PHENOMENOLOGICAL RANDOM WALK MODEL OF QUASI ELASTIC AND DAMPED COLLISIONS

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A model is presented describing quasi elastic and damped heavy ion collisions, at energies larger than 10 MeV/u, as a process of stochastic nucleon and momentum transfer. Mass, energy, and angular distributions are calculated. The model allows also to evaluate dissipation of the entrance channel angular momentum. The effect of subsequent evaporation is included.

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1. Introduction

The aim of this work is to describe heavy ion reactions at intermediate energies ($>$ 10 MeV/u) in which a projectile-like fragment, PLF, is observed. Similarly as at lower energies ($<$ 10 MeV/u) the quasi elastic and the damped heavy ion collision scenarios are assumed. This is the reaction mechanism suggested recently for non central collisions at intermediate energies [1, 2]. We would like to calculate from our model the mass, energy, and angular distributions of emitted particles. Such predictions are important in investigations of heavy ion reaction mechanism and also for planning of complicated and expensive multi-detector experiments.

At low energies (damped) deep inelastic collisions, DIC, are observed for impact parameters which are too large for a formation of a compound

nucleus but which still involve a considerable overlap of projectile and target nuclei. The colliding ions reseparate after some exchange of nucleons, momentum (energy), and angular momentum. The relative kinetic energy may be dissipated down to the Coulomb barrier, but the participating ions do not completely loose their identity. In the collision process the attractive mean field of the system governs the effective flow of nucleons through a window between the target-nucleus and the projectile-nucleus [3, 4]. The individual nucleon-nucleon interaction is highly suppressed by the Pauli principle. Consequently, the energy damping seems to proceed mainly by one body dissipation [5]. The flow of nucleons through the window between the target nucleus and the projectile undergoes fluctuations which can be included in the picture by solving the Fokker–Planck equation, within the transport theory of dissipative collisions [6]. In order to get a proper isotope composition of ejectiles different random walk procedures are being incorporated in the transport calculations [7].

The reaction picture becomes more complicated at intermediate energies where individual nucleon-nucleon interactions, residual Pauli blocking, and mean field effects have to be included. To solve the problem one commonly uses semi-classical kinetic equations based on the collision term derived for quantum gases by Uehling and Uhlenbeck [8]. Further improvements have been taken from Landau and Vlasow [9], who were solving similar problems for plasmas, where both collisions and long range electric fields have to be accounted for. Different methods used to solve this problem are referred to as B.U.U. [10] or V.U.U. [11], and L.V.[12]. As an alternative one can use the molecular dynamics formalism [13]. Here the classical equations of motion are used to describe individual interacting nucleons and both Pauli blocking and mean field effects are included in Monte Carlo simulations.

The above calculations, based on general kinetic equations, are performed on the microscopic level. Due to numerical and computer time difficulties they are frequently not able to answer detailed questions concerning the dynamics of heavy-ion nuclear collisions and the production of heavy fragments. In particular they have difficulties in describing peripheral collisions, whereas simpler phenomenological models appear to be quite successful in explaining some of these aspects [14–18]. Most of the latter models are based on the Randrup assumption that in a heavy ion collision the energy dissipation proceeds mainly through stochastic transfer of nucleons [14]. In the model of Tassan-Got and Stéphan [15] the random transfer of nucleons is influenced by the Coulomb barrier between colliding nuclei and by corresponding densities of states. In the models of Harvey [16] and Cole [17] use is made of the observation that yield of fragments at collision energies higher than about 10 MeV/u is influenced by individual nucleon-nucleon collisions. These collisions are responsible for stochastic transfer

of nucleons between colliding nuclei. The "event generator" of Durand [18] is based on experimental findings known in the intermediate energy range. A geometrical limit picture corresponding to a straight trajectory for the incoming particle, and a simple nucleon-nucleon cascade model are used to estimate the number of nucleons randomly transferred between colliding nuclei and nucleons emitted as preequilibrium particles.

In the present work we present a model which allows to calculate the mass, energy, and angular distributions of emitted particles in a consistent way. Our work was inspired by that of Cole [17] who proposed a model for peripheral heavy ion reactions based on two main postulates:

i) The mechanism is mediated by the number of nucleon-nucleon collisions taking place along the trajectory describing the relative motion of projectile and target.

ii) The deflection of the projectile-like fragment is produced in part by the potential acting between the ions and in part by recoil effects due to the mass exchange.

With these two assumptions the model of Refs [17] and [19] predicts mass distributions and also angular distributions of projectile-like fragments (PLF's). Concerning the energy spectra, only the mean energy of the fragment as a function of laboratory angle is calculated.

In the present work the following modifications were made. It is assumed that the reaction is described by a number of random walk processes, undefined as yet, which determine the mass and momentum transfer. The processes are not necessarily limited to $N - N$ collisions. The assumption remains that these processes can be considered as statistically independent. In this model random walk in the mass transfer as well as in three dimensions of the momentum transfer is taken into account. In Refs [17, 19] the random walk was limited to the mass transfer only, while the momentum transfer was treated in an average way. Because of extension to the momentum space we are able to calculate both the energy and angular distributions in a self consistent way.

Assuming that the reaction occurs at a point of closest approach it is also possible to determine the final relative angular momentum and spins of fragments in the exit channel.

Another refinement of the present model is inclusion of the secondary sequential emission. It is known experimentally that the primary PLF's can be excited and undergo sequential binary decay, SBD [20]. This secondary evaporation has to be included if one wants to compare predictions of the model to experimental data. A short and preliminary presentation of our model was given elsewhere [2].

2. Description of the model

The model assumes the following reaction picture. A projectile with an initial mass number A_P, momentum $\vec{K}_P$ and spin zero is moving along a classical trajectory given by the impact parameter, and the nucleus-nucleus potential. When ions are close enough to feel a nuclear interaction, processes occur which modify the potential and the linear and angular momentum, according to the nucleon and momentum exchange. Next, the projectile-like fragment with a final mass number A_{PLF}, and spin $\vec{s}_{\mathrm{PLF}}$ is moving along a trajectory given by new exit channel parameters. The final momentum of the PLF is $\vec{K}_{\mathrm{PLF}}$. For calculational purposes it is assumed that the reaction parameters that determine the trajectories change at the distance of closest approach, R_i^0. If in addition one assumes that the reaction is binary, the target-like fragment spin $\vec{s}_{\mathrm{TLF}}$, and the final relative angular momentum, $\vec{l}_f$ can be predicted in a straightforward way. The model does not distinguish between protons and neutrons, therefore for simplicity, we assume $Z_{\mathrm{PLF}} = \frac{1}{2} A_{\mathrm{PLF}}$.

In order to describe the exit reaction channel, the probability for mass and momentum transfer is calculated. The mass transfer is treated as a random walk process occurring in the overlap region. The momentum transfer is due to the potential interaction along the corresponding trajectory, and due to random interactions taking place in the overlap region. A recoil due to the mass transfer is taken into account.

