THERMODYNAMIC-TYPE ASPECT OF QUANTUM TIME EVOLUTION*

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To the memory of Jan Rzewuski

The familiar analogy between the quantum time evolution of an isolated system and the thermal equilibrium of an open system with a thermostat is considered as a signal of a profound thermodynamic-type aspect of the physical time, suggesting small nonunitarity corrections to the conventional quantum theory. These imply, as their characteristic consequence, a tiny unitarity defect of S matrix. Another surprising consequence is a slight time dependence of mass of any matter system localized close to an experimental device such as a running supercollider or a big laser in action, where an intensive particle flow is produced. This suggests a described gedankenexperiment.

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1. Introduction

As is well-known, the relativistic physics discovered and established the geometrical aspect of physical time, unifying it with physical space into one $3+1$ dimensional spacetime being generally a Riemannian manifold. On the other hand, the familiar analogy between the time evolution of an isolated system in the quantum theory and the thermal equilibrium of an open system with a thermostat or heat reservoir [1] may be not a formal coincidence but rather an important signal of an underlying thermodynamic-type aspect of the physical time, somehow hidden in the conventional quantum theory.

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Let us first recall briefly our contribution to this point of view [2]. In fact, in the spirit of the familiar correspondence $it/\hbar \to 1/kT$ taken *à rebours*, $1/kT \to it/\hbar$, we put forward the hypothesis that any so called isolated quantum system persists in a *temporal equilibrium* with the physical spacetime which plays then the role of a *chronostat* or reservoir of a new thermodynamic-type quantity being an analogue of heat Q. Call this quantity *energy width* Γ.

Such a chronostat defines time t running equally at all space points $\vec{r}$ (in a Minkowski frame) and causes the time evolution in the quantum theory to be described by the familiar unitary operator $\exp(-iHt/\hbar)$. This is an analogue of the thermal-equilibrium distribution $\exp(-H/kT)$ conditioned by a thermostat defining absolute temperature T equal everywhere within the complex of the system and thermostat.

The above hypothesis, although in the case of conventional quantum theory it may seem only a phrase of semantic character, becomes physically significant when we ask the question whether some small deviations from such a temporal equilibrium may occur, implying tiny corrections to the conventional time evolution. In this case, in analogy with the thermal-nonequilibrium temperature field $T(\vec{r})$), we can speak of the temporal-nonequilibrium *time field* $t = t(\vec{r})$ having small nonzero gradient (of course, an important phenomenological difference is that at any space point $\vec{r}$ the temperature field may be fixed, while the time field is always running). Denoting by t_0 time running at a particular space point $\vec{r} = \vec{r}_0$, we can use t_0 as a time parameter and thus write $t(\vec{r}, t_0)$ (obviously, $t(\vec{r}_0, t_0) = t_0$). Then, $t(\vec{r}, t_0) \equiv t_0$ in the temporal equilibrium.

The analogy $kT \leftrightarrow -i\hbar t^{-1}$ or, in consequence, its shifted form $k(T - T_0) \leftrightarrow -i\hbar(t^{-1} - t_0^{-1})$, if considered as a physically profound correspondence, suggests for the inverse-time field

$$\varphi(\vec{r}, t_0) \equiv t^{-1}(\vec{r}, t_0) - t_0^{-1} \tag{1}$$

(in the case of a homogeneous matter medium) a conductivity equation of the form

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right) \varphi(\vec{r}, t_0) = 0\,. \tag{2}$$

This is an analogue of the familiar conductivity equation for the temperature field $T(\vec{r}, t) - T_0$. Here, $\lambda_\Gamma > 0$ plays the role of an unknown length-dimensional conductivity constant. Of course, in the temporal equilibrium the identity $\varphi(\vec{r}, t_0) \equiv 0$ holds.

It is interesting to note that the conductivity equation (2) can be considered as a nonrelativistic approximation for a tachyonic-type Klein–Gordon equation of the form

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2} + \frac{1}{4\lambda_\Gamma^2}\right)\chi(\vec{r}, t_0) = 0\,, \tag{3}$$

where

$$\chi(\vec{r}, t_0) \equiv \varphi(\vec{r}, t_0)\exp\left(\frac{c\,t_0}{2\lambda_\Gamma}\right)\,. \tag{4}$$

In fact,

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2} + \frac{1}{4\lambda_\Gamma^2}\right)\chi = \exp\left(\frac{c\,t_0}{2\lambda_\Gamma}\right)\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0} - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2}\right)\varphi$$

$$\simeq \exp\left(\frac{c\,t_0}{2\lambda_\Gamma}\right)\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\varphi \tag{5}$$

if φ changes slowly enough.

