INTERMITTENCY IN CLUSTER MODELS; CORRELATION AND FLUCTUATION APPROACHES

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Intermittent correlations/fluctuations in the particle spectra of high-energy collisions are studied using correlation and fluctuation descriptions of cluster models. It is shown that leading contribution to intermittency in both methods may be connected with a cluster structure of multiparticle process.

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1. Introduction

The study of correlations of final state particles emitted at various positions of rapidity in hadron-hadron interactions has revealed a tendency for particles to be grouped in clusters over a range of rapidity of about 1 to 2 units [1–4]. These short range correlations for hadron-hadron collisions were interpreted in terms of cluster models [5–8] in which the observed hadrons are decay products of clusters. The e^+e^- data shown similar effect [9]. Moreover, the cluster scheme is useful for Monte Carlo simulation of hadronization in e^+e^--annihilation (cluster fragmentation model [10]).

In this paper we shall analyse the intermittent behaviour in particle spectra (for short review see [11, 12]) under the general assumption that multiparticle production can be described by a two-step cluster mechanism. By this we mean that the clusters are formed on the first stage of multiparticle process and all final hadrons are created by the decays of clusters on the second stage. The descriptions of cluster models via semi-inclusive densities and multiplicity distributions enable one to examine the intermittency using correlation (Section 2) and fluctuation (Sections 3, 4) approaches, respectively.

The purpose of this paper is to examine the contributions from both stages of multiparticle production to intermittency phenomenon. We discuss how self-similar patterns can arise in the cluster models. It is shown that the investigations possible without detailed understanding of structure of particle semi-inclusive densities and multiplicity distributions for full rapidity. The main result of the paper is that specific intermittent correlations/fluctuations can arise among particles coming from the decay of the same cluster. Since our analysis of the cluster models possesses large generality, the conclusion makes it appropriate to speak about general features of the models leading to intermittency in which the clusters decay into final hadrons independently. In our opinion, this result is important because it can be interpreted in any particular dynamical scheme.

Note that throughout this paper we consider only identical clusters and identical final hadrons.

2. Intermittent correlations in cluster models

Over last years, there has been a growing interest in intermittent behaviour in particle physics [11, 12]. In the terms of normalized factorial moments the intermittent behaviour in spectra of particles means the existence of a power-law singularity at small intervals of phase space [13] (see Section 3). Such behaviour is a straightforward manifestation of nonstatistical multiplicity fluctuations of secondary particles produced in high-energy interactions. However, other way of studying of such fluctuations is possible also. The power-like singularity in the normalized factorial moments of second order corresponds to the power-like singularity in two-particle inclusive density [14, 15] for $y_2 \to y_1$. Then the two-particle inclusive density can be expressed in the form

$$\rho(y_1, y_2) = \frac{1}{\sigma_I} \frac{d^2\sigma}{dy_1 dy_2} \simeq a \mid y_2 - y_1 \mid^{-d_2} + L(y_1, y_2), \tag{1}$$

where σ_I is the total inelastic cross section, a is some constant, d_2 denotes an anomalous fractal dimension (AFD) [16] and $L(y_1, y_2)$ is some translation invariant function. The last term in (1) corresponds to the possible nonsingular correlations.

The expression (1) implies that two-particle semi-inclusive density has the form

$$\rho_n(y_1, y_2) = \frac{1}{\sigma_n} \frac{d^2\sigma_n}{dy_1 dy_2} \simeq a_n \mid y_2 - y_1 \mid^{-d_2} + L_n(y_1, y_2), \tag{2}$$

recalling that

$$\rho(y_1, y_2) = \langle \rho_n(y_1, y_2) \rangle_n, \tag{3}$$

where σ_n is the cross section for the production of n charged particles, $\langle \ldots \rangle_n$ denotes the average over all events, and $L(y_1, y_2) = \langle L_n(y_1, y_2) \rangle_n$, $a = \langle a_n \rangle_n$. Here we assume that the AFD d_2 is independent of how many particles are being produced in full rapidity interval for simplicity.