Let us consider a reaction for a given initial angular momentum $\vec{l}_i$. The goal is to calculate the probability that the reaction will lead from an initial state $\vec{l}_i, \vec{K}_P, A_P$ to a final state $\vec{l}_f, \vec{K}_{\mathrm{PLF}}, A_{\mathrm{PLF}}$. The entrance trajectory plane is given by a location of the target nucleus and by the $\vec{K}_P$ vector. Orientation of this plane in space depends on an azimuthal angle φ. Due to the random walk process occurring in the overlap region the final state $\vec{K}_{\mathrm{PLF}}, A_{\mathrm{PLF}}$ will contain contributions from different entrance channel trajectory planes (different angles φ). By definition, the angle φ is zero when entrance and exit channel trajectories are coplanar.

The total momentum transfer defined as

$$\vec{Q} = \vec{K}_{\mathrm{PLF}} - \vec{K}_P \,, \tag{1}$$

is a sum of two parts

$$\vec{Q} = \vec{Q}_R + \vec{Q}_P \,. \tag{2}$$

The random walk momentum transfer

$$\vec{Q}_R = \vec{K}^0_{\mathrm{PLF}} - \vec{K}^0_P \,, \tag{3}$$

where $\vec{K}_P^0$, is the momentum at the end of the projectile trajectory, and $\vec{K}_{\mathrm{PLF}}^0$ is the momentum at the beginning of the PLF trajectory. $\vec{Q}_P$ denotes a momentum transfer induced by the potential interaction acting along the entrance and the exit channel trajectories. For clarity all defined quantities are shown in Fig. 1a.

A particular momentum transfer $\vec{Q}_R$ leading from an entrance channel plane at an angle φ to a given final state $\vec{K}_{\mathrm{PLF}}, A_{\mathrm{PLF}}$, is labeled $\vec{Q}_R(\varphi)$. For given $\vec{K}_P, A_P, l_i$ and $\vec{K}_{PLF}, A_{\mathrm{PLF}}$ the final value of $\vec{Q}_P$ is uniquely determined by $\vec{Q}_R(\varphi)$:

$$\vec{Q}_P = f(\vec{Q}_R(\varphi)) . \tag{4}$$

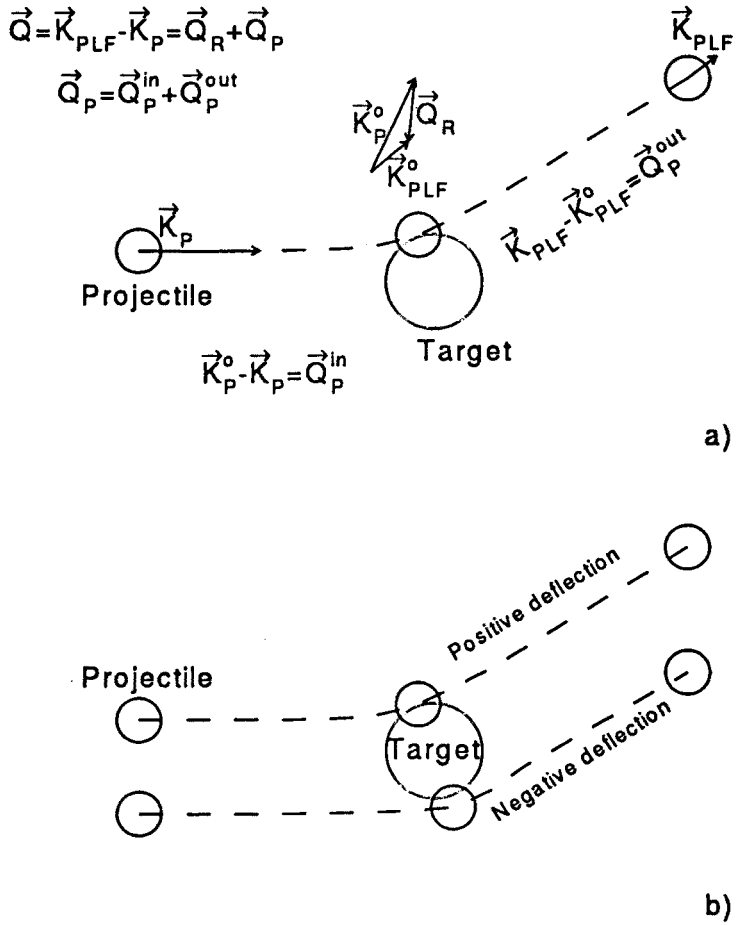

Fig. 1. a) Vectors describing the input and output reaction channels. b) Schematic view of the positive and negative deflection angle.

The prescription for $f(\vec{Q}_R(\varphi))$ contains a numerical calculation of the entrance and exit channel trajectories with an interaction taken as a sum of the Coulomb and the proximity potential [21].

According to Eqs (1)–(4), the final momentum $\vec{K}_{\mathrm{PLF}}$ of the projectile-like fragment is

$$\vec{K}_{\mathrm{PLF}} = \vec{Q}_R(\varphi) + f(\vec{Q}_R(\varphi)) + \vec{K}_P\,. \tag{5}$$

The energy and angular distributions are given by a probability that the random walk momentum transfer will have a value and direction corresponding to $\vec{Q}_R(\varphi)$. The probability has to be integrated over all possible values of φ.

2.1. The random walk mass and momentum transfer

According to the assumptions outlined in the introduction the reaction contains a number of random walk steps leading to the mass and momentum transfer. As in [17] we define P^+ to be a probability for a one nucleon pick-up by the projectile, P^-a probability for a one nucleon stripping from the projectile, and P^0 denotes a probability of a reaction step without the mass transfer. In stripping the lost nucleon is not necessarily transferred to the target nucleus but can be also emitted into the continuum (the preequilibrium emission). We assume that

$$\sum_{\lambda} P^{\lambda} = 1\,, \tag{6}$$

where $\lambda = +, -$ or 0.

With each reaction step a momentum transfer $\vec{q}$ is associated. We replace a possible momentum transfer $\vec{q}$ which is a continuous quantity by a sufficiently fine grid. Now vector $\vec{q}_i$ is a one of all the possible momentum transfers with a momentum transfer probability $p^{\lambda}(\vec{q}_i)$. For every λ, $\sum_i p^{\lambda}(\vec{q}_i) = 1$.