2. Basic equations

Taking over the form of Eq. (3) in the case of inverse-time field in the vacuum (then $\lambda_\Gamma > 0$ becomes an unknown universal conductivity constant of the vacuum) and introducing there matter sources, we may construct a universal equation for the inverse-time field interacting with the matter. Specifically, we propose in this way an inhomogeneous equation of the form

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t_0^2} + \frac{1}{4\lambda_\Gamma^2}\right)\chi = -4\pi\, g_\Gamma\lambda_\Gamma\left(\frac{\partial\rho}{\partial t_0} + \mathrm{div}\vec{j}\right)\,, \tag{6}$$

corresponding to the homogeneous equation (3). Here, $g_\Gamma > 0$ is an unknown dimensionless coupling constant and $(c\rho, \vec{j}) \equiv (j^\mu)$ denotes the matter 4-current with

$$j^\mu(\vec{r}, t_0) \equiv \langle\Psi(t_0)|\, J^\mu(\vec{r})\, |\Psi(t_0)\rangle_{\mathrm{av}} \tag{7}$$

being the spin-averaged expectation value (in the state $\Psi(t_0)$) of the operator of total particle-number 4-current $J^\mu(\vec{r})$ (in the Schrödinger picture, identical with the Heisenberg and interaction pictures at an initial instant $t_0 = t_{00}$).

In the nonrelativistic approximation, Eq. (6) gives through Eq. (5) an inhomogeneous conductivity equation of the form

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\varphi = -4\pi g_\Gamma\lambda_\Gamma\left(\frac{\partial\rho}{\partial t_0} + \mathrm{div}\vec{j}\right)\exp\left(\frac{-c\,t_0}{2\lambda_\Gamma}\right)\,, \tag{8}$$

corresponding to the homogeneous conductivity equation (2).

Since T is the absolute temperature varying in the interval $0 \leq T < \infty$, the analogy $kT \leftrightarrow -i\hbar t^{-1}$, if taken earnestly, suggests that also the physical time t gets in the realistic universe an absolute meaning in the sense that $0 \leq t < \infty$,where the limits $t \to 0$ and $t \to \infty$ are analogues of the limits $T \to \infty$ and $T \to 0$, respectively. Obviously, it would be natural to ascribe the limit $t \to 0$ to the hypothetic Big Bang of the universe, which thus would determine the absolute zero of cosmic time, an analogue of the infinite absolute temperature. In this limit, the deviations of the universe from an overall temporal equilibrium $\varphi \equiv 0$ would be singular. In contrast, in the limit $t \to \infty$ the universe would approach an overall temporal equilibrium $\varphi \equiv 0$ because for $t \to \infty$ the source term in Eq. (8) would switch off. So, for our "old" universe (if it is old enough) its deviations from an overall temporal equilibrium should be small, and then $g_\Gamma \exp(-ct_0/2\lambda_\Gamma)$ in Eq. (8) could be expected as small (depending, however, on the unknown constants $g_\Gamma > 0$ and $\lambda_\Gamma > 0$ as well as the actual age t_0 of the universe, popularly estimated as 1.5×10^{10} yr). Further on, we will accept this point of view.

Then, for the length-dimensional λ_Γ two opposite options are interesting: (i) the case of "microscopic" $\lambda_\Gamma \ll ct_0$, where $\exp(-ct_0/2\lambda_\Gamma) \lll 1$, and (ii) the case of "macroscopic" $\lambda_\Gamma \lesssim ct_0$, where $\exp(-ct_0/2\lambda_\Gamma) \lesssim 1$ or $\ll 1$. In the case (ii) there might be some chances for experimental detection of the hypothetic temporal-nonequilibrium deviations, even in our "old" universe (*cf.* Eq. (15) later on). Further, we will assume this option.