Now let us consider the generating functional (GF) for the semi-inclusive densities of final hadrons in cluster models. In terms of the GF $\tilde{G}[\phi(\tilde{y})]$ of clusters and the GF $\tilde{\tilde{G}}[\phi(y); \tilde{y}]$ of hadrons emitted by one cluster that is formed at rapidity $\tilde{y}$, we can write the GF for final hadrons as follows (using [8] with some slight change of definitions)

$$G[\phi(y)] = \tilde{G}\left[\tilde{\tilde{G}}[\phi(y); \tilde{y}]\right], \tag{4}$$

where y is rapidity of final hadrons and $\phi(y)$ is auxiliary function. The generating functional for m-particle semi-inclusive densities $\rho_n(y_1, \ldots y_m)$ is

$$G[\phi(y)] = \sum_{m=0}^{\infty} \prod_{i=1}^{m} \frac{1}{m!} \int dy_i \phi(y_i) \rho_n(y_1, \ldots y_m), \tag{5}$$

where the semi-inclusive densities are given by the successive functional derivatives of the GF with respect to $\phi(y)$ at $\phi(y) = 0$.

By applying formulas (4), (5) the two-particle semi-inclusive rapidity density $\rho_n(y_1, y_2)$ of final hadrons can be expressed as

$$\begin{aligned} \rho_n(y_1, y_2) = &\int_{\Omega_1} \tilde{\rho}_m(\tilde{y}) \tilde{\tilde{\rho}}_k(y_1, y_2; \tilde{y}) d\tilde{y} \\ &+ \int_{\Omega_2} \tilde{\rho}_m(\tilde{y}_1, \tilde{y}_2) \tilde{\tilde{\rho}}_k(y_1; \tilde{y}_1) \tilde{\tilde{\rho}}_k(y_2; \tilde{y}_2) d\tilde{y}_1 d\tilde{y}_2, \end{aligned} \tag{6}$$

where $\tilde{\rho}_m(\tilde{y})$, $\tilde{\rho}_m(\tilde{y}_1, \tilde{y}_2)$ are cluster semi-inclusive rapidity densities with fixed multiplicity m and $\tilde{\tilde{\rho}}_k(y; \tilde{y})$, $\tilde{\tilde{\rho}}_k(y_1, y_2; \tilde{y})$ are semi-inclusive rapidity densities of particles emitted by one cluster which is formed at rapidity $\tilde{y}$ (k is number of particles per cluster). As we see, the density $\rho_n(y_1, y_2)$ splits into two terms according to whether both particles come from the decay of the same cluster or from the decay of different clusters. Integration of the semi-inclusive densities over the full domain of the rapidity space gives the identities

$$\int_{\Omega_1} \tilde{\rho}_m(\tilde{y}) d\tilde{y} = m, \qquad \int_{\Omega_2} \tilde{\rho}_m(\tilde{y}_1, \tilde{y}_2) d\tilde{y}_1 d\tilde{y}_2 = m(m-1), \tag{7}$$

$$\int_{\Omega_1} \tilde{\rho}_k(y;\tilde{y})dy = k, \qquad \int_{\Omega_2} \tilde{\rho}_k(y_1,y_2;\tilde{y})dy_1dy_2 = k(k-1). \tag{8}$$

Let us show that the second term in (6) does not lead to the power-law singularity in the form $\mid y_2 - y_1 \mid^{-d_2}$.

For the case when the clusters are uncorrelated in rapidity space, this statement is trivial. Indeed, if clusters would be emitted independently, $\tilde{\rho}_m(\tilde{y}_1,\tilde{y}_2)$ would be the product of the two single cluster distribution functions $\tilde{\rho}_m(\tilde{y}_1)$, $\tilde{\rho}_m(\tilde{y}_2)$, and second term in (6) reduces to $F(y_1)F(y_2)$, where $F(y) = \int_{\Omega_1} \tilde{\rho}_m(\tilde{y})\tilde{\rho}_k(y;\tilde{y})d\tilde{y}$. For the general case, when clusters are correlated, the proof of the statement will hinge on the fact that the relation

