THE PSEUDO–SU(3) SYMMETRY SCHEME FOR DEFORMED SINGLE–PARTICLE LEVELS*

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The transformation formula to pseudo-SU(3) space for arbitrary one-body operators is derived using Algebraic Generator Coordinate Method (AGCM). The single-particle energies of the Nilsson and pseudo-Nilsson Hamiltonians are compared.

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The concept of pseudo-spin $\tilde{s}$, pseudo-orbital angular momentum $\tilde{l}$ and, as follows, pseudo-SU(3) group was introduced [1–4] in order to describe the observed approximate degeneracy of the nuclear single-particle levels. Late successes of this approach, especially in description of some superdeformed bands in neighboring nuclei [5], reopened the detailed discussion on the physical meaning of pseudo-SU(3) symmetry [6] as well as on its mathematical foundations [7, 8].

An interesting problem in the pseudo-SU(3) formalism was to find out a simple form of the transformation between the single-particle space $\mathcal{K}$ and its "pseudo" counterpart $\tilde{\mathcal{K}}$. If the considered single-particle space $\mathcal{K}$ is spanned by the eigenstates of the spherical harmonic oscillator H_0:

$$H_0|N(ls)jm\rangle = E_0|N(ls)jm\rangle \tag{1}$$

with usual notation for its quantum numbers, then the transformation to pseudo-oscillator space $\tilde{\mathcal{K}}$ can be formally written as $\phi : \mathcal{K} \longrightarrow \tilde{\mathcal{K}}$:

$$\phi|N(ls)jm\rangle = |\tilde{N}(\tilde{l}\tilde{s})\tilde{j}\tilde{m}\rangle\,. \tag{2}$$

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Here $|\tilde{N}(\tilde{l}\tilde{s})\tilde{j}\tilde{m}\rangle$ is an eigenstate of the pseudo-oscillator $\tilde{H}_0$ with quantum numbers: $\tilde{N} = N - 1$, $\tilde{s} = s$, $\tilde{j} = j$, $\tilde{m} = m$ and

$$\tilde{l} = \begin{cases} l+1, & \text{for } j = l + 1/2\,; \\ l-1, & \text{for } j = l - 1/2\,. \end{cases} \tag{3}$$

The transformation $\phi = UP$ should be considered as a product of a unitary transformation U relabelling states (1) and a projection P of $\mathcal{K}$ on $\tilde{\mathcal{K}}$ which excludes the intruders *i.e.*, the states $|N(Ns)j = N + \frac{1}{2}\ m\rangle$. This form of ϕ implies that the standard transformation scheme for any arbitrary operator O *i.e.*, $\tilde{O} = \phi O \phi^{-1}$, cannot be here directly applied because the transformation ϕ is irreversible.

In the papers [7, 8] the analytic form of the unitary transformation U was derived. Unfortunately, the formulas presented in [7] are complicated and, in practice, they were used only to operators diagonal in the spherical harmonic oscillator basis (1).

The majority of difficulties connected with explicit construction of the transformation (2), especially of the projection operator P, can be omitted with the aid the Algebraic Generator Coordinate Method (AGCM) [9, 10] which has many advantages while projecting operators or states onto a subspace with definite symmetry. It allows to construct the state space of the pseudo harmonic oscillator $\tilde{H}_0$, in a single step without consecutive applications of the operators P and U, as the orthogonal sum of carrier spaces of irreducible representations of the pseudo-SU(3) group.

For this purpose let us consider the operators $T(g)$ representing $g \in \mathrm{SU}(3)$ in $\mathcal{K}$ and define a new family of operators $\tilde{T}(g)$ by the formula :

$$\langle N'(l's')j'm'|\tilde{T}(g)|N(ls)jm\rangle = \langle \tilde{N}'(\tilde{l}'\tilde{s}')\tilde{j}'\tilde{m}'|T(g)|\tilde{N}(\tilde{l}\tilde{s})\tilde{j}\tilde{m}\rangle\,, \tag{4a}$$

