EXCITATION OF A QUANTAL GAS IN A TIME-DEPENDENT POTENTIAL*

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Numerical studies of the excitation of 112 fermions in oscillating nucleus-like Woods–Saxon potentials are compared with analogous classical calculations in infinitely deep cavities. For oscillation frequencies such that $\hbar\omega$ is large compared to the level spacings, and for excitations small compared to the separation energy, a close correspondence is observed. For small frequencies a suppression of the excitation (relative to the classical result) is found.

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1. Introduction

Numerous calculations [1–3] have established a relationship between the order-to-chaos transition in the dynamics of classical particles in a container, and the transition from an elastic to a dissipative response of the container to

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shape changes. In this paper we report on preliminary results obtained when the particles in the container are quantized. The objective is to establish approximate contact with the classical results in that range of parameter values where a semi-classical treatment should be valid, and then to explore deviations due to specifically quantal effects.

2. The oscillating Woods–Saxon potential

As in Ref. [4] we took a Woods-Saxon potential $V(r,t)$ with a size corresponding approximately to a nucleus with mass number $A = 184$, with a depth V_0 equal to 50 (or 48.65), 100 or 200 MeV, and with a diffuseness parameter a which was varied from 0.6 fm down to 0.1 fm. By using a large depth and extrapolating to zero diffuseness, one approaches the infinite square well potential (a "billiard") for which many classical results are available.

Explicitly, we have

$$V(r,t) = -\frac{V_0}{1+\exp\left(\frac{r-R(t)}{a}\right)}, \tag{1}$$

$$R(t) = \frac{R_0}{\lambda(t)}\Big(1+\alpha_n(t)P_n(\cos\theta)\Big), \tag{2}$$

$R_0 = 1.16\,A^{1/3}\,\mathrm{fm} = 6.5978\,\mathrm{fm}$.

$$\alpha_n(t) = \sqrt{\frac{2n+1}{5}}(c_0 + c_1\cos(\omega t)), \tag{3}$$

where $n = 2, 4, 6$ determines the order of the Legendre polynomial $P_n(\cos\theta)$, and the coefficient c_1 specifies the amplitude of the oscillation around the shape specified by c_0. The factor $\sqrt{(2n+1)/5}$ ensures that the RMS deviation of the surface from the spherical shape is the same for all n. The factor $\lambda(t)$ ensures volume conservation.

The potential well was filled with 112 uncharged fermions, meant to represent the neutrons in a typical nucleus with $A = 184$. Owing to the axial and reflection symmetries of the potential and the neutrons' spin degeneracy (there is no spin-orbit potential) there are typically 34 intrinsically different single-particle wave functions that accommodate the 112 neutrons.

The time development is started at $t = 0$ when the well is at its maximum deformation and the total energy of the 112 particles is E_0, say. The relative excitation energy $(E(t)-E_0)/E_0$ is followed as a function of time by solving the time-dependent Schroedinger equation using an oscillator basis

with 20 major oscillator shells. (Increasing the basis to 22 shells had a negligible effect on Figs 1, 2. Concerning Figs 3, 4 see below. In the evaluation of matrix elements numerical integrations with up to 22 integration points in both radial and axial directions were used. This was adequate except for the smallest diffuseness, $a = 0.1$ fm, where a significant inaccuracy may be present).

3. A sample of results

The solid curves in Fig. 1 show E_{exc}/E_0 as functions of time for one period of oscillation specified by $n = 4$, $c_0 = 0$, $c_1 = 0.2$, $V_0 = -48.65$ MeV, $a = 0.1, \ldots 0.6$ fm, and a frequency specified by the adiabaticity parameter η of Ref. [1]. It is equal to the ratio of the maximum surface speed of the deforming well, given by $c_1 \omega R_0$, to the speed v of the most energetic particle in the well:

$$\eta = \frac{c_1 \omega R_0}{v} . \tag{4}$$

It follows that the characteristic energy $\hbar\omega$ is related to η by

$$\hbar\omega = \frac{\hbar P}{m R_0} \left(\frac{\eta}{c_1} \right) , \tag{5}$$

where P is the (Fermi) momentum of the most energetic particle and m the nucleon mass. The relation between the Fermi wave number $\hbar/P$ and the nuclear radius constant r_0 is

$$\frac{\hbar}{P} = \left(\frac{8}{9\pi} \right)^{1/3} r_0 ,$$

from which we deduce that

$$\hbar\omega = \frac{\hbar^2}{m r_0^2} \left(\frac{\eta}{c_1} \right) A^{-1/3} \simeq 8.3 \left(\frac{\eta}{c_1} \right) \mathrm{MeV} , \tag{6}$$

for $A = 184$.