$$K\tilde{\rho}_k(y_1;\tilde{y}_1) \equiv \int_{\Omega_1} \tilde{\rho}_m(\tilde{y}_1,\tilde{y}_2)\tilde{\rho}_k(y_1;\tilde{y}_1)d\tilde{y}_1 \tag{9}$$

is linear integral transformation K of real square-integrable function $\tilde{\rho}_k(y_1;\tilde{y}_1)$ with kernel $\tilde{\rho}_m(\tilde{y}_1,\tilde{y}_2)$. For identical clusters the kernel is a symmetric function with respect to exchange of any of its arguments, *i.e.* $\tilde{\rho}_m(\tilde{y}_1,\tilde{y}_2) = \tilde{\rho}_m(\tilde{y}_2,\tilde{y}_1)$. Let us remind from theory of linear integral equations [17] that the quadratic integral form with symmetric kernel

$$(K\tilde{\rho}_k,\tilde{\rho}_k) = \int_{\Omega_2} \tilde{\rho}_m(\tilde{y}_1,\tilde{y}_2)\tilde{\rho}_k(y_1;\tilde{y}_1)\tilde{\rho}_k(y_2;\tilde{y}_2)d\tilde{y}_1d\tilde{y}_2\,, \tag{10}$$

can be expressed via the Hilbert formula as

$$\sum_{\nu=1}^{\infty} \varphi_\nu(y_1)\varphi_\nu(y_2)\lambda_\nu^{-1}\,, \tag{11}$$

where

$$\varphi_\nu(y_{1,2}) = \int_{\Omega_1} \tilde{\rho}_k(y_{1,2};s)\phi_\nu(y_{1,2},s)ds\,, \tag{12}$$

where $\{\phi_\nu(y,s)\}$ is complete set of eigenfunctions of the kernel and the λ_ν are eigenvalues. It is easy to see that the expression (11) does not lead to the power-like singularity in Eq. (2) and, hence, the intermittent behaviour in semi-inclusive density may be connected only with the first term in Eq. (6).

As is well known, the cluster semi-inclusive densities can be approximated by expression [8, 18]

$$\tilde{\rho}_k(y_1,\ldots y_r;\tilde{y}) = \frac{1}{\alpha^r}\frac{\partial^r}{\partial y_1\ldots\partial y_r}\left[1+\alpha\sum_{i=1}^{k}\int_0^{y_i} D(y-\tilde{y})dy\right]^k_{\alpha=0}, \tag{13}$$

where $D(y - \tilde{y})$ is probability density to have one particle at y emitted by the cluster which is formed at rapidity $\tilde{y}$. The $D(y-\tilde{y})$ is well approximated by a Gaussian distribution $D(y-\tilde{y}) \propto \exp[-(y-\tilde{y})^2/2\delta^2]$, where δ is decay width in rapidity space of the clusters. The expression (13) corresponds to the idea about absence of any correlations between particles. To see this, we shall consider the following *probability densities* in the usual statistical sense [19]

$$\tilde{\rho}'_k(y_1,\dots y_r;\tilde{y}) = \frac{(k-r)!}{k!}\tilde{\rho}_k(y_1,\dots y_r;\tilde{y}) \tag{14}$$

with the normalization condition

$$\int_{\Omega_r} \tilde{\rho}'_k(y_1,\dots y_r;\tilde{y})dy_1 \dots dy_r = 1\,. \tag{15}$$

These quantities define the probability densities that r particles from one cluster, successively chosen from an event with k particles ($r \geq k$), have respectively rapidities $y_1, y_2, \dots y_r$. The expression (13) can be rewritten in terms of these densities as

$$\tilde{\rho}'_k(y_1,\dots y_r;\tilde{y}) = \prod_{i=1}^{r} \tilde{\rho}'_k(y_i;\tilde{y})\,, \tag{16}$$

where $\tilde{\rho}'_k(y_i;\tilde{y}) = D(y_i - \tilde{y})$. Hence, due to the factorization property, the continuous random variables y_i, $i = 1\dots r$ are statistically independent.

