NON-UNIFORM PAIRING GAP OF THE SUPERFLUID NEUTRONS IN THE CRUST OF NEUTRON STARS

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(Received June 1, 1994)

We study the neutron superfluidity in the inner crust of a neutron star where a gas of unbound nucleons coexists with a lattice of neutron rich nuclei, using a Wigner–Seitz approximation and a well tested nuclear force. We found a neutron pairing energy gap which is generally larger outside the nuclear region. In order to clarify the relevance of our results we evaluate the pinning energy of vortices to nuclei and compare it with previous calculations.

PACS numbers: 21.60. Ev

1. Introduction

The crust of a neutron star is, in its outer part (10^6 g/cm^3 $< \rho <$ 10^{11} g/cm^3) made of nuclei disposed in a coulomb lattice and of a homogeneous background of relativistic electrons. At $\rho \approx 4.3 \times 10^{11}$ g/cm^3, the nuclei are so neutron rich that with increasing density the neutron states lying in the continuum begin to be filled and the lattice of neutron–rich nuclei becomes permeated by a sea of neutrons [1]. At larger depths more peculiar nuclear shapes can be expected on the basis of energy minimization [2–3].

Taking into account that, in the range of density corresponding to the inner crust, the nuclear interaction for pairs of neutrons moving in time reversal states is attractive, and that the crustal temperature of the star is much lower than the critical temperature, one expects to have neutron superfluidity in the inner crust of neutron stars [4].

The effects of superfluidity of the neutron component on the global dynamics of the star are very important. The crust of the star spins around its axis and as a consequence the neutron superfluid component inside rotates by forming vortex lines parallel to the rotation axis, around which the superfluid circulation is quantized. Since the rotational properties of a superfluid are determined by the density and distribution of vortices in it, pinning of the vortices on the nuclei in the crust has important consequences for pulsar dynamics and in the pulsar glitches phenomena [5]. We point out that superfluidity has also influence in the cooling rate of the star.

These problems as well as the question of nuclear forces in a high density medium demanded a lot of theoretical work on the superfluidity properties of neutron and nuclear matter. Microscopic calculations concentrated on the problem of superfluidity in uniform system in terms of fundamental nucleon–nucleon forces and correlations. However, the non-uniform system as in the inner crust of a neutron star, is still to be worked out from a microscopical point of view. In the present paper we study within the framework of the local density approximation and using an effective two body nucleon–nucleon force, the system of nucleus in the center of the Wigner–Seitz cell embedded in a sea of unbound neutrons.

In Section 2 we apply the basic BCS formalism with Gogny effective force, to treat uniform systems, considering only singlet pairing ($^1S_0$). The Section 3 is devoted to the study of the problem of the nucleus in a neutron superfluid gas treated in the Wigner–Seitz approximation. In Section 4 we apply the results to the calculation of the pinning energy for a vortex-nucleus system in the range of densities corresponding to the inner crust of neutron stars.

2. Uniform neutron system

According to the BCS model, the pairing properties of uniform neutron superfluid are characterized by the pairing energy gap, satisfying the integral equation

$$\Delta(p) = -\frac{1}{(2\pi)^3} \int d^3k v(p-k) \frac{\Delta(k)}{2\sqrt{(\varepsilon(k)-\varepsilon_F)^2 + \Delta(k)^2}}, \tag{1}$$

where $v(p-k)$ is the matrix element of the interaction in the superfluid channel calculated between plane waves, $\varepsilon(k)$ is the single-particle energy as a function of the momentum while ε_F is the neutron Fermi energy. Single particle energies can be calculated in Hartree–Fock approximation. Following Kuchareck *et al.* [6–7], we adopt the effective-mass approximation and treat the difference between the single particle energy and the Fermi energy in (1) in proximity of the neutron Fermi wavenumber k_N as

$$\varepsilon(k) - \varepsilon_F \approx \hbar^2 \frac{(k^2 - k_N^2)}{2m^*(k_N)},$$

where the effective mass at the Fermi surface is defined as

$$m^*(k_N) = \frac{m\hbar k_N}{(d\varepsilon(k)/dk)_{k=k_N}}.$$

In the present paper, we consider the D1 Gogny force [8] consisting in a sum of two gaussians in the coordinate space, one repulsive of radius $r_1 = 0.8$ fm, one attractive $r_2 = 1.2$ fm, a density-dependent part, necessary to give saturation in finite nuclei, and a spin-orbit part. This force gives a good description of the pairing properties in finite nuclei [8], and has also been used for calculations in nuclear matter [6] as well as in pure neutron matter [9].

