COMPOSITION OF DENSE MATTER AND NEUTRINO COOLING OF NEUTRON STARS*, **

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Composition of the neutron star matter above nuclear density is discussed within various models of superdense matter. Weak interaction processes, leading to neutrino cooling of dense, hot neutron star interior are reviewed. Neutrino emissivities of hot dense matter are shown to depend dramatically on the composition of matter. Cooling scenarios for young neutron stars, under various assumptions concerning the composition of the neutron star interior, are presented.

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1. Introduction

Neutron stars are the final products of thermonuclear evolution of massive stars of mass $\gtrsim 8M_\odot$ ($M_\odot = 1.989 \times 10^{33}$ g — mass of the Sun). Neutron stars are also expected to be produced, under specific conditions, in the gravitational collapse of white dwarfs.

Both the theory of stellar evolution, as well as astronomical observations indicate, that a typical neutron star mass is $\sim 1.4\ M_\odot$. However, theoretical

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calculations show, that the neutron star mass can be larger, up to some maximum value of $M_{\max} \simeq 2-2.5\ M_\odot$. Typical radius of a massive neutron star is expected to be ~ 10 km. These estimates imply, that the baryon number density in the neutron star interior can significantly exceed the saturation density of nuclear matter, $n_0 = 0.16\ \mathrm{fm}^{-3}$. We expect that the maximum density in the interior of neutron star can be as high as $\sim 8-10\ n_0$ (for an introduction to the physics and astrophysics of neutron stars, see [1]).

Neutron stars were observationally discovered in 1967 as radio pulsars. Since then, they were observed as X-ray pulsars and X-ray bursters. Neutron star constitute unique cosmic laboratories, in which the theory of dense matter can be confronted, through astrophysical scenarios, with astronomical observations.

Neutron stars are born as very hot objects, with internal temperature exceeding 10^{10} K. Even then, however, the constituents of the superdense interior of neutron star with $n > n_0$ form strongly degenerate Fermi gas. In the simplest model, in which the neutron star matter is approximated by the free Fermi gas of neutrons, the neutron Fermi energy is $E_{\mathrm{Fn}} = 58(n/n_0)^{2/3}$ MeV, and $kT \ll E_{\mathrm{Fn}}$ even for the newly born neutron star, with $T \sim 10^{10}$ K. In view of this, the structure and composition of the interior of neutron star can be calculated neglecting thermal effects.

Since its formation, neutron star cools by radiating neutrinos and photons. Thermal neutrinos can be detected on Earth only during the first few tens of seconds after the neutron star birth; their flux and mean energy decrease very rapidly with time. On the contrary, the photons emitted from the surface of a neutron star at a distance of a few kiloparsecs (1 kpc= 3260 light years) can be detected by the X-ray detectors on the satellites orbiting around Earth, as long as the surface temperature does not fall below some 10^6 K. However, while the photons are the carriers of information about the surface temperature, the temperature itself is determined by the neutrino emission from the neutron star interior. During first $10^5 - 10^6$ years of the neutron star life, neutrino cooling dominates over the photon one.

Neutrino emissivity of dense matter with $n > n_0$, which typically constitutes more than 95% of the mass of a massive neutron star, depends in a dramatic manner on the composition of the matter (percentage of neutrons, protons, electrons, hyperons *etc.*). In view of this, observational "measuring" of the surface temperature of a neutron star of a known age can yield precious information about the properties of matter at ultrahigh density.

In the present paper we restrict ourselves to non-exotic phases of superdense matter; we do not consider hypothetical phases resulting from pion condensation or deconfinement of quarks. There seem to be at least two arguments in favor of this limitation. Firstly, no experimental evidence for pion condensation has been found in "terrestrial" nuclear physics. Secondly, it is quite probable, that the low temperature deconfinement of quarks occurs at the densities higher than maximum ones encountered in the neutron stars.

In Section 2 we present results of calculations of the composition of superdense matter, composed of baryons and leptons. Weak interaction processes in dense matter are discussed in Section 3, where we calculate also the neutrino emissivity of hot, dense neutron star matter. A significant part of Section 3 is devoted to the description of very recent progress in this field, which took place in 1991–92. In Section 4 we present cooling scenarios for young neutron stars, under various assumptions on the composition of the neutron star interior. Theoretical cooling curves are confronted with existing X-ray observations. Finally, Section 5 contains conclusions.

2. Composition of matter at supranuclear density

At the baryon density $n_b > n_h = 0.1\ \mathrm{fm}^{-3}$ nucleons cannot be localized in nuclei and matter can only exist as a homogeneous, superdense plasma of baryons and leptons. At the densities close to n_h matter consists mainly of neutrons, protons, and electrons — the elementary constituents of ordinary "terrestrial" matter; at $n_b \gtrsim n_0$, the lepton component of matter will include some admixture of muons. This is the simplest model of neutron star matter, valid at not too high a density. This model will be described in some detail in Sec. 2.1. However, at higher density, say, at $n_b > 3n_0$, the composition of matter may be expected to be more complicated. In particular, it may be a *baryon matter*, containing not only nucleons, but also hyperons. Such a model of dense matter will be discussed in Sec. 2.2.

2.1. Nucleon matter: the npe and the npeμ models

The simplest model of the dense matter at $n_b > n_h$ is the *npe* plasma, composed of neutrons, protons, and electrons of number densities n_n, n_p, and n_e, respectively. Electric neutrality implies $n_p = n_e$, and so it is suitable to use two independent variables: baryon (in this case: nucleon) density, $n_b = n_n + n_p$, and the proton fraction, $x = n_p/n_b$.

For such a model, the energy per nucleon, E, is the sum of the nucleon, E_N, and the electron, E_e, contributions, $E = E_N + E_e$. The energy density, ϵ, which includes the rest energy of matter constituents, is related to E by $\epsilon = \sum_j n_j m_j c^2 + n_b E$. Many-body calculations of asymmetric nuclear matter with realistic hamiltonians show, that E_N can be, to a very good approximation, written as

$$E_N(n_b,\ x) = E_{N,kin}(n_b,\ x) + V_0(n_b) + V_2(n_b)(1-2x)^2\ . \qquad (2.1)$$

Here, $E_{\mathrm{N,kin}}$ is the free Fermi gas contribution to E_{N},

$$E_{\mathrm{N,kin}} = \tfrac{3}{5} E_{\mathrm{F0}}(n_{\mathrm{b}}) \left[x^{5/3} + (1-x)^{5/3} \right] , \tag{2.2}$$

where $E_{\mathrm{F0}} = \hbar^2/(2m) \left(3\pi^2 n_{\mathrm{b}}\right)^{2/3} = 58.4 (n_{\mathrm{b}}/n_0)^{2/3}$ MeV is the Fermi energy of a free Fermi gas of neutrons, of density n_{b}. For simplicity, we neglect the difference between proton and neutron masses, and put $m =$ 939 MeV.

The validity of formula (2.1) means that the strong interaction contribution to E_{N} is to a very good approximation quadratic in the neutron excess parameter $\alpha = (n_{\mathrm{n}} - n_{\mathrm{p}})/n_{\mathrm{b}} = 1 - 2x$ ([2, 3]). The formula (2.1) tells us, that the quantity E_{N}, which is *par excellence* a function of two independent variables, n_{b} and x, has a very simple and universal x dependence, and its overall structure is determined by two functions of n_{b}: V_0 and V_2.

For small α, the energy per nucleon can be approximated by the first two terms of the expansion in α,

$$E_{\mathrm{N}}(n_{\mathrm{b}}, \alpha) = E_0(n_{\mathrm{b}}) + S(n_{\mathrm{b}})\alpha^2 , \tag{2.3}$$

where S is the symmetry energy of nuclear matter, which is determined by V_2 alone,

$$S = \frac{1}{8} \left(\frac{\partial^2 E_{\mathrm{N}}}{\partial x^2} \right)_{x=1/2} = \frac{1}{3 \cdot 2^{2/3}} E_{\mathrm{F0}}(n_{\mathrm{b}}) + V_2(n_{\mathrm{b}}) . \tag{2.4}$$

While the absence of odd powers of α in expansion (2.1) is a direct consequence of charge symmetry of nuclear interactions, the fact that the quadratic approximation for the interaction part of E_{N} is very good up to $\alpha = 1$ (or $x = 0$) is a non-trivial property of the nuclear hamiltonian. Actually, this property leads to a significant simplification of the problem of the determination of the composition of the neutron star matter.

Under usual conditions, neutron star matter is — to a very good approximation - in equilibrium with respect to weak interaction processes. In the case of the *npe* model these processes are:

$$n \longrightarrow p + e^- + \bar{\nu}_e , \tag{2.5}$$

$$p + e^- \longrightarrow n + \nu_e . \tag{2.6}$$

More precisely, under usual conditions the timescales of the beta reactions, (2.5), (2.6), are much shorter than those related to the processes of the neutron star evolution.