In Fig. 1 the value of η is 0.24, *i.e.* $\hbar\omega \simeq 10$ MeV. The dashed curves show the predictions of the wall formula for dissipation (for a well with zero diffuseness) [5], *i.e.*

$$\frac{E_{\mathrm{exc}}}{E_0} = \tau + \frac{\tau^2}{5} , \tag{7}$$

where

$$\tau = \tfrac{3}{4} c_1 \eta (\omega t - \tfrac{1}{2} \sin 2\omega t) . \tag{8}$$

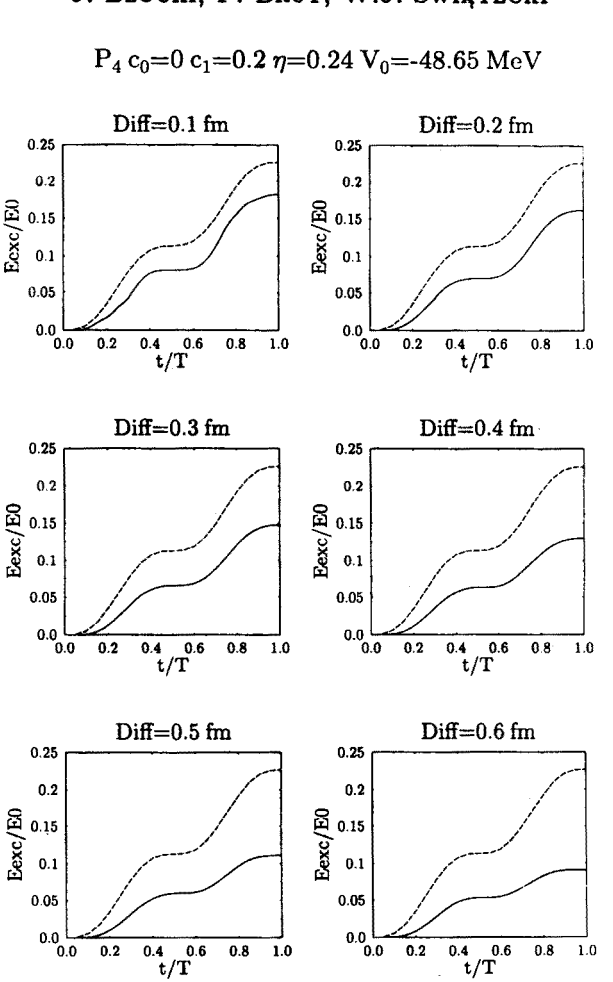

Fig. 1. The time dependence of the relative excitation energy for one period of oscillation of a Woods-Saxon well filled with 112 fermions (solid curves) is compared with the wall formula (dashed curves). The diffuseness parameter a varies from 0.1 fm to 0.6 fm.

The quantal results for the diffuse potentials are seen to be consistently lower than for the classical billiard. Most of the deviation appears to be related to the effect of diffuseness. Thus, the values of E_{exc}/E_0 after one period are 0.090, 0.111, 0.130, 0.148, 0.162, 0.182 for $a = 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$ fm. This defines an almost perfectly linear relation, which extrapolates to $E_{\mathrm{exc}}/E_0 \simeq 0.20$ for $a = 0$, to be compared with the wall formula prediction $E_{\mathrm{exc}}/E_0 \simeq \frac{3}{4}(0.2)(0.24)(2\pi) = 0.226$

The trend of the results is, in fact, consistent with the early studies of Koonin, Hatch and Randrup [6], which predicted that the heating up

of a semi-infinite quantal gas bounded by a rippling plane diffuse surface should agree with the wall formula for ripples of infinite wavelengths (and any degree of surface diffuseness), but that for finite wavelengths the results should fall below the wall formula, the deviations increasing with increasing diffuseness. Our studies with P_6 type oscillations- not displayed here- are also consistent with the expectation that the deviations (for a given diffuseness) should increase with decreasing wavelength of the ripples.