Let us consider a reaction for one given entrance angular momentum. If $\overline{n}$ is an average number of steps, we can define the average number of steps in a λ direction as:

$$\overline{n}^{\lambda} = P^{\lambda}\overline{n}\,. \tag{7}$$

The average number of processes associated with a transfer $\vec{q}_i$ is

$$\overline{n}_i^{\lambda} = \overline{n}^{\lambda} p^{\lambda}(\vec{q}_i) = \overline{n} P^{\lambda} p^{\lambda}(\vec{q}_i)\,. \tag{8}$$

Since the elementary interaction processes are treated as statisticall y independent the corresponding distributions of n_i^{λ}(n_i^{λ} is the number of steps

with a momentum transfer $\vec{q}_i$ and with a direction λ) are described by the Poisson law:

$$\mathcal{P}(n_i^\lambda) = \exp(-\overline{n}_i^\lambda)\frac{(\overline{n}_i^\lambda)^{n_i^\lambda}}{(n_i^\lambda)!}. \tag{9}$$

Using a notation $\Delta A = A_{\mathrm{PLF}} - A_P$, the total momentum and mass transfer which take place during the reaction can be written as

$$\vec{Q}_R(\varphi) = \sum_i (n_i^+ + n_i^- + n_i^0)\vec{q}_i\,, \tag{10}$$

$$\Delta A = \sum_i (n_i^+ - n_i^-)\,. \tag{11}$$

With Eqs (9)–(11) the probability density for a final state characterized by $\vec{Q}$ and ΔA is given as

$$\begin{aligned} P_{l_i}(\Delta A, \vec{Q}) = \frac{1}{2\pi}\int_0^{2\Pi} d\varphi \sum_{\text{all } n_i^\lambda} \Bigg[& \prod_j \mathcal{P}(n_j^+) \prod_k \mathcal{P}(n_k^-) \prod_l \mathcal{P}(n_l^0) \\ & \times \delta_D\left(-\vec{Q}_R(\varphi) + \sum_i (n_i^+ + n_i^- + n_i^0)\vec{q}_i\right) \\ & \times \delta_K\left(\Delta A - \sum_i (n_i^+ - n_i^-)\right) J(\vec{Q}_R(\varphi) \mid \vec{Q})\Bigg], \end{aligned} \tag{12}$$

where δ_D is the Dirac δ, δ_K is the Kronecker δ, and J denotes a Jacobian for a transformation $\vec{Q}_R(\varphi) \rightarrow \vec{Q}$. The summation is over all possible combinations $\{n_i^\lambda\}$ of the momentum a nd mass transfer. The products contain all probabilities for each momentum transfer occurring in a certain random walk. The δ_D and δ_K operators pick out only those combinations which lead to the random momentum transfer $\vec{Q}_R(\varphi)$, and to the net mass transfer ΔA, respectively.

Using for the δ functions the integral representations:

$$\delta_D(\vec{y}) = \frac{1}{(2\pi)^3}\int d\vec{x} \mathrm{e}^{i\vec{y}\vec{x}}\,, \tag{13}$$

$$\delta_K(m) = \frac{1}{2\pi}\int_0^{2\pi} dt \mathrm{e}^{imt} \tag{14}$$

expression (12) can be rewritten as

$$P_{l_i}(\Delta A,\vec{Q}) = \frac{1}{(2\pi)^5}\int d\varphi \int d\vec{x}\int dt \sum_{all\ n_i^\lambda}\left[\prod_j\left(e^{-\overline{n}_j^+}\frac{(\overline{n}_j^+)^{n_j^+}}{(n_j^+)!}\right)\right.$$
$$\times\prod_k\left(e^{-\overline{n}_k^-}\frac{(\overline{n}_k^-)^{n_k^-}}{(n_k^-)!}\right)\prod_l\left(e^{-\overline{n}_l^0}\frac{(\overline{n}_l^0)^{n_l^0}}{(n_l^0)!}\right)e^{i\vec{x}\sum_k(n_k^+ + n_k^- + n_k^0)\vec{q}_k}$$
$$\left.\times\, e^{it\sum_k(n_k^+ - n_k^-)}\right]e^{-i\vec{x}\vec{Q}_R(\varphi)}e^{-it\Delta A}J(\vec{Q}_R(\varphi)\mid\vec{Q}). \qquad (15)$$

We now rearrange Eq. (15) and using $\overline{n}=\sum_i\left(\overline{n}_i^+ + \overline{n}_i^- + \overline{n}_i^0\right)$ obtain:

$$P_{l_i}(\Delta A,\vec{Q}) = \frac{e^{-\overline{n}}}{(2\pi)^5}\int d\varphi\int d\vec{x}\int dt\sum_{\text{all } n_i^\lambda}\left[\prod_j\frac{(\overline{n}_j^+ e^{i\vec{x}\vec{q}_j}e^{it})^{n_j^+}}{(n_j^+)!}\right.$$
$$\left.\times\prod_k\frac{(\overline{n}_k^- e^{i\vec{x}\vec{q}_k}e^{-it})^{n_k^-}}{(n_k^-)!}\prod_l\frac{(\overline{n}_l^0 e^{i\vec{x}\vec{q}_l})^{n_l^0}}{(n_l^0)!}\right]$$
$$\times\, e^{-i\vec{x}\vec{Q}_R(\varphi)}e^{-it\Delta A}J(\vec{Q}_R(\varphi)\mid\vec{Q}). \qquad (16)$$

After summation over all possible $\{n_i^\lambda\}$ one finds that

$$P_{l_i}(\Delta A,\vec{Q}) = \frac{e^{-\overline{n}}}{(2\pi)^5}\int d\varphi\int d\vec{x}\int dt\exp\left[\sum_j(\overline{n}_j^+ e^{it}+\overline{n}_j^- e^{-it}+\overline{n}_j^0)e^{i\vec{x}\vec{q}_j}\right]$$
$$\times e^{-i\vec{x}\vec{Q}_R(\varphi)}e^{-it\Delta A}J(\vec{Q}_R(\varphi)\mid\vec{Q}). \qquad (17)$$

Next, using (8) the sum over discrete vectors $\vec{q}_j$ is converted to an integral over $d\vec{q}$

$$P_{l_i}(\Delta A,\vec{Q}) = \frac{e^{-\overline{n}}}{(2\pi)^5}\int d\varphi\int d\vec{x}\int dt\exp\left[\overline{n}\int d\vec{q}\left(P^+p^+(\vec{q})e^{it}\right.\right.$$
$$\left.\left.+P^-p^-(\vec{q})e^{-it}+P^0p^0(\vec{q})\right)e^{i\vec{x}\vec{q}}\right]e^{-i\vec{x}\vec{Q}_R(\varphi)}e^{-it\Delta A}J(\vec{Q}_R(\varphi)\mid\vec{Q}). \qquad (18)$$

This is the final result for the probability density . The transition from a probability to reaction cross-section is straightforward. We obtain

$$\frac{d^2\sigma}{d\Omega dE} = \frac{\pi\hbar^2 m_P m_{\mathrm{PLF}}^{3/2}(2E_{\mathrm{PLF}})^{1/2}}{2E_P\mu_i^2}\sum_{l_i}\left[(2l_i+1)P_{l_i}(\Delta A,\vec{Q})\right], \qquad (19)$$

where m, E are the mass and lab energy of the projectile P and PLF, respectively, μ_i is the reduced mass in the input channel and summation is over all entrance angular momenta l_i.

2.2. Relative angular momentum and spins

With the present model one can also obtain the relative angular momentum and spins. For simplicity we assume that the primary reaction is binary, what in the considered energy region and for no central collisions is not a bad approximation [22].