As we mentioned before, the first term in (6) may be related to the power-law singularity in question. Let us define the second order cluster correlation function as

$$\tilde{\tilde{C}}'_k(y_1,y_2;\tilde{y}) = \tilde{\rho}'_k(y_1,y_2;\tilde{y}) - \tilde{\rho}'_k(y_1;\tilde{y})\tilde{\rho}'_k(y_2;\tilde{y})\,. \tag{17}$$

The correlation function, as defined above, vanishes if the densities have factorization property (16) and, hence, for (13) one gets $\tilde{\tilde{C}}'_k(y_2,y_1;\tilde{y}) = 0$ also. When the correlation function is singular for $y_2 \to y_1$, then the result follows that the final semi-inclusive density $\tilde{\rho}_n(y_1,y_2)$ in (6) has singularity too, *i.e.*, if

$$\tilde{\tilde{C}}'_k(y_1,y_2;\tilde{y}) \propto |\, y_2 - y_1\,|^{-d_2}\,, \tag{18}$$

we have

$$\tilde{\rho}_n(y_1,y_2;\tilde{y}) \propto |\, y_2 - y_1\,|^{-d_2}\;. \tag{19}$$

Note that the correlation function does not depend on $\tilde{y}$ when the particle-separation in rapidity space tends to zero. Thus the intermittent behaviour implies that the particles produced by one cluster are correlated in rapidity space and, hence, the models described by Eq.(13) are unlikely to be realized in nature.

3. Bunching projection method and intermittent fluctuations

Up to now we have considered a simplest case of cluster model with a *fixed* number of particles on each stage for all events. In order to discuss the intermittency in models with non-fixed number of particles, we shall consider the cluster models in terms of generation functions (GFns) $Q(z, Y)$ for the probabilities $P_n(Y)$ that n particles be produced altogether in the whole rapidity interval Y

$$Q(z,Y) \equiv \sum_{n=0}^{\infty} z^n P_n(Y) = Q^1(Q^2(z,\Delta), Y - \Delta), \tag{20}$$

$$Q^1(z, Y - \Delta) \equiv \sum_{n=0}^{\infty} z^n P_n^1(Y - \Delta), \tag{21}$$

$$Q^2(z,\Delta) \equiv \sum_{n=0}^{\infty} z^n P_n^2(\Delta), \tag{22}$$

where z is auxiliary variable. The $Q^1(z, Y - \Delta)$ is GFn for cluster multiplicity distribution $P_n^1(Y - \Delta)$. The $Q^2(z, \Delta)$ describes the multiplicity distribution $P_n^2(\Delta)$ of final hadrons which are emitted by the cluster, Δ is the characteristic size of rapidity interval containing almost all final hadrons that are emitted by one cluster. There are many ways in which the Δ and decay width δ resemble each other. However, one must expect that $\Delta > \delta$. Concrete realizations of the convolution (20) of cluster models are well known in high energy physics (see for example [20–27]). Here we shall utilize only general statistical formalism to analyse the intermittency phenomenon in such models.

As already mentioned in the previous section, the intermittent behaviour is defined as a power-like increase of normalized factorial moments (NFMs) $F_k(\delta y)$ in small rapidity interval δy so that in terms of GFn one should have

$$F_k(\delta y) = \frac{Q^{(k)}(z, \delta y)\mid_{z=1}}{(Q'(z, \delta y)\mid_{z=1})^k} \propto \delta y^{-d_k(k-1)}, \tag{23}$$

where d_k are the AFDs. To calculate the NFMs, we must know GFn $Q(z, \delta y)$ for small bin δy. On the theoretical side of the question, this task is fenomenological one. It may be solved using the projection method (see

[27, 19] where the method was used in particle physics). The GFn $Q(z,\delta y)$ is given in terms of the projection GFn $G_n(z,\delta y)$ and the probabilities $P_n(Y)$ by

$$Q(z,\delta y) = \sum_{n=0}^{\infty} P_n(Y) G_n(z,\delta y) \equiv \langle G_n(z,\delta y)\rangle_n \,, \tag{24}$$

$$G_n(z,\delta y) \equiv \sum_{l=n}^{\infty} z^l W_{l,n}(\delta y) \,, \tag{25}$$

where $W_{l,n}(\delta y)$ is the probability to have exactly l particles in the bin δy when there are n in full interval Y.