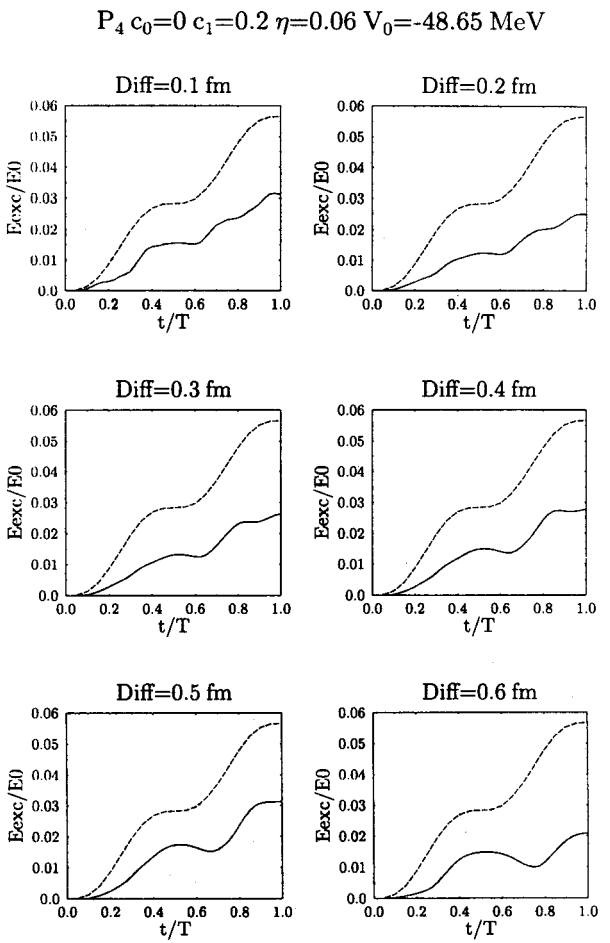

Fig. 2. This is like Fig. 1 but for a frequency of oscillation four times smaller.

Fig. 2 is similar to Fig. 1 but the frequency is much lower: $\eta = 0.06, \hbar\omega \simeq 2.5$ MeV. The quantal excitation curves are now considerably below the wall formula for all values of the diffuseness, and they begin to show structures not present in the classical results. The explanation is likely to be related

to the fact that $\hbar\omega$ is no longer large compared to the spacings between levels and one is not in the regime where a semi-classical treatment would be accurate. For still lower values of $\hbar\omega$ the excitation would be dominated by the distance to the lowest unoccupied level and would show structures associated with avoided level crossings, having no counterpart in a classical treatment.

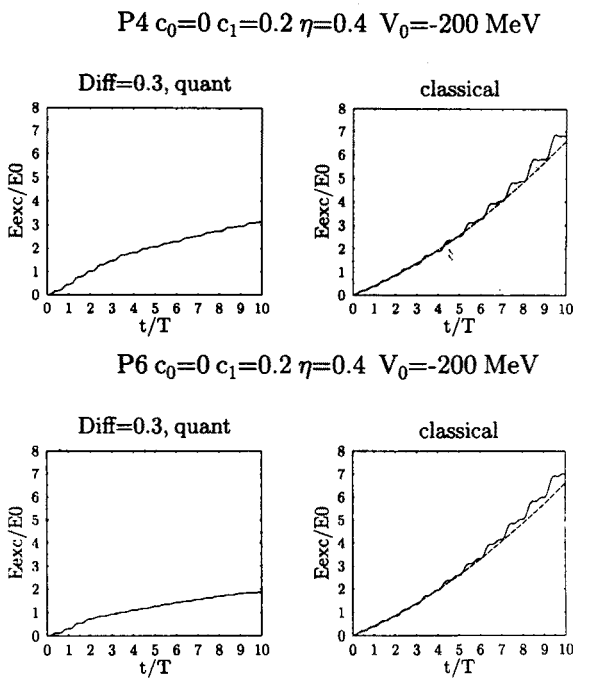

Fig. 3. The left part of the figure shows quantal excitation curves for P_4 and P_6 modes, extended to 10 periods of oscillation. The right part shows analogous classical calculations (in a cavity), which are compared with the smooth part of the wall formula, Eqs (7),(8), (dashed curves).

