SELFORGANIZATION AND DISORDER IN AN OPEN QUANTUM SYSTEM*,**

M. MÜLLER, I. ROTTER,

Forschungszentrum Rossendorf, Institut für Kern-und Hadronenphysik
D-01314 Dresden, Germany

AND

W. ISKRA

Sołtan Institute for Nuclear Studies
Hoża 69, 00-681 Warszawa, Poland

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The properties of the atomic nucleus are investigated from the point of view of selforganization. The transition from low to high level density is traced as a function of the coupling strength between the discrete nuclear states and the environment of decay channels. A redistribution inside the nucleus takes place in a small region around some critical value of the coupling strength. Beyond the critical value, a few relevant short-lived modes exist together with long-lived states (*slaving principle of synergetics*). In the critical region of the coupling strength, information entropy, calculated from the resonant states in relation to the discrete states of the closed system, is created (*maximum information entropy principle*). The long lived states are characterized by disorder, which is expressed by a large information entropy. The relevant modes have a high order and take, correspondingly, a small part of the information entropy of the whole system.

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Open systems are expected to be selforganized. Their properties are determined by the interplay between internal stability and interaction with

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the environment. The evolution of such a system can be traced best by investigating its behaviour near instability points in dependence of some control parameter [1]. Here, a selforganizing system, which is described by classical methods, is shown to form stable modes together with a few unstable modes. The unstable modes determine the behaviour of the system while the stable ones are suppressed ("slaving principle"). By that, the complexity of the system is reduced. Nevertheless, the information entropy of the whole system increases up to a certain maximal value as a function of the increasing control parameter, in full analogy to the second law of thermodynamics for closed systems ("maximum information entropy principle") [1]. Our aim is to trace the redistribution in the nucleus, taking place at high level density, and to compare the results in detail with those obtained in synergetics [2]. This gives us a qualitative analogy between classically decribed and quantum mechanical systems.

The model used in our investigations is the *continuum shell model* which describes the nucleus as an open quantum mechanical system [3]. In this model, the Schrödinger equation $(H - E)\Psi = 0$ is solved with an ansatz for Ψ which contains both bound and unbound states. We are interested in the properties of the bound states, embedded in the continuum of decay channels, which follow from

$$(H^{\text{eff}}_{QQ} - \tilde{\mathcal{E}}_R)\tilde{\Phi}_R = 0\,. \tag{1}$$

H^{eff}_{QQ} denotes the complex, energy dependent, effective Hamiltonian which has the following form:

$$H^{\text{eff}}_{QQ} = QHQ + QHP \cdot G^{(+)}_P \cdot PHQ\,. \tag{2}$$

Here, the operators Q and P denote the projectors on the subspaces of bound and scattering wave functions and $G^{(+)}_P$ stands for the Green function for a one particle motion in the continuum. The first term in Eq. (2) represents the shell model Hamiltonian and the second term contains the coupling of the bound states to the open decay channels. Therefore, H^{eff}_{QQ} acts on the subspace of bound states, but contains information about the subspace of scattering wavefunctions. Due to the coupling term, it is non-hermitean and energy dependent, with complex eigenvalues:

$$\tilde{\mathcal{E}}_R(E) = \tilde{E}_R(E) - \tfrac{i}{2}\tilde{\Gamma}_R(E)\,. \tag{3}$$

$\tilde{E}_R$ and $\tilde{\Gamma}_R$ are the energies and widths of the bound states embedded in the continuum ("resonance states"). In order to study the behaviour of the system under the influence of the environment, we are varying the coupling strength via a control parameter α^{ex}. The two-body interaction reads:

$$V^{\text{ex}}_{12} = -\alpha^{\text{ex}} V_0(a + bP^{\sigma}_{12})\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\,. \tag{4}$$

Here P_{12}^{σ} denotes the spin exchange operator, $V_0 = 500$ MeV fm^3, $a = 0.73$ and $b = 0.27$. Details of the model can be found in [3].

One of the results obtained [3] is the *trapping effect*: At a critical value of the coupling strength between system and environment of decay channels $\alpha^{ex} = \alpha_{cr}^{ex}$, a redistribution inside the nucleus takes place. Most states of the system become long-lived (trapped), while a few of them are distinguished by large widths. The number of these states, the widths of which are large as a result of the redistribution (trapping effect), is exactly equal to the number of open decay channels [5].

In order to illustrate the trapping effect, $\sum_{R=1}^{n} \tilde{\Gamma}_R$ is drawn as a function of the coupling strength α^{ex} in Fig. 1 for $1 \leq n \leq N$. The calculations are performed for two open decay channels, and $N = 70$ is the number of $J^{\pi} = 1^-$ resonances considered. The widths $\tilde{\Gamma}_R$ are ordered according to $\tilde{\Gamma}_1 \leq \tilde{\Gamma}_2 \leq ... \leq \tilde{\Gamma}_N$.

