text{d}}), \quad \mu_3 = \tfrac{1}{2}\Delta m\,, \tag{9}$$

where $\Delta m = m_{\text{d}} - m_{\text{u}}$.

Expanding the effective action up to terms of the order of m_s, Δm, m_s^2, $\Delta m\, m_s$, $m_s\,\Omega$ and $\Delta m\ \Omega$ one gets:

$$\begin{aligned} L_m &= -\,\sigma\, m_s + \sigma \left(m_s D_{88}^{(8)} + \frac{\sqrt{3}}{2}\,\Delta m D_{38}^{(8)} \right), \\ L_{m\Omega} &= -\,\frac{2}{\sqrt{3}} m_s\ D_{8a}^{(8)} K_{ab}\Omega_b - \Delta m D_{3a}^{(8)} K_{ab}\Omega_b\,, \\ L_{m^2} &= \frac{2}{9} m_s^2 \left(N_0 (1 - D_{88}^{(8)})^2 + 3 N_{ab} D_{8a}^{(8)} D_{8b}^{(8)} \right) \\ &\quad + \frac{2}{3\sqrt{3}} m_s\ \Delta m \left(N_0 (D_{38}^{(8)} D_{88}^{(8)} - D_{38}^{(8)}) + 3 N_{ab} D_{3a}^{(8)} D_{8b}^{(8)} \right), \end{aligned} \tag{10}$$

where the constant σ is related to the nucleon sigma term $\Sigma = 3/2\,(m_u + m_d)\,\sigma$ and $D^{(8)}_{ab} = 1/2\ \mathrm{Tr}(A^\dagger \lambda_a A \lambda_b)$. The mass spectrum obtained with the help of $L_0 + L_m + L_{m\Omega}$ was discussed in Refs. [4, 9]; there one can also find explicit formulae for $K_{ab} = \mathrm{diag}(K_1, K_1, K_1, K_2, K_2, K_2, K_2, 0)$. Let us here only remind that the *anomalous* moments of inertia K_i are nearly entirely given by the valence part, whereas the contribution of the valence level to I_i amounts to approximately 60%. The quantities $N_{ab} = \mathrm{diag}(N_1, N_1, N_1, N_2, N_2, N_2, N_2, N_0/3)$ have been derived in Ref. [8]. Their values together with the values of $I_{1,2}$ and $K_{1,2}$ for different constituent masses are listed in Table I.

TABLE I

Moments of inertia for different constituent masses

M [MeV]	Σ [SU(2)] [MeV]	I_1 [fm]	I_2 [fm]	K_1 [fm]	K_2 [fm]	N_0 [fm]	N_1 [fm]	N_2 [fm]
363.	60.32	1.512	0.720	0.606	0.372	0.765	0.647	0.496
395.	58.14	1.285	0.618	0.438	0.290	0.704	0.500	0.408
419.	56.14	1.178	0.569	0.369	0.255	0.668	0.438	0.370
423.	55.52	1.156	0.560	0.357	0.250	0.658	0.426	0.362
442.	52.52	1.070	0.521	0.315	0.229	0.603	0.379	0.329

The Lagrangian of Eq. (10) reminds the one of the Skyrmion. The quantization proceeds as in the Skyrme model and has been described in detail in the literature [16]. Let us here remind that at first one defines the quantities:

$$J_a = I_{ab}\Omega_b - \mu_i D_{ib} K_{ba} - \delta_{a8}\frac{N_c}{2\sqrt{3}} \tag{11}$$