It is easy to see that the problem of possible power-like behaviour of NFMs for small bin is one of non-trivial peculiarity of the projection distribution for small δy. As an example, let us consider the case when there are no any correlations between particles in rapidity space. Then the projection distribution is positive binomial one with GFn in the form

$$G_n^{\rm PB}(z,\delta y) = (p(\delta y,n)z + q(\delta y,n))^n, \qquad p(\delta y,n) + q(\delta y,n) = 1 \,, \tag{26}$$

where $p(\delta y,n) \simeq \lambda(n)(\delta y/Y)$, $\lambda(n)$ is some function depending only on n. Using (23)–(26), for NFM one gets

$$F_k^{\rm PB} = \left\langle \lambda^k(n) n^{[k]} \right\rangle_n \langle \lambda(n) n\rangle_n^{-k} \,, \tag{27}$$

where $n^{[k]} \equiv n(n-1)\ldots(n-k+1)$. These NFMs have no tendency to increase with decreasing interval of rapidity and can not lead to intermittency.

To overcome this difficulty, we must turn to a more general projection distribution taking into account a possible correlations between particles. For this, we will use bunching projection distribution which was introduced for the first time in [28]

$$G_n^{\rm bunch}(z,\delta y) = G_n^{\rm PB}(z,\delta y) + \sum_{k=2}^{n} c(k,n) \left(\frac{\delta y}{Y}\right)^{s(k)} (z-1)^k \,, \tag{28}$$

where $c(k,n)$, $s(k)$ are positive constants. As was shown in [28], this distribution reflects a clustering of particles, *i.e.* it leads to a fluctuations of particles in rapidity space. Then for NFMs of any distribution $P_n(Y)$ after projection with (28) one obtains

$$F_k(\delta y) = F_k^{\rm PB} + \frac{\xi(k)}{\langle\lambda(n)n\rangle_n^k} \left(\frac{Y}{\delta y}\right)^{k-s(k)} \,, \tag{29}$$

where $\xi(k) = k! \sum_{i=k}^{\infty} c(k,i)P_i(Y)$. The AFDs d_k are given by $d_k = (k - s(k))/(k-1)$. They depend only on a free parameter $s(k)$ of the bunching projection distribution.

Let us now turn back to the two-step mechanism of multiparticle production with GFn (20). Consider the case $\delta y \ll \Delta$. There is a probability for final hadrons emitted by one cluster to be detected in the restricted interval δy. Moreover, in the restricted bin of size δy may be emitted hadrons by different clusters which are formed at limited rapidity interval of size $\sim \Delta$. So, we shall make two projection in Δ and δy in both multiplicity distributions of the first and the second stages, respectively. For this, we shall use the bunching projection distributions taking into account not only that clusters can be grouped in bunches on rapidity in the end of the first stage but also fluctuations of hadrons emitted by the clusters on the second one. On the other hand, it is easy to see that we must make only one projection in δy for Q^1, if $\delta y \gg \Delta$. The corresponding GFns are given by the expressions

$$Q(z,\delta y) \simeq Q^1(Q^2(z,\delta y),\Delta), \qquad \delta y \ll \Delta\,, \tag{30}$$

$$Q(z,\delta y) \simeq Q^1(Q^2(z,\Delta),\delta y), \qquad \delta y \gg \Delta\,, \tag{31}$$

$$Q^1(z,\Delta) = \sum_{n=0}^{\infty} P_n^1(Y-\Delta)G_n^1(z,\Delta)\,, \tag{32}$$

$$Q^1(z,\delta y) = \sum_{n=0}^{\infty} P_n^1(Y-\Delta)G_n^1(z,\delta y)\,, \tag{33}$$

$$Q^2(z,\delta y) = \sum_{n=0}^{\infty} P_n^2(\Delta)G_n^2(z,\delta y)\,, \tag{34}$$

where G_n^1, G_n^2 denote the bunching projection distributions in the form (28).

