AN APPLICATION OF THE DIRECT INTEGRALS IN THE AGCM APPROACH*

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In the paper the idea of direct integrals of Hilbert spaces applied to the algebraical generator coordinate method is shown. The rotational-spectrum of ^{8}Be is obtained within this method.

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1. The algebraic generator coordinate method (AGCM)

The Generator Coordinate Method (GCM) allows to construct the nuclear collective spaces by means of a very general ansatz for a trial function [1]:

$$|\Psi\rangle = \int dq u(q)|q\rangle \,. \tag{1}$$

These states are pure states in quantum mechanical sense. The AGCM [2] is an extension of the GCM approach to the mixed states generated from a given density matrix. Within this approach the collective variables are introduced by means of the appropriate group of motions which defines the possible types of excitations in the physical system.

The main object in the procedure of construction of the collective space generated by a density operator ρ and a given group of motions G is an *-algebra $\mathrm{L}^2(\mathrm{G})$ and the space of functionals (metastates) on it [3]. The metastate on the algebra $\mathrm{L}^2(\mathrm{G})$ is chosen as the following integral:

$$\langle\rho; u\rangle = \int_G dg u(g) \mathrm{Tr}(\rho T(g)) \,, \tag{2}$$

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where $T(g)$ is a unitary representation of the group of motion G. After some group theoretical considerations one can obtain the collective space $\mathcal{K}$ with the nonlocal scalar product defined by the metastate:

$$(u|v)_{\mathcal{K}} \equiv \langle \rho; u^{\#} * v \rangle = \int\limits_{G} dg' \int\limits_{g} dg u^{*}(g') Tr(\rho T(g'^{-1} g)) v(g) . \quad (3)$$

The standard GCM approach corresponds to the AGCM approach with the density matrix representing a pure state $\rho = |-\rangle\langle -|$.

This algebraical approach allows to consider cases, where statistical approach is required. It also gives a tool for classification of the energy spectra with respect to subgroups of the group of motions giving, in addition, information about the states internal structure [4].

2. Direct integrals of Hilbert spaces

For completeness we recall shortly the notion of the direct integral [5] of a family of separable Hilbert spaces K_λ , where λ belongs to a locally compact separable topological space Λ with non-negative measure μ: The direct integral of Hilbert spaces, denoted by

$$\int\limits_{\Lambda} \oplus K_\lambda d\mu(\lambda) , \quad (4)$$

is the set of all measured vector functions $u = (u_\lambda)$, where u_λ belongs to K_λ, which satisfy the following integrability condition

$$\int\limits_{\Lambda} \|u_\lambda\|^2 d\mu(\lambda) < \infty . \quad (5)$$

The linear operations in this new Hilbert space are defined pointwise:

$$(u+v)_\lambda \equiv u_\lambda + v_\lambda, \quad (au)_\lambda \equiv a u_\lambda , \quad (6)$$

and the scalar product is given by the formula

$$(u|v) \equiv \int\limits_{\Lambda} (u_\lambda | v_\lambda)_\lambda d\mu(\lambda) . \quad (7)$$

3. The AGCM and direct integrals. Application to ^{8}Be

Using of the density matrices dependent on some external parameters like temperature, deformations, external fields *etc.* is very useful for generating metastates of the physical system (the functionals on the algebra of motions). A natural way of introducing a dynamical description in respect to these external variables is, of course, an appropriate extension of the group of motion for the system. However, this method is, because of practical difficulties, not always possible leading sometimes to a very large (even infinite dimensional) group manifolds.

In this report, we present a concept of approximate dynamical dependence of physical systems on the external parameters making use of the idea of the direct integral of Hilbert spaces [5] for the nucleus ^{8}Be. This nucleus consists of two α-particles and forms a dumb-bell shape. Its excitations should have both rotational and vibrational spectra.

The rotational excitations one can easily investigate considering the SO(3) group of motion with static deformation as the external parameter. The vibrations require "switching" on a dynamics by extension of the group of motion to *e.g.* GL(3,R) group. However, the structure of this group is rather complicated and we propose below a simplified method which is an approximation to the exact solution.

In the paper [6] the SO(3) basis $|JM,\lambda\rangle$ diagonal for the arbitrary hamiltonian H, where λ denotes a nuclear deformation parameter, has been constructed. To built the shape vibrations (in the deformation λ) on top of pure rotational states $|JM,\lambda\rangle$ with the energies $E_J(\lambda)$ we construct the direct integral of the spaces K_λ spanned by $|JM,\lambda\rangle$ for fixed λ. The coupling between the rotational and vibrational degrees of freedom results here directly from the nuclear many-body hamiltonian H. The matrix elements of the hamiltonian in the resulting state space are the following:

$$(\nu, JM|H|\nu', J'M') = \int_{\mathbf{R}} d\lambda \langle JM,\lambda|u_\nu^*(\lambda) H u_{\nu'}(\lambda)|J'M',\lambda\rangle$$
$$= \delta_{JJ'}\delta_{MM'} \int_{\mathbf{R}} d\lambda u_\nu^*(\lambda) u_{\nu'}(\lambda) E_J(\lambda)\,, \qquad (8)$$

where $u_\nu(\lambda)$ are basic states describing the λ-vibrations (here we assumed as the basis the eigenfunctions of the 1-dimensional harmonic oscillator), and ν is the vibrational quantum number.

Making use of the Brink–Boeker forces, as in the paper [7], and using the direct integral + AGCM method, we have calculated rotational+vibrational energies for ^{8}Be nucleus which are in good agreement with experimental data. The results are shown in Fig. 1, where one can notice the additional band (*e.g.* 0^+6.6 MeV, 2^+9.1 MeV, 4^+15.1 MeV) which arises due to vibrational degrees of freedom and the coupling to the rotational motion. The direct integral method is a kind of diagonal approximation in the metastate kernel function (overlap function). This provides rather weak coupling between rotational and vibrational motions. For the nucleus ^{8}Be this is the case.

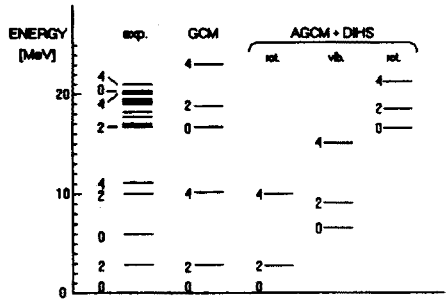

Fig. 1. The comparison of experimental spectrum of ^{8}Be [8] with the theoretical ones given by GCM [7], and by AGCM plus direct integrals of Hilbert spaces.

A better test of the direct integral approximation is in progress. The calculations of electromagnetic transition probabilities within the obtained space are in preparation. However, even now it seems that this approximation is valuable tool for evaluations of some effects in the AGCM approach.

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