Equilibrium with respect to beta processes, which establish the composition of the *npe* matter, implies a relation between the chemical potentials of matter constituents. This relation can be obtained by minimizing the energy density, under the condition of charge neutrality, with respect to x. This gives

$$\mu_{\rm n} = \mu_{\rm p} + \mu_{\rm e} \ . \tag{2.7}$$

Here $\mu_{\rm j} = (\partial\epsilon/\partial n_{\rm j})_{n_{\rm k}\neq n_{\rm j}}$ is the chemical potential of species "j", and includes the rest energy.

Relation (2.7) deserves a comment. Under prevailing conditions, neutron star matter is transparent to neutrinos, which leave neutron star. Their instantaneous number density in matter is negligible, compared with those of matter constituents (baryons, leptons). The neutrino interaction with matter constituents can be neglected. Therefore, under prevailing conditions neutrinos do not contribute to the thermodynamic quantities of the neutron star matter.

Using Eq. (2.1) we obtain

$$\mu_{\rm n} - \mu_{\rm p} = E_{\rm F0}(n_{\rm b}) \left[(1-x)^{2/3} - x^{2/3}\right] + 4(1-2x)\, V_2(n_{\rm b}) \ . \tag{2.8}$$

At the densities under consideration, electrons are ultrarelativistic ($\mu_{\rm e} \gg m_{\rm e}c^2$), so that

$$\mu_{\rm e} = \hbar c \left(3\pi^2 n_{\rm b} x\right)^{1/3} = 331.4\ (x n_{\rm b}/n_0)^{1/3}\ {\rm MeV} \ , \tag{2.9}$$

where we used the condition of charge neutrality, $n_{\rm e} = n_{\rm b}x$. At given $n_{\rm b}$, the beta equilibrium condition determines the equilibrium proton fraction, $x_{\rm eq}(n_{\rm b})$, as a solution of Eq. (2.7),

$$E_{\rm F0}(n_{\rm b})\ \left[(1-x)^{2/3} - x^{2/3}\right] + 4(1-2x)\ V_2(n_{\rm b}) = \hbar c \left(3\pi^2 n_{\rm b} x\right)^{1/3} \ . \tag{2.10}$$

The value of $x_{\rm eq}$ is thus determined by the $V_2(n_{\rm b})$ function [or, equivalently, the nuclear symmetry energy $S(n_{\rm b})$], and does not depend on the quantity $V_0(n_{\rm b})$.

The threshold density for the appearance of muons in dense matter, $n_{{\rm b}\mu}$, is slightly greater than $n_{\rm h}$. At $n_{\rm b} = n_{{\rm b}\mu}$ the electron chemical potential is equal to the muon rest energy, $\mu_{\rm e} = m_\mu c^2 = 105.7$ MeV, so that for $n_{\rm b} > n_{{\rm b}\mu}$ the reaction $n \longrightarrow p + \mu^- + \bar{\nu}_\mu$ becomes energetically allowed: the $npe\mu$ phase of matter is then energetically preferred over the npe one. The muon fraction $x_\mu = n_\mu/n_{\rm b}$. In the presence of muons, the charge neutrality condition reads $x = x_{\rm e} + x_\mu$. The condition of equilibrium with respect to weak interaction processes,

$$n \longrightarrow p + \mu^- + \bar{\nu}_\mu \ , \tag{2.11}$$

$$p + \mu^- \longrightarrow n + \nu_\mu \ , \tag{2.12}$$

leads to $\mu_e = \mu_\mu$. Consequently, the equilibrium values of x, x_e, x_μ can be determined from:

$$E_{F0}(n_b)\left[(1-x)^{2/3} - x^{2/3}\right] + 4(1-2x)\,V_2(n_b) = \hbar c\left(3\pi^2 n_b x_e\right)^{1/3}\,, \tag{2.13}$$

$$\hbar c\left(3\pi^2 n_b x_e\right)^{1/3} = \{m_\mu^2 c^4 + \hbar^2 c^2\left[3\pi^2 n_b x_\mu\right]^{2/3}\}^{1/2}\,, \quad x = x_e + x_\mu\,. \tag{2.14}$$

As in the case of the npe matter, the equilibrium composition of the $npe\mu$ matter is determined solely by the $V_2(n_b)$ function [or, equivalently, by the symmetry energy, $S(n_b)$)].

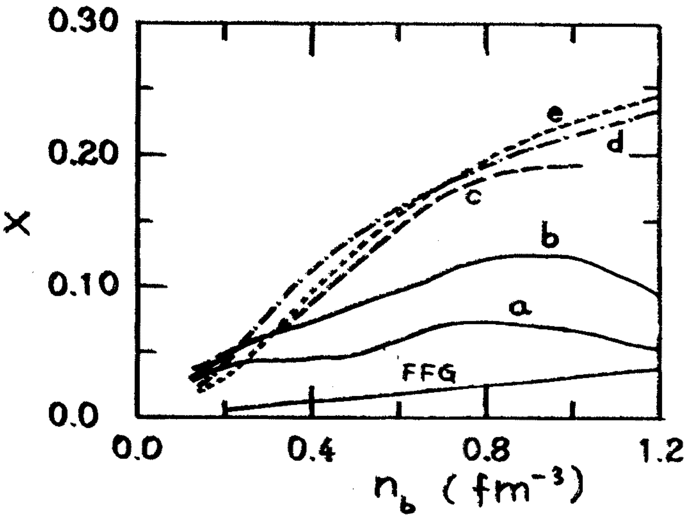

Fig. 1. Equilibrium proton fraction in the $npe\mu$ matter versus baryon density. Labels correspond to different models of dense matter. References: a, b [3]; c [4]; d [5]; e [6]. FFG: free Fermi gas model.

The density dependence of the nuclear symmetry energy is rather poorly known, and this results in a large uncertainty in the composition of the $npe\mu$ matter above n_0. In Fig. 1 we give the plots of the equilibrium proton fraction, versus baryon density, for several many-body theories of dense matter. For the sake of comparison, we show also results obtained for the free Fermi gas model of the $npe\mu$ matter.

We see that the proton fraction is determined essentially by the nuclear interaction term, and turns out to be strongly model dependent. Curves c, d, e, obtained within relativistic models of dense matter [4–6] show monotonic increase of x with density, in contrast to curves a, b, obtained for non-relativistic nuclear hamiltonians [3]. The proton fraction for the models a, b is, to a large extent, determined by the three-nucleon interaction term in the nuclear hamiltonian [3]. The three-nucleon interaction term tends to increase the proton fraction as compared to the case of the purely two-body interaction.

2.2. *Baryon matter: the general case*

At sufficiently high density, it is energetically advantageous to convert some nucleons into hyperons.

Let us consider the composition of the $npe\mu$ matter as a function of increasing baryon density, $n_{\rm b}$, assuming a very crude free Fermi gas model for all constituents. The first hyperon to appear is the lightest one, Λ. The threshold for the appearance of Λ is $n_{\rm t}(\Lambda)$, determined from the condition $\mu_{\rm n} = m_{\Lambda}c^2 = 1116$ MeV. For the free Fermi gas model one gets $n_{\rm t}(\Lambda) = 0.62\ {\rm fm}^{-3}$.

The inclusion of the nucleon-nucleon $(N - N)$ and nucleon-hyperon $(N - H)$ interaction lowers significantly the value of $n_{\rm t}(\Lambda)$. Typically, for relativistic mean field theory models of dense matter $n_{\rm t}(\Lambda) \sim 2 - 3n_0$ [7]. Depending on the assumed model of the baryon-baryon $(B - B)$ interactions, the hyperon with lowest threshold density can be Σ^- $(m_{\Sigma^-} = 1197$ MeV) or Λ (see below).

At $n_{\rm b} > n_{\rm t}(\Lambda)$ the energy of the $npe\mu$ matter can be *lowered* by converting some nucleons into Λ via weak interaction processes, *e.g.*, via $p + e^- \longrightarrow \Lambda + \nu_e$. Above the Σ^- threshold, the relevant reaction will be: $n + e^- \longrightarrow \Sigma^- + \nu_e$. It is clear, that the hyperonisation of matter will lead to a relative decrease of the electron fraction (see below).