We have seen that in real physical situation the value of Δ is about one unit in rapidity. Then the intermittent behaviour of NFM must be determined by GFn in the form of Eq. (30). Using (23), (30) for the leading term of NFM in limited rapidity interval $\delta y \to 0$ one gets

$$F_k^{\mathrm{lead}}(\delta y) \propto \left(\frac{Y}{\delta y}\right)^{k-s_2(k)}, \qquad k > 1\,, \tag{35}$$

where $s_2(k)$ are constants of the bunching projection distribution $G_n^2(z,\delta y)$ for the second stage. As we see, if rapidity interval is small enough then leading contribution to intermittency is determined by the last stage as we also illustrated in the second section.

If we suppose that there exist a maximum invariant mass of clusters $M_{\max}$, then the number of the terms in sum (34) is restricted by $N \sim M_{\max}/m_\pi$ because of the law of conservation of energy (m_π is mass of pion). For example, this case takes place in the cluster fragmentation model in which the clusters are parton colourless objects possessing properties of simple hadron resonances [10]. Then, as shown in [28, 29], expression (35) is correct as long as $k \leq N$. If $k > N$, the leading terms of NFMs have form

$$F_k^{\text{lead}}(\delta y) \propto \left(\frac{Y}{\delta y}\right)^{f_k}, \qquad f_k = \left\{ k - \sum_{\{m_l\}} m_l s_2(l) \right\}_{\max}, \tag{36}$$

where $\sum\limits_{\{m_l\}}$ is the sum over all species $\{m_l\}$ which satisfy the condition $\sum\limits_{l=2}^{N} m_l l = k$, $(N > 2)$. After that, we must choose the maximum value among the species $\{f_k\}$ with fixed k. For multifractal behaviour $d_k \simeq ak$ (a is some constant), we have

$$d_k = \frac{1}{k-1} \left\{ \sum_{\{m_l\}} al(l-1)m_l \right\}_{\max}, \qquad k > N > 2. \tag{37}$$

From (37) it can be shown that AFD has a tendency to decrease with the increasing rank k if its value is greater then N. For instance, if $N = 4$, we have $d_4/d_5 = 2$ and for $k > 5$, d_k has a complicated oscillatory decreasing form. This observation can be revealed in high-energy experiments. The tendency to decrease in behaviour of AFM after some values of k was observed so far only in some nucleus–nucleus reactions [30, 31]. It is possible that this is the result of some analogous of two-step cluster mechanism. Now we have no clear evidence to consider this experimental results as dynamical effect in view of low statistics in the experiments.

4. Clan structure as a source of intermittency

As an example of our approach, let us consider negative binomial distributions (NBDs) which give a good fit to most multiparticle production data [32–39]. The analysis of the NBD may be carried out in terms "clans" for groups of particles of common ancestry [20–23]. For full rapidity interval, the NBD is given by the convolution of Poisson GFn $Q^P(z)$ and logarithmic GFn $Q^L(z)$

$$Q^{\mathrm{NBD}}(z,Y) = \left(1 + \frac{\bar{n}}{\omega}(1-z)\right)^{-\omega} = Q^{\mathrm{P}}(Q^{\mathrm{L}}(z)), \tag{38}$$

$$Q^{\mathrm{P}}(z) = \mathrm{e}^{\bar{N}(z-1)}, \qquad Q^{\mathrm{L}}(z) = \frac{\ln(1-bz)}{\ln(1-b)}, \tag{39}$$

where $\bar{N} = \omega \ln(1+\bar{n}/\omega)$, $b = \bar{n}/(\bar{n}+\omega)$, $\bar{n}$ is average multiplicity of final hadrons in full rapidity interval Y and ω is a positive parameter. The Q^{P} is GFn for the distribution of the number of clans (or ancestors), and Q^{L} is GFn for the multiplicity distribution of particles in one clan.