Let us consider a projectile and a target nucleus at the distance of closest approach $\vec{R}_i^0$. Now, in the CM system, positions of the projectile, the target nucleus, and the overlap region are given by the vectors $\vec{r}_P, \vec{r}_T$, and $\vec{r}_r$, respectively,

$$\vec{r}_P = \frac{A_T \vec{R}_i^0}{A_P + A_T}, \tag{20}$$

$$\vec{r}_T = -\frac{A_P \vec{R}_i^0}{A_P + A_T}. \tag{21}$$

For $\vec{r}_r$ the center of the overlap region evaluated at the distance $\vec{R}_i^0$ is used. A_T is the target nucleus mass number. After some straightforward algebra one gets for spins of the PLF and TLF (target like fragment):

$$\vec{s}_{\mathrm{PLF}} = (\vec{r}_r - \vec{r}_P) \times \left(\vec{Q}_R - \frac{\Delta A}{A_{\mathrm{PLF}}} \vec{K}_f^0 \right), \tag{22}$$

$$\vec{s}_{\mathrm{TLF}} = (\vec{r}_r - \vec{r}_T) \times \left(-\vec{Q}_R - \frac{\Delta A}{A_{\mathrm{TLF}}} \vec{K}_f^0 \right). \tag{23}$$

The relative angular momentum is simply

$$\vec{l}_f = \vec{R}_f^0 \times \vec{K}_f^0, \tag{24}$$

where

$$\vec{R}_f^0 = \vec{r}_P \frac{A_P}{A_{\mathrm{PLF}}} - \vec{r}_T \frac{A_T}{A_{\mathrm{TLF}}} + \vec{r}_r \left(\frac{A_T}{A_{\mathrm{TLF}}} - \frac{A_P}{A_{\mathrm{PLF}}} \right). \tag{25}$$

In formulas (22)–(24) $\vec{Q}_R$ and $\vec{K}_f^0$ are taken in the CM system.

For peripheral collisions where the size of the overlap region is small localization of the reaction at a point $\vec{r}_r$ is a good approximation. The situation becomes more complicated for smaller impact parameters where,

in principle, mass transfer can take place anywhere in a large overlap region. Consequently, the magnitude and direction of the spins and angular momentum are less certain. From inspection of Eqs (22)–(25) one may find that the relative angular momentum, l_f, is the least sensitive to the transfer location. The largest sensitivity is found for the spin of the lighter fragment. In any case Eqs (22)–(24) provide average values only for $\vec{s}$ and $\vec{l}_f$.

2.3. Model parameters

In order to use the final formula (18) one has to insert values for the average number of reaction steps $\bar{n}$, and the probabilities P^+, P^- ($P^0 = 1 - P^+ - P^-$). The momentum transfer distributions $p^\lambda(q)$ ($\lambda = (+), (-), (0)$) have to be also specified.

For sufficiently high collision energies, where the reaction cross section is mainly determined by the average nucleon-nucleon cross section $\bar{\sigma}_{NN}$, one can interpret $\bar{n}(l_i)$ as an average number of nucleon- nucleon collisions taking place in the overlap region. A simple analytic formula for $\overline{n}$ was given by Karol [23]. In his formalism the average number of nucleon-nucleon collisions is obtained in the optical limit of the Glauber theory [24] as:

$$\overline{n} = \overline{\sigma}_{NN} \int_{-\infty}^{\infty} dz \left[\int \rho_1 \rho_2 dv \right] , \tag{26}$$

where the integral in square brackets represents the convolution of projectile and target densities which in turn is integrated over the (supposed) stright line trajectory in the beam direction z. For simplification Karol used an approximate Gaussian form for the projectile and target density distributions (tail regions).

For impact parameters $b = b(l_i)$ corresponding to very peripheral collisions one obtains:

$$\overline{n} = \pi^2 \overline{\sigma}_{NN} \rho_T(0) \rho_P(0) \frac{a_T^3 a_P^3}{a_T^2 + a_P^2} \exp\left(- \frac{b^2}{a_T^2 + a_P^2} \right) , \tag{27}$$

where symbols $a_P, \rho_P(0)$, and $a_T, \rho_T(0)$ represent Gaussian shape density distributions of the projectile and target, respectively. Numerical values and more details are given in Ref. [23].

For smaller impact parameters we use a parameterization which takes into account the Pauli blocking effects (see Ref. [19]):

$$\frac{d\overline{n}}{\overline{n}} = \frac{db^2}{a_T^2 + a_P^2} \exp\left(\frac{-\kappa \overline{n}}{\overline{\sigma}_{NN}} \right) . \tag{28}$$

Here the parameter κ represents the amount of Pauli blocking. The two parameterizations (26) and (27) are matched at an impact parameter where the average number of interactions is one per trajectory.

The random walk momentum transfer associated with the nucleon transfer reflects the internal motion of nucleons inside nuclei. It has a distribution $p^{\lambda}(\vec{q}\,)$ in which such complicated factors as the Pauli principle, the density of states, *etc.* play also some role. Here we simulate $p^{\lambda}(\vec{q}\,)$ by a 3-dimensional Gaussian distribution:

$$p^{\lambda}(\vec{q}\,) = \frac{1}{(2\pi\sigma_0)^{3/2}} \exp\left(-\frac{\vec{q}-\vec{q}_0^{\,\lambda}}{2\sigma_0^2}\right), \tag{29}$$

where $\vec{q}_0^{\,\lambda}$ is a center of the Gaussian taken in an appropriate coordinate system. For instance in the LAB system

$$\vec{q}_0^{\,(-)} = \frac{\vec{K}_P^{\,0}}{A_P}, \tag{30}$$

$$\vec{q}_0^{\,(-)} = \frac{\vec{K}_T^{\,0}}{A_T}. \tag{31}$$

For σ_0 we take values suggested by the Stokstad systematics [25].

In most cases the momentum transfer without the mass transfer ($\lambda = 0$) is much smaller then the momentum transfer due to the mass transfer ($\lambda = +, -$). For simplicity we take for $p^0(\vec{q}\,)$ a Dirac function centered at the zero momentum.

The most difficult is to find proper values for the P^{λ}. In calculations of Cole the P^{λ} are independent from the impact parameter. For $A_P < A_T$ he uses $P^+ < P^- < P^0$. The above relation can be justified by noting that at large impact parameters excitations due to inelastic scattering dominate ($\lambda = 0$). For large impact parameters P^+ is smaller than P^- as is evidenced by a dominance of stripping over pick-up in asymmetric reactions ($A_P \ll A_T$). Situation becomes more complicated at smaller impact parameters which are considered in this work. For smaller impact parameters the overlap region of two nuclei increases together with a corresponding phase space. Now both P^+ and P^- should increase and the difference between them should be smaller. We adopted here the following prescription. For peripheral collisions the values of P^+, P^- were taken from Cole [19]. For less peripheral collisions P^+ and P^- have been increased. Their values were found in an error and trial procedure. The transition from large to small values of P^+, P^- was assumed to have a Fermi shape.

It should be pointed out that the average number of steps in the direction $\lambda, \overline{n}^{\lambda}$, (Eq. (7)) should be zero at large distances between the colliding

nuclei. It is fulfilled by the fact that the average number of steps $\overline{n}$ goes to zero at these distances (see Eq. (27)).