In general, the composition of the baryon matter, composed of leptons (index $j_{\rm L}$ =e,μ,...) and baryons (index $j_{\rm B}$ = n, p, Λ, Σ^-,...) is determined from the conditions of the charge neutrality and that of thermodynamic equilibrium. This is a straightforward generalisation of the procedure for the npe matter, described in Section 2.1. Consider a mixture of leptons and baryons, at $T = 0$ and a fixed baryon density $n_{\rm b}$. The energy of a unit volume of matter, ϵ, is then a function of the densities n_j. However, the number densities n_j are subject to constraints of charge neutrality and of fixed baryon density:

$$\sum_j n_j q_j = 0\ , \tag{2.15}$$

$$\sum_{j_{\rm B}} n_{j_{\rm B}} = n_{\rm b}\ . \tag{2.16}$$

The calculation of the minimum of ϵ with respect to the set of n_j, subject to conditions (2.15), (2.16), can be then replaced by the calculation of the minimum of $\epsilon' = \epsilon + \alpha \sum_j n_j q_j + \beta \sum_{j_{\rm B}} n_{j_{\rm B}}$, where n_j are treated as independent variables, and α and β are the Lagrange multipliers, to be determined from the conditions (2.15) and (2.16). The resulting relations between the chemical potentials of leptons are:

$$\begin{aligned} q_{j_{\rm L}} &= -1 & \mu_{j_{\rm L}} &= \mu_e \ , \\ q_{j_{\rm L}} &= \ \ 0 & \mu_{j_{\rm L}} &= 0 \ , \\ q_{j_{\rm L}} &= +1 & \mu_{j_{\rm L}} &= -\mu_e \ . \end{aligned} \tag{2.17}$$

At the densities under considerations leptons can be approximated by free Fermi gases, and therefore only $q = -1$ leptons can be present in cold matter (*i.e.*, e, μ^-). The relations between the chemical potentials of baryons read:

$$\begin{aligned} q_{j_{\rm B}} &= -1 & \mu_{j_{\rm B}} &= \mu_{\rm n} + \mu_e \ , \\ q_{j_{\rm B}} &= \ \ 0 & \mu_{j_{\rm B}} &= \mu_{\rm n} \ , \\ q_{j_{\rm B}} &= +1 & \mu_{j_{\rm B}} &= \mu_{\rm p} \ . \end{aligned} \tag{2.18}$$

One sees, that the first hyperon to appear is Σ^- or Λ, that Σ^- will appear before Σ^0, and that the last hyperon from the Σ triplet to appear in dense matter is Σ^+.

The existing models of the multicomponent dense baryon matter are even less reliable, than those of dense nucleon matter. This results not only from the deficiencies of the many-body theories of dense baryon matter, but also from the lack of knowledge of the $N - H$ and $H - H$ interaction. Typically, the $N - H$ and $H - H$ interaction model is derived using some symmetry arguments, within a specific model of hadronic structure (based, *e.g.*, on the quark model — see, *e.g.* [8]) and/or constrained by experimental data on hypernuclei [9]. The calculations based on the nonrelativistic $B - B$ potential, in which the ground state of matter has been determined using a simplified version of the variational principle, were done in [10]. More recently, the equation of state and composition of dense baryon matter have been calculated using relativistic mean field theory model. The interaction of baryons has been described there by the exchange of the neutral scalar and neutral vector mesons, pions, and rho mesons [5–7, 11]. The parameters of the model have been adjusted to the saturation properties of nuclear matter, as well as to the existing data on hypernuclei, and on the hyperon-nucleon interaction. The ground state energy of the system has been calculated using the self-consistent mean field approximation, introduced originally by Walecka [12].

In Fig. 2 we show the fractions of leptons and baryons in dense, cold matter, calculated by Glendenning [7], using his "case 2" baryon Lagrangian. For this model, the hyperon couplings to meson field have been reduced as compared with the nucleon and isobar couplings. The threshold for the appearance of hyperons is about $2n_0$, the first hyperon to appear is Σ^-. At $n_b \simeq 1\ \mathrm{fm}^{-3}$ dense matter is a real "baryon soup". One should stress large uncertainty, resulting from the lack of knowledge of the $N-H$ and $H-H$ interaction. This is seen when comparing Fig. 2 with Fig. 3, which shows the results of [7] obtained using his "case 3" baryon Lagrangian, with universal couplings for all baryons. Notice, that in both cases the Δ isobars and pions do not appear in the matter at the densities considered.

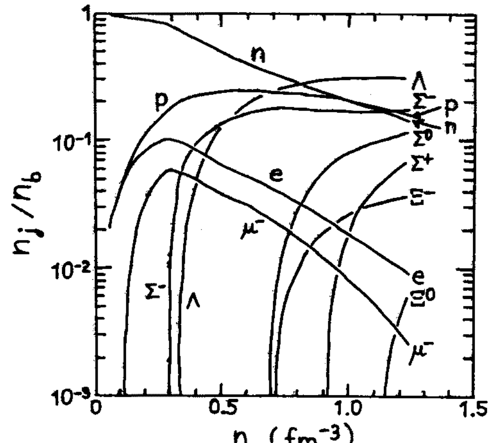

Fig. 2. The equilibrium baryon and lepton fractions as a function of baryon density, for "case 2" model of [7].

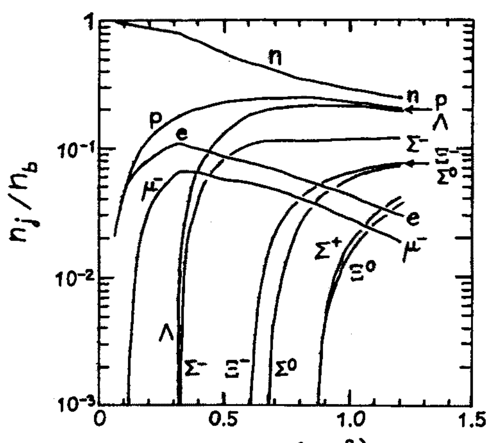

Fig. 3. The equilibrium baryon and lepton fractions as a function of baryon density, for "case 3" model of [7].

3. Neutrino emissivity of dense hot matter

All results, discussed in Sec. 2, were obtained neglecting thermal effects, and assuming, that matter is at $T = 0$. Under such conditions, matter is in its *ground state*, and neutrino emissivity of matter vanishes. Consider the simplest β processes involving nucleons,

$$\begin{aligned} \beta- &: \quad n \longrightarrow p + e^- + \bar{\nu}_e \ , \\ \beta+ &: \quad p + e^- \longrightarrow n + \nu_e \ . \end{aligned} \tag{3.1}$$

Baryon matter is a strongly interacting many-body system. However, we can still speak about individual "neutrons" and "protons", and their energies and momenta, as long as we replace them by the corresponding "quasiparticles".

Quasiparticles are elementary objects, which are useful in describing the low lying excited states of the degenerate, strongly interacting Fermi liquid (see, *e.g.*, [13]). For instance, simplest version of beta decay of a neutron in dense nuclear matter is equivalent to the particle-hole excitation (neutron-hole proton-particle). A notion of a quasiparticle is useful, when the corresponding states are formed by creation and annihilation of quasiparticle states close to the corresponding Fermi surface; only then quasiparticles have a sufficiently long lifetime, and can be considered as objects of a reasonably well defined energy and momentum. While "bare" nucleons are strongly interacting, the quasiparticles form a "weakly" interacting Fermi gas. Many of the notions referring to the *non-interacting* Fermi gas can be used to describe the gas of quasiparticles. In particular, the Fermi momentum for nucleon-quasiparticles corresponding to a real Fermi liquid of density n_j is, neglecting thermal corrections, the same as for the free Fermi gas of the same density: $p_{\rm Fj} = \hbar(n_j 3\pi^2)^{1/3}$. In what follows, we will, for the sake of simplicity, omit the prefix "quasi" when speaking about neutrons and protons in dense neutron star matter.

Consider the $\beta-$ process at $T = 0$. The conservation of energy requires, that $\mathcal{E}_{\rm i} = E_{\rm n}$ be equal to the energy of the final state, $\mathcal{E}_{\rm f} = E_{\rm p} + E_{\rm e} + E_\nu$. Decaying neutron occupied a state within the neutron Fermi sphere, so that $E_{\rm n} \leq E_{\rm Fn} \equiv \mu_{\rm n}$. On the other hand, all momentum states within the Fermi surfaces for protons and electrons are occupied, so that the Pauli principle implies $E_{\rm p} > E_{\rm Fp} \equiv \mu_{\rm p}$, $E_{\rm e} > E_{\rm Fe} \equiv \mu_{\rm e}$. However, in the beta equilibrium $\mu_{\rm n} = \mu_{\rm p} + \mu_{\rm e}$, so that $\mathcal{E}_{\rm i} = \mathcal{E}_{\rm f}$ *cannot* be satisfied. Inclusion of interactions with other quasiparticles does not help. Consider more complicated reaction $n + n \longrightarrow n + p + e^- + \bar{\nu}_e$. Once again, the Pauli blocking in the final state implies zero rate.

Let us consider now the effects of finite temperature of matter. In such a case, the Fermi surface is no longer "sharp": within a shell of the thickness

$\sim kT$ around μ_n, the neutron states are only partially occupied, and the condition $\mathcal{E}_i = \mathcal{E}_f$ can now be fulfilled. Clearly, the energies of the neutrinos emitted in the beta processes are at most $E_\nu \sim kT$, but the rate of the emission is finite.

Neutron stars are born as very hot objects, with initial temperatures $\sim 10^{11}$ K. Except for a short period just after their birth (shorter than 1 hour) they are transparent to neutrinos, which leave neutron star, taking away its thermal energy. Beta equilibrium of matter implies the equality of the rates of the $\beta+$ and $\beta-$ reactions, $\Gamma(\beta+) = \Gamma(\beta-)$, and the rate of neutrino emission is thus $\Gamma_\nu(\beta) = 2\Gamma(\beta-)$. Neutrinos leave neutron star interior at the velocity of light (in contrast to photons, which diffuse slowly through the stellar interior, to be eventually emitted from the neutron star "surface"), and thus the beta processes, Eq. (3.1), can represent a very efficient sink for the thermal energy of the star.

