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QUADRUPOLE COLLECTIVE STATES IN A LARGE SINGLE-j SHELL*,**

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Quadrupole collective states of the pairing-plus-quadrupole hamiltonian acting in a degenerate single-j shell are determined within the generator coordinate method. The particle-number and angular-momentum symmetries are exactly conserved. The collective space is constructed by acting with the single-particle quadrupole operator on a condensate of monopole fermion pairs.

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We discuss collective excitations in a degenerate single-j shell filled with even number of particles N. The schematic pairing-plus-quadrupole hamiltonian [1–4] is used in order to describe the interaction between particles

$$\hat{H} = -G\hat{P}^{+}\hat{P} - \chi\hat{Q}\cdot\hat{Q}\,,$$

where $\hat{P}^{+} = \sum_{m>0}(-1)^{j-m}a_{m}^{+}a_{-m}^{+}$ is the pair transfer operator, $\hat{Q}$ is the quadrupole moment operator with its five components defined as $\hat{Q}_{\mu} = \sum_{mm'}(jmjm'|2\mu)(-1)^{j+m'}a_{m}^{+}a_{-m'}$, and a_{m}^{+} is the operator creating a particle with the magnetic quantum number m. This model is known to have many interesting features. It incorporates some of the basic properties of collective states by taking into account effects of the quadrupole deformation as well as those of the monopole pairing. Due to its simplicity, the exact solutions can be obtained for small values of j, since in this case the dimension of the complete Hilbert space is still not too large [1–4].

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However, collective effects become more pronounced for large particle numbers N. In the frame of the single-j shell model, this requires studying the case of large values of j for which the exact solutions cannot be obtained. The method presented in this paper allows for determining low-energy collective states within a reasonable numerical effort. In fact, in our method the computation time grows as the third power of $2j+1$, whereas the exact diagonalization would require times growing as $(2j+1)!$. This enables solutions for j as large as 31/2.

The method is based on the single-particle coherent excitation model (SCEM) [5], which assumes that the quadrupole collective excitations can be constructed using the single-particle quadrupole excitation operator $\hat{F}^+_\mu = \hat{Q}_\mu$. This allows for constructing the generating functions which are then used in the generator coordinate method (GCM) [6] with exact angular-momentum and particle-number projections. The SCEM basis states for quadrupole excitations can be represented as coherent states

$$|t_2\rangle = \exp\{t_2 \cdot \hat{F}^+\} \exp\{\hat{S}^+\}|0\rangle ,$$

where $\hat{S}^+ = \sum_{mm'}(jmjm'|00)a^+_m a^+_{m'}$ is the operator creating a pair of particles coupled to spin $I = 0$ and t_2 is a quadrupole tensor. The five magnetic components of t_2 constitute our set of the discrete collective generating coordinates. We use the states $|t_2\rangle$ to construct the spin- and particle-number projected generating states

$$|\Psi^I_M\rangle = \sum_n \sum_K f^I_K(t_2^{(n)}) \hat{P}^I_{MK} \hat{P}_N |t_2^{(n)}\rangle ,$$

where $f^I_K(t_2^{(n)})$ denotes the GCM weight function while $t_2^{(n)}$ stands for an nth value of the t_2 tensor in the intrinsic frame of reference. The projections are done exactly which requires the three-dimensional numerical integration over the Euler angles and the one-dimensional integration over the gauge angle. The discretized Hill–Wheeler equation [6]

$$\sum_{n'K'} \langle t_2^{(n)}| \left(\hat{H} - E^I\right) \hat{P}^I_{KK'} \hat{P}_N |t_2^{(n')}\rangle f^I_{K'}(t_2^{(n')}) = 0 ,$$

is solved for each value of the spin I. The calculations are performed on a mesh of points (see black dots in Fig. 1) distributed over the 0° to 60° $\beta - \gamma$ sector of the collective coordinates $\beta_0 = t_{20}$ and $\beta_2 = \sqrt{2}t_{22}$ in the intrinsic frame. Note that here β is not identical to the usual Bohr coordinate corresponding to the spatial shape elongation.