From the standpoint of phenomenological thermodynamics, the new conductivity equation (8) implies that the first law of thermodynamics [3] should be now extended to the form

$$dU = \delta W + \delta Q - i\delta\Gamma \tag{9}$$

including an imaginary term $-i\delta\Gamma$ with $-i\Gamma$ being an analogue of heat Q (when $-i\hbar t$ takes over the role of kT). Thus, a new thermodynamic-type quantity Γ — called by us energy width — is transferred to the system from the physical spacetime (that is included in the system's surroundings). In general, this transfer makes the system's internal energy U complex. Only in the conventional case, where the temporal equilibrium $\varphi \equiv 0$ persists, the internal energy U can be taken as real, since only then Eq. (9) is reduced to its conventional form with $\delta\Gamma \equiv 0$.

Thus, in general, it is natural to consider for a so called isolated system a quantum state equation in the nonunitarity form

$$i\hbar\frac{d\Psi(t_0)}{dt_0} = [H - i\mathbf{1}\Gamma(t_0)]\,\Psi(t_0) \tag{10}$$

(in the Schrödinger picture), where $\mathbf{1}$ is the unit operator and $\Gamma(t_0)$ denotes the total energy width of the system ($\Gamma(t_0)$ is a c-number-valued function of t_0). Specifically, we propose that in the nonrelativistic approximation

$$\Gamma(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r}\, \rho(\vec{r}, t_0)\varphi(\vec{r}, t_0) \tag{11}$$

with φ satisfying Eq. (8) and $\rho = c^{-1}j^0$ as given in Eq. (7).Then, as it should be, $\Gamma(t_0) \equiv 0$ in the temporal equilibrium $\varphi \equiv 0$, what reduces in this case Eq. (10) to the conventional quantum state equation [4] having the unitarity form

$$i\hbar \frac{d\Psi(t)}{dt} = H\Psi(t) \tag{12}$$

with $t \equiv t_0$. Evidently, Eq. (10) leads to a nonunitary quantum time evolution, including small nonunitarity corrections to the conventional quantum time evolution described in Eq. (12).

The mixed set of coupled equations (6) or (8) and (10) (with (11)) for $\varphi(\vec{r}, t_0)$ and $\Psi(t_0)$ gives us a quantum theory of hypothetic small deviations from the temporal equilibrium. Call such a theory phenomenological *chronodynamics* in analogy with the phenomenological thermodynamics. Note that, when treating the physical spacetime as a chronostat or energy-width reservoir, we do not discuss the physical nature of spacetime which is certainly connected, to some extent at least, with the theory of gravitation (or supergravitation) and its expected quantization. Analogically, in phenomenological thermodynamics, we do not discuss the physical nature of thermostat or heat reservoir.

3. The first approximation

Strictly speaking, the set of equations (8) and (10) is nonlinear (and nonlocal) in $\Psi(t_0)$, slightly perturbing the superposition principle, fundamental in the conventional quantum theory (valid in the temporal equilibrium). However, this set becomes linear (and local) in the approximation, where in Eq. (7) defining j^μ, the state $\Psi(t_0)$ is replaced in the zero approximation by the state vector $\Psi^{(0)}(t_0)$ satisfying the temporal-equilibrium state equation (12). Then, in the first approximation, Eqs. (8) and (10) take the forms

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right) \varphi^{(1)}(\vec{r}, t_0) = -4\pi g_\Gamma \lambda_\Gamma \left(\frac{\partial \rho^{(0)}}{\partial t_0} + \operatorname{div}\vec{j}^{(0)}\right) \exp\left(\frac{-c\,t_0}{2\lambda_\Gamma}\right) \tag{13}$$

and

$$i\hbar \frac{d\Psi^{(1)}(t_0)}{dt_0} = \left[H - i1\Gamma^{(1)}(t_0)\right]\Psi^{(1)}(t_0) \tag{14}$$

with

$$\Gamma^{(1)}(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r}\, \rho^{(0)}(\vec{r}, t_0)\varphi^{(1)}(\vec{r}, t_0) = O\left[g_\Gamma^2 \exp\left(\frac{-c\, t_0}{2\lambda_\Gamma}\right)\right] , \quad (15)$$

where

$$\vec{\jmath}^{\mu(0)}(\vec{r}, t_0) \equiv \langle \Psi^{(0)}(t_0)|\, \vec{J}^\mu(\vec{r})\, |\Psi^{(0)}(t_0)\rangle_{\rm av} . \quad (16)$$

