A SIMPLE MICROSCOPIC THEORY OF THE OPTICAL POTENTIAL FOR HEAVY-ION SCATTERING*,**

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The optical potential $\mathcal{V}$ for heavy-ion scattering is derived from the properties of nuclear matter and the NN cross section. The proximity approximation for the complex energy-dependent $\mathcal{V}$ is discussed.

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In my talk, I shall present the nuclear matter (NM) approach to the heavy-ion (HI) optical potential $\mathcal{V} = \mathcal{V}_R + i\mathcal{V}_I$, worked out in collaboration with Sigurd Köhler of the University of Arizona [1–4], and its relation to the proximity approximation [5, 6].

Our approach is based on the local density approximation. The two nuclei 1 (target) and 2 (projectile) [with masses $M_i = A_i m$ (m = nucleon mass)] colliding with the relative momentum $\boldsymbol{K}_{\mathrm{REL}}$ are described locally as two interpenetrating slabs of NM moving against each other. For the nucleon density and momentum distributions at $\mathbf{r}$, $\rho(\boldsymbol{r})$ and $n(\boldsymbol{k})$, we apply the frozen density approximation in which $\rho(\boldsymbol{r}) = \rho_1(\boldsymbol{r}) + \rho_2(|\boldsymbol{r} - \boldsymbol{R}|)$, where ρ_1 and ρ_2 are the empirical density distributions of nuclei 1 and 2, and $\boldsymbol{R}$ is the relative position vector between the centers of mass of 1 and 2. For n, we assume the distribution consisting of two Fermi spheres with Fermi momenta $k_{F1} = [3\pi^2\rho_1(\boldsymbol{r})/2]^{1/3}$ and $k_{F2} = [3\pi^2\rho_2(|\boldsymbol{r} - \boldsymbol{R}|)/2]^{1/3}$, whose centers are separated by the average relative nucleon momentum $\boldsymbol{K}_r = (m/\mu)\boldsymbol{K}_{\mathrm{REL}}$), where $\mu = M_1M_2/(M_1 + M_2)$. If $K_r < k_{F1} + k_{F2}$ the two Fermi spheres

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overlap. We resolve the double occupancy problem in the overlap region by increasing k_{F1} and k_{F2}, which leads to an increase $\Delta\tau$ in the kinetic energy density $\tau = \Omega^{-1}\sum_k n(k)\varepsilon(k)$, where $\varepsilon = \hbar^2k^2/2m$ and Ω is the volume of the system.

We define $\mathcal{V}$ as the difference between the energy $\mathcal{E}$ of the overlapping and that of spatially separated nuclei:

$$\mathcal{V} = \mathcal{E}(K_{\mathrm{REL}}, R) - \frac{\hbar^2 K_{\mathrm{REL}}^2}{2\mu} - \mathcal{E}_1 - \mathcal{E}_2\,, \tag{1}$$

where $\mathcal{E}_i$ are intrinsic nuclear energies of nuclei 1 and 2, and

$$E = \hbar^2 K_{\mathrm{REL}}^2/2\mu\,.$$

In our local density approximation,

$$\mathcal{E}(K_{\mathrm{REL}}, R) = \int dr(H^{\mathrm{NM}}[n(k,\rho(r)] + H_\nabla(\rho(r)))\,, \tag{2}$$

where H^{NM} is the energy density of the local NM, and the density gradient correction $H_\nabla = (\hbar^2/72m)(\nabla\rho)^2/\rho + \eta(\nabla\rho)^2$. For $\mathcal{E}_i$, we also apply expression (2) with $\rho = \rho_i(r)$ and $n = n_0(k,\rho_i(r)) = \theta(k_{Fi} - k)$, and adjust η to the experimental values of $\mathcal{E}_i$.

One way of calculating H^{NM}, followed by Faessler and his collaborators [7–9], is to start from the NN interaction and apply the Brueckner theory. We bypass the difficult task of calculating self-consistently the Brueckner reaction matrix in the case of the two-sphere momentum distribution, and determine H^{NM} directly from the properties of NM and from the NN cross section σ.

For the real part $\mathrm{Re}\,H^{\mathrm{NM}}$, we apply the approximate expression (which is exact in the case of Skyrme interaction [4]):

$$\begin{aligned}\mathrm{Re}\,H^{\mathrm{NM}} &\simeq H^{\mathrm{NM}}[n_0(k,\rho)] + \Omega^{-1}\sum_k (n(k,\rho) - n_0(k,\rho))e_0(k,\rho) \\ &= f\rho + (\tau - \tau_0)/\nu\,,\end{aligned} \tag{3}$$

where $H^{\mathrm{NM}}[n_0] = (E_{\mathrm{NM}}/A)\rho = f\rho$ is the energy density, τ_0 the kinetic energy density, and e_0 the s.p. energy in he ground state of NM of density ρ. In the last step, the effective mass approximation is assumed for e_0, with $m^*/m = \nu = (1 + (\nu_0^{-1} - 1)\rho/\rho_0)^{-1}$, where ρ_0 is the saturation density of NM. For f, we assume the form $f = (3/5)\varepsilon(k_F) + \sum_{l=3,5} a_l(k_F/k_{F0})^l$, where $k_F(k_{F0})$ is the Fermi momentum in the ground state of NM of density $\rho(\rho_0)$. The coefficients a_l are determined by the empirical values of ρ_0, $f(\rho_0) = \varepsilon_{\mathrm{vol}}$, and $K_c = k_{F0}(d^2f/dk_F^2)_0$.

