SIMPLE MODEL OF THE RAPIDLY ROTATING HOT NUCLEUS*

W. NAWROCKA

Institute of Theoretical Physics, University of Wrocław
Pl. M. Borna 9, 50-205 Wrocław, Poland

L. JACAK, A. WÓJS

Institute of Physics, Technical University of Wrocław
Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland

R.G. NAZMITDINOV

Laboratory of Theoretical Physics, Joint Institute for Nuclear Research
Dubna, Head Post Office, P.O.Box 79, Moscow, Russia

(Received December 20, 1993)

The cranked anisotropic harmonic oscillator is examined in the grand canonical ensemble as a simple model of hot and rapidly rotating nuclei. The shape of the nucleus and its phase transitions are studied as functions of temperature and angular momentum of the system. The results are graphically presented and discussed.

PACS numbers: 21.60. -n

The leading factors which determine a behaviour of rotating nucleus are: an effective, average, strongly deformed single-particle nuclear field, Coriolis and centrifugal forces, residual two-body interactions (like pairing, D-D, Q-Q *etc.*) and nuclear temperature. Because of the theoretical and numerical complexity of this problem many model approaches have been considered. One of such simple models is the cranked anisotropic harmonic oscillator which allows for far-reaching analytical calculus. It has been solved in coordinate space by Glas *et al.* [1]. Ripka [2], Bohr and Mottelson [3], Zelevinsky [4] have treated the same problem in the Fock space. More recently, this

* Presented at the XXIII Mazurian Lakes Summer School on Nuclear Physics, Piaski, Poland, August 18–28, 1993.

problem has been analyzed also by Arvieu and Troudet [5]. In all these considerations the nucleus was treated as an isolated, energy conserving system. Taking into account, however, experimental conditions it is interesting to discuss the thermalization of this model.

In the present work we consider the system of A nucleons in cranked harmonic oscillator potential as a grand canonical ensemble described by the temperature T and the chemical potential μ. We study the anisotropic oscillator frequencies $\omega_x, \omega_y, \omega_z$ (connected with the deformation parameters, *cf. e.g.*, [5]) as functions of the temperature, particle number and the angular momentum.

The Hamiltonian of the considered system has the following form:

$$H^\omega = \sum_{i=1}^{A} (h^\omega)_i , \tag{1}$$

where

$$h^\omega = h_0 - \hbar\omega l_x , \tag{2}$$

$$h_0 = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + \frac{1}{2} m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) , \tag{3}$$

ω — cranking angular velocity, l_x — x-component of the one-particle angular momentum.

The one-particle Hamiltonian (2) can be diagonalized and the one-particle energies look like:

$$\varepsilon_\alpha^\omega = \hbar\omega_x(n_x + \tfrac{1}{2}) + \hbar\omega_+(n_+ + \tfrac{1}{2}) + \hbar\omega_-(n_- + \tfrac{1}{2}) , \tag{4}$$

where $\{\alpha\} = \{n_x, n_+, n_-\}$, and

$$\omega_\pm^2 = \tfrac{1}{2}(\omega_y^2 + \omega_z^2) + \omega^2 \pm \tfrac{1}{2}\sqrt{(\omega_y^2 - \omega_z^2)^2 + 8\omega^2(\omega_y^2 + \omega_z^2)} . \tag{5}$$

In the framework of the thermodynamic approach (including temperature) we have to average the dynamical variables over the grand canonical ensemble. This procedure removes the dependence on the fixed quantum configuration.

The grand canonical potential $\Omega(T, \omega, \mu)$ is expressed by the grand partition function:

$$\mathcal{Z} = Tr(\exp(-\beta(\hat{H}^\omega - \mu\hat{N}))) , \tag{6}$$

$$\Omega = -\tfrac{1}{\beta} \ln \mathcal{Z} , \tag{7}$$

where

$$\beta = \frac{1}{k_B T},$$

and k_B is the Boltzmann constant.