The pairs of reactions such as (3.1) were first considered by Gamow and Schoenberg as early as in 1941 [14], who discussed a more general case $(A,Z) \longrightarrow (A,Z+1) + e^- + \bar{\nu}_e$, $(A,Z+1) + e^- \longrightarrow (A,Z) + \nu_e$. Gamow and Schoenberg pointed out, that such pairs of reactions in hot dense stellar matter, at the late stage of stellar evolution (*e.g.*, red giant stage) could be responsible for very efficient thermal energy loss, leading eventually to stellar collapse. In their paper they refer to such processes using a mysterious name of "Urca processes". The name was in fact that of a casino in the Urca district of Rio de Janeiro (closed down in 1955). As Gamow explained afterwards, they introduced this name to commemorate the casino, in which they first met, and also because the Urca process results in a rapid disappearance of thermal energy of the star, similarly to the rapid disappearance of money of gamblers in the Casino da Urca [15].

The Urca mechanism for the neutrino emission results from the *charged current* processes of the standard Weinberg–Salam–Glashow (WSG) theory of weak interactions (see, *e.g.* [16]). The WSG theory predicts also the existence of *neutral current* weak interactions, which could lead to production of neutrinos in the nucleon Bremsstrahlung processes,

$$\begin{aligned} n+n &\longrightarrow n+n+\bar{\nu}+\nu\ , \\ n+p &\longrightarrow n+p+\bar{\nu}+\nu\ , \\ p+p &\longrightarrow p+p+\bar{\nu}+\nu\ , \end{aligned} \tag{3.2}$$

where ν is neutrino of any flavor (e, μ, or τ).

Neutrino emissivity of hot and dense neutron star matter, resulting from both charged current and neutral current reactions, will be discussed in the subsequent subsections.

3.1. Baryon matter: nucleons

Let us consider simple *npe* model of neutron star matter. Each of the constituents separately is a Fermi liquid in thermodynamic equilibrium, with chemical potential μ_i (i=n, p, e). We assume, that matter is strongly degenerate ($E_{Fi} \gg kT$, i=n, p, e). Neglecting thermal corrections, the Fermi momenta are $p_{Fi} = \hbar \left(3\pi^2 n_i\right)^{1/3}$, where n_i is the number density.

Our calculations will be done under a simplifying assumption, that all constituents of dense matter form normal Fermi liquids. While this is true for electrons, neutrons are most probably superfluid, and protons are most probably superconducting above nuclear density, for temperatures below $T_c \sim 10^9$ K, with energy gaps in the excitation spectra $\Delta \sim kT_c$. Therefore, our results will be valid for $T > T_c$. Effects of superfluidity and/or superconductivity will be discussed in Section 4.

Due to strong degeneracy of the *npe* liquids, the energy conservation implies, that the momenta of the particles participating in the Urca reactions be restricted to the vicinity of the corresponding Fermi surfaces. Actually, the *moduli* of p_j can differ from p_{Fj} by $\sim kT/c$, and the neutrino momentum $p_\nu \sim kT/c$. Under usual conditions, prevailing in the neutron star interior, $p_{Fj} \gg kT/c$.

The momentum conservation in the Urca process $\beta-$, (3.1) requires, that $\boldsymbol{p}_n = \boldsymbol{p}_p + \boldsymbol{p}_e + \boldsymbol{p}_\nu$. Putting all momenta at the corresponding Fermi surfaces, and neglecting the neutrino momentum, we see, that momentum conservation in the $\beta-$ process can be satisfied only if the triangle inequality is valid: $p_{Fn} \leq p_{Fp} + p_{Fe}$. For our model this corresponds to $p_{Fn} \leq 2p_{Fe}$ and thus $n_p/n_b \geq x_{crit} = 1/9 = 11.1\%$. Only in such a case β processes in their simplest form (called "direct Urca" process) are kinematically allowed in degenerate *npe* matter.

For more than 26 years since the classical paper of Chiu and Salpeter [17], who were first to consider the Urca process as an efficient mechanism for neutron star cooling, it was believed, that at the baryon densities, prevailing in the interiors of neutron stars, the equilibrium proton fraction, $x = n_p/n_b$, is so low that the simple "direct Urca processes", Eq. (3.1), cannot proceed. This standard assumption was corroborated by the simplest free Fermi gas model of the *npe* plasma, for which $x < 0.01$ at $n \sim n_0$ - much less, than the threshold value x_{crit} (see Fig. 1). The paper of Boguta [18], who pointed out that for some models of the *npe* matter proton fraction could be quite large, was unnoticed by the neutron star theorists. It view of this, the standard assumption was, that the beta equilibrium is maintained through the *modified Urca* processes,

$$n + n \longrightarrow n + p + e^- + \bar{\nu}_e$$
$$n + p + e^- \longrightarrow n + n + \nu_e\ . \tag{3.5}$$

The participation of an additional "active spectator" nucleon in the neutron decay or electron capture reactions is necessary to allow for the simultaneous conservation of energy and momentum in the degenerate neutron star matter, in which neutron, proton and electron Fermi momenta are assumed to violate the inequality $p_{\rm Fn} < p_{\rm Fp} + p_{\rm Fe}$ [1, 17].

Recently, it has been shown that for numerous models of dense nucleon matter the momentum condition $p_{\rm Fn} < p_{\rm Fp} + p_{\rm Fe}$ *is* actually satisfied at a sufficiently high $n_{\rm b}$, allowing thus for the *direct* Urca processes in the neutron star matter: $n \longrightarrow p + e^- + \bar{\nu}_e$, $p + e^- \longrightarrow n + \nu_e$ [19]. As it will be shown, this would dramatically increase (by many orders of magnitude) the neutrino emissivity of the neutron star interior [19], implying therefore a very rapid cooling of young neutron stars.

The rate of energy loss of a unit volume of the *npe* matter, implied by the direct $\beta-$ process, and resulting from the emission of $\bar{\nu}_e$, can be calculated using Fermi's golden rule, and taking proper account of the statistical factor in dense *npe* medium. For simplicity, we will omit all the $\hbar$ and c factors. We get then

$$\begin{aligned} Q_\nu(\mathrm{dir}.\beta-) = 2\pi\, 2 \int \frac{d^3p_\nu}{(2\pi)^3} E_\nu \int \frac{d^3p_e}{(2\pi)^3} \int \frac{d^3p_{\rm n}}{(2\pi)^3} \int \frac{d^3p_{\rm p}}{(2\pi)^3} \\ \times\, \overline{|M_{\mathrm{dir}.\beta}|^2}\, \delta(E_{\rm n} - E_{\rm p} - E_e - E_\nu) \\ \times\, \delta(\boldsymbol{p}_{\rm n} - \boldsymbol{p}_e - \boldsymbol{p}_{\rm p} - \boldsymbol{p}_\nu) \\ \times\, f_{\rm n}(E_{\rm n})[1 - f_{\rm p}(E_{\rm p})][1 - f_e(E_e)]\,. \end{aligned} \tag{3.4}$$

Here, $\overline{|M_{\mathrm{dir}.\beta}|^2} = {}^1\!/_2 \sum_{\rm spin} |M_{\mathrm{dir}.\beta}|^2$ is squared transition amplitude for the *direct* β process (3.1), averaged over initial and summed over final spin states, $f_i(E_i) = \{1 + \exp[(E_i - \mu_i)/kT]\}^{-1}$, and $E_\nu = p_\nu c$ is the neutrino energy.

For the strongly degenerate *npe* matter, the expression for $Q_\nu(\mathrm{dir}.\beta-)$, Eq. (3.4), can be calculated analytically using standard methods of the theory of normal Fermi liquids (see, *e.g.*, [13]). Neglecting neutrino momentum in $\delta(\boldsymbol{P}_i - \boldsymbol{P}_{\rm f})$ and in $\overline{|M_{\mathrm{dir}.\beta}|^2}$, one obtains a fortunate decoupling of the angular and dp integrations. Restricting then to lowest order in thermal corrections, we make a replacement $d^3p_j = m^*_j p_{\rm Fj} {\rm d}E {\rm d}\hat{\boldsymbol{p}}_j$, where $\hat{\boldsymbol{p}} = \boldsymbol{p}/p$, and where m^*_j is the quasiparticle effective mass at the corresponding Fermi surface. After angular integration, we are left with angular average of $\overline{|M_{\mathrm{dir}.\beta}|^2} = G_{\rm F}^2\, \cos^2\theta_{\rm C}\, (1 + 3g_{\rm A}^2)$, where $G_{\rm F}$ is the weak coupling constant, $\theta_{\rm C} = 0.239$ is the Cabibbo angle, and $g_{\rm A} = 1.261$ is the axial vector coupling constant. In the c.g.s. units, we have $G_{\rm F} = 1.436 \times 10^{-49}$ erg cm^{-3}. Keeping only the leading terms in T, and putting back all the $\hbar$, c factors, we get [19]:

$$Q_\nu(\mathrm{dir.}\beta-) = \frac{457\pi}{20160} G_\mathrm{F}^2 \cos^2\theta_\mathrm{C}\,(1+3g_\mathrm{A}^2)\,\frac{m_\mathrm{n}^* m_\mathrm{p}^* \mu_\mathrm{e}}{\hbar^{10} c^5}(kT)^6 \Theta_\mathrm{t}\,, \quad (3.5)$$

where $\Theta_\mathrm{t} = \theta(p_{\mathrm{Fp}} + p_{\mathrm{Fe}} - p_{\mathrm{Fn}})$ is the threshold factor [$\theta(x) = 1$ for $x > 0$, and is zero otherwise]. The effective masses are defined by $p_{\mathrm{Fj}}/m_j^* = (\mathrm{d}E_j/\mathrm{d}p)_{p=p_{\mathrm{Fj}}}$.