($i = 3$ and 8, $a, b = 1\ldots8$) which, as a result of the quantization procedure, are promoted to the spin operators $\hat{J}_a$. Note that the relation (11) depends on the quark masses. The wave function of the baryon state $B = Y, T, T_3, J, J_3$ belonging to the SU(3) representation $\mathcal{R}$ reads (see Appendix A of Ref. [8]:

$$|\,\mathcal{R}, B\rangle = \sqrt{\mathrm{dim}\mathcal{R}}\langle Y, I, I_3 \mid D^{(\mathcal{R})}(A) \mid -Y', J, -J_3\rangle^*\,, \tag{12}$$

where the right hypercharge Y' is in fact constrained to be -1. The lowest SU(3) representations which contain states with $Y = 1$ are: $\mathcal{R} = \mathbf{8}$ and $\mathcal{R} = \mathbf{10}$. The quantized Hamiltonian from Eq. (8) reads:

$$H^{(0)} = M_{\mathrm{cl}} + H_{\mathrm{SU(2)}} + H_{\mathrm{SU(3)}}\,,$$

$$H_{SU(2)} = \frac{1}{2I_1} C_2(SU(2)) ,$$

$$H_{SU(3)} = \frac{1}{2I_2} \left[C_2(SU(3)) - C_2(SU(2)) - \frac{N_c^2}{12} \right] .$$

Here C_2 denote the Casimir operators of the spin SU(2) and flavor SU(3). M_{cl} is the classical soliton mass. It has been calculated by many authors and its value turns out to be relatively large: $M_{cl} \approx 1.2$ GeV. This is a common problem for all chiral models. There are, however, some negative corrections to it, like Casimir energy or rotational band corrections which might bring M_{cl} to the right value. In this paper, instead on insisting on the calculation of the absolute masses, we will concentrate on the mass splittings.

The Hamiltonian up to terms linear and quadratic in m_s and linear in Δm and $m_s \Delta m$ reads:

$$\begin{aligned}
H^{(1)} &= \left\{ \sigma - r_2 Y - (\sigma - r_2) D_{88} + \frac{2}{\sqrt{3}} (r_1 - r_2) \sum_{A=1}^{3} D_{8A} \hat{J}_A \right\} m_s , \\
H^{(2)}_{\text{kin}} &= \frac{2}{3} \left\{ r_2 K_2 (1 - D_{88}^2) + (r_1 K_1 - r_2 K_2) \sum_{A=1}^{3} D_{8A}^2 \right\} m_s^2 , \\
H^{(2)}_{\text{dyn}} &= -\frac{2}{9} \{ (N_0 + 3N_2) - 2N_0 D_{88} + (N_0 - 3N_2) D_{88}^2 \\
&\quad + 3(N_1 - N_2) \sum_{A=1}^{3} D_{8A}^2 \Big\} m_s^2 , \\
h^{(1)} &= \left\{ -r_2 T_3 - \frac{\sqrt{3}}{2} (\sigma - r_2) D_{38} + (r_1 - r_2) \sum_{A=1}^{3} D_{8A} \hat{J}_A \right\} \Delta m , \\
h^{(2)}_{\text{dyn}} &= \frac{2}{3\sqrt{3}} \{ N_0 D_{38} (3N_2 - N_0) D_{38} D_{88} \\
&\quad + 3(N_2 - N_1) \sum_{A=1}^{3} D_{38} D_{8A} \Big\} m_s \Delta m , \\
h^{(2)}_{\text{kin}} &= \frac{2}{3} \left\{ -r_2 K_2 D_{38} D_{88} + (r_1 K_1 - r_2 K_2) \sum_{A=1}^{3} D_{38} D_{8A} \right\} m_s \Delta m ,
\end{aligned} \tag{14}$$

where $r_i = K_i / I_i$ ($i = 1, 2$) and T_3 stands for isospin. We have split the $O(m_s^2)$ and $O(m_s \Delta m)$ Hamiltonian into the *kinematical* part which appears due to the fact that the quantization relation between the angular velocity and the spin operators $\vec{J}$ is m_s and Δm dependent (see Eq.(11)) and the *dynamical* part which comes from the expansion of the effective action in terms of m.