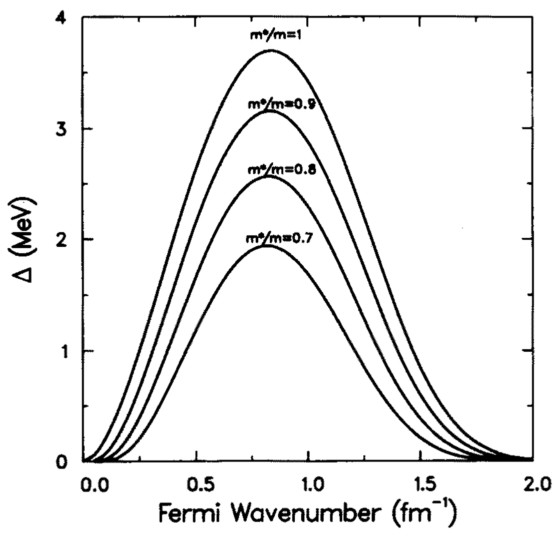

Fig. 1. Pairing gap calculated at the Fermi wavenumber for a uniform system with the D1 Gogny force. Different values for the effective mass parameter have been used.

Since the matrix element of the interaction in the superfluid channel in this plane-waves approximation is simply the Fourier transform of the nucleon-nucleon interaction in coordinate space, $v(q)$ assumes a simple gaussian form also in momentum space. In figure 1, we plot the pairing energy gap for pure neutron matter as a function of neutron Fermi wavenumber obtained with different fixed values for the effective mass parameter ranging from $m^*/m = 0.7$, to $m^*/m = 1$. We notice that for all the curves the pairing gap is maximum for $k_F \approx 0.8$ fm^{-1} and then decreases as a consequence of the finite range of the interaction and of the repulsion at large densities. We notice also that the gap at the saturation ($k_F \approx 1.4$ fm^{-1}) is quite high respect to most of neutron matter calculations, however in most of those works it has not been checked if correct pairing properties of finite nuclei were obtained.

3. Non–uniform neutron system

In the following the system consisting of the lattice of nuclear clusters and free neutrons is treated in the Wigner–Seitz approximation, that is the lattice is assumed divided into unit spherical cells of given radius containing a nucleus at the centre and filled by a gas of neutrons. Detailed studies of such a system has been carried out by Negele and Vautherin in Ref. [10].

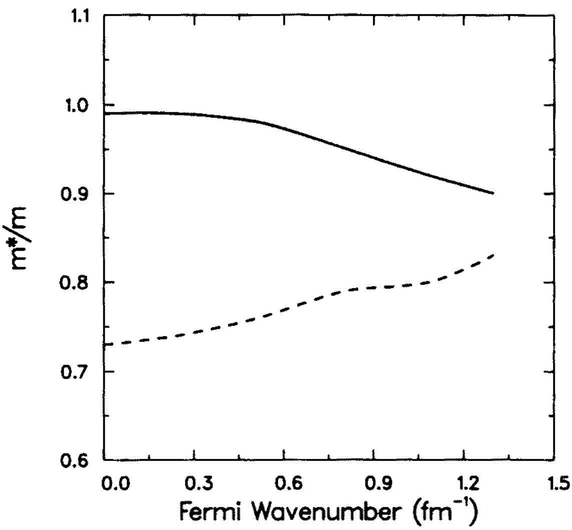

Fig. 2. Effective mass parameters as a function of the Fermi wavenumber. The full line represents the value outside the nuclear cluster, while the dashed one is the value inside.