2.4. Some results for primary fragments lighter than projectile

In order to simplify formula (18) which includes the 8-fold integral one can notice that usually a momentum transfer perpendicular to the entrance channel plane is relatively small compared to the total transfer. Therefore, only a small range of angles φ contribute. Consequently, one can use a single plane approximation. Instead of integrating over φ we add contributions from two trajectories only, corresponding to positive and negative deflections ($\varphi = 0°, 180°$, see Fig. 1b). In such an approximation Eqs (18) and (19) can be rewritten as:

$$P_{l_i}(\Delta A, \vec{Q}) = \frac{e^{-\overline{n}}}{(2\pi)^3} \sum_{\varphi=0°}^{\varphi=180°} \int d\vec{x} \int dt \exp\Big[\overline{n} \int d\vec{q}\Big(P^+p^+(\vec{q})e^{it} + P^-p^-(\vec{q})e^{-it} + P^0p^0(\vec{q})\Big)e^{i\vec{x}\vec{q}}\Big] e^{-i\vec{x}\vec{Q}_R(\varphi)} e^{-it\Delta A} J(\vec{Q}_R(\varphi) \mid \vec{Q}), \quad (32)$$

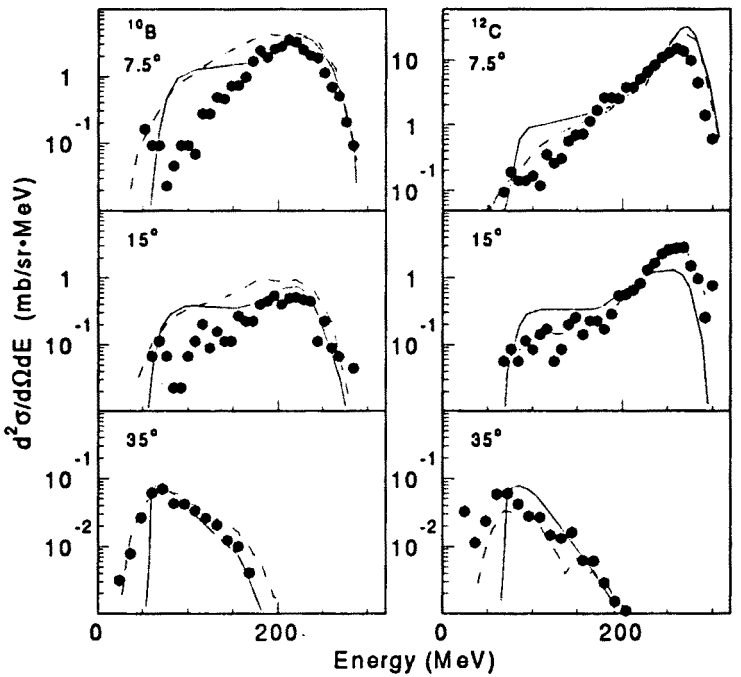

Fig. 2. Energy spectra (solid points) of B and C, and O ions from the reaction N + Tb. The lines are the random walk model calculations without (solid line) and with (broken line) secondary evaporation included.

$$\frac{d^2\sigma}{d\Omega dE} = \frac{\hbar^2 m_P m_{\mathrm{PLF}}}{4E_{\mathrm{beam}}\mu_i^2 \sin(\theta)} \sum_{l_i} \left[(2l_i+1)P_{l_i}(\Delta A, \vec{Q})\right] . \qquad (33)$$

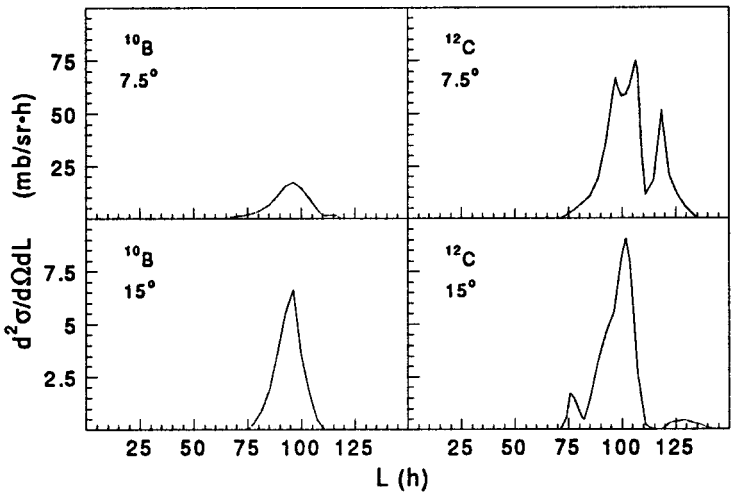

Fig. 3. The angular momentum partition of the cross-section calculated for the B and C ejectiles (N + Tb).

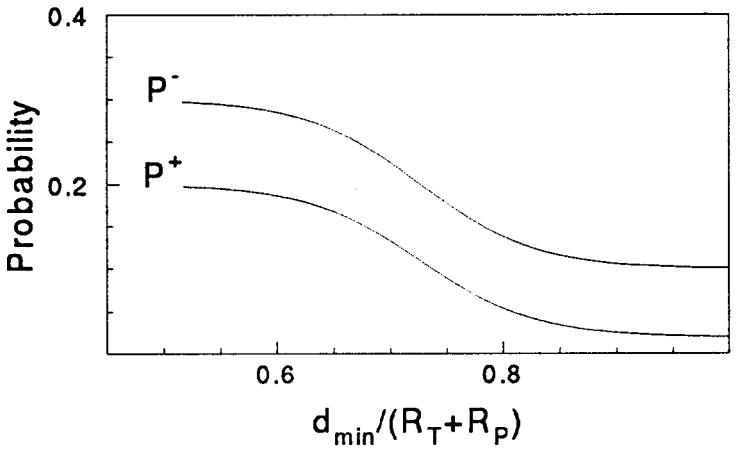

Fig. 4. The P^+, P^- probabilities as a function of the distance of the closest approach for the N + Tb reaction.

Next we can assume a Gaussian for $p^\lambda(\vec{q})$ and a δ_D function for $\lambda = 0$ and the integral over $\vec{q}$ can be analytically calculated. Also the integral over t

can be carried out (see Gradshteyn and Ryzhik, formula (3.937) [26]). In this way Eq. (32) can be reduced to the 2-fold integral. In Eq. (33), ϑ denotes an angle between the beam direction and the ejectile velocity.

For the purpose of testing our model we have selected the reaction $^{14}N + ^{159}Tb$ at 309 MeV. Experimental data were taken from [2]. Fig. 2 presents energy spectra of ^{10}B and ^{12}C ions at laboratory angles of 7.5°, 15° and 30° calculated from the model (solid line) together with experimental points. Fig. 3 shows the corresponding values of $d^2\sigma/(d\Omega dl)$. The strong variation as a function of L for ^{12}C is due to the contribution of positive and negative deflection angles. For the present calculation, we have $\sigma_0^+ = \sigma_0^- = 54$ MeV/c from the Stokstad systematics [25]. For κ the value of 0.565 suggested by Cole was used [19]. The P^+, P^- angular momentum dependence is presented in Fig. 4. As one can see from Fig. 2 the model describes energy distributions of fragments reasonably well. In particular a characteristic transition from the quasi elastic scattering at 7.5° to the deep inelastic scattering at 35° is properly reproduced. It should be pointed out that not only shapes but also the magnitude of cross sections is reproduced. Note, that the model does not include any normalization factor because the total calculated reaction cross section (σ_R) depends only on the number of the nucleon-nucleon collisions. On the other hand P^+, P^- decide how σ_R is partioned over the different exit channels.