The Hamiltonians $H^{(1)}$ and $h^{(1)}$ mix states of different SU(3) representations. The corresponding $O(m_s^2)$ contribution to the energy reads:

$$\begin{aligned} E_{\rm wf}^{(2)} = & - \left\{ \tfrac{1}{60} \left(1 + Y - X + \tfrac{1}{2}Y^2\right) \ (\sigma - r_1)^2 \right. \\ & \left. + \tfrac{1}{250} \left(\tfrac{13}{2} + \tfrac{5}{2}X - \tfrac{7}{4}Y^2\right) \tfrac{1}{9}(3\sigma + r_1 - 4r_2)^2 \right\} I_2 m_s^2 \end{aligned} \tag{15}$$

for the octet and for the decuplet:

$$\begin{aligned} E_{\rm wf}^{(2)} = & - \left\{ \tfrac{1}{16} \left(1 + \tfrac{3}{4}Y + \tfrac{1}{8}Y^2\right) \tfrac{1}{9}(3\sigma - 5r_1 + 2r_2)^2 \right. \\ & \left. + \tfrac{5}{336} \left(1 - \tfrac{1}{4}Y - \tfrac{1}{8}Y^2\right) (\sigma + r_1 - 2r_2)^2 \right\} I_2 m_s^2 \, . \end{aligned} \tag{16}$$

Here $X = 1 - T(T+1) + 1/4\ Y^2$ is the usual combination entering Gell-Mann–Okubo mass relations (T stands for isospin).

Similarly one can write the $O(m_s \Delta m)$ wave function contribution to the octet states:

$$\begin{aligned} e_{\rm wf}^{(2)} = & \tfrac{1}{30}(\sigma - r_1)^2 \ (1 + Y)\, T_3 \ \ I_2 m_s \Delta m \\ & - \tfrac{2}{125}(\sigma + \tfrac{1}{3}r_1 - \tfrac{4}{3}r_2)^2 \ Y\, T_3 \ \ I_2 m_s \Delta m \, . \end{aligned} \tag{17}$$

4. Hyperon splittings

With the help of the matrix elements of the D functions and spin operators one arrives at the following result for the hyperon splittings:

$$\begin{aligned} \Delta M^{(8)} &= A - \frac{F}{2}\,Y - \frac{D}{\sqrt{5}}\,X - G\,Y^2 \, , \\ \Delta M^{(10)} &= B - \frac{C}{2\sqrt{2}}\,Y - H\,Y^2 \, . \end{aligned} \tag{18}$$

Constants A and B do not contribute to the splittings within the multiplets, however they shift the mass of the centers and contribute to the **10-8** mass difference. Constants G and H, not present in the first order Gell-Mann–Okubo mass formula, are of the order of m_s^2.

Hyperon splittings obtained with the help of Eq.(18) have been discussed in detail in Ref. [8]. Here for completeness we repeat only the main points.

Experimentally one gets:

$$\begin{aligned} F &= \Xi - \mathrm{N} = 379 \text{ MeV}, \\ D &= \tfrac{\sqrt{5}}{2}(\Sigma - \Lambda) = 86 \text{ MeV}, \\ G &= \tfrac{1}{4}(3\Lambda + \Sigma) - \tfrac{1}{2}(\mathrm{N} + \Xi) = 6.75 \text{ MeV} \end{aligned} \tag{19}$$

for the octet. For the decuplet the three operators: $1, Y$ and Y^2 do not form a complete basis and therefore there are two independent relations which determine constants C and H with small uncertainty:

$$\begin{aligned} C &= \sqrt{2}(\Xi^* - \Delta) = \tfrac{1}{\sqrt{2}}(\Omega - 2\Delta + \Sigma^*) = 422.5 \pm 3.5 \text{ MeV}, \\ H &= \tfrac{1}{2}(2\Sigma^* - \Xi^* - \Delta) = \tfrac{1}{6}(3\Sigma^* - 2\Delta - \Omega) = 2.83 \pm 0.33 \text{ MeV}. \end{aligned} \tag{20}$$

In Table II we list the coefficients $A \dots H$ for a typical value of $m_s =$ 180 MeV as functions of the constituent mass M. It can be seen that in order to reproduce the experimental numbers of Eqs.(19), (20) one has to take the constituent mass of the order of 400 MeV. Then all constants $A \dots H$ are roughly reproduced. The constant G and H being of the order $O(m_s^2)$ are small. For reasonable strange quark masses $O(m_s^2)$ corrections to A, B, C and D are of the order of 20% of the leading $O(m_s)$ terms with the exception of F for which $O(m_s^2)$ corrections are almost zero. This is illustrated in Fig. 1, where $O(m_s^2)/O(m_s)$ contributions to constants $A \dots F$ are plotted as functions of m_s for the fixed value of $M = 420$ MeV.