From Eq. (14) we can see that

$$\Psi^{(1)}(t_0) = \Psi^{(0)}(t_0) \exp\left[-\frac{1}{\hbar}\int_{t_{00}}^{t_0} dt_0'\, \Gamma^{(1)}(t_0')\right] , \quad (17)$$

where

$$\Psi^{(0)}(t_0) = \exp\left[-\frac{1}{\hbar}H(t_0 - t_{00})\right] \Psi_{\rm H} , \quad (18)$$

if at $t_0 = t_{00}$ the Schrödinger picture coincides with the Heisenberg picture (and interaction picture): $\Psi^{(1)}(t_{00}) = \Psi_{\rm H}(= \Psi_I^{(1)}(t_{00}))$. When both instants t_{00} and t_0 (reckoned from the Big Bang) belong to our "old" universe, the approximation proposed above can be expected as excellent because we accepted $g_\Gamma \exp(-c\, t_0/2\lambda_\Gamma) \ll 1$ in Eq. (8).

In spite of the varying-in-time norm of the state $\Psi^{(1)}(t_0)$ obeying the nonunitary time evolution (17),

$$\langle \Psi^{(1)}(t_0)|\Psi^{(1)}(t_0)\rangle = \langle \Psi_{\rm H}|\Psi_{\rm H}\rangle \exp\left[-\frac{2}{\hbar}\int_{t_{00}}^{t_0} dt_0'\, \Gamma^{(1)}(t_0')\right] , \quad (19)$$

we will take for granted in our further considerations the probability interpretation of $\Psi^{(1)}(t_0)$, thus maintaining in the first approximation the conventional probabilistic interpretation of $\Psi^{(0)}(t_0)$. Such an interpretation of $\Psi^{(1)}(t_0)$ is consistent with the linearity and homogeneity of Eq. (14) as well as with the fact that Eq. (14) can be rewritten in the form

$$i\hbar \frac{d\Psi_\nu^{(1)}(t_0)}{dt_0} = \left[H - i\Gamma^{(1)}(t_0)\right] \Psi_\nu^{(1)}(t_0) \quad (20)$$

for any state $\Psi^{(1)}(t_0)$ related to an eigenstate

$$\Psi_\nu^{(0)}(t_0) = |\nu\rangle \exp\left[-\frac{i}{\hbar}E_\nu(t_0 - t_{00})\right]$$

of the energy operator H. Here, the energy width-operator

$$\Gamma^{(1)}(t_0) \equiv \sum_{\nu'} |\nu'\rangle \Gamma^{(1)}_{\nu'} \langle \nu'| \tag{21}$$

(with $\langle \nu' \mid \nu \rangle = \delta_{\nu'\nu}$) is state-independent, while the particular $\Psi^{(1)}(t_0)$ and $\Gamma^{(1)}(t_0)$ are denoted by $\Psi^{(1)}_\nu(t_0)$ and $\Gamma^{(1)}_\nu(t_0)$. Thus, the superposition principle holds for the states $\Psi^{(1)}_\nu(t_0)$ and so, for all states being their linear combinations.

When interpreted probabilistically, the varying-in-time norm of $\Psi^{(1)}(t_0)$ is a thermodynamic-type manifestation of the exciting physical fact that our so called isolated system is dynamically not complete, interacting — on an underlying quantum level — with the physical spacetime (whose consistent quantum structure may be topologically more involved than its familiar classical counterpart). So, this spacetime plays the role of an "enormous" dynamic complement of the system, being a chronostat in an excellent zero approximation corresponding to the temporal equilibrium. Of course, a consistent quantum theory of gravitation (or supergravitation) is invited here as a natural ingredient of the theory of this dynamical complement.

From the formal point of view, our nonunitarity quantum state equation (10) for a so called isolated system may be considered as an averaged result of projecting the states [5] of a future dynamically complete quantum theory (including the physical spacetime as an "enormous" dynamic complement) onto the Hilbert subspace corresponding to the system. This remark characterizes the difference between our basic thermodynamic-type concept of energy width Γ, connected formally with the act of projecting onto the whole Hilbert space of the system, and the familiar notion of the width of an energy level, following formally from projecting onto a distinguished direction in this space, corresponding to the energy level of the system. Of course, another component of our thermodynamic-type theory is the phenomenological field equation (6) or (8) for the parameter-valued inverse-time field. Such an equation should hopefully follow from the future complete quantum theory as an averaged result valid in the case of small deviations from the temporal equilibrium.

