ECONOMY MASS FORMULA FOR LEPTONS AND QUARKS: PART TWO*

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An economy mass formula, found recently for leptons and quarks as a semiempirical relationship based on a Dirac square-root model proposed previously for three fermion families, is supplemented by an ansatz concerning four mass scales appearing in this formula (in four cases of neutrinos, charged leptons, up quarks and down quarks). Then, in the mass formula there remain *only three* free parameters that can be determined from experimental values of m_e, m_μ and $|V_{us}|$. Nevertheless, it predicts *all other* lepton and quark masses as well as quark mixing parameters in nice agreement with existing experimental data.

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In a recent paper [1] an economy semiempirical mass formula for leptons and quarks was found, well reproducing all their masses (in terms of experimental values of m_e, m_μ and m_c, m_b) as well as all quark mixing parameters (in terms of experimental values of m_e/m_μ and $|V_{us}|$). In particular, it predicted $m_\tau = 1776.80$ MeV and $m_t \simeq 170$ GeV successfully (the latter if $m_c \simeq 1.5$ GeV) as well as $|V_{cb}| = 0.0455$ nicely (if $|V_{us}| = 0.218$), though such a value of $|V_{cb}|$ is perhaps by 10% too large.

The theoretical base for this formula was a *Dirac square-root model* for three families of leptons and quarks [2], making use of new (generally) reducible representations of Dirac algebra defined by means of some Clifford algebras. These representations act on wave functions carrying $N = 1, 3, 5$ Dirac bispinor indices of which $N - 1 = 0, 2, 4$ are physically undistinguishable and so antisymmetrized (such an *intrinsic Pauli principle* excludes all $N = 7, 9, 11, \ldots$). Then, these wave functions labelled by $N = 1, 3, 5$ are reduced to three Dirac bispinors which are conjectured to correspond to

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three fermion families. The familiar standard-model signature is given by SU(3) × SU(2) × U(1) additional indices.

This mass formula describes the 3×3 mass matrices $\hat{M}^{(f)}$ for four kinds $f = \nu\,,\ e\,,\ u\,,\ d$ of fundamental fermions: neutrinos $(\nu_e\,,\ \nu_\mu\,,\ \nu_\tau)$, charged leptons $(e^-\,,\ \mu^-\,,\ \tau^-)$, up quarks $(u,\ c,\ t)$ and down quarks $(d,\ s,\ b)$, and can be put in the form

$$\hat{M}^{(f)} = \frac{\mu^{(f)}}{N_C^{(f)2}}\,\hat{\rho}\left[g^{(f)2}\,\hat{N}^{(f)2} - \frac{g^{(f)2} - \varepsilon^2(N_C^{(f)}\,B^{(f)} - L^{(f)})}{\hat{N}^{(f)2}} \right.$$
$$\left. +\varepsilon g^{(f)}\,N_C^{(f)2}\,B^{(f)}\left(\hat{a}\,e^{i\varphi^{(f)}} + \hat{a}^\dagger\,e^{-i\varphi^{(f)}}\right)\right]\hat{\rho}\,. \quad (1)$$

In Eq. (1) there appear the following 3 × 3 matrices defined in the fermion-family space:

$$\hat{\rho} = \frac{1}{\sqrt{29}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24} \end{pmatrix} \quad (2)$$

and

$$\hat{N}^{(f)} = \hat{N} + g^{(f)}\,N_C^{(f)}\,B^{(f)}\,\hat{n}(\hat{n} - \hat{1}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 + 2g^{(f)}\,N_C^{(f)}\,B^{(f)} \end{pmatrix} \quad (3)$$

with

$$\hat{N} = \hat{1} + 2\hat{n} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}, \quad \hat{n} = \hat{a}^\dagger\hat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad (4)$$

while

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}. \quad (5)$$

Here, $\hat{a}$ and $\hat{a}^\dagger$ play the role of (restricted) annihilation and creation operators acting intrinsically within leptons and quarks (note that $\hat{a}^3 = 0$ and $\hat{a}^{\dagger 3} = 0$). They change by -2 and $+2$, respectively, the fermion-family label $N = 1, 3, 5$ equal to an eigenvalue of the matrix $\hat{N}$. More precisely, they change by -1 and $+1$, respectively, the modified fermion-family label $n = (N-1)/2 = 0, 1, 2$ equal to an eigenvalue of the matrix $\hat{n}$ and being the number of pairs of the $N - 1$ antisymmetrized Dirac bispinor indices of wave functions for three fermion families.