It should be stressed, that the numerical factor in Eq. (3.5) was obtained assuming the vacuum value of the weak interaction matrix element. The effective weak interaction matrix element is expected to be modified by the strong interactions in dense matter. We expect, that these effects will somewhat reduce the value of Q_ν (but by less than a factor of 10) [19].

The specific power of temperature (sixth) in formula (3.5) deserves a comment. It results from the integration over energies, restricted to the shell of a thickness $\sim kT$. Every integration over degenerate species (*npe*) gives then a factor of T, which yields T^3. Integration over neutrino phase space gives an additional factor T^3, E_ν yields T, while the delta function of the conservation of energy removes one T factor. This leaves an overall T^6 factor.

The total luminosity from the direct Urca processes, Eq. (3.1), is $Q_\nu(\mathrm{dir.Urca}) = 2Q_\nu(\mathrm{dir.}\beta-)$. Inserting numerical values of all factors, we get [19]

$$Q_\nu(\mathrm{dir.Urca}) = 4.0\times 10^{27}\,\frac{m_\mathrm{n}^*}{m_\mathrm{n}}\frac{m_\mathrm{p}^*}{m_\mathrm{p}}\left(x_\mathrm{e}\frac{n_\mathrm{b}}{n_0}\right)^{1/3}(T_9)^6\;\Theta_\mathrm{t}\;\frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{s}}\,, \quad (3.6)$$

where $T_9 = T/(10^9\ \mathrm{K})$, and the electron fraction $x_\mathrm{e} = x$. The astrophysical significance of the above equation, which implies very rapid neutrino cooling of a young neutron star, will be discussed in Sec. 4.

Let us notice, that $Q_\nu(\mathrm{dir.Urca})$ exceeds, by orders of magnitude, the neutrino emissivity of hypothetical, exotic phases of dense matter, such as pion condensate or deconfined quark matter [19]. The emissivities from these exotic phases of matter have similar T^6 temperature dependence, as $Q_\nu(\mathrm{dir.Urca})$, and therefore lead to an enhanced cooling of dense matter, as compared to that resulting from modified Urca and nucleon Bremsstrahlung mechanisms. In fact, before 1991, such an enhanced cooling was believed to be a signature of presence of exotic phase of matter inside neutron star!

It is interesting to note, that the formulae (3.5)–(3.6) were first published as recently as in 1991. The widely accepted view before 1991, resulting - as it seemed - from genuine properties of neutron star matter, was that direct Urca processes do not occur in *npe* matter (see, *e.g.*, [1]). The reason for this was a too low proton fraction in the matter. In such a situation, the only allowed charged current neutrino processes in the *npe* matter were the modified Urca processes, involving "an active spectator" nucleon [17]. Let us consider the modified $\beta-$ process, with a participation of an additional neutron [first line of Eq. (3.3)]. The neutrino emissivity from this process can be calculated using the same techniques as those used for the direct process. Omitting for simplicity all the $\hbar$, c factors, we obtain

$$\begin{aligned} Q_\nu(\text{mod.}\beta-) = \\ 2\pi\, 4 \int \frac{d^3p_\nu}{(2\pi)^3} E_\nu \int \frac{d^3p_e}{(2\pi)^3} \int \frac{d^3p_n}{(2\pi)^3} \int \frac{d^3p_{n'}}{(2\pi)^3} \int \frac{d^3p_{n''}}{(2\pi)^3} \int \frac{d^3p_p}{(2\pi)^3} \\ \times \overline{|M_{\text{mod.}\beta}|^2}\, \delta(E_n + E_{n'} - E_{n''} - E_p - E_e - E_\nu) \\ \times \delta(\mathbf{p}_n + \mathbf{p}_{n'} - \mathbf{p}_e - \mathbf{p}_{n''} - \mathbf{p}_p - \mathbf{p}_\nu) \\ \times f_n(E_n) f_n(E_{n'})[1 - f_p(E_p)][1 - f_n(E_{n''})][1 - f_e(E_e)] \,. \end{aligned} \tag{3.7}$$

where $\overline{|M_{\text{mod.}\beta}|^2} = {}^1\!/\!_4 \sum_{\text{spin}} |M_{\text{mod.}\beta}|^2$ is the squared transition amplitude, averaged over spins of initial states and summed over spins of the final states. In view of the participation of an additional neutron, both transition amplitude for the beta process and the phase space integration are more complicated than in the case of the direct beta processes. In particular, a large uncertainty arises from the approximate treatment of the nuclear interactions between the nucleons in the initial and final state. Nuclear (strong) interaction enters in a crucial manner into the transition amplitude for the weak interaction process: $M_{\text{mod.}\beta}$ vanishes for a non-interacting nucleon gas. We will use the transition amplitude $M_{\text{mod.}\beta}$ calculated, using the methods of the Fermi liquid theory, by Friman and Maxwell [20]; for a critique of their approach see Migdal et al [21]. Rewriting the results of Friman and Maxwell [20] in a generalized form, we get an explicit formula:

$$Q_\nu(\text{mod.Urca}) \approx 10^{22} \left(\frac{m_n^*}{m_n}\right)^3 \frac{m_p^*}{m_p} \left(x_e \frac{n_b}{n_0}\right)^{1/3} (T_9)^8 \; \frac{\text{erg}}{\text{cm}^3\ \text{s}} \,. \tag{3.8}$$

The additional T^2 factor results from the presence of an additional (degenerate) neutron in the initial and final state. It should be stressed, that the value of the numerical prefactor, and the density dependence of expression (3.8) may be a very poor representation of reality [21]. In general, we expect a rather large uncertainty resulting from the lack of knowledge of the (strong) nucleon-nucleon interaction in high-density *npe* matter, as well as from the deficiencies and approximations of the many-body theory of dense matter.

One notices a huge difference between the neutrino emissivities from the direct Urca, Eq. (3.5), and the modified Urca process, Eq. (3.8). If

$x > x_{\rm crit}$, then for $n_{\rm b} \sim 2 - 3n_0$ $Q_\nu(\text{dir.Urca})/Q_\nu(\text{mod.Urca}) \sim 10^6\ T_9^{-2}$. For $x > x_{\rm crit}$, the modified Urca process should be treated as a negligibly small correction to the direct Urca one. This result can be understood using the phase space arguments. Two additional degenerate neutrons, appearing in the modified Urca process, lead to an additional phase space factor of $(T/T_{\rm Fn})^2$, where $T_{\rm Fn}$ is the characteristic Fermi temperature of the neutron component, $T_{\rm Fn} \equiv E_{\rm Fn}/k$. At $n_{\rm b} \sim 2 - 3n_0$, we have $T_{\rm Fn} \sim 10^{12}$ K, so that $Q_\nu(\text{dir.Urca})/Q_\nu(\text{mod.Urca}) \sim (T_{\rm Fn}/T)^2 \sim 10^6\ T_9^{-2}$, which is consistent with results of detailed calculation. It is clear, that the presence of a core with $x > x_{\rm crit}$ in the interior of neutron star would have dramatic consequences for the rate of the neutron star cooling.

The presence of muons in the neutron star matter leads to two modifications with respect to the simplest *npe* model. Let us consider the *npeμ* model. At a given $n_{\rm b}$, the equilibrium value of x will be higher than in the case of the *npe* matter, while $x_{\rm e}$ will be somewhat smaller. The critical value of x for the direct *electron* Urca processes, $n \longrightarrow p + e^- + \bar{\nu}_{\rm e}$, $p + e^- \longrightarrow n + \nu_{\rm e}$, will be somewhat higher than 0.111. However, it will be always lower, than the extremal value of 0.148, reached in the limiting case of the ultrarelativistic muons. Also, $x_{\rm crit,e}$ will be reached at a value of baryon density $n_{\rm crit,e}$, lowered with respect to that of the *npe* model.