4. Some consequences

For scattering processes $i \to f$, we impose on $\Psi^{(0)}_I(t_0)$ and $\Psi^{(1)}_I(t_0)$ in the interaction picture the initial conditions $\Psi^{(0)}_I(t_{00} - \delta/2) = | i\rangle$ and $\Psi^{(1)}_I(t_{00} - \delta/2) = | i\rangle$, where δ is a time interval sufficiently large to cover the (microscopic) effective interaction time for the collision at $t_0 = t_{00}$, but extremely small when compared with the age $t_0 \sim t_{00}$ of our "old" universe. Then, when transformed to the interaction picture, Eq. (17) implies that in the first approximation S-matrix elements $S^{(1)}_{fi} = \langle f \mid \Psi^{(1)}_I(t_{00} + \delta/2)\rangle$ are equal to

$$S^{(1)}_{fi} = S^{(0)}_{fi} \exp(-\Delta^{(1)}_i)\,, \tag{22}$$

where $S^{(0)}_{fi} = \langle f \mid \Psi^{(0)}_I(t_{00} + \delta/2)\rangle$ describe elements of the conventional unitary S matrix and

$$\Delta^{(1)}_i \equiv \frac{1}{\hbar} \int\limits_{t_{00}-\delta/2}^{t_{00}+\delta/2} dt_0\, \Gamma^{(1)}(t_0) \tag{23}$$

are small decrement exponents dependent on initial states i and the age t_{00} of the universe at the instant of collision (of course, $t_0 \sim t_{00}$). For $c\delta/2\lambda_\Gamma$ small enough (what is always true in the case of "macroscopic" λ_Γ)

$$\Delta^{(1)}_i \simeq \frac{1}{\hbar} \Gamma^{(1)}(t_{00})\, \delta \tag{24}$$

due to Eq. (15).

We can see from Eq. (22) that in the first approximation the S matrix avails a tiny unitarity defect (determined by $\Delta^{(1)}_i$), conditioned by the hypothetic small nonunitarity corrections to the conventional time evolution. In principle, $\Delta^{(1)}_i$ can be measured (or estimated) by experimentally testing [6] the corresponding deviations from the conventional optical theorem, where, in the first approximation, the additional factor $\exp(-\Delta^{(1)}_i)$ appears at the imaginary part of forward scattering amplitude.

The possible nonunitarity of time evolution may be also examined by comparing the norms of a quantum state in the temporal equilibrium and in a situation when the temporal equilibrium is perturbed by an external inverse-time field. To set an academic example for such an experiment, consider a container with gas of hydrogen atoms in their ground states, put close to a big collider, say, the Tevatron. Producing particles during its stationary run, the collider induces the nonzero matter-current 4-divergence

$$\frac{\partial \rho^{\rm ex}(\vec{r})}{\partial t_0} + {\rm div}\vec{\jmath}^{\,\rm ex}(\vec{r}) \equiv {\rm div}\vec{\jmath}^{\,\rm ex}(\vec{r}) = f_S(\vec{r}) \tag{25}$$

and so, due to Eq. (8), creates the inverse-time field

$$\varphi^{\rm ex}(\vec{r}, t_0) = \varphi^{\rm ex}(\vec{r}) \exp\left(\frac{-ct_0}{2\lambda_\Gamma}\right)\,,$$

where

$$\varphi^{\rm ex}(\vec{r}) = -4\pi g_\Gamma \lambda_\Gamma \left(\Delta + \frac{1}{2\lambda_\Gamma^2}\right)^{-1} f_{\rm S}(\vec{r}) \tag{26}$$

(more accurately, using Eq. (6) for $\chi^{\rm ex}(\vec{r}, t_0) \equiv \varphi^{\rm ex}(\vec{r})$ one gets Eq. (26) with $(\lambda_\Gamma\sqrt{2})^{-2}$ replaced by $(2\lambda_\Gamma)^{-2}$, what is irrelevant in our argument).