$$\tilde{T}(g)|N(ls)j = N + 1/2\ m\rangle = 0\,. \tag{4b}$$

It can be shown that the family $\tilde{T}(g)$ is a unitary representation of the group SU(3) in the subspace $\tilde{\mathcal{K}} \subset \mathcal{K}$. This representation can be identified with the pseudo-SU(3) group and allows us to define the metastate kernel function $\langle \rho; \cdot \rangle$:

$$\begin{aligned} \langle \rho; g\rangle &= \sum_{Nljm} \langle N(ls)jm|\tilde{T}(g)|N(ls)jm\rangle n_{Nlj} \\ &= \sum_{Nljm} D^{\tilde{N}}_{\tilde{l}\tilde{j}\tilde{m},\tilde{l}\tilde{j}\tilde{m}}(g) n_{Nlj}\,, \end{aligned} \tag{5}$$

where $D^{N}_{ljm,l'j'm'}(g)$ are matrix elements of $dim[N]$-dimensional irreducible representation of the SU(3) group, and $n_{Nlj} \geq 0$ stand for arbitrary weights

with the normalization condition $\sum_{Nlj} n_{Nlj} = 1$. With this metastate kernel function, which more detailed specification is not needed for further purpose, we can solve the overlap eigenequation:

$$(\mathcal{N}u^N_{ljm})(g) = \Lambda(Nlj)u^N_{ljm}(g)\,, \tag{6}$$

where

$$(\mathcal{N}u)(g) = \int_{SU(3)} dg' \langle \rho; g^{-1}g' \rangle u(g') \tag{7a}$$

and

$$u^N_{ljm}(g) = \sqrt{dim[N]} D^N_{ljm;ljm}(g)\,. \tag{7b}$$

The eigenfunctions of the overlap operator which belong to nonzero eigenvalues furnish the complete set of normalized functions (see [9, 10] and references therein)

$$\psi^N_{ljm}(g) = \frac{1}{\sqrt{\Lambda(Nlj)}} u^N_{ljm}(g) \tag{8}$$

orthonormal with respect to the scalar product:

$$(\psi|\psi')_{\tilde{\mathcal{K}}} = \int\limits_{SU(3)} dg \int\limits_{SU(3)} dg' \psi^*(g) \langle \rho; g^{-1}g' \rangle \psi'(g')\,. \tag{9}$$

The space spanned by the basis (8) is unitarily isomorphic to the space $\tilde{\mathcal{K}}$ and we shall identify them.

Now, using the formula (9) one can calculate matrix elements of any single-particle operator $\tilde{O}$ acting in $\tilde{\mathcal{K}}$, as follows:

$$\begin{aligned}(\psi^{N'}_{l'j'm'}|\tilde{O}\psi^N_{ljm})_{\tilde{\mathcal{K}}} &= \int\limits_{SU(3)} dg \int\limits_{SU(3)} dg' \psi^{N'}_{l'j'm'}(g)^* \langle \rho; g^{-1}Og' \rangle \psi^N_{ljm}(g') \\ &= \delta_{N',N} \langle N+1(\tilde{l}'s')j'm'|O|N+1(\tilde{l}s)jm \rangle\,. \end{aligned} \tag{10}$$

The equation (10) says that the representation $\tilde{O}$ of any operator O in the 'pseudo' space $\tilde{\mathcal{K}}$ is built from some matrix elements of the original operator O in the usual harmonic oscillator basis (1). On the left-hand side of the formula (10) we have states and quantum numbers of the pseudo harmonic oscillator. On the right-hand side of (10) we have the usual harmonic oscillator eigenvectors with proper quantum numbers. The quantum numbers $\tilde{l}'$ and $\tilde{l}$ are given by the equation (3).

An interesting feature of the transformation (10) is that the image $\tilde{O}$ of the operator O does not couple the pseudo-oscillator shells even if its original O does.