When $p_{\rm Fn} < p_{{\rm F}\mu} + p_{\rm Fp}$ (which occurs above some $x_{{\rm crit},\mu}$), the direct *muon* Urca processes can take place,

$$\begin{aligned} n &\longrightarrow p + \mu^- + \bar{\nu}_\mu\ , \\ p + \mu^- &\longrightarrow n + \nu_\mu\ . \end{aligned} \tag{3.9}$$

This occurs above some $n_{{\rm crit},\mu}$.

In order to give an example of the effect of muons, let us consider a specific model of the symmetry energy, $S(n_{\rm b})$ ("model I" used in [22]). For this model, we get for the *npe* matter $n_{\rm crit,e} = 0.44\ {\rm fm}^{-3}$. The presence of muons lowers the value of the $n_{\rm crit,e}$ to $n_{\rm crit,e} = 0.42\ {\rm fm}^{-3}$; the critical value of the proton fraction is $x_{\rm crit} = 0.135$. The critical density for the direct muon Urca process is somewhat higher than $n_{\rm crit,e}$, namely $n_{{\rm crit},\mu} = 0.52\ {\rm fm}^{-3}$, and corresponds to $x_{{\rm crit},\mu} = 0.16$.

The neutrino emissivity, resulting from the neutral current processes in dense matter, has a weak dependence on the neutron star matter composition. For the sake of comparison with Urca processes, we quote the result of Friman and Maxwell [20] for the *nn* Bremsstrahlung:

$$Q_\nu(\text{Brem.}nn) = 2 \times 10^{19} \left(\frac{m_{\rm n}^*}{m_{\rm n}}\right)^4 \left(\frac{n_{\rm b}}{n_0}\right)^{1/3} (T_9)^8 \frac{\rm erg}{\rm cm^3\ s}\ . \tag{3.10}$$

The temperature dependence of $Q_\nu(\text{Brem.})$ deserves a comment. The straightforward application of the phase space-factor rule, explained in the case of the Urca processes, yields a T^{10} factor for the Bremsstrahlung processes (T^4 from the degenerate nucleons, T^7 from the neutrino pair, T^{-1} from the energy conservation delta function). However, the transition amplitude contains a characteristic singularity, reflecting the infrared divergence of the rate of the $\bar{\nu}\nu$ pair emission, characteristic of Bremsstrahlung processes (also the photon ones), which yields an additional T^{-2} factor. This gives an overall T^8 factor in (3.10).

The neutrino emissivity from the nucleon Bremsstrahlung processes is significantly lower than that from the modified Urca process.

It should be stressed, once again, that the numerical prefactors in Eqs (3.6), (3.8), and (3.10), and their density dependence, are subject to a large uncertainty. This results mainly from the approximate treatment of the strong interaction effects in dense nucleon medium (see, *e.g.*, [21]).

3.2. Baryon matter: hyperons

At sufficiently high density ($n_{\rm b} > 2 - 3n_0$), neutron star matter may contain an admixture of hyperons. In view of this, we should consider also neutrino emissivity resulting from weak interaction processes involving hyperons.

The simplest neutrino emitting processes, involving hyperons, are

$$\begin{aligned} B_1 &\longrightarrow B_2 + l + \bar{\nu}_l\ , \\ B_2 + l &\longrightarrow B_1 + \nu_l\ , \end{aligned} \tag{3.11}$$

where B_1 and B_2 are baryons, and l is a lepton ($l = e,\ \mu$). The non-exotic case with $B_1 = n$, $B_2 = p$, was considered in Sec. 3.1. In this Section we will consider the processes, in which B_j are hyperons ($B = \Lambda,\ \Sigma,\ \Xi$).

We assume, that hyperons form degenerate Fermi gases. Therefore, all hyperons participating in the hyperon-Urca processes, Eq. (3.11), must have momenta close to their respective Fermi momenta, $p_{\rm Fj}$. Matter is very close to beta equilibrium, and therefore the chemical potentials of baryons and leptons satisfy the equilibrium condition of Section 2.2:

$$\mu_{B_1} = \mu_{B_2} + \mu_l\ . \tag{3.12}$$

The condition for momentum conservation in the hyperon-Urca processes, which is essential for neutrino emissivity, is equivalent to triangle inequalities:

$$|p_{{\rm F}B_2} - p_{{\rm F}l}| \le p_{{\rm F}B_1} \le p_{{\rm F}B_2} + p_{{\rm F}l}\ . \tag{3.13}$$

When these inequalities are not satisfied, the direct hyperon-Urca processes are not allowed, and the relevant neutrino emission processes are the modified hyperon-Urca processes,

$$\begin{aligned} B' + B_1 &\longrightarrow B' + B_2 + l + \bar{\nu}_l\ , \\ B' + B_2 + l &\longrightarrow B' + B_1 + \nu_l\ . \end{aligned} \tag{3.14}$$

The modified hyperon-Urca processes were studied in 1987 by Maxwell [23]. It is interesting to note, that the direct hyperon-Urca processes were first considered very recently, in 1992 [24].

The calculation of the neutrino emissivity from the direct hyperon-Urca processes can be done using the same methods, as those applied for the direct nucleon-Urca process. The difference concerns the quantity $\overline{|M|^2}$. The squared matrix elements for various hyperon direct-Urca processes were calculated in [24]. Baryons were treated in the non-relativistic limit, and the SU(3) symmetry has been assumed when calculating the relevant matrix elements. The general result, obtained assuming that the effective masses at the Fermi surfaces are equal to the bare (vacuum) masses, is given by the formula [24]

$$Q_\nu(\text{dir.hyp.Urca}) = 4.0 \times 10^{27} \frac{m_{B_1} m_{B_2}}{m_n^2} \left(x_e \frac{n_b}{n_0} \right)^{1/3} \mathcal{R} \cdot (T_9)^6 \, \Theta_t \, \frac{\text{erg}}{\text{cm}^3 \text{ s}}, \quad (3.15)$$

where Θ_t is the threshold factor, equal 1 if the triangle inequality, Eq. (3.13), is satisfied, and zero otherwise. The factors $\mathcal{R}$ depend on the charged current process involved, and are given in Table I, based on [24]. By definition, $\mathcal{R} = 1$ for the direct nucleon-Urca process, discussed in the preceding Section.

TABLE I

The $\mathcal{R}$ factor for the direct Urca processes

Process	$\mathcal{R}$
$n \longrightarrow p + l + \bar{\nu}_l$	1.0000
$\Lambda \longrightarrow p + l + \bar{\nu}_l$	0.0394
$\Sigma^- \longrightarrow n + l + \bar{\nu}_l$	0.0125
$\Sigma^- \longrightarrow \Lambda + l + \bar{\nu}_l$	0.2055
$\Sigma^- \longrightarrow \Sigma^0 + l + \bar{\nu}_l$	0.6052
$\Xi^- \longrightarrow \Lambda + l + \bar{\nu}_l$	0.0175
$\Xi^- \longrightarrow \Sigma^0 + l + \bar{\nu}_l$	0.0282
$\Xi^0 \longrightarrow \Sigma^+ + l + \bar{\nu}_l$	0.0564
$\Xi^- \longrightarrow \Xi^0 + l + \bar{\nu}_l$	0.2218

It should be stressed, that the numerical values of the coefficients in Eq. (3.15), and their density dependence, are rather uncertain. This results from the approximate treatment of strong interactions in dense matter, as well as from the lack of knowledge of the $N - H$ and $H - H$ interactions.

The neutrino emissivities from direct hyperon-Urca processes are suppressed in comparison with the nucleon ones, because they have smaller matrix elements for the relevant charged current processes (Table I). However, direct hyperon-Urca emissivities are larger by a factor of order $(T_{Fj}/T)^2 \sim 10^6\ (T_9)^{-2}$ than those of the modified hyperon-Urca processes, considered by Maxwell [23].

The expression for the direct baryon-Urca processes, Eq. (3.15), was obtained assuming, that the lepton l is the electron. In the presence of muons, with $\mu_\mu = \mu_e$, one can get an additional contribution to Q_ν(dir.Urca), provided the relevant triangle inequality, Eq. (3.13), is satisfied.

Let us consider the threshold concentrations for the direct hyperon-Urca processes. The hyperon composition of dense matter has been discussed in Sec. 2.2. Typically, the hyperon Λ, with a vacuum (bare) mass of 1116 MeV, and Σ^-, with mass 1197 MeV, appear at a similar density, because the somewhat higher mass of Σ^- is compensated by the presence of the electron chemical potential in the equilibrium condition for the Σ^-: $\mu_{\Sigma^-} = \mu_n + \mu_e$ (for the Λ the condition reads: $\mu_\Lambda = \mu_n$). In the case of a multicomponent baryon-lepton plasma, the derivation of the threshold conditions in terms of the fractions $x_{B_j} = n_{B_j}/n_b$ is a rather complicated task. However, some general statements can be made. It is easy to show, that if the direct nucleon-Urca process is *forbidden*, then the direct hyperon-Urca processes $\Sigma^- \longrightarrow n + l + \bar{\nu}$ are also forbidden, because $x_{\Sigma^-} < x_p$. In such a situation, the only direct Urca processes, that could be allowed, are those involving Λ's: $\Lambda \longrightarrow p + l + \bar{\nu}$ and $\Sigma^- \longrightarrow \Lambda + l + \bar{\nu}$. The threshold concentration of Λ's can be very low (it can be shown, that it is less than 0.032 [24]).