In Eq. (1) there are also real constants of two sorts: six free parameters $\mu^{(f)}$, ε^2 and (effectively) $\varphi \equiv \varphi^{(u)} - \varphi^{(d)}$ which are to be fitted to experimental data, and others given as the simple numbers

$$g^{(f)} = \left(N_C^{(f)} Q^{(f)}\right)^2 = \begin{cases} 0 \\ 1 \\ 4 \\ 1 \end{cases}, \quad N_C^{(f)} = \begin{cases} 1 \\ 1 \\ 3 \\ 3 \end{cases}, \quad Q^{(f)} = \begin{cases} 0 \\ -1 \\ 2/3 \\ -1/3 \end{cases} \tag{6}$$

and

$$B^{(f)} = \begin{cases} 0 \\ 0 \\ 1/3 \\ 1/3 \end{cases}, \quad L^{(f)} = \begin{cases} 1 \\ 1 \\ 0 \\ 0 \end{cases}, \tag{7}$$

for $f = \nu, e, u, d$, respectively. Note that $N_C^{(f)} B^{(f)} + L^{(f)} = 1$ and $B^{(f)} + L^{(f)} = 1/N_C^{(f)}$ for each f. The physical meaning of the numbers $N_C^{(f)}$, $Q^{(f)}$, $B^{(f)}$ and $L^{(f)}$ is transparent.

For leptons the nondiagonal term mixing fermion families in Eq. (1) vanishes, in contrast to the situation for quarks when the mass matrices require a diagonalizing procedure.

In the case of neutrinos, Eq. (1) yields [1]

$$\begin{aligned} m_{\nu_e} &\equiv M_1^{(\nu)} = \frac{\mu^{(\nu)}}{29} \varepsilon^2 , \\ m_{\nu_\mu} &\equiv M_3^{(\nu)} = \frac{4}{9} \frac{\mu^{(\nu)}}{29} \varepsilon^2 , \\ m_{\nu_\tau} &\equiv M_5^{(\nu)} = \frac{24}{25} \frac{\mu^{(\nu)}}{29} \varepsilon^2 , \end{aligned} \tag{8}$$

thus $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$ if $\mu^{(\nu)} = 0$, but generally $m_{\nu_\mu} \leq m_{\nu_\tau} \leq m_{\nu_e}$.

In the case of charged leptons, Eq. (1) gives [1]

$$\begin{aligned} m_e &\equiv M_1^{(e)} = \frac{\mu^{(e)}}{29} \varepsilon^2 , \\ m_\mu &\equiv M_3^{(e)} = \frac{4}{9} \frac{\mu^{(e)}}{29} \left(80 + \varepsilon^2\right) , \\ m_\tau &\equiv M_5^{(e)} = \frac{24}{25} \frac{\mu^{(e)}}{29} \left(624 + \varepsilon^2\right) . \end{aligned} \tag{9}$$

Hence, using the experimental values of m_e and m_μ we obtain

$$m_\tau = 1776.80\,\text{MeV} \tag{10}$$

and

$$\mu^{(e)} = 85.9924\,\mathrm{MeV}\,, \qquad \varepsilon^2 = 0.172329\,. \tag{11}$$

The agreement of the prediction (10) with the recent experimental value $m_\tau = 1776.96^{+0.18+0.20}_{-0.19-0.16}$ MeV [3] is very good.

In the case of up and down quarks, numerical diagonalization performed in Eq. (1) with the use of the value (11) for ε^2 and the experimental tips

$$m_c \equiv M_3^{(u)} \simeq (1.2 \text{ to } 1.5)\,\mathrm{GeV}\,,\; m_b \equiv M_5^{(d)} \simeq (4.5 \text{ to } 5.0)\,\mathrm{GeV} \tag{12}$$

provides [1]

$$\begin{aligned} m_u \equiv -M_1^{(u)} &\simeq (3.7 \text{ to } 4.6)\,\mathrm{MeV},\; m_d \equiv -M_1^{(d)} \simeq (6.8 \text{ to } 7.5)\,\mathrm{MeV}\,, \\ m_t \equiv M_5^{(u)} &\simeq (140 \text{ to } 170)\,\mathrm{GeV},\; m_s \equiv M_3^{(d)} \simeq (130 \text{ to } 150)\,\mathrm{MeV} \end{aligned} \tag{13}$$

(here, the lowest masses m_u and m_d are taken to be nonnegative, as always for Dirac masses). Then, the quark mass scales in Eq. (1) turn out to be

$$\mu^{(u)} \simeq 9 \times (61 \text{ to } 76)\,\mathrm{MeV}\,, \quad \mu^{(d)} \simeq 9 \times (110 \text{ to } 120)\,\mathrm{MeV}\,. \tag{14}$$

The predicted mass ratios within triplets u, c, t and d, s, b are independent of the input (12) as being determined by ε^2 only (thus, only by the experimental mass ratio m_e/m_μ, due to Eq. (9)). They are

$$\begin{aligned} \frac{m_u}{m_c} &= 0.00305405\,, \quad \frac{m_d}{m_s} = 0.0509344\,, \\ \frac{m_c}{m_t} &= 0.00877909\,, \quad \frac{m_s}{m_b} = 0.0294872\,, \end{aligned} \tag{15}$$