We assume a local dependence of the effective mass, according to the proton and neutron local Fermi momenta. Calculating explicitly the Hartree–Fock exchange potential we get the expression for the effective mass pa-

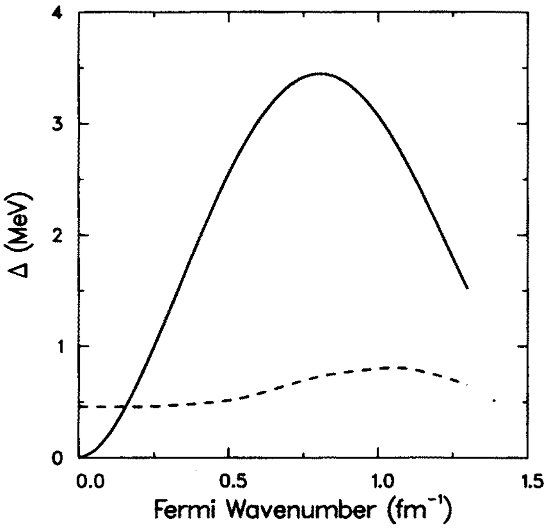

Fig. 3. Pairing gap as a function of Fermi wavenumber plotted for two different regions. Dashed line represents the pairing gap inside the nuclear clusters, while full line is the pairing gap outside.

rameter (the ratio between the effective and the bare mass) at the Fermi surface [11]

$$\frac{m^*(k_N)}{m} = \left(1 + \frac{mk_N}{4\pi^2\hbar^2}\sum y_i^N v_i^N c_i + \frac{m}{\hbar^2 k_N}\sum y_i^P v_i^P c_i\right)^{-1} \tag{2}$$

with

$$v_i^N = \left(\frac{2}{r_i k_N}\right)^4 \left[\frac{r_i^2 k_N^2}{2} - 1 + \left(1 + \frac{r_i^2 k_N^2}{2}\right)\exp\left(-r_i k_N\right)^2\right] \tag{3}$$

and

$$\begin{aligned} v_i^P = &\exp\left[-\frac{(k_N + k_P)^2 r_i^2}{4}\right]\left[\frac{1}{2\pi^2 r_i^2} - \frac{1}{\pi^2 r_i^2}\left(\frac{1}{k_N^2 r_i^2} + \frac{1}{2} + \frac{k_P}{2k_N}\right)\right] \\ &+ \exp\left[-\frac{(k_N - k_P)^2 r_i^2}{4}\right]\left[-\frac{1}{2\pi^2 r_i^2} + \frac{1}{\pi^2 r_i^2}\left(\frac{1}{k_N^2 r_i^2} + \frac{1}{2} - \frac{k_P}{2k_N}\right)\right] \end{aligned} \tag{4}$$

and $c_i = \pi^{3/2} r_i^3/4$, while

$$y_i^N = v_3^i - 2v_4^i + v_1^i + 2v_2^i$$

and

$$y_i^P = v_3^i - 2v_4^i \,.$$

The parameters v_j^i $(j = 1,4)$ are given in Table I. The values of k_N and k_P are obtained from the density profiles given in Ref. [10], using the relations

$$k_N = (3\pi^2 \rho_N)^{1/3}$$

and

$$k_P = (3\pi^2 \rho_P)^{1/3} \,.$$

The first term in (2) comes from neutron-neutron interaction, while the second represents the neutron-proton channel. The presence of protons decreases the value of the effective mass.

TABLE I

The parameter set D1 for the Gogny force

i	$v1$	$v2$	$v3$	$v4$
1	−402.4	−100.0	496.2	23.56
2	− 21.3	− 11.77	−37.27	68.81

The Wigner–Seitz cell is roughly divided into two parts, one (inner) corresponding to the nucleus and the other one (outer) corresponding to the remaining part of the cell. In figure 2 we report the effective mass parameter as a function of the Fermi wavenumber of the neutrons of the external gas. In the inner zone, the effective mass parameters ranges from $m^*/m = 0.73$ to $m^*/m = 0.83$, while in the outer zone, the effective mass parameter has values between $m^*/m = 0.9$ and $m^*/m = 1$. The corresponding pairing gaps are plotted in figure 3. Apart from region at low Fermi wavenumbers, corresponding to the lower part of the inner crust, the pairing gap in the region outside is larger respect to the region inside, reaching the maximum value for a density corresponding to $\rho/\rho_0 = 0.12$.