Calculations with phenomenological $N - N$, $N - H$, $H - H$ potentials lead to conclusion, that the hyperons appear at rather high density. Pandharipande and Garde [25] find only the Σ^- hyperon in dense matter. Bethe and Johnson [10] found that the hyperons appear above $3 - 6n_0$, depending on the assumed baryon-baryon potential model. In more recent calculations with relativistic mean field models of dense matter (cf. Sec. 2.2), hyperons appear at $2 - 3n_0$, and their abundances (fractions) grow rapidly with density [7, 9, 11]. For these most recent models, the proton fraction (in the presence of hyperons) exceeds the critical one, and therefore the direct hyperon-Urca emissivities just add to the dominating direct nucleon-Urca emissivity.

It should be stressed, that the hyperon concentrations in dense matter are even more uncertain than the proton abundance. Also, all most recent calculations of the hyperon abundances have been done using the relativistic mean field theories, in which correlations are neglected. Therefore, it is difficult to make a conclusive statement about *actual* neutrino luminosity of dense baryon matter.

4. Cooling of neutron stars

The initial temperature in the interior of neutron star, just after its formation, exceeds 10^{10} K. An important feature of hot neutron star, which determines its subsequent cooling, is the structure of the stellar interior. Let us discuss the structure of neutron star, of a typical mass of $\sim 1.4\ M_{\odot}$. Let us notice, that $1.4\ M_{\odot}$ is a typical mass of *observed* neutron stars. A radial cross-section through a $1.4\ M_{\odot}$ neutron star model, calculated for a specific equation of state of dense matter, is shown in Fig. 4. Basically, neutron star is composed of the "crust" of the density ranging from ~ 10 g cm^{-3} to $\sim 10^{14}$ g cm^{-3}, and of a liquid interior. The potentially detectable thermal radiation (photons) from cooling neutron star is emitted from a photosphere, situated in a few centimeters thin gaseous *atmosphere* - the outermost layer of an idealized model of neutron star. Neutrino emissivity of the crust is much lower than that of the liquid interior. Also, the thermal conductivity of the neutron star crust is much lower than that of the liquid, superdense core. Let us notice, that the crust contains only a few percent of the neutron star mass, but due to its relatively low thermal conductivity constitutes a very efficient "insulator", separating the liquid core from the photosphere.

Before discussing detailed scenarios of the neutron star cooling, let us estimate the timescale of cooling of small mass element of the liquid interior. A characteristic time for cooling by neutrino emission, $\tau_{\rm cool} = T/\dot{T}$, can be estimated by equating the neutrino energy loss rate, Q_{ν}, to the rate of change of the thermal energy, $c_V\dot{T}$, where c_V is the specific heat of matter (per volume) at a fixed volume. This gives $\tau_{\rm cool} = c_V T/Q_{\nu}$. If we approximate c_V by that of a free Fermi gas of neutrons at $n_{\rm b} = n_0$, we get then:

$$\tau_{\rm cool}^{\rm dir.Urca} \simeq \frac{1\ {\rm min}}{T_9^4}\ , \quad \tau_{\rm cool}^{\rm mod.Urca} \simeq \frac{1\ {\rm year}}{T_9^6}\ . \tag{5.1}$$

These simple estimates imply, that if the direct nucleon-Urca processes are operating, the liquid core will cool down to 10^9 K in a minute, and to 10^8 K in about a week. Actually, cooling to 10^9 K will be somewhat delayed by the ν_e, $\bar{\nu}_e$ absorption within the direct Urca core, the actual timescale being a few tens of minutes [22]. In the case when the direct Urca processes are not allowed, it takes a year for the core to cool down to 10^9 K, while cooling to 10^8 K will take some $\sim 10^6$ years.

These simple estimates show, that the operating direct Urca process will speed-up in a dramatic way the cooling of neutron star interior. However, from the *observational* point of view, the relevant timescale is that needed for the propagation of the thermal signal of this rapid cooling (the "cooling wave") to the neutron star photosphere, from which photons are

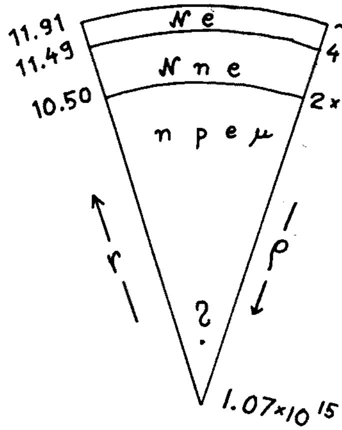

Fig. 4. Cross section of a neutron star model with $M = 1.4\ M_\odot$. This specific neutron star model has been calculated for the model I of Bethe and Johnson [10] for the inner core, and the standard (BPS + BBP) model for the crust [1]. The outer crust is composed of nuclei embedded in electron gas, the inner crust contains nuclei immersed in electron gas and neutron gas. The inner, liquid core consists of a baryon-lepton plasma. The numbers on the right give the density, in g cm^{-3}. The numbers on the left give the distance from the star center, in km. The question mark is put to remind, that the composition of the central core with $\rho \sim 10^{15}$ g cm^{-3} is very uncertain. The total baryon number of the star is 1.84×10^{57}.

emitted. This requires solving the problem of the heat transport in neutron star, starting from some initial, high-temperature, conditions. The thermal evolution of neutron star is then determined by the neutrino losses from the stellar interior, thermal conduction within neutron star (in particular, through the neutron star crust), and photon emission from the neutron star photosphere.

Detailed calculations show, that the timescale needed to reach the steady thermal state within the neutron star crust is some tens of years (see, *e.g.*, [26]); for earlier times crust and liquid interior cool quite independently, and surface photons cannot give us information about the thermal state of the liquid interior.

From the observational point of view, the most interesting aspect of the thermal evolution of neutron star is the "cooling curve": surface (effective) temperature, T_s, versus neutron star age. The photon luminosity of neutron star is then given by $L_\gamma = 4\pi R^2 \sigma T_s^4$. A set of cooling curves for a specific sequence of the neutron star models is given in Fig. 5 [27]. The neutron star models have been calculated using a specific model of the liquid interior. The relevant parameters of the neutron star models are given in Table II. Stars with $M > 1.35\ M_\odot$ have a core, in which direct Urca processes are allowed;

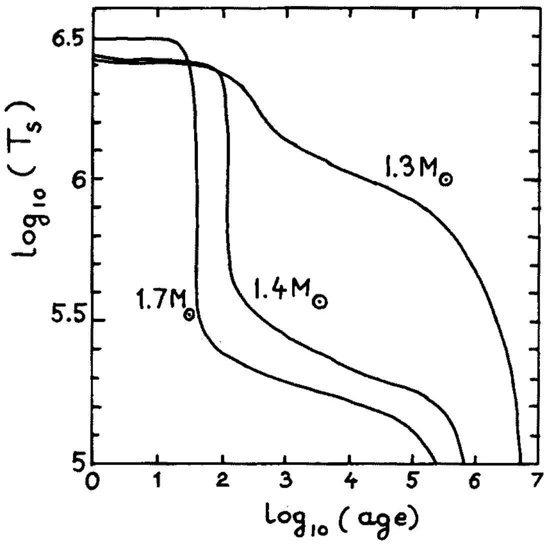

Fig. 5. Surface temperature, as measured by a distant observer (in K), versus age (in years), for some of the neutron star models of Table II. Normal nucleon liquids.

stars with lower masses cool exclusively by the modified Urca processes. Both neutrons and protons are treated as normal liquids.

TABLE II

Parameters of the neutron star models

M $(M_\odot)$	R (km)	$M_{\rm liq.}/M$	$R_{\rm liq.}$ (km)	$M_{\rm d.Urca}$ $(M_\odot)$	$R_{\rm d.Urca}$ (km)	$\rho_{\rm centr.}$ $(10^{15}$ g cm$^{-3})$
1.2	11.63	0.930	9.99	0.000	0.000	1.03
1.3	11.46	0.940	10.03	0.000	0.000	1.16
1.4	11.25	0.960	10.02	0.038	2.400	1.33
1.5	10.97	0.965	9.93	0.212	4.170	1.56
1.6	10.57	0.975	9.72	0.489	5.345	1.93
1.7	9.47	0.980	8.91	1.003	6.240	3.38

Qualitatively, the cooling curves can be divided into segments (stages), corresponding to specific *dominating* mechanisms of cooling. The sequence starts with the earliest stage, corresponding to a very flat segment of the cooling curve. Surface temperature is then determined by a rather slow cooling of the very outer layers via neutrino emission. At some $\sim$ 100 years the cooling wave from the center reaches the surface (somewhat earlier, if direct Urca mechanism operates). In the case when direct Urca mechanism operates, the surface temperature falls by an order of magnitude within a few years. Notice, that this is equivalent to a 10^4 drop in the surface photon luminosity, L_γ ! If direct Urca is not allowed, the cooling is less dramatic, and extended within tens of years.