Fig. 3 shows the result of continuing the above type of calculation for 10 complete periods of oscillation. The diffuseness is $a = 0.3$ fm, $\eta = 0.4$ ($\hbar\omega \simeq 16.5$ MeV) and both classical and quantal excitation curves are shown for P_4 and P_6 modes. The classical results agree quite well with the wall formula (of which only the smooth trend is indicated by the dashed curves). By contrast, the quantal results begin to flatten out after a couple of periods. Such a saturation could be an indication of a quantal "localization" phenomenon [7], but at this time the significance of the quantal curves in Fig. 3 is quite unclear. Thus, the huge excitation energies (several times E_0, itself of the order of 20 MeV per particle) imply that quantal states in excess of the depth of the Woods–Saxon potential would be excited. These would be

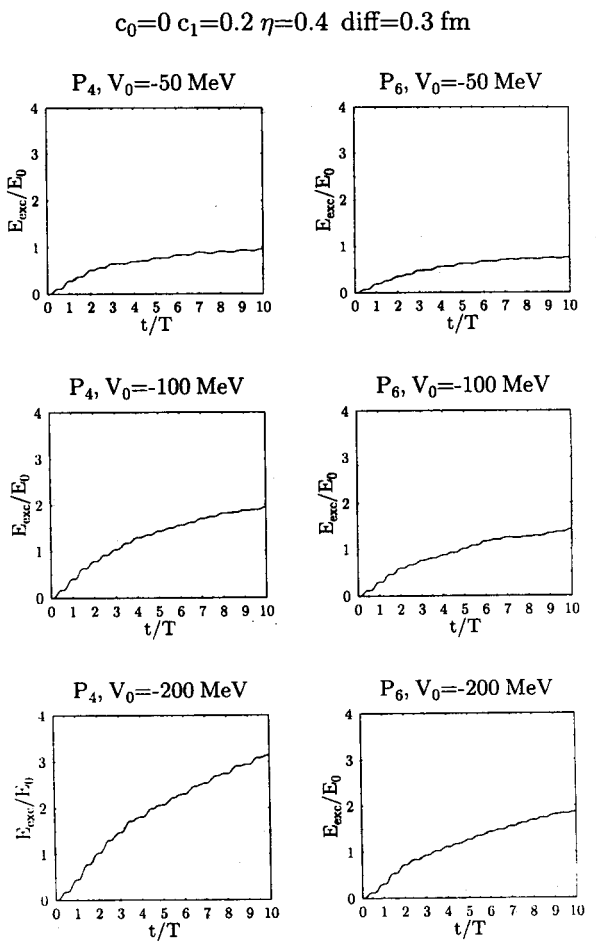

Fig. 4. These are quantal calculations similar to those in Fig. 3, but the depth of the potential was $V_0 = 50$, 100, 200 MeV.

states in the continuum, but since the evolution equations are being solved in a finite oscillator basis, the continuum is artificially suppressed and the significance of the resulting curves is obscure. Thus the saturation effect could be partly — or even entirely — an artifact of the finite oscillator basis used. Fig. 4 illustrates the seriousness of this problem. It shows that the "saturation" is a function of the potential depth and does not appear to have reached any well-defined limiting trend at $V_0 = 200$ MeV.

The results discussed earlier in connection with Figs 1 and 2 are not subject to the ambiguity associated with the continuum, since the excitation energies there are only of the order of 3 % of E_0, *i.e.* of the order of

0.6 MeV per particle, much less than the separation energy in a well of even 48.65 MeV depth.

4. Summary

We have established a numerical correspondence between classical and quantal dissipation curves for independent particles in a time-dependent potential representing an idealized nucleus — at least for a certain choice of parameters($\hbar\omega$ large compared to level spacings, $E_{\rm exc}$ small compared to the separation energy). Using this regime as a kind of anchorage, we shall now explore systematically the specifically quantal modifications of the semi-classical one-body dissipation mechanism.

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