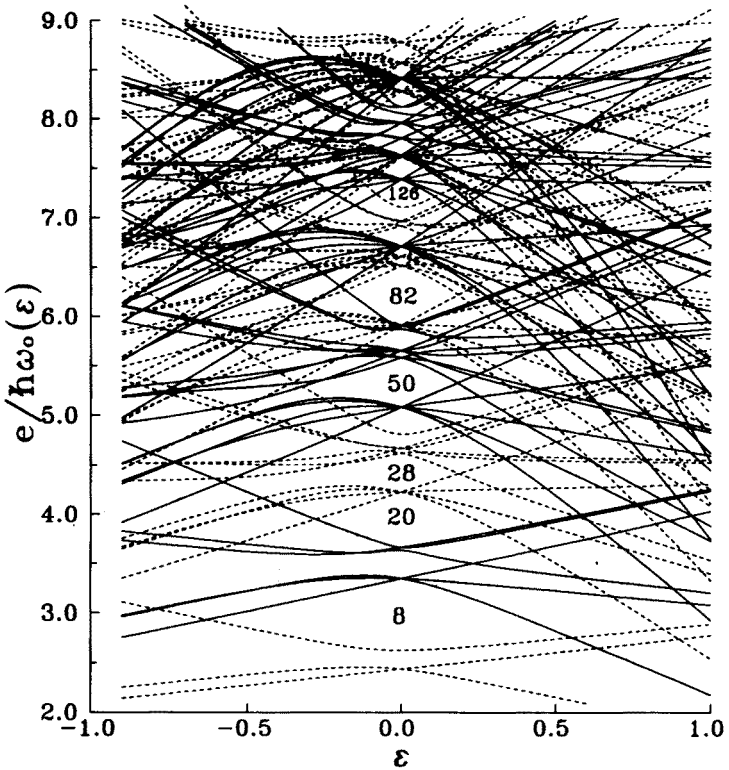

Fig. 1. The single particle energies of the axially symmetric Nilsson Hamiltonian for neutrons, plotted as a function of the quadrupole deformation parameter ε. The solid lines indicate the positive parity levels and dashed ones the negative parity levels.

The transformation rule (10) allows us not only to repeat all results presented in [7] but to find out the matrix representation of any pseudo-observable whenever it is diagonal in the spherical basis (1) or not. Thus we have got the practical and exact method to apply the pseudo-SU(3) symmetry to an arbitrary single-particle problem, even for deformed Nilsson or Saxon-Woods potentials.

In order to investigate the influence of the pseudo-SU(3) symmetry on the deformed Nilsson orbits we have constructed the representation of deformed Nilsson Hamiltonian in the space $\tilde{\mathcal{K}}$ with the aid of its known matrix elements in the spherical basis (1), as it was described in (10), and then we have diagonalized the obtained matrix.

In Fig. 1 the standard single-particle spectrum of the Nilsson Hamiltonian [11] for neutrons is shown to compare it with the corresponding spectrum of the pseudo-Nilsson Hamiltonian in Fig. 2.

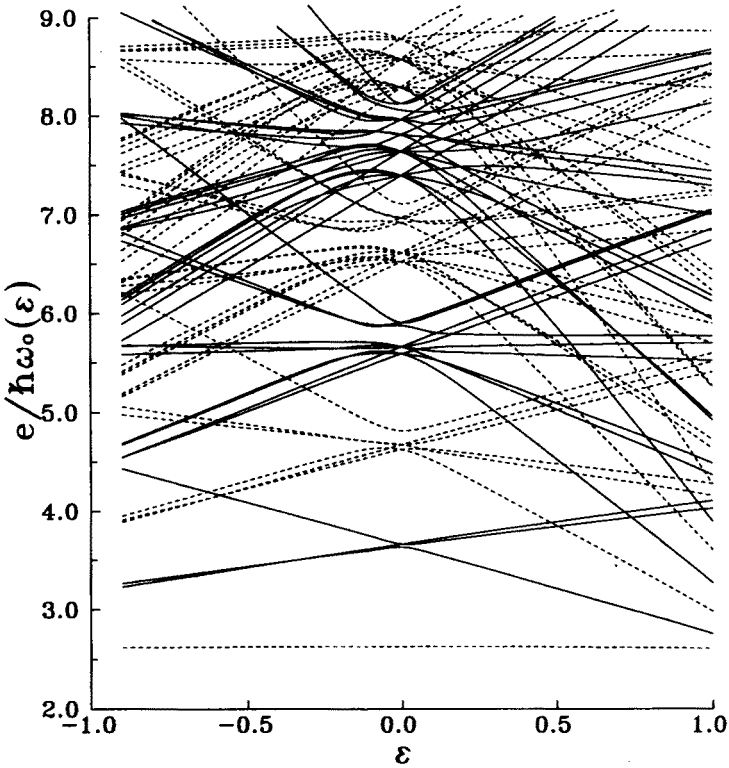

Fig. 2. The spectrum of the pseudo–Nilsson Hamiltonian obtained after transformation (10) from the Nilsson Hamiltonian spectrum displayed in Fig. 1.