2.5. Excitation of fragments

As it was mentioned in Section 1 primary PLF's can be excited and undergo sequential binary decays. Energy partition between fragments in heavy ion induced binary reactions was studied experimentally with a general conclusion that *energy goes to where the mass floes* [26]. However one has to remember that a multiple exchange of nucleons may take place in two directions, and both PLF and TLF may be excited. Nevertheless one expects larger excitation energies for PLF's which are heavier than the projectile, and *vice versa*. Therefore sequential decay will be more important for heavier fragments. Indeed, the agreement sofar was for predictions of the model without excitation energy of the PLF and without secondary evaporation. However the different cross section of oxygen ions measured in the same reaction are very much overestimated by the model predictions for primary ejectiles (Fig. 5, the solid line). In the following we take this into account.

In our reaction picture a projectile and a target nucleus move along a classical trajectory and exchange some number of nucleons in the overlap region. The net mass transferred in the collision is:

$$\Delta m = \Delta m^{(+)} - \Delta m^{(-)}, \tag{34}$$

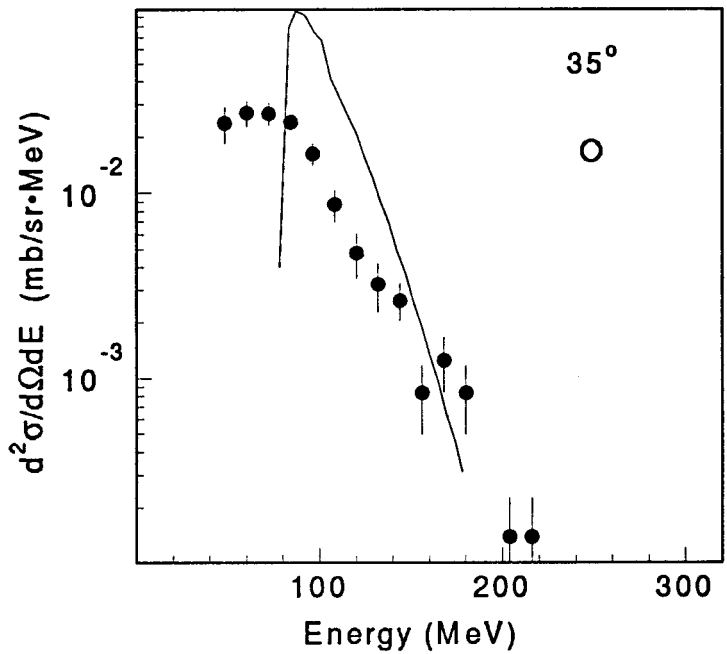

Fig. 5. Energy spectrum of oxygen ions (experimental points),with the random walk model prediction for primary reaction (solid line).

where $\Delta m^{(+)}$and $\Delta m^{(-)}$ is the mass of a group of nucleons acquired by the projectile, and of a group of nucleons lost by the projectile, respectively. The resulting PLF, and TLF have excitation energy $E^*_{\rm PLF}$ and $E^*_{\rm TLF}$, respectively.

The excitation energy is generated by a dissipation of the kinetic energy and binding energy of captured nucleons. The *ansatz* for the calculation of the excitation energy partition is similar as in Ref. [28] *i.e.* the stripped nucleons are assumed to be at rest with respect to the donor nucleus.

According to these assumptions the excitation energy is given as:

$$E^*_{\rm PLF} = \frac{\left(\vec{K}^0_P + \vec{Q}^{(-)}_R\right)^2}{2(m_P - \Delta m^{(-)})} + \frac{\left(\vec{K}^0_T\right)^2}{2m_T} - \frac{\left(\vec{K}^0_P + \vec{Q}_R\right)^2}{2(m_P + \Delta m)} - \frac{\left(\vec{K}^0_T - \vec{Q}^{(+)}_R\right)^2}{2(m_T - \Delta m^{(+)})} + \Delta V^{\rm PLF} + Q^{\rm PLF}_{00}. \quad (35)$$

Here m_P and m_T denotes the mass of the projectile and target, respectively, and Q^+_R and Q^-_R are the momentum transfers due to mass transfers

$\Delta m^{(+)}$and $\Delta m^{(-)}$, respectively. The Q-value Q_{00}^{PLF} is:

$$Q_{00}^{\mathrm{PLF}} = c^2(m_T + m_{P'} - m_{T'} - m_{\mathrm{PLF}}), \tag{36}$$

where $m_{p'}$, is the mass of the projectile after a loss of $\Delta m^{(-)}$, and $m_{T'}$, is the mass of the target nucleus after a loss of $\Delta m^{(+)}$. Similarly ΔV^{PLF} is a difference of the potential in the entrance and exit channel of a "reaction" defined by (35) (see also Ref. [28]).

For the target-like fragment one obtains similarly

$$E^*_{\mathrm{TLF}} = \frac{\left(\vec{K}^0_T - \vec{Q}^{(+)}_R\right)^2}{2(m_T - \Delta m^{(+)})} + \frac{\left(\vec{K}^0_P\right)^2}{2m_P} - \frac{\left(\vec{K}^0_T - \vec{Q}_R\right)^2}{2(m_T - \Delta m)} - \frac{\left(\vec{K}^0_P + \vec{Q}^{(-)}_R\right)^2}{2(m_P - \Delta m^{(-)})} + \Delta V^{\mathrm{TLF}} + Q_{00}^{\mathrm{TLF}}. \tag{37}$$

With a similar meaning for ΔV^{TLF} and Q_{00}^{TLF}, thus

$$Q_{00}^{\mathrm{TLF}} = c^2(m_p + m_{T'} - m_{p'} - m_{\mathrm{TLF}}). \tag{38}$$

Here $Q_{00}^{\mathrm{PLF}} + Q_{00}^{\mathrm{TLF}} = Q_{00}$ is a ground state to ground state reaction Q-value. For the overall change of the potential between the entrance and the exit channel $\Delta V = \Delta V^{\mathrm{PLF}} + \Delta V^{\mathrm{TLF}}$ must hold.

We would like to point out that instead of Eqs (35)–(38) one could try to use a different *ansatz*, for example one that also allows for a hole excitation of the stripped nucleus.

Due to the random walk character of the reaction mechanism the excitation energies E^*_{PLF} and E^*_{TLF} are given by probability density distributions that are related to the probability distributions for $\Delta m^{(+)}, \Delta m^{(-)}$, and probability density distributions for $\vec{Q}^{(+)}_R$, $\vec{Q}^{(-)}_R$.