In particular, consider as an analytically calculable model the inverse-time field

$$\varphi^{\rm ex}(\vec{r}) = \Omega_{\rm S} \frac{g_\Gamma \lambda_\Gamma}{|\vec{r} - \vec{r}_{\rm S}|} \cos \frac{|\vec{r} - \vec{r}_{\rm S}|}{\lambda_\Gamma \sqrt{2}} \sum_{l\, m_l} c_{l\, m_l} Y_{l\, m_l}(\theta, \phi) \tag{27}$$

with $\sum_{l\, m_l} c_{l\, m_l} Y_{l\, m_l}(0,0) = 1$, corresponding through Eq. (26) to the matter-source distribution

$$f_{\rm S}(\vec{r}) = \Omega_{\rm S} \left[\delta^3(\vec{r} - \vec{r}_{\rm S}) + \sum_{l\, m_l} c_{l\, m_l} \frac{l(l+1)}{4\pi\, |\vec{r} - \vec{r}_{\rm S}|^3} \cos \frac{|\vec{r} - \vec{r}_{\rm S}|}{\lambda_\Gamma \sqrt{2}} Y_{l\, m_l}(\theta, \phi)\right], \tag{28}$$

where θ, ϕ are spherical angles of the distance vector $\vec{r} - \vec{r}_{\rm S}$ from a well localized particle-production centre $\vec{r}_{\rm S}$. Here, the constant $\Omega_{\rm S} > 0$ is the number of particles produced by the collider per unit of time. In fact, from Eq. (28)

$$\int d^3\vec{r}\, f_{\rm S}(\vec{r}) = \Omega_{\rm S} \tag{29}$$

in consequence of

$$l(l+1) \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi\, Y_{l\, m_l}(\theta, \phi) = 0\,. \tag{30}$$

Note that $\varphi^{\rm ex}(\vec{r}, t_0) > 0$ and so $t^{\rm ex}(\vec{r}, t_0) < t_0$ for $|\, \vec{r} - \vec{r}_{\rm S}\,| < \infty$, where $t_0 = t^{\rm ex}(\vec{r}_0, t_0)$ for $|\vec{r} - \vec{r}_{\rm S}| = \infty$ (*cf.* Eq. (1)).

The external inverse-time field $\varphi^{\rm ex}(\vec{r}, t_0)$ modifies the temporal-equilibrium wave function

$$\psi_{\rm H}^{(0)}(\vec{r}_e, \vec{r}_p) \exp\left[-\frac{i}{\hbar} E\, (t_0 - t_{00})\right] \tag{31}$$

of any hydrogen atom in the container. If switched on at the instant $t_0 = t_{00}$, this field leads in the first approximation to the modified wave function

$$\psi_{\rm H}^{(0)}(\vec{r}_e, \vec{r}_p) \exp\left[-\frac{1}{\hbar} E\, (t_0 - t_{00}) - \frac{1}{\hbar\, N_{\rm H}} \int_{t_{00}}^{t_0} dt_0'\, \Gamma^{(1)}(t_0')\right] \tag{32}$$

consistent with Eq. (17), where now

$$\Gamma^{(1)}(t_0) = 2g_\Gamma \hbar \frac{N_\mathrm{H}}{V} \left[\int\limits_V d^3\vec{r}\, \varphi^{\mathrm{ex}}(\vec{r}) \right] \exp\left(\frac{-c\, t_0}{2\lambda_\Gamma} \right) \tag{33}$$

according to Eq. (15). In Eq. (33), consistently with Eq. (16), the hydrogen-gas particle density is given in the zero approximation by

$$\begin{aligned} \rho^{(0)}(\vec{r}) &= N_\mathrm{H} \int\limits_V d^3\vec{r}_e \int\limits_V d^3\vec{r}_p \; |\psi_\mathrm{H}^{(0)}(\vec{r}_e, \vec{r}_p)|^2 \; [\delta^3(\vec{r} - \vec{r}_e) + \delta^3(\vec{r} - \vec{r}_p)] \\ &= \frac{2N_\mathrm{H}}{V} \end{aligned} \tag{34}$$

with N_H denoting the number of hydrogen atoms in the container (notice that $|\psi_\mathrm{H}^{(0)}(\vec{r}_e, \vec{r}_p)|^2 = |\psi_\mathrm{H}^{(0)}(\vec{r}_e - \vec{r}_p)|^2 / V$ as $|\exp(i\vec{P} \cdot \vec{R}/\hbar)/\sqrt{V}|^2 = 1/V$). Hence, due to Eq. (32), the hydrogen-gas particle density in the first approximation becomes

$$\rho^{(1)}(\vec{r}, t_0) = \frac{2N_\mathrm{H}}{V} \exp\left[-d_\mathrm{H}^{(1)}(t_0) \right] , \tag{35}$$

where

$$d_\mathrm{H}^{(1)}(t_0) \equiv \frac{2}{\hbar} \int\limits_{t_{00}}^{t_0} dt_0' \, \Gamma^{(1)}(t_0') \, . \tag{36}$$