At first sight one can notice the difference between Figs. 1 and 2. In the spherical case (for deformation parameter ε equal to zero) the Nilsson single-particle levels form the closed shells shown in Fig. 1, and for higher shells ($N \geq 4$), the positive parity levels (solid lines) and the negative parity levels (dashed lines) are mixed. The spectrum of the pseudo–Nilsson Hamiltonian, see Fig. 2, is much simpler because all orbitals with the maximal total angular momentum $j = N + \frac{1}{2}$ in each shell (intruder levels) are removed and, in consequence, each pseudo-shell contains levels of the same parity.

The Figs. 1 and 2 can be compared only qualitatively because of their rather complicated structure. More details can be seen in Fig. 3 and Fig. 4.

The fragment of Fig. 1 with only one $N = 5$ neutron shell is displayed in Fig. 3. The Nilsson states are labelled by the asymptotic quantum numbers $[Nn_z\Lambda]\Omega = m = \Lambda \pm \frac{1}{2}$ and, like before, by the parity ($\pi = +$, solid lines; $\pi = -$, dashed lines). Between normal parity orbitals (dashed) one can see a unique parity ($i_{\frac{13}{2}}$ in the spectroscopic notation) intruder orbital from the shell above ($N = 6$). In the lower part of Fig. 3, the orbital $h_{\frac{11}{2}}$ with

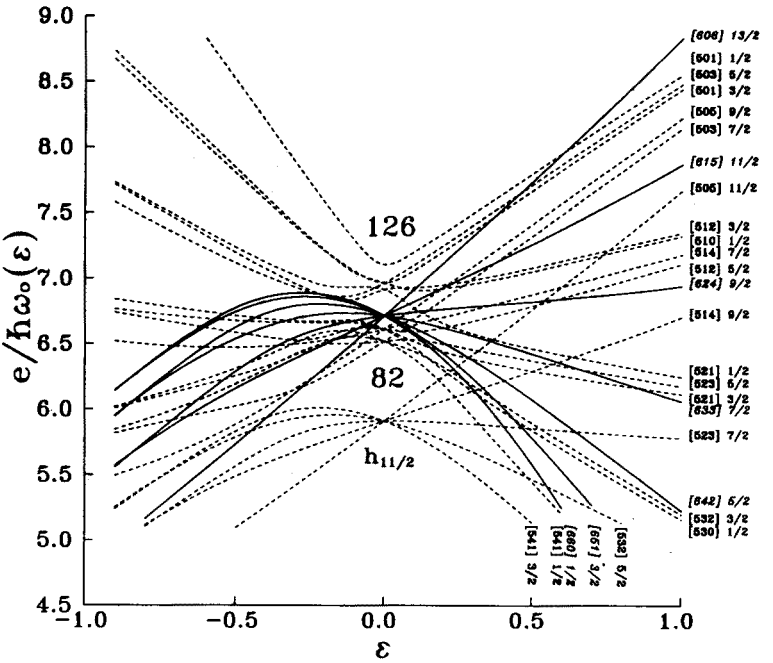

Fig. 3. The Nilsson neutron single-particle levels for $82 < N < 126$ ($N = 5$ shell) from Fig. 1. The states are characterized by the asymptotic quantum numbers $[Nn_z\Lambda]\Omega$, where N is the principal quantum number of the oscillator shell, n_z the number of oscillator quanta along the z-axes, Λ and Ω the components of the orbital and total angular momenta along the symmetry axes , respectively.

$j_{max} = N + \frac{1}{2}$ is presented. This orbital, pushed down by the spin-orbit interaction, belongs (as intruder) to the next lower $N = 4$ shell.