TABLE II

Different contributions to coefficients of Eqs. (18) for $M = 423$ MeV and $m_s = 180$ MeV

	$O(m_s)$	$O(m_s^2)$				total	exp.
		kin.	dyn.	w.f.	total		
A	546.10	10.94	−64.64	−0.15	−53.85	492.25	–
B	546.10	10.85	−64.55	−53.81	−107.50	438.60	–
F	381.20	1.18	−27.67	22.76	−3.73	377.47	379.00
D	120.76	−0.02	−11.78	−0.07	−11.87	108.89	86.00
C	348.16	1.20	−19.97	90.92	72.15	420.32	422.00
G	0.00	0.61	−0.66	1.53	1.48	1.48	6.75
H	0.00	-0.29	0.41	4.57	4.70	4.70	2.83

In order to make phenomenological statements we adopt the following procedure: first for given M we find the optimal m_s which reproduces **10-8** splitting. To this end we define the mean octet and decuplet values:

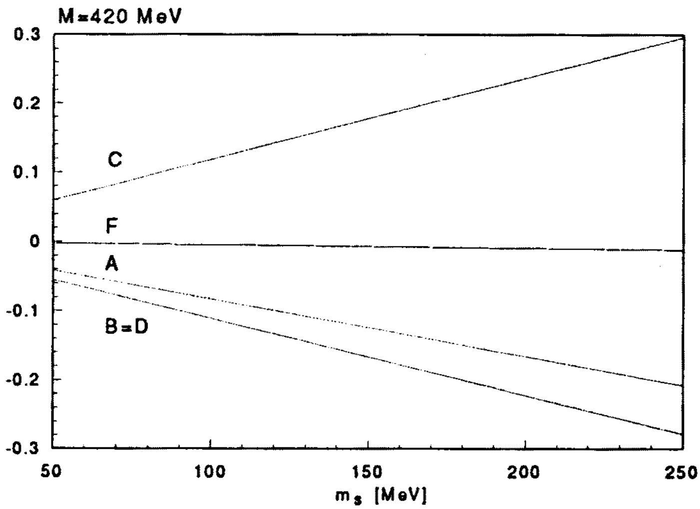

Fig. 1. Ratios of $O(m_s^2)/O(m_s)$ contributions to $A \ldots D$, F as functions of m_s.

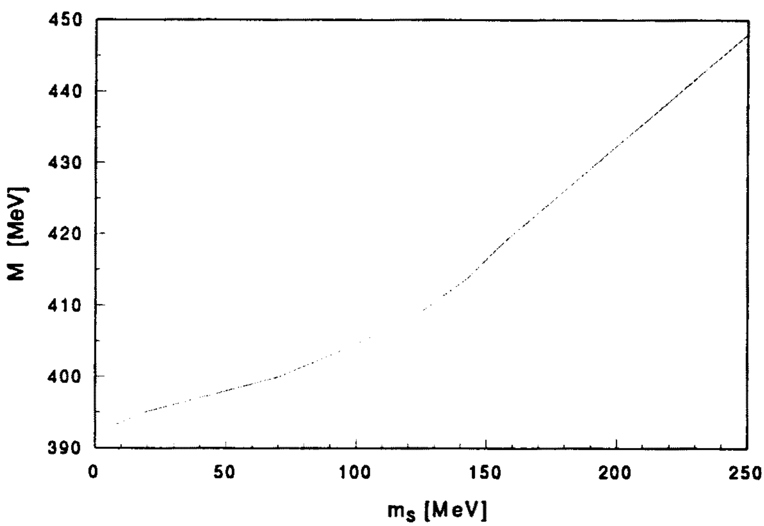

Fig. 2. M *vs.* m_s dependence induced by the condition $\Delta_{10-8} = 230$ MeV.