In order to illustrate the problem related to the behaviour of the convolution (38) for small rapidity intervals, we shall make projection of the NBD. For this, we will use PB projection distribution (26). For the sake of simplicity we suppose that $p(\delta y, n)$ is independent of n, *i.e.* $p(\delta y, n) = p(\delta y) \equiv p$. (this case see, for example, in [27]). Then one gets the same definitions of projections, as those of Eqs (30)–(34) written for bunching projection distributions, substituting $G_n^{\mathrm{P}} = (p^{\mathrm{P}} z + q^{\mathrm{P}})^n$ to G_n^1 and $G_n^{\mathrm{L}} = (p^{\mathrm{L}} z + q^{\mathrm{L}})^n$ to G_n^2. The GFn for NBD in small rapidity interval is then

$$Q^{\mathrm{NBD}}(z,\delta y) = \left(1 + \frac{\bar{n}(\delta y)}{\omega(\delta y)}(1-z)\right)^{-\omega(\delta y)},$$
$$\bar{n}(\delta y) = p^{\mathrm{P}} p^{\mathrm{L}} \bar{n}, \quad \omega(\delta y) = p^{\mathrm{P}} \omega, \tag{40}$$

where

$$p^{\mathrm{P}} \simeq p^{\mathrm{P}}(\Delta) = \mathrm{const.}, \quad p^{\mathrm{L}} \propto \delta y, \qquad \text{for} \quad \delta y \ll \Delta, \tag{41}$$

$$p^{\mathrm{P}} \propto \delta y, \quad p^{\mathrm{L}} \simeq 1, \qquad \text{for} \quad \delta y \gg \Delta. \tag{42}$$

The NFMs are

$$F_k^{\mathrm{NBD}}(\delta y \ll \Delta) \simeq \mathrm{const.}, \qquad F_k^{\mathrm{NBD}}(\delta y \gg \Delta) \propto \delta y^{1-k}. \tag{43}$$

Note that for both cases $\bar{n} \propto \delta y$ while the behaviour of ω is different for different values of rapidity intervals. It seems that behaviour $\omega(\delta y) \to$ const. for $\delta y \to 0$ is not improbable for high energy collisions [32, 34, 38]. If the value of the decay width Δ in rapidity space is not very small, we have no any intermittent effect.

Let us note that this two-projection method differs radically from the one discussed in Ref.[40], where $\omega(\delta y) \propto \delta y^{\alpha}$ $(0 < \alpha \ll 1)$ for all δy. In this case, the intermittency with a monofractal behaviour of AFD $d_k = \alpha =$ const. does exist. However, the existing experimental data hardly provide any direct evidence for such behaviour of $\omega(\delta y)$ [32, 34, 38]. In addition, the monofractal behaviour of AFD is very uncommon one for most particle collisions for which the multifractal behaviour is typical [30].

In the case of bunching projection distributions, we can describe intermittency with an arbitrary AFD $d_k = (k - s_L(k))/(k-1)$, where $s_L(k)$ is determined by rapidity fluctuations of particles in one clan. The value of $s_L(k)$, of course, must be calculated in a dynamical production theory or models.

5. Conclusion

We considered the simple two-stage scheme of multiparticle production using both correlation and fluctuation approaches for the description of intermittency. The most important result of our analysis is that if rapidity interval is small enough then leading contribution to intermittency is determined by the last stage. The influence of correlation between clusters at the end of the first stage in intermittent spectra of final hadrons must be slight for $\delta y \to 0$. The correlations between clusters determine only non-leading terms in the NFM of multiplicity distribution for final particles.

In conclusion, it should be emphasized that the behaviour $\delta y^{-d_2} = | y_2 - y_1 |^{-d_2}$ of semi-inclusive density (or power-law behaviour of NFM), when rapidity difference tends to zero, is not one in usual experimental sense, since this power-law is not valid for arbitrary small δy (δy)in real multiparticle processes. A generalization this approach for case of existence of some critical value $\delta y_{\min}$ determining the scale beyond which the process is no longer scale invariant must be subject of future investigation.

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