For the imaginary part of H^{NM}, we use the approximation:

$$\mathrm{Im}\, H^{\mathrm{NM}} \simeq -\nu(\hbar^2/2m)\Omega^{-2} \sum_{k_1 k_2} n(k_1,\rho)n(k_2,\rho)Q|(k_1-k_2)/2)|\sigma\,, \quad (4)$$

where the exclusion principle operator $Q = 1$ when the scattering of the two nucleons with momenta k_1, k_2 ends with final momenta outside our two Fermi spheres, and $Q = 0$ otherwise.

With the help of expressions (3) and (4) for H^{NM}, we may calculate $\mathcal{E}$, Eq. (2), and thus determine $\mathcal{V}$, Eq. (1), in terms of NM parameters and σ. The results to be presented were obtained with the following input: $k_{F0} = 1.35$ fm^{-1} ($\rho_0 = 0.166$ fm^{-3}, $\varepsilon_{\mathrm{vol}} = -15.8$ MeV, $K_c = 235$ MeV, $\nu_0 = 0.7 - 0.83$, and the experimental values of σ.

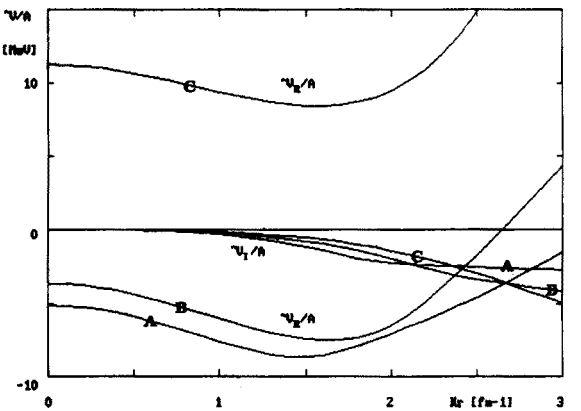

Fig. 1. $\mathcal{V}/A$ for two slabs of NM of the same density as function of K_r for total density $\rho = \rho_0/2$ (A), $\rho = \rho_0$ (B), and $\rho = 2\rho_0$ (C).

Results for two infinite slabs of NM of equal (constant) density $\rho_1 = \rho_2 = \rho/2$ (here $H_\nabla = 0$) are shown in Fig. 1. The potential energy contribution to $\mathcal{V}_{\mathrm{R}}/A$ ($A = A_1 + A_2$) increases (algebraically) with K_r (which may be traced back to the short range NN repulsion), and the repulsive kinetic energy contribution $\Delta\tau/\rho$ decreases with K_r (and vanishes for $K_r > k_{F1} + k_{F2}$). The net effect is the initial decrease in $\mathcal{V}_{\mathrm{R}}/A$ and the appearance of a "window" of a minimum in $\mathcal{V}_{\mathrm{R}}/A$ at $K_r \simeq 1.5 - 2$ fm^{-1}. If $\rho_1 = \rho_2 < \rho_0/2$ (cases A and B), one gains energy by merging the two slabs, and $\mathcal{V}_{\mathrm{R}}/A < 0$ at $K_r = 0$. The depth of $\mathcal{V}_{\mathrm{I}}/A$ increases with K_r, because more and more phase space allowed by Q is available for the final states of the NN scattering.

Results obtained for $\mathcal{V}(R)$ for ^{40}Ca–^{40}Ca are shown in Fig. 2. [The kinetic energy of the projectile nucleus per projectile nucleon $E_{\mathrm{LAB}}/A_2 =$

$(\hbar^2/2m)K_r^2 = 20.7K_r^2$.] The behavior of these results reflects the behavior of the results for two slabs in Fig. 1, supplemented by the density gradient correction which is attractive at large and repulsive at small distances R. Both $\mathcal{V}_R$ and $\mathcal{V}_I$ show a strong energy dependence. The absorptive potential $\mathcal{V}_I$ is concentrated at small distances R, and $\mathcal{V}_R$ reveals an increasing repulsion at small distances with increasing energy. [The occurrence of the short range repulsion has been demonstrated recently [10] in the analysis of the carbon-carbon data.] In general, neither $\mathcal{V}_R$ nor $\mathcal{V}_I$ has the Woods–Saxon (WS) shape often used in phenomenological analyses.