In the following we use $\Delta A^{\pm}$ to indicate the number of nucleons associated with the transferred mass $\Delta m^{\pm}$. The net exchanged number of nucleons is:

$$\Delta A = \Delta A^{(+)} + \Delta A^{(-)}, \tag{39}$$

and net transferred momentum is:

$$\vec{Q}_R = \vec{Q}^{(+)}_R + \vec{Q}^{(-)}_R. \tag{40}$$

From Eq. (35) it is seen that E^*_{PLF} depends on $\vec{Q}_R$, $\vec{Q}^{(+)}_R$, and ΔA, each depending also on the impact parameter and a related angular momentum

l_i. However, only the net transfer, $\vec{Q}_R$ and ΔA, can be observed. Therefore, we need to integrate over all contributions $\vec{Q}_R^{(+)}$ and $\Delta A^{(+)}$ and finally to sum over l_i.

We proceed by first integrating over $\vec{Q}_R^{(+)}$. The excitation energy of the PLF is then given by the probability density distribution

$$E^*_{\mathrm{PLF}}(\vec{Q}_R, \Delta A^{(+)}, \Delta A) = \int d^3\vec{Q}_R^{(+)} \left[P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A^{(+)}, \Delta A) \times E^*_{\mathrm{PLF}}(\vec{Q}_R, \vec{Q}_R^{(+)}, \Delta A^{(+)}, \Delta A) \right], \quad (41)$$

where $P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A^{(+)}, \Delta A)$ denotes a conditional probability density with a requirement that $\vec{Q}_R, \Delta A^{(+)}$ and ΔA are kept fixed. Next we sum over all possible values of $\Delta A^{(+)}$, obtaining the final expression

$$\begin{aligned} E^*_{\mathrm{PLF}}(\vec{Q}_R, \Delta A) &= \sum_{\Delta A^{(+)}} P(\Delta A^{(+)} \mid \vec{Q}_R, \Delta A) E^*(\vec{Q}_R, \Delta A^{(+)}, \Delta A) \\ &= \sum_{\Delta A^{(+)}} \left(P(\Delta A^{(+)} \mid \vec{Q}_R, \Delta A) \int d^3\vec{Q}_R^{(+)} \left[P(\vec{Q}_R^{(+)} \mid \right. \right. \\ &\quad \left. \left. \times \vec{Q}_R, \Delta A^{(+)}, \Delta A) E^*(\vec{Q}_R, \vec{Q}_R^{(+)}, \Delta A^{(+)}, \Delta A) \right] \right) \quad (42) \end{aligned}$$

What remains to be calculated are the conditional density distributions $P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A^{(+)}, \Delta A)$ and $P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A)$. Because of (38) and (39) we may write

$$P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A^{(+)}, \Delta A) \propto P(\vec{Q}_R^{(+)} \mid \Delta A^{(+)}) P(\vec{Q}_R^{(-)} \mid \Delta A^{(-)}), \quad (43)$$

and utilizing Eqs. (28), (38), and (39) we obtain

$$\begin{aligned} P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A^{(+)}, \Delta A) &\propto \exp\left[\frac{-(\vec{Q}_R^{(+)} - \Delta A^{(+)} \vec{q}_0^{(+)})^2}{2(\Delta A^{(+)}(\sigma_0)^2)} \right] \\ &\quad \times \exp\left[\frac{-\left(\vec{Q}_R - \vec{Q}_R^{(+)} + (\Delta A - \Delta A^{(+)}) \vec{q}_0^{(-)} \right)^2}{2(\Delta A^{(-)}(\sigma_0)^2)} \right]. \quad (44) \end{aligned}$$

For $P(\vec{Q}_R^{(+)} \mid \vec{Q}_R, \Delta A)$ we write (see *e.g.* [29]):

$$\begin{aligned} P(\Delta A^{(+)} \mid \vec{Q}_R, \Delta A) &\propto P(\Delta A^{(+)}, \vec{Q}_R, \Delta A) \\ &= P(\Delta A^{(+)} \mid \Delta A) P(\vec{Q}_R \mid \Delta A^{(+)}, \Delta A). \quad (45) \end{aligned}$$

Here

$$P(\Delta A^{(+)} \mid \Delta A) = \frac{\mathcal{P}(\Delta A^{(+)})\mathcal{P}(\Delta A^{(-)})}{\sum_{\Delta A^{(+)}} \mathcal{P}(\Delta A^{(-)})\mathcal{P}(\Delta A^{(+)})}, \tag{46}$$

where $\mathcal{P}$ denotes a Poisson distribution

$$\mathcal{P}(\Delta A^{(+)}) = \exp(-\overline{\Delta A}^{(+)})\frac{(\overline{\Delta A}^{(+)})^{\Delta A^{(+)}}}{(\Delta A^{(+)})!}, \tag{47}$$

and

$$\overline{\Delta A}^{(+)} = \bar{n}P^{(+)}. \tag{48}$$

The last factor in formula (44) can be expressed as:

$$P(\vec{Q}_R \mid \Delta A^{(+)}, \Delta A) = \int P(\vec{Q}_R^{(+)} \mid \Delta A^{(+)})P\left((\vec{Q}_R - \vec{Q}_R^{(+)}) \mid (\Delta A - \Delta A^{(+)})\right) d^3\vec{Q}_R^{(+)}$$
$$= \frac{1}{(2\pi s^2)^{3/2}} \exp\left[-\frac{\left(\vec{Q}_R + \Delta A^{(-)}\vec{q}_0^{(-)} + \Delta A^{(+)}\vec{q}_0^{(+)}\right)^2}{2s^2}\right], \tag{49}$$

where

$$s^2 = \left(\Delta A^{(-)} + \Delta A^{(+)}\right)(\sigma_0)^2. \tag{50}$$

Using the probabilities given by Eqs (43)–(49) the excitation energy of the PLF can now be evaluated from Eq. (42).

The excitation energy of the targetlike fragment can be calculated in the same way but it is more convenient to make use of the energy conservation:

$$E^*_{\mathrm{PLF}} + E^*_{\mathrm{TLF}} = \frac{\left(\vec{K}_P^0\right)^2}{m_P} + \frac{\left(\vec{K}_T^0\right)^2}{m_T} - \frac{\left(\vec{K}_{\mathrm{PLF}}\right)^2}{m_{\mathrm{PLF}}} - \frac{\left(\vec{K}_{\mathrm{TLF}}\right)^2}{m_{\mathrm{TLF}}} + Q_{00}. \tag{51}$$

2.6. The deexcitation process

Following the philosophy explained in the introduction we divide the reaction process into two steps. In the first step, the mass and the excitation energy of primary fragments, PLF and TLF is calculated according to the random walk mechanism, in the second step the excited fragments undergo deexcitation.