$\overline{M}^{(8)} = 1/2\ (\Lambda + \Sigma) = 1155$ MeV and $\overline{M}^{(10)} = \Sigma^* = 1385$ MeV. Then $\Delta_{10-8} \equiv \overline{M}^{(10)} - \overline{M}^{(8)} = 230$ MeV is given by:

$$\Delta_{10-8} = \frac{3}{2I_1} + B - A\,. \tag{21}$$

Since $A - B = \text{const.} \times m_s^2$ one can numerically solve Eq. (21) for m_s. The result is plotted in Fig. 2 (see also Table III).

In Fig. 3 we show the m_s dependence of the deviations *theory – experiment* for each hyperon. One should remember that for each m_s the *optimal* constituent quark mass M was used, so that Δ_{10-8} was automatically reproduced for each m_s. The smallest deviations ± 7 MeV for all splittings correspond to $m_s \simeq 185$ MeV, *i.e.* $M \simeq$ 426 MeV.

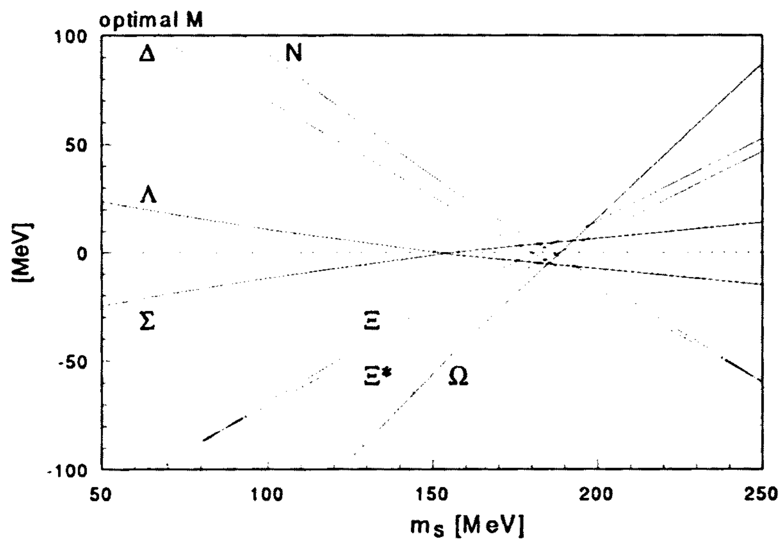

Fig. 3. The m_s dependence of the deviations *theory – experiment* for each hyperon for M chosen according to Fig. 2.

It is constructive to plot the dependence of constants F, D and C entering the Gell-Mann–Okubo mass relations as functions of m_s (and *optimal* M). Fig. 4 shows the influence of the $O(m_s^2)$ terms on these quantities. Dashed lines represent the first order results and solid lines the full result. It can be seen that $O(m_s^2)$ corrections substantially improve model predictions. F hits its experimental value for $m_s = 181$ MeV and the corresponding $M \simeq 424$ MeV. For these values C is within the experimental errors and D overshoots slightly the experimental value.

In this way we have fixed M — the only remaining free parameter of the model to the value of about 425 MeV. The corresponding value of $m_s \simeq$ 180 MeV required to fit the **10-8** splitting overshoots the value deducted from the meson sector $m_s \simeq 150$ MeV. However, in an exact treatment of the vacuum sector and perturbation theory around this exact vacuum for the soliton sector, the crucial quantity is $M_s - M_u$, the difference of