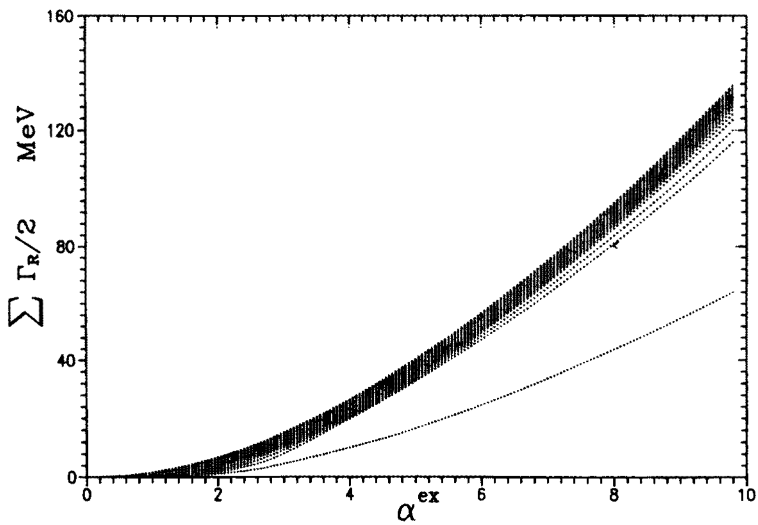

Fig. 1. $\sum_{R=1}^{n} \tilde{\Gamma}_R$ *versus* α^{ex} for $n = 1, \ldots, N$ with $\tilde{\Gamma}_1 \geq \tilde{\Gamma}_2 \geq \ldots \geq \tilde{\Gamma}_N$

One sees clearly from Fig. 1, that there exists a critical value $\alpha_{cr}^{ex} \approx 2.5$ beyond which the main part of the sum is given by the widths of two resonance states. Their widths are well separated from the widths of the remaining $N - 2$ trapped ones.

That means that the effective number of degrees of freedom of the system is strongly reduced beyond α_{cr}^{ex}. At low α^{ex}, 70 resonances contribute to the decay process more or less equally. The conditions change dramatically as soon as $\alpha^{ex} > \alpha_{cr}^{ex}$. Here, the decay process is determined mainly by two resonances. The difference in the widths is up to 5 orders of magnitude.

Mathematically, the redistribution taking place at $\alpha_{\rm cr}^{\rm ex}$ is an interference effect to which all N resonance states contribute.

Such a result is in full accordance with the *slaving principle of synergetics*. Beyond $\alpha_{\rm cr}^{\rm ex}$, most resonances are enslaved by a few resonance states which are relevant for the decay process. The stable trapped modes are suggested to correspond to the slaved modes. They determine the long-time scale.

The wavefunctions $\tilde{\Phi}_R$ in Eq. (1) are complex. They are eigenfunctions of the nonhermitean Hamiltonian $H_{QQ}^{\rm eff}$. The transformation which diagonalizes $H_{QQ}^{\rm eff}$ is not unitary but orthogonal. Therefore, the scalar products $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle$ are *not* normalised but $(|\tilde{\Phi}_R\rangle)^2 = 1$. As a consequence, the following equation holds [3, 6]:

$$\tilde{\Gamma}_R\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle = \sum_c |\tilde{\gamma}_{Rc}| \,. \tag{5}$$

Here $\tilde{\gamma}_{Rc}$ are the partial widths, the sum of which is generally larger than the total width $\tilde{\Gamma}_R$. Only in the case of isolated resonances, where the $\tilde{\Phi}_R$ are real, it follows from Eq. (5) that $\tilde{\Gamma}_R = \sum_c |\tilde{\gamma}_{Rc}|$.

In Fig. 2, the squared norms $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle$ are shown for the two broad states as well as for the three broadest trapped ones as a function of the control parameter $\alpha^{\rm ex}$. In a region around $\alpha_{\rm cr}^{\rm ex}$, where the redistribution takes place, $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle > 1$ for almost all states, while outside of the critical region for $\alpha^{\rm ex}$, $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle \approx 1$ for the broad states, but $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle > 1$ for the trapped ones. This means, that the broad modes behave like isolated resonances in contrast to the trapped ones.

Although the structure of the wavefunctions of the states and hence their degree of mixing depends on the basic set of wavefunctions and on the size of the configuration space, *e.g.*, in the shell model approach, it is possible to draw some general conclusions from these values. An information entropy I_β for the complex wavefunctions $\tilde{\Phi}_R$ characterizes their purity in relation to, *e.g.*, the shell model wavefunctions. It can be defined in an analogous manner as for real wavefunctions [4]. It is

$$I_\beta = \sum_R I_\beta^R \tag{6}$$

with

$$I_\beta^R = -\sum_{i=1}^{N} |\hat{\beta}_{Ri}|^2 \ln |\hat{\beta}_{Ri}|^2 \,, \tag{7}$$

where the $\hat{\beta}_{Ri}$ are the normalized expansion coefficients in relation to the shell model basis and the I_β^R are the mixing coefficients.