4. Pinning energy

The data presented in the previous section can be used to estimate pinning energy [12, 13] of a vortex core to a nucleus. The energy cost per particle of the normal matter in a vortex core is Δ^2/E_F where Δ is the pairing gap of the neutron superfluid and E_F is its Fermi energy. The neutrons of the external gas have different local densities inside and outside the potential well corresponding to the nuclei at the lattice sites. The difference in the corresponding pairing gap gives out a difference between the energy cost of a vortex line going through a nucleus and one that threads through the interstitial region. This leads to forces that pin vortex lines to nuclei or repel them from nuclei depending on which situation is much more favorable.

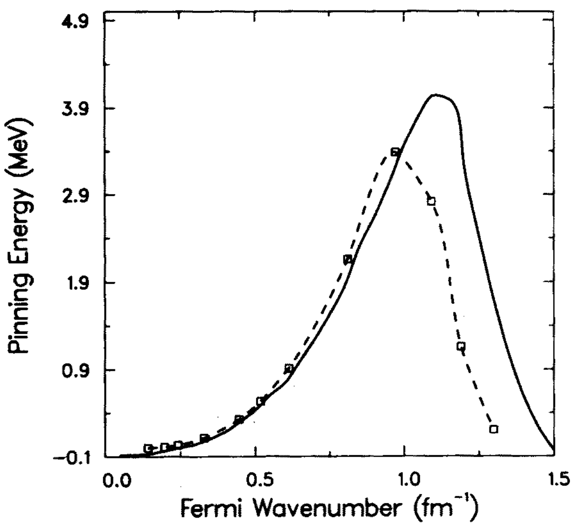

Fig. 4. Pinning energy E_p as a function of the Fermi wavenumber. Solid line shows the result of our calculation, dashed line corresponds to data taken from Ref. [9].

Pinning energy can be written as [12]

$$E_p = \frac{3}{8}\left[\left(\frac{\Delta^2}{E_F}\rho_N\right)_{\rm out} - \left(\frac{\Delta^2}{E_F}\rho_N\right)_{\rm in}\right]V\,. \tag{5}$$

The quantity in square brackets is the energy cost per volume to create a vortex line core, in, out refer to neutrons inside and outside the nuclear cluster, respectively. V is the overlap volume of the vortex line and a nucleus (with radius R)

$$V = \frac{4\pi}{3}R^3\left[1 - \left(1 - \frac{\xi^2}{R^2}\right)^{3/2}\right] \quad (\xi < R)\,,$$

$$V = \frac{4\pi}{3}R^3 \quad (\xi > R) \tag{6}$$

and

$$\xi = \frac{2}{\pi}\frac{E_F}{k_N \Delta}\,,$$

corresponding to the coherence length of the neutron superfluid is a vortex core radius of a cylindrical vortex. In figure 4 the resulting pinning energy E_p as a function of the Fermi wavenumber of the external neutrons, is plotted and compared with previous results obtained by Takatsuka [13]. The region of pinning found is much larger than the one found in [13] (see Fig. 1). The formula (5) follows from the approximation made in the paper to treat pairing properties locally, according to the value of the local density and proton fraction. On the other hand, the theory of vortex pinning in neutron stars is not well-developed and it is possible that a full solution of the problem, still to be worked out, might give quite different results to the local model, as found in the case of vortices in superconductors [14].

5. Conclusions

In this work, extending previous investigations about the paring properties in nuclear and neutron matter and using a well tested nuclear force, we have studied in detail a non–uniform system existing in the inner crust of a neutron star. We have found a strong spatial dependence of the pairing energy gap, leading in the lower part of the crust to a value of pairing energy gap smaller in the region outside the nuclei. An opposite situation was found for densities larger than $\rho/\rho_0 \approx 3. \times 10^{-3}$ We expect that this behavior strongly affects the pinning properties of a vortex–nucleus system.

We would like to thank J.A. Sauls, C.J. Pethick, R.A. Broglia, P.M. Pizzochero and M. Baldo for useful discussions.

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