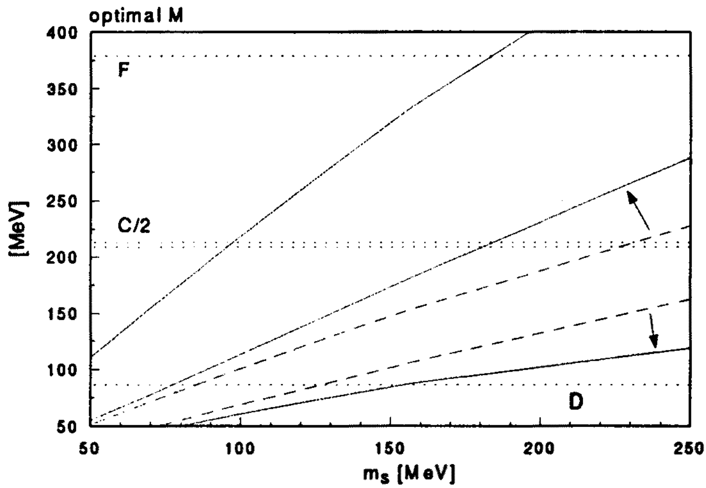

Fig. 4. F, C and D as functions of m_s; dashed lines – first order contributions, solid lines – full $O(m_s^2)$ result, dotted lines – experimental values, note almost zero correction to F.

the constituent quark masses, instead of m_s and this turns out to be $\simeq$ 180 – 200 MeV $\gg m_s$. Within such a treatment the discrepancy between meson and baryon sectors disappears [17].

TABLE III

Constants f and d as functions of M and m_s chosen according to Fig. 2

M	m_s	$f^{(1)}$	$f^{(2)}_{\rm wf}$	$f^{(2)}_{\rm dyn}$	$f^{(2)}_{\rm kin}$	$\sum f$	$d^{(1)}$	$d^{(2)}_{\rm wf}$	$d^{(2)}_{\rm dyn}$	$d^{(2)}_{\rm kin}$	$\sum d$
395.	19.	3.34	−0.05	−0.01	−0.00	3.28	0.46	0.01	−0.00	−0.001	0.47
419.	157.	3.21	−0.34	−0.12	−0.02	2.72	0.45	0.07	−0.02	−0.006	0.49
423.	177.	3.18	−0.37	−0.14	−0.02	2.65	0.45	0.07	−0.02	−0.007	0.49
428.	209.	3.11	−0.41	−0.16	−0.02	2.53	0.44	0.08	−0.03	−0.007	0.48
442.	258.	3.03	−0.45	−0.19	−0.02	2.37	0.43	0.09	−0.03	−0.008	0.48

5. Isospin splittings

Hadronic parts of the isospin splittings have been estimated in [18]:

$$(n-p)_{\rm H} = 2.05 \pm 0.3, \quad (\Sigma^- - \Sigma^+)_{\rm H} = 7.89 \pm 0.3, \quad (\Xi^- - \Xi^0)_{\rm H} = 5.5 \pm 0.7 \quad (22)$$

(in MeV) for the octet states. In Ref. [9] we have also estimated the splittings for the decuplet on the basis of the simple Dashen parametrization, which has proven to work equally well for both decuplet and octet particles. For the purpose of this work, however, we will concentrate entirely on the octet, since the numbers of Eq. (22) are experimental, whereas the decuplet estimations, in view of the lack of the data, were based upon the theoretical guess, as mentioned above.

From theoretical point of view the isospin splittings are described by the formula analogous to the Gell-Mann–Okubo mass formula for the hyperon splittings, namely [9]:

$$(\Delta M)_{\rm H} = -\tfrac{1}{3} f\, T_3 + d\, Y\, T_3\,, \tag{23}$$

where, taking into account Eq. (22), one gets:

$$f = 11.33 \pm 1.14 \text{ MeV}, \qquad d = 1.73 \pm 0.38 \text{ MeV}. \tag{24}$$

On the other hand f and d can be directly evaluated in the present model with the help of Eqs. (14), (17):

$$\begin{aligned}
f^{(1)} &= \tfrac{3}{4}(\sigma + r_1 + 2r_2), \quad f^{(2)}_{\rm kin} = \tfrac{1}{5}(2r_2K_2 - 3r_1K_1)\, m_s\,,\\
f^{(2)}_{\rm dyn} &= -\tfrac{1}{5}(2N_0 - 3N_1 + 2N_2)\, m_s, \quad f^{(2)}_{\rm wf} = -\tfrac{1}{10}(\sigma - r_1)^2\, I_2\, m_s\,,\\
d^{(1)} &= \tfrac{3}{20}(\sigma - 3r_1 + 2r_2), \quad d^{(2)}_{\rm kin} = \tfrac{1}{45}(2r_2K_2 - 5r_1K_1)\, m_s,\\
d^{(2)}_{\rm dyn} &= -\tfrac{1}{45}(4N_0 - 5N_1 + 2N_2)\, m_s,\\
d^{(2)}_{\rm wf} &= \left[\tfrac{1}{30}(\sigma - r_1)^2 - \tfrac{2}{125}\left(\sigma + \tfrac{r_1}{3} - \tfrac{4r_2}{3}\right)^2\right] I_2\, m_s
\end{aligned} \tag{25}$$