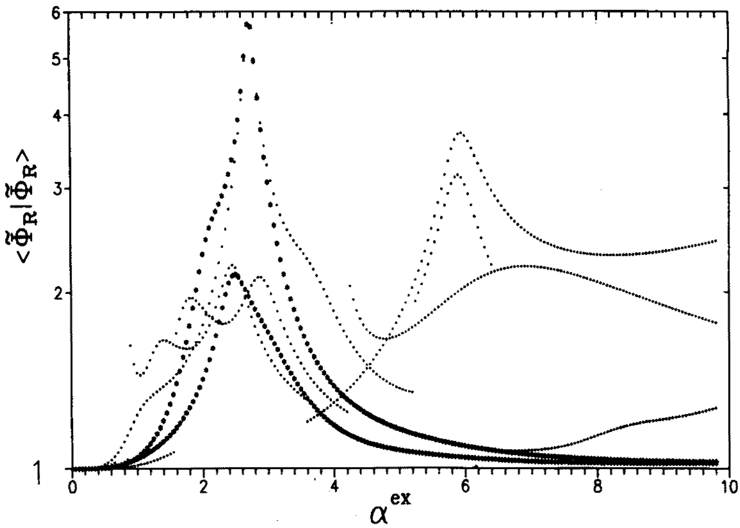

Fig. 2. The $\langle\tilde{\Phi}_R|\tilde{\Phi}_R\rangle$ as a function of α^{ex} for the two fast modes (thick points) and the three broadest trapped modes (thin points). The gaps arise because the ordering of the resonances according to the value of their widths sometimes changes with α^{ex}.

Most interesting is the question whether I_β increases as a function of the coupling strength α^{ex} between the basic shell model states and the decay channels. In Fig. 3, the mixing coefficients I_β^R are drawn for 70 resonances versus α^{ex}. In the instability region around α_{cr}^{ex}, the I_β^R of the two broad states (thick points) increase much stronger than those of most trapped ones (thin points). By this, the driving role of the open decay channels in the redistribution process is reflected. This behaviour of the mixing coefficients is well known in synergetics [1]. The results given in Fig. 3 show further that the redistribution takes place according to the *maximum information entropy principle*, *i.e.*, I_β increases up to a certain maximal value.

If one takes into account the fact that only two of the resonance states of the system are relevant (at time and energy scales characteristic of the system), then the sum in Eq. (7) runs over these two states only. As a consequence, the redistribution at α_{cr}^{ex} is accompanied by a reduction of the entropy of the relevant part of the system. This corresponds to the formation of a new order. A large part of the entropy (summation in Eq. (7) over 68 states) belongs to the trapped states. According to the large entropy, these states resemble disorder. Such an interpretation is in full agreement with the results obtained in [7].

Summarizing, it can be stated that the nuclear system behaves like a selforganising system. A redistribution inside the system takes place at a

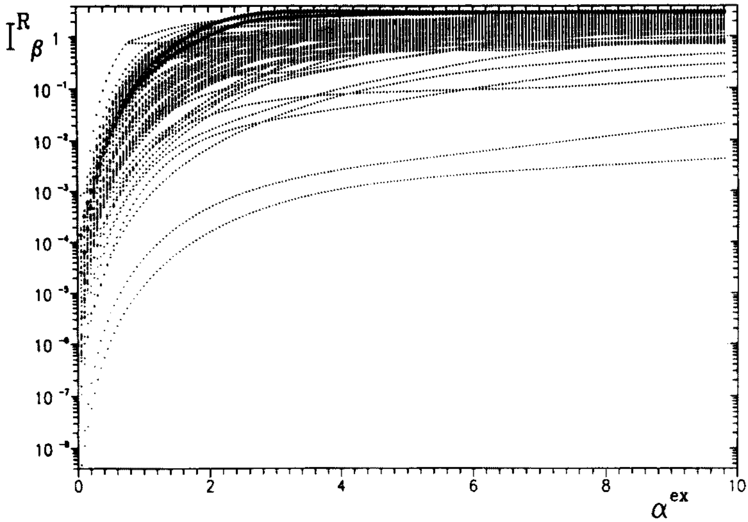

Fig. 3. The mixing coefficients I^R_β of all 70 states R in relation to the basic set of the shell model wavefunctions versus α^{ex}. The I^R_β of the two states with the largest widths are displayed by thick points.

certain critical value of the "control parameter" α^{ex} in accordance with the *slaving principle* as well with the *maximal information entropy principle of synergetics*. The two scenarios below and beyond α^{ex}_{cr} are very well known in nuclear physics and described by phenomenological methods.

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