On the ground of a numerical experience, let us consider the following ansatz for the quark mass scales $\mu^{(u)}$ and $\mu^{(d)}$, consistent with their approximate values (14):

$$\begin{aligned} \mu^{(u)} &= 9 \times \tfrac{8}{9}\mu^{(e)} = 9 \times 76.4377\,\mathrm{MeV}\,, \\ \mu^{(d)} &= 9 \times \tfrac{4}{3}\mu^{(e)} = 9 \times 114.657\,\mathrm{MeV}\,. \end{aligned} \tag{16}$$

Of course, for the mass scales (16) the scale–independent ratios (15) hold. Together with the value (11) for ε^2 determined by the experimental mass ratio m_e/m_μ, the figures (16) for the mass scales $\mu^{(u)}$ and $\mu^{(d)}$ lead to

$$\begin{aligned} m_u \equiv -M_1^{(u)} &= 4.58608\,\mathrm{MeV}\,,\; m_d \equiv -M_1^{(d)} = 6.99371\,\mathrm{MeV}\,, \\ m_c \equiv M_3^{(u)} &= 1.50164\,\mathrm{GeV}\,, \quad m_s \equiv M_3^{(d)} = 137.308\,\mathrm{MeV}\,, \\ m_t \equiv M_5^{(u)} &= 171.047\,\mathrm{GeV}\,, \quad m_b \equiv M_5^{(d)} = 4.65653\,\mathrm{GeV}\,. \end{aligned} \tag{17}$$

Here, $m_u/m_d = 0.655744$. We can see an overall consistency of the predicted quark masses (17) with their experimental tips, in particular for the top quark reported recently to be seen with the mass $m_t = 174 \pm 10\ ^{+13}_{-12}$ GeV [4].

It is interesting to note that, assuming the neutrino mass scale $\mu^{(\nu)} = 0$, we can write in the case of the mass-scale ansatz (16) the formula

$$\mu^{(f)} = \begin{cases} 0 \\ \mu^{(e)} \\ 8\mu^{(e)} \\ 12\mu^{(e)} \end{cases} = \left(1 + B^{(f)}\right) N_C^{(f)} \left(N_C^{(f)} - I_3^{(f)} - \tfrac{1}{2}\right) \mu^{(e)}, \qquad (18)$$

where

$$I_3^{(f)} = \begin{cases} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{cases} \qquad (19)$$

is the eigenvalue of the third component of weak isospin for lefthanded fermions of the kind $f = \nu\,,\ e\,,\ u\,,\ d$, respectively.

The quark mixing parameters evaluated in Ref. [1] — as being scale-independent — hold for the mass scales (16). As is shown in Ref. [1], these predictions are in an overall consistency with existing experimental data.

In conclusion, we can see that the semiempirical mass formula (1), if considered *jointly* with the mass-scale ansatz (18), includes *only three* free parameters: the charged-lepton mass scale $\mu^{(e)}$, the pure-number constant ε^2 and (effectively) the phase difference $\varphi \equiv \varphi^{(u)} - \varphi^{(d)}$. The first two, $\mu^{(e)}$ and ε^2, can be determined (as in Eq. (11)) from two experimental lepton masses m_e and m_μ , and then *one* lepton mass m_τ as well as *all six* quark masses are predicted (as in Eqs (10) and (17)). With ε^2 evaluated (as in Eq. (11)) from the experimental mass ratio m_e/m_μ, the last parameter, φ, was determined in Ref. [1] from the magnitude of experimental CKM-matrix element $|V_{us}|$, and then *all other* CKM-matrix elements (including their CP-violating phase) were predicted. The agreement of all these predictions with existing experimental data for lepton and quark masses as well as quark mixing parameters [5] is nice.

If one wants to diminish a bit the prediction $|V_{cb}| = 0.0455$ and then also $|V_{ub}| = 0.00240$ (with the input $|V_{us}| = 0.218$) [1], one may introduce in place of the constant ε, when appearing in the nondiagonal term in Eq. (1), the *fourth* free parameter $\eta = \zeta\varepsilon$ where $\zeta = O(1)$. Then, with $\zeta \simeq 0.90$ (and $|V_{us}| = 0.218$) one obtains $|V_{cb}| \simeq 0.041$ and also $|V_{ub}| \simeq 0.0019$. In this case with the mass-scale ansatz (16), one gets $m_u \simeq 3.8$ MeV and

$m_d \simeq 5.8$ MeV giving the ratio $m_u/m_d \simeq 0.66$. Such masses, especially m_d, are perhaps a bit too small. Other quark masses do not change if considered in the two–digital accuracy.

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