and

$$f = (f^{(1)} + f^{(2)}_{\rm kin} + f^{(2)}_{\rm dyn} + f^{(2)}_{\rm wf})\Delta m, \quad d = (d^{(1)} + d^{(2)}_{\rm kin} + d^{(2)}_{\rm dyn} + d^{(2)}_{\rm wf})\Delta m. \tag{26}$$

It is worth noting that in the order $m_s \Delta m$ the Gell-Mann–Okubo mass formula of Eq. (23) is not violated, but the coefficients f and d depend on m_s. In Table III we list the values of f and d for different constituent masses M and for m_s chosen according to Fig. 2.

It is clearly seen from Table III that the second order corrections to f and d are dominated by the wave function contribution, however the dynamical part, although smaller, is by no means negligible. This is in contrast to the hadronic splittings where the dynamical corrections were equally important as the wave function ones [8].

Finally in Figs 5–7 we show the Δm dependence of the isospin splittings for three constituent masses M = 419, 423 and 428 MeV and three

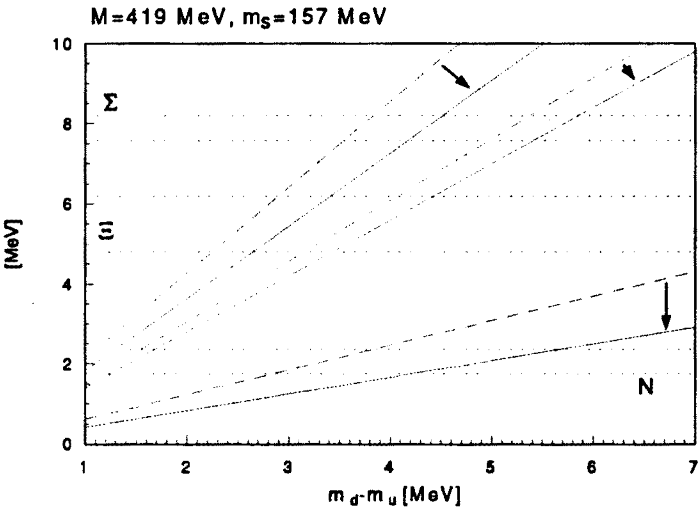

Fig. 5. Δm dependence of the isospin splittings for $M = 419$ MeV and $m_s = 157$ MeV. Horizontal dotted lines correspond to experimental error bars.

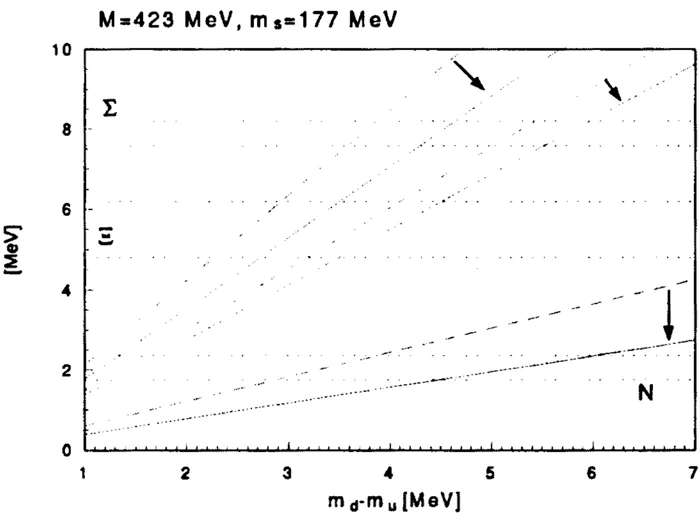

Fig. 6. Same as Fig. 5 for $M = 423$ MeV and $m_s = 177$ MeV.