For $j = 15/2$ the exact solutions can be obtained and reduced E2 transition probabilities can be calculated. In this case quadrupole collective

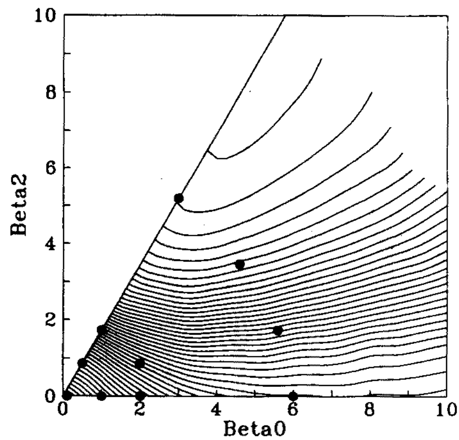

Fig. 1. The potential energy surface for the system of $N = 10$ particles filling the $j = 31/2$ shell and interacting with the quadrupole-quadrupole force. The energy difference between two contour lines is 0.07 in units of the force strength χ. The oblate minimum corresponds to energy value -5.54 in the same units. Full circles denote values of the collective coordinates for which the intrinsic states have been constructed and used in the GCM.

bands can be identified using the rule, that a state with spin I is connected with a state with spin $I - 2$ to which it has the largest reduced E2 transition probability. States belonging to those bands can be judged as strongly collective. Our calculations show, that the GCM applied to the SCEM correctly reproduces all of those states. We must admit that for smaller values of j the GCM states exhaust the complete Hilbert space and the SCEM is able to reproduce all exact states. This is possible because the SCEM is a fully fermionic model.

Here we present the GCM results for the $j = 31/2$ shell with $N = 10$ particles. An exact solution of this problem would require the diagonalization of a 1028172×1028172 matrix, which is a rather difficult task.

In Fig. 1 we show the potential energy surface in case of the pure quadrupole force. The oblate valley is clearly seen and therefore the system is expected to behave as an oblate rotor.

Fig. 2 presents the low-energy part of the GCM spectrum for spins from $I = 0$ up to $I = 18$. The five lowest collective bands can be interpreted as those of the oblate rotor coupled to the γ-vibrations. The first excited band can be called the γ-vibrational band with $K = 2$. Also the $K = 4$ and $K = 6$ γ-bands can be identified and ascribed to the two- and three-phonon γ-vibrations. The excited $K = 0$ band has probably the γ-vibrational two-phonon character too, in which case the β-vibrational band is missing from the spectrum. These assignments have been based on the

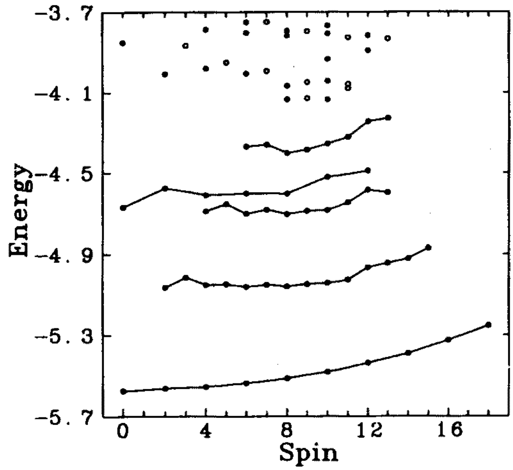

Fig. 2. The low-energy part of the GCM spectrum for $j = 31/2$ and $N = 10$ as a function of the angular momentum I. The strength χ of the quadrupole-quadrupole force serves as an energy unit. The lines connect states which are tentatively assigned to the five lowest collective bands.

energy values only and should be confirmed by calculations of the reduced transition probabilities, which are now being performed. The properties of the GCM approach derived from $j = 15/2$ analysys suggest that also in the case of $j = 31/2$ the obtained spectrum contains mostly states with strong quadrupole collectivity.

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