For $c(t_0 - t_{00})/2\lambda_\Gamma$ small enough (as it is always in the case of "macroscopic" λ_Γ)

$$\begin{aligned} d_\mathrm{H}^{(1)}(t_0) &\simeq \frac{2}{\hbar} \Gamma^{(1)}(t_{00})(t_0 - t_{00}) \\ &= 4g_\Gamma \frac{N_\mathrm{H}}{V} \left[\int\limits_V d^3\vec{r} \varphi^{\mathrm{ex}}(\vec{r}) \right] \exp\left(\frac{-c\, t_{00}}{2\lambda_\Gamma} \right) (t_0 - t_{00}) \end{aligned} \tag{37}$$

according to Eq. (33). Note that a tiny unitarity defect of time evolution of hydrogen wave function (32) is determined by $d_\mathrm{H}^{(1)}(t_0)/2N_\mathrm{H}$.

If in the considered experiment the model (27) of $\varphi^{\mathrm{ex}}(\vec{r})$ can be approximately used in Eq. (37), then we obtain

$$\begin{aligned} d_\mathrm{H}^{(1)}(t_0) &\simeq 4N_\mathrm{H} g_\Gamma^2 \left(\frac{\lambda_\Gamma}{L} \right) \cos\left(\frac{L}{\lambda_\Gamma \sqrt{2}} \right) \exp\left(\frac{-c\, t_{00}}{2\lambda_\Gamma} \right) \Omega_\mathrm{S}(t_0 - t_{00}) \\ &\simeq 4N_\mathrm{H} g_\Gamma^2 \left(\frac{\lambda_\Gamma}{L} \right) \exp\left(\frac{-c\, t_{00}}{2\lambda_\Gamma} \right) \Omega_\mathrm{S}(t_0 - t_{00}) \, , \end{aligned} \tag{38}$$

when we apply the mean-value theorem to the integral in Eq. (37) and so replace there the distance $|\vec{r} - \vec{r}_S|$ by the mean value $L = |\vec{r}_{av} - \vec{r}_S|$ over the container. The last approximation above is always justified in the case of "macroscopic" λ_Γ, where certainly $2\lambda_\Gamma \gg L$. In Eq. (38), the value of Loschmidt number gives $N_H \simeq 2.69 \times 10^{19} V/\,\mathrm{cm}^3$ (in the so called normal conditions), while $t_0 \sim t_{00} \simeq 1.5 \times 10^{10}$ yr is the actual age of the universe.

Taking, as for the Tevatron, the luminosity $\sim 10^{31}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$, the $\bar{p}p$ total cross-section $\sim$ 100 mb and the average particle multiplicity $\sim$ 100 we can get $\Omega_S \sim 10^8\mathrm{sec}^{-1}$ for the particle-production rate. Then, Eq. (38) implies

$$d_H^{(1)}(t_0) \sim 10^{28} g_\Gamma^2 \left(\frac{\lambda_\Gamma}{L}\right) \exp\left(\frac{-c\,t_{00}}{2\lambda_\Gamma}\right) \frac{V}{\mathrm{cm}^3} \frac{t_0 - t_{00}}{\mathrm{sec}}\,, \tag{39}$$

where $ct_{00} \simeq 1.4 \times 10^{28}$ cm and, for example, $V/L \sim 10\,\mathrm{cm}^2$. Unfortunately, the constants $g_\Gamma > 0$ and $\lambda_\Gamma > 0$ are unknown from the very beginning. We only assumed the option of "macroscopic" λ_Γ (otherwise there are no chances for experimental detection of the hypothetic unitarity defect of quantum time evolution). In such a situation, to make a numerical exercise, put $g_\Gamma^2 \sim 1$ and, specifically, $ct_{00}/2\lambda_\Gamma \simeq 140$ to 150, what gives $\lambda_\Gamma \simeq (5.1\,\mathrm{to}\,4.7) \times 10^{25}$ cm and

$$d_H^{(1)}(t_0) \sim (10^{-7}\,\mathrm{to}\,10^{-12}) \frac{\mathrm{cm}}{L} \frac{V}{\mathrm{cm}^3} \frac{t_0 - t_{00}}{\mathrm{sec}} \tag{40}$$