corresponding $m_s = 157$, 177 and 209 MeV respectively. The first set corresponds to the strange quark mass as required by the meson sector. For this values, however, the hyperon spectrum is not correctly reproduced (see Figs. 3 and 4). The second set corresponds to the best fit to the hyperon

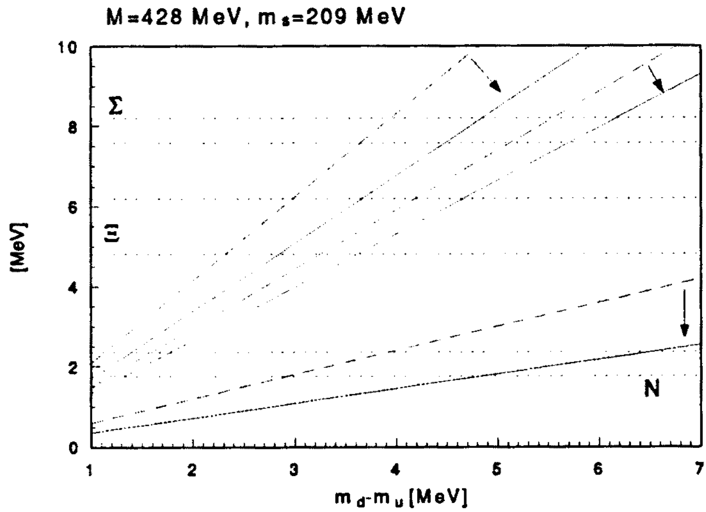

Fig. 7. Same as Fig. 5 for $M = 428$ MeV and $m_s = 209$ MeV.

spectra. The third set has been chosen to show what happens if m_s starts overshooting the optimal value.

The figures clearly show, that the slopes of the splittings as functions of Δm uniformly decrease with m_s. Without the $O(m_s)$ corrections all splittings fall within the experimental error bars for $\Delta m \simeq 3.5$ MeV (dashed lines on Figs. 5–7). When the $O(m_s)$ corrections are included the Δm range for which theoretical curves fall into experimental error bars shifts towards higher values and shrinks at the same time, so that for $M = 428$ MeV (Fig. 7) there is no common value of Δm which would describe all splittings. For $M = 423$ MeV (Fig. 6) it is still possible to describe all splittings with $\Delta m \simeq 4.4$ MeV. This value should be compared with the $\Delta m = 2.6 - 4.2$ MeV which is needed to reproduce meson masses.

6. Concluding remarks

Almost all chiral models have problems in reproducing isospin splittings (see Ref. [11] and references therein). In our previous work [9] we have shown that in the semibosonized NJL *all* isospin splitting within the octet and decuplet of baryons are reproduced within surprisingly good accuracy in the zeroth order in m_s. In the Skyrme model the $O(m_s)$ corrections to the isospin splittings are large and decrease the splittings. It has been therefore of importance to see how this corrections influence the mass differences in the present model. We have shown that the slope of the isospin splittings as functions of Δm decreases with increasing m_s. This is in principle what happens also in the SU(3) Skyrme model, however here the effect is much less pronounced. This is due to the existence of the *anomalous* parts associated with the quantities $K_{1,2}$. Already at the zeroth order, where $f^{(1)} = 3.18$ for $M = 423$ MeV the *anomalous* contribution amounts to 28%. The wave function corrections decrease f as seen from Table III by -0.37, however without the *anomalous* part the decrease would be much stronger: -0.46 for $m_s = 177$ MeV. Altogether the final value of $f = 2.65$ would be decreased to 1.68 (*i.e.* by 37%) if the anomalous part was absent. On the contrary, the influence of *anomalous* terms on the coefficient d is small; it amounts to 4%.

Let us stress once more that the successful phenomenology of the isospin splittings could not have been achieved within the SU(2) NJL model, where, similarly to the Skyrme model, the matrix elements of the relevant operator simply vanish.

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