In Fig. 4 one can compare normal parity Nilsson levels from Fig. 3 (dashed lines) with their pseudo-Nilsson partners obtained after transformation (10) (solid lines) and presented before in Fig. 2. The pseudo–Nilsson orbitals are labelled by the pseudo-asymptotic quantum numbers $[\tilde{N} = N - 1, \tilde{n}_z = n_z, \tilde{\Lambda} = \Lambda \pm 1]\tilde{\Omega} = \Omega$. It is easy to find that Nilsson levels labelled by $(\Lambda, \Omega = \Lambda + \frac{1}{2})$ and $(\Lambda + 2, \Omega = \Lambda + \frac{3}{2})$ form pairs of close lying levels. These pairs can be viewed as pseudo-spin doublets with pseudo-asymptotic quantum numbers $\tilde{\Lambda} = \Lambda + 1$ and $\Omega = \tilde{\Lambda} \pm \frac{1}{2}$. In addition, pseudo-shell $\tilde{N} = 4$ contains three pseudo-spin singlets with $\Omega = \frac{1}{2}$ and $\tilde{\Lambda} = 0$, and these singlets correspond to $\Lambda = 1$ and $\Omega = \frac{1}{2}$ Nilsson orbitals.

In Fig. 4 it can also be seen that in spherical limit ($\varepsilon = 0$), when Nilsson Hamiltonian [11] is diagonal in the spherical harmonic oscillator basis (1), the normal parity levels and their pseudo-partners are exactly the same.

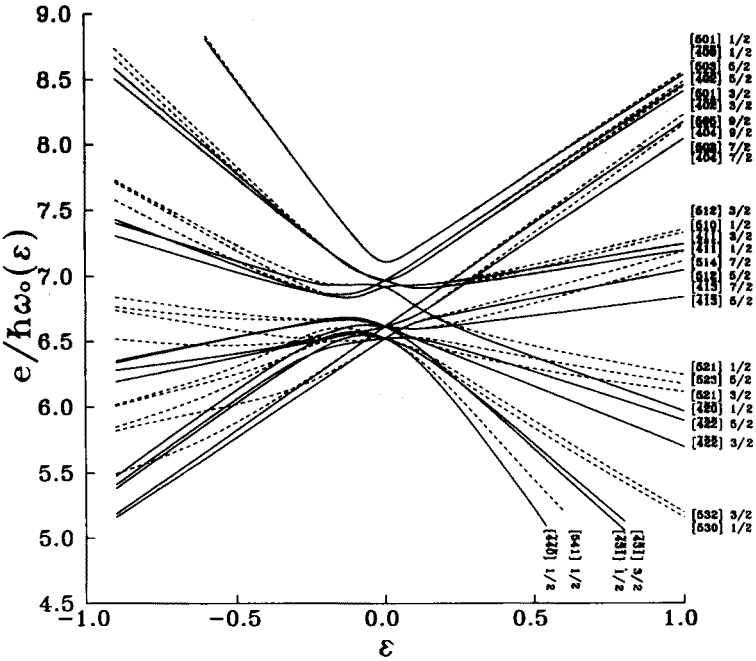

Fig. 4. Comparison of the normal parity single-particle levels from Fig. 3 (dashed lines) and their pseudo-partners from $\tilde{N} = 4$ pseudo-shell of the pseudo-Nilsson Hamiltonian (solid lines). The pseudo-Nilsson levels are labelled by the pseudo-asymptotic quantum numbers $[\tilde{N} = N - 1 \;\; \tilde{n}_z = n_z \;\; \tilde{\Lambda} = \Lambda \pm 1]\tilde{\Omega} = \Omega$.

At non-zero deformations pseudo-Nilsson levels differ from normal Nilsson orbitals and discrepancies are proportional to the absolute value of the deformation parameter. These differences, among other reasons, are caused by the projection of the original single-particle space $\mathcal{K}$ on the pseudo-subspace $\tilde{\mathcal{K}}$ without high-j intruder orbitals. In the case of finite deformations, for prolate as well as oblate shapes, the pseudo-Nilsson levels are pushed down with respect to the normal-Nilsson levels and deviation grows with j for orbitals in one shell.

We have shown here the method which allows to use the pseudo-SU(3) scheme for arbitrary single-particle problem. Our calculations for the Nilsson potential allow to expect similar results for Woods-Saxon model.

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