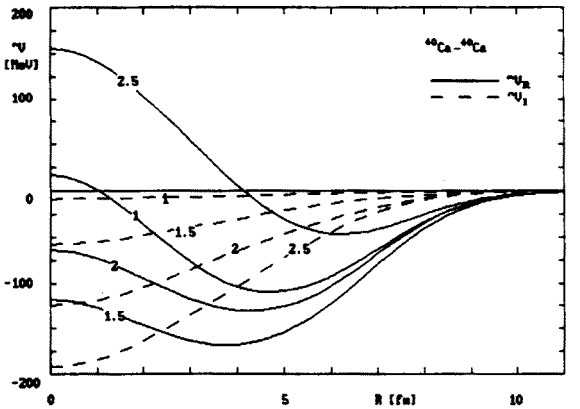

Fig. 2. $\mathcal{V}(R)$ for ^{40}Ca–^{40}Ca at the indicated values of K_r (in fm^{-1}).

The results of our simple theory agree nicely with the results of the laborious calculations of Faessler and his group [8], and the cross sections for HI scattering calculated in [8] reproduce reasonably well the experimental data.

Let me say a few words about the proximity approximation (PA) [11] in which the interaction $\mathcal{V}$ between two gently curved leptodermous objects in close proximity is approximated by $\mathcal{V}^{\mathrm{PA}}$, a superposition of interaction potentials $\mathcal{U}(Z)$ per unit area between two flat surfaces at distance Z made of the same material. In the case of two spherical nuclei with WS densities $\rho_i(r) = \rho_0/[1 + \exp((r - R_i)/a))]$, we have:

$$\mathcal{V}^{PA} = 2\pi \int_0^\infty dr_\perp r_\perp \mathcal{U}(E, Z) = 4\pi a\gamma R_{12} \int_{s_0}^\infty ds\phi(E, s)\,, \tag{5}$$

where $R_{12} = R_1R_2/(R_1 + R_2)$, $s = Z/a$, $s_0 = (R - R_1 - R_2)/a$, γ is the surface energy per unit surface, and the dimensionless proximity function $\phi(E, s) = \mathcal{U}(E, Z)/2\gamma$, where $\mathcal{U}$ is the interaction potential of two

semi-infinite slabs of NM with WS profiles, with Z equal to the distance between the half-fall planes of the two slabs. Approximation (5) may be easily derived from our expression for $\mathcal{V}(E,R)$ for such distances R for which $r_\perp \ll R_i$ in the overlap region of the two nuclei ($r_\perp$ is the distance from the line connecting the centers of the two nuclei).

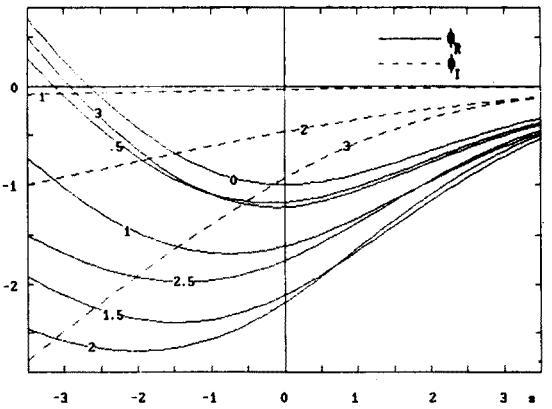

Fig. 3. $\phi(s)$ for the indicated values of K_τ (in fm^{-1}).

Whereas the original PA applied to $\mathcal{V}_R$ at zero energy, our expression (5) shows the energy dependence of $\mathcal{V}^{\rm PA}$. Furthermore, for $E > 0$ our $\mathcal{V}^{\rm PA}$ becomes complex, similarly as our proximity function $\phi = \phi_{\rm R} + i\phi_{\rm I}$. Our results for ϕ, obtained with $a = 0.57$ fm (with the corresponding calculated value of $\gamma = 0.98$ fm^{-2}), are shown in Fig. 3. The behavior of $\phi \sim \mathcal{U}$ is similar to that of $\mathcal{V}$ (see Fig. 2) and reflects the behavior of $\mathcal{V}/A$ for two infinite slabs (Fig. 1), supplemented by the density gradient correction to $\mathcal{U}$, which reaches its minimum value of -2γ at $Z = 0$.

A comparison of our results for $\mathcal{V}^{\rm PA}$ with those for $\mathcal{V}$ permits an estimate of the accuracy of PA, which turns out to be satisfactory for not too light nuclei ($A \gtrsim 16$) for $R - R_1 - R_2 \gtrsim -3$ fm. One consequence of PA is the scaling law: $\mathcal{V}^{\rm PA}/R_{12}$ is a universal function of $R - R_1 - R_2$ and energy. The experimental evidence for the scaling law for $\mathcal{V}_{\rm R}$ was presented in [11]. As discussed in [6], there is some experimental evidence that also $\mathcal{V}_{\rm I}$ obeys this law.

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