We describe the deexcitation of primary fragments as a statistical, binary decay of an equilibrated system [30], [31]. Starting from an excited

primary fragment with an excitation energy E^*, and angular momentum J, a Monte Carlo program selects consecutive partitionings along the evaporation path. A probability for a given binary splitting into nuclei labeled i and j is written as [32]:

$$P_{\text{split}} \propto \frac{\exp\left[2\sqrt{a(E^* - E_{\text{sep}}^{ij} - E_B^{ij} - E_{\text{rot}}^{ij} - 2T)}\right]}{\exp\left(2\sqrt{aE^*}\right)}, \tag{52}$$

where E_{sep}^{ij} is the ground state separation energy into i and j. E_B^{ij} is the mutual Coulomb barrier and $T = \sqrt{E^*/a}$ is the temperature with the level density parameter $a = \frac{A}{8}$ /MeV. The rotational energy is defined by

$$E_{\text{rot}}^{ij} = \tfrac{1}{2}\omega^2[\tfrac{2}{5}M_iR_i^2 + \tfrac{2}{5}M_jR_j^2 + \mu_{ij}(R_i + R_j)^2]. \tag{53}$$

The scission configuration is assumed to be that of two touching spheres rotating about their center of mass with angular velocity ω given by the parent spin J and its moment of inertia, *i.e.* $\omega = J/(\frac{2}{5}MR^2)$, and μ_{ij} is the reduced mass of the system *i-j*. The initial angular momentum J is calculated assuming that the mass transfer must occur in the overlap region (see Section 2.2). The total kinetic energy is randomized after each partition according to the method of Moretto [32] in order to simulate the CM energy distribution of fission (evaporation) fragments. In the randomization process energy is conserved on the average.

Our calculation is of the "spin-off" type, because all fragments are emitted in the plane perpendicular to the initial angular momentum which, in turn, is taken to be uniformly oriented in the plane perpendicular to the beam direction. It should be emphasized that this "spin-off" approach represents a major simplification compared to a Hauser–Feshbach calculation, HF, and is invoked to avoid the considerable complexity inherent in the HF model for emission of intermediate-mass fragments.

This evaporation model is also an "equal-temperature" one, insofar that the available thermal energy is shared between the products in ratio of their masses, which seems a reasonable approximation for compound nucleus emission. In the case of "elementary" ejectiles ($A < 5$) all the thermal energy was assigned to the heavier partner.

The computer code FASTIMF combines calculations of the primary and of the secondary (post evaporation) event data. The calculated values are without any normalizing factor.

2.7. *Some results for fragments heavier than projectile*

The importance of the secondary evaporation of ejectiles and their subsequent decay can be deduced from Figs 2 and 5.

As shown in Fig. 2 the random walk model predictions for primary events agree well with energy distributions of boron and carbon ejectiles (solid lines). Inclusion of secondary evaporation (broken lines) has very little effect. The situation is different for oxygen, an ejectile which is heavier than the projectile.

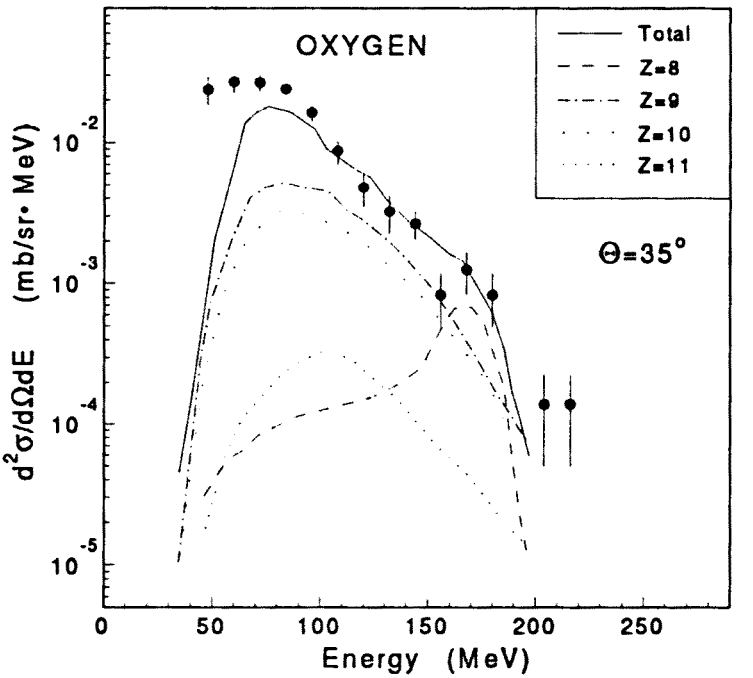

Fig. 6. Energy spectrum of oxygen ions (from $N+Tb$), calculated from the random walk model with the secondary evaporation included (solid line). Contribution of different primary ejectiles ($Z = 8 - 11$) is also presented. Experimental data are given by black dots.

In Fig. 5 the calculated spectrum of the primary oxygen at $\vartheta = 35°$ is shown (solid line) together with experimental points. With the exception of the quasielastic region the random walk model (no excitation and deexcitation included) overestimates experimental data up to two orders of magnitude. Inclusion of the excitation and deexcitation effects provides a reasonable agreement with experiment Fig. 6 (thick solid line). Different lines in Fig.6 represent contributions of various parent fragments to the

oxygen yield. In particular fluorine ($Z = 9$) and neon ($Z = 10$) contribute, whereas the primary oxygen contributes little due to the secondary evaporation.

3. Summary and conclusions

We have presented a model for the nucleus-nucleus collisions, at energies larger then 10 MeV/u, where both mean field effects and individual nucleon-nucleon interactions have to be included. The main ingredient in the model is a random walk process in the dimensions of mass and momentum. The model predicts absolute values of cross sections for various ejectiles. It allows to calculate dissipation of an initial angular momentum into the spins of the projectile- and the target-like fragments, and orientation of the spins. The model permits quite arbitrary forms for functions describing the momentum and mass transfer, $p^{\lambda}(\vec{q})$ and $P^{\lambda}(l_i)$, opening a way for studying the dependence on the initial energy and the geometrical factors of these reactions. By treating explicitly the nucleus-nucleus interaction, as it is done here, it is possible that for the same impact parameter different trajectories may evolve, e.g. the positive and the negative angle scattering. As the model is analytical also very small cross sections can easily be calculated. A Monte Carlo version of the model which treats neutrons and protons separately is being prepared. It can be used as an event generator.

Treating a nucleus-nucleus collision as a number of independent steps one obtains the excitation probability of fragments as a function of the amount of the transferred mass and momentum in both directions. The final result depends on the average number of reaction steps $\bar{n}$, the stripping, pick-up, and inelastic scattering probabilities P^{λ}, and on the momentum transfer distributions $p^{\lambda}(\vec{q})$. It is assumed that the excitation is generated by dissipation of the kinetic energy and of the binding energy of the captured nucleons.

When the excitation energy of primary products is sufficiently high it leads to evaporation of particles. This second reaction step, simulated by a Monte Carlo code is treated as a sequential evaporation from equilibrated primary fragments. The change of mass and the corresponding recoil effects are calculated step by step along the evaporation chain. For a given ejectile mass the final energy distributions contain important (and in the deep inelastic region dominating) contributions from heavier primary event decays, whereas the primary contribution is strongly reduced by evaporation.

The model has been tested by comparing its predictions with experimental energy, angular, and elemental distributions in the reactions: $^{14}\mathrm{N} + {}^{159}\mathrm{Tb}/{}^{\mathrm{nat}}Ag/{}^{\mathrm{nat}}\mathrm{Cu}$, at 22 MeV/$u$ [2].

As the primary binary reaction and the deexcitation stage are treated independently, different deexcitation scenarios may be included in the model.

In particular the deexcitation process of TLF can be taken into account as well as different modes of deexcitation as *e.g.* the statistical multifragmentation [33].

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