Then comes a long stage, when the star is in the steady thermal state, and surface temperature decreases at the rate dictated by the neutrino losses in the liquid interior. The internal temperature, $T_{\rm int}$, is about an order of magnitude lower, than the surface one. However, the temperature drop takes place essentially within the very outer layer of neutron star, where the heat conductivity is relatively low. The liquid interior of neutron star is to a very good approximation isothermal. This stage lasts for some 10^5 years in the case of direct Urca cooling, and some 10^6 years, if direct Urca does not operate. As the internal temperature decreases, neutrino losses decrease, too (let us remind, that $Q_\nu(\text{dir.Urca}) \propto T^6$, $Q_\nu(\text{mod.Urca}) \propto T^8$), and the last stage of cooling is determined by the photon losses from the surface, which dominate over the neutrino losses.

Below some critical temperature, $T_{\rm c,p}$, protons in the liquid interior of neutron star are expected to be superconducting. The proton superconductivity results from the isotropic pairing in the $^1S_0$ state, and the value of $T_{\rm c,p}$ depends on the density. Neutrons are expected to be paired in the $^1S_0$ state below nuclear density, and their superfluid properties are believed to play an essential role in the "neutron star quakes", taking place in the neutron star crust, which are thought to be responsible for "glitches" observed in the timing of the radio pulsars [28]. Above n_0 it ceases to be energetically favourable for neutrons to pair in the $^1S_0$ state; this is due to the increasing role of the short-range repulsive $n-n$ interactions. As a result of the increasingly important attractive tensor force, neutrons tend to pair in a $^3P_2$ state, for which the influence of the strong short range repulsion is less significant. Below some (density dependent) critical temperature, $T_{\rm c,n}$, neutrons in the liquid interior of neutron star are thus expected to be superfluid.

It should be stressed, that theoretical estimates of $T_{\rm c}$ are very uncertain; actually, even within the simplest BCS weak-coupling approximation, their values depend dramatically (exponentially!) on the relevant matrix elements of the pairing interaction. Typically, however, $T_{\rm c} \sim 10^9$ K.

For $T < T_{\rm c}$ the spectrum of the particle - hole excitations near the Fermi energy is characterized by an energy gap, Δ. In the case of the isotropic $^1S_0$ pairing, $\Delta = 1.76kT_{\rm c}$; in the case of the $^3P_2$ pairing the energy gap is anisotropic.

We can qualitatively discuss the effect of the non-zero energy gaps on the neutron star cooling, using a simple estimate of the characteristic cooling timescale, $\tau_{\rm cool} = c_V T/Q_\nu$. The existence of the energy gaps has a strong effect on the phase space available for the weak interaction processes discussed in Sec. 3; for $T \ll T_{\rm c}$ the available phase space is dramatically reduced, which implies a strong damping of the corresponding neutrino emissivity Q_ν. This would lead to a slowing down of the neutron star cooling. However, at the same time, the existence of energy gap leads to a decrease of the specific heat of the corresponding component of matter, which tends to speed up the cooling.

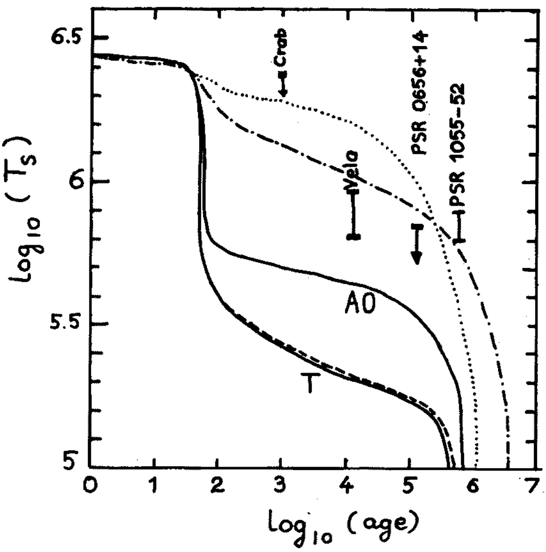

Fig. 6. Surface temperature versus age, as measured by a distant observer, for the 1.4 $M_\odot$ and 1.3 $M_\odot$ neutron star models of Table II, with various models of proton and/or neutron pairing. All curves include proton $^1S_0$ [29] and neutron $^1S_0$ [33] energy gaps. Solid lines correspond to a 1.4 $M_\odot$ star (small direct Urca core), with $^3P_2$ neutron energy gap of Takatsuka (T) [29], and Amundsen and Ostgaard (AO) [30]. Dashed line: 1.4 $M_\odot$ star, no neutron $^3P_2$ pairing. Dotted and dash-dotted lines correspond to 1.3 $M_\odot$ model (no direct Urca core). Dotted line: $^3P_2$ neutron pairing present (0.3HGRR model of [27]). Dash-dotted line: no $^3P_2$ neutron pairing. Observational data are described in the text.

Results of detailed calculations [28], showing the effect of the energy gaps on the cooling curve of neutron star are shown in Fig. 6. The 1.4 $M_\odot$ neutron star has a small core, in which direct Urca process is allowed. All curves include proton $^1S_0$ [29] and neutron $^1S_0$ [33] pairing , which have a small effect on cooling. Three cases of the neutron $^2P_2$ energy gap were considered: energy gap calculated by Takatsuka [29], that of Amundsen and Ostgaard [30], and a zero neutron gap case. Generally, the larger gaps give slower cooling (higher surface temperature at a give neutron star age). For the sake of comparison, the cooling curves for a 1.3 $M_\odot$ star, which cools via modified Urca processes, are also shown.

Thermal radiation from the surfaces of cooling neutron stars could be detected by the X-ray detectors on the dedicated satellites (EXOSAT, ROSAT). To be detectable, neutron stars should not be too distant (at most at a few kiloparcecs), and not too old, to be sufficiently powerful emitters in the X-ray domain. The age of neutron star is determined from the known date of its formation, or from the pulsar slowing down (assuming a specific model for the magnetic braking). In Fig. 6 we plotted the inferred surface temperature (or the upper limits to it) for four best candidates for observed cooling neutron stars. Let us notice, that the Crab pulsar, the Vela pulsar, PSR 0656+16, PSR 1055-52 are observed as radio pulsars, and therefore are neutron stars. We conclude from Fig. 6, that the Vela pulsar and PSR 0656+14 lie below the standard cooling curve without the direct Urca cooling. Therefore, both pulsars require some enhanced cooling, which could be provided by a direct Urca mechanism, damped to some extent by the sufficiently high energy gaps of paired nucleons.

As far as PSR 1055-52 is concerned, its high temperature could be due to internal heating, resulting from dissipative processes in its rotating interior. Several such mechanisms were proposed (see, *e.g.* [31, 32]). Generally, the fact that an observed neutron star temperature lies *above* the calculated cooling curve is not conclusive (because this can be explained by some additional energy sources). On the contrary, the fact, that observed temperature lies *below* the calculated cooling curve is conclusive: it rules out the theoretical cooling curve and the dense matter model used, and requires a novel mechanism of cooling.

5. Conclusion

Composition of dense matter at the densities exceeding significantly saturation density of nuclear matter ($n_b > n_0$ or $\rho > 2.7 \times 10^{14}$ g cm^{-3}) is rather uncertain. In particular, the relative proton fraction, which is determined by the density dependence of the nuclear symmetry energy, strongly depends on the assumed model of the nuclear hamiltonian, and on the many-body theory of dense matter. The uncertainty in the hyperon composition of dense matter at $\rho > 5 - 7 \times 10^{14}$ g cm^{-3} is even larger.

Particularly important is the density dependence of the proton fraction in dense matter. If it exceeds a critical value (which is 0.11-0.15 in the absence of hyperons) direct Urca processes are allowed, and the neutrino emissivity increases by many orders of magnitude as compared to that for lower proton fraction. The presence of hyperons can also strongly increase the neutrino emissivity of hot, dense matter.

While the bulk properties of neutron stars, such as radius and moment of inertia at a given mass, depend only on the equation of state of dense matter (relation between pressure and density), the cooling rate of young neutron stars is very sensitive to the composition of the stellar liquid interior. In particular, if neutron star contains a central core in which the direct Urca process is operative, the cooling timescale shortens by many orders of magnitude. Energy gaps due to superfluidity/superconductivity of nucleons tend to decrease this dramatic effect.

Some X-ray observations of pulsars indicate a need of an additional cooling mechanism, as compared to the "standard cooling curves". This could be explained by the specific composition of the dense baryon interior, which allows for a direct Urca core, without introducing such speculative phases of dense matter, like deconfined quarks and/or pion condensate.

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