(notice that if alternatively $c\,t_{00}/2\lambda_\Gamma \sim 1$, the estimate (40) requires $g_\Gamma^2 \sim 10^{-63}$ to 10^{-68}). Hence, for $V/L \sim 10\mathrm{cm}^2$ and the run-time $t_0 - t_{00} =$ 1 month $\sim 10^6$ sec, our exercise leads to

$$d_H^{(1)}(t_{00} + 1\,\mathrm{month}) \sim 1 \text{ to } 10^{-5}\,. \tag{41}$$

Note that then $\exp[-d_H^{(1)}(t_{00} + 1 \text{ month})] \sim 0.37$ to $1 - 10^{-5}$. In general, $d_H^{(1)}(t_0)$ decreases violently with decreasing λ_Γ. Of course, only a small $d_H^{(1)}(t_0)$ is physically acceptable, but how small and for what run-times $t_0 - t_{00}$, is an experimental question.

In connection with this question, it is surprising to realize that, after the stationary run of the collider is switched on at the instant $t_0 = t_{00}$, the mass of hydrogen gas in the container is

$$M_V^{(1)}(t_0) = N_H m_H \exp\left[-d_H^{(1)}(t_0)\right]\,, \tag{42}$$

although before this switching on the mass was

$$M_V^{(0)} = N_H m_H\,. \tag{43}$$

In fact, making use of the probabilistic interpretation of the state (17) and so of the resulting wave function (32), we calculate :

$$\begin{aligned} M_V^{(1)}(t_0) &= N_{\mathrm{H}} \int\limits_V d^3\vec{r} \int\limits_V d^3\vec{r}_e \int\limits_V d^3\vec{r}_p \left\{ |\psi_{\mathrm{H}}^{(0)}(\vec{r}_e, \vec{r}_p)|^2 \right. \\ &\quad \left. \times \exp\left[-d_{\mathrm{H}}^{(1)}(t_0)\right] \left[m_e^{\mathrm{eff}} \delta^3(\vec{r}-\vec{r}_e) + m_p^{\mathrm{eff}} \delta^3(\vec{r}-\vec{r}_p)\right] \right\} \\ &= N_{\mathrm{H}}(m_e^{\mathrm{eff}} + m_p^{\mathrm{eff}}) \exp\left[-d_{\mathrm{H}}^{(1)}(t_0)\right] , \end{aligned} \tag{44}$$

where $m_{\mathrm{H}} = m_e^{\mathrm{eff}} + m_p^{\mathrm{eff}}$ is the mass of hydrogen atom. An analogical remark pertains also to any other global physical quantity ascribed to the hydrogen gas in the container (*e.g.* its polarized magnetic moment).

We can see from Eqs. (42) and (43) that the ratio $M_V^{(1)}(t_0) : M_V^{(0)}$ is equal to $\exp\left[-d_{\mathrm{H}}^{(1)}(t_0)\right] \lesssim 1$ or

$$\ln M_V^{(1)}(t_0) - \ln M_V^{(0)} = -d_{\mathrm{H}}^{(1)}(t_0) \lesssim 0 \,. \tag{45}$$

In principle, such a new effect of slight change of matter mass in an external inverse-time field can be experimentally tested, offering to us an interesting method of measuring (or estimating) the hypothetic unitarity defect of quantum time evolution (determined in our academic example by $d_{\mathrm{H}}^{(1)}(t_0)$). This completes the gedankenexperiment proposed here to examine the possible nonunitarity corrections to the conventional quantum theory.

As a final remark, let us note that if in our nonunitarity quantum state equation (10) we take $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$ (*i.e.*, the free Dirac Hamiltonian) and then $\Gamma = \beta\varepsilon c^2$ with an infinitesimal constant $\varepsilon > 0$, we obtain automatically the Feynman-propagator prescription $m \to m - i\varepsilon$ when $\varepsilon \to +0$. Thus, the Feynman's (or Stückelberg's casual) choice for spin $-\frac{1}{2}$ fermion propagator, correct in the Dyson–Wick perturbation procedure in the conventional quantum field theory, may be interpreted as a signal of a tiny nonunitarity deviation of the quantum state equation from its conventional unitarity form (12). Hence, the infinitesimal $\varepsilon > 0$ may be a physically necessary remnant of our more basic thermodynamic-type theory when its other nonconventional, thermodynamic-type features can be approximately neglected. A remark that the Feynman's ε might be of a statistical origin, the author had an opportunity to hear from Wolfgang Pauli in ETH many years ago.

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