The occupation numbers of the fermion levels are

$$f_\alpha = \frac{1}{\exp(\beta(\varepsilon_\alpha^\omega - \mu)) + 1}, \tag{8}$$

and the chemical potential μ is determined by the condition

$$\langle 1 \rangle = A, \tag{9}$$

where $\langle \ldots \rangle$ denotes the statistical averaging, *i.e.*

$$\langle \ldots \rangle = \sum_\alpha [\exp[\beta(\varepsilon_\alpha^\omega - \mu)] + 1]^{-1} \ldots . \tag{10}$$

It is convenient to perform the Legendre transformation and to use the following new potential (as it is desirable to fix the total angular momentum L instead of the angular velocity ω):

$$\Phi = \Omega + \omega \langle \hat{L}_x \rangle + \mu \langle \hat{N} \rangle, \quad \hat{L}_x = \sum_i (\hat{l}_x)_i . \tag{11}$$

Therefore, we deal with three fixed parameters T, L, A (instead of T, ω, μ). The internal variables are $\omega_x, \omega_y, \omega_z, \omega, \mu$ — they are, however, not independent since the incompressibility condition has been imposed (it corresponds to conservation of the nucleus volume)

$$\omega_x \omega_y \omega_z = \omega_0^3 = \text{const}. \tag{12}$$

Assuming the system to be in equilibrium state we derive the basic equations as the necessary conditions for the minimum of the Φ-potential:

$$\frac{\partial \Phi}{\partial \omega_x} = 0, \quad \frac{\partial \Phi}{\partial \omega_y} = 0, \quad \frac{\partial \Phi}{\partial \omega_z} = 0, \quad \frac{\partial \Phi}{\partial \omega} = 0, \quad \frac{\partial \Phi}{\partial \mu} = 0, \tag{13}$$

supplemented with the condition (12).

If we choose four independent variables: $\omega_z, \omega_y, \omega, \mu$, the above equations attain the following form:

$$\omega_x^2 < n_x + \frac{1}{2} > = \omega_y^2 \frac{1}{2} [\frac{< n_+ + \frac{1}{2} >}{\omega_+} + \frac{< n_- + \frac{1}{2} >}{\omega_-}$$

$$+\frac{\omega_y^2+\omega_z^2+4\omega^2}{\omega_+^2-\omega_-^2}\left(\frac{<n_+ + \frac{1}{2}>}{\omega_+} - \frac{<n_- + \frac{1}{2}>}{\omega_-}\right)],$$

$$(\omega_y^2-\omega_z^2)[(1+\frac{\omega_y^2+\omega_z^2+4\omega^2}{\omega_+^2-\omega_-^2})\frac{<n_+ + \frac{1}{2}>}{\omega_+}$$

$$+(1-\frac{\omega_y^2+\omega_z^2+4\omega^2}{\omega_+^2-\omega_-^2})\frac{<n_- + \frac{1}{2}>}{\omega_-}] = 0, \qquad (14)$$

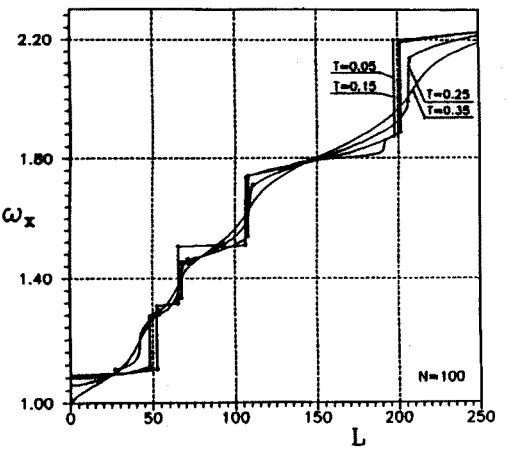
$$<1>= A,$$

$$L = m\omega(\langle y^2\rangle + \langle z^2\rangle) + \frac{4\omega}{\omega_+^2-\omega_-^2}(\langle n_+ + \tfrac{1}{2}\rangle\omega_+ - \langle n_- + \tfrac{1}{2}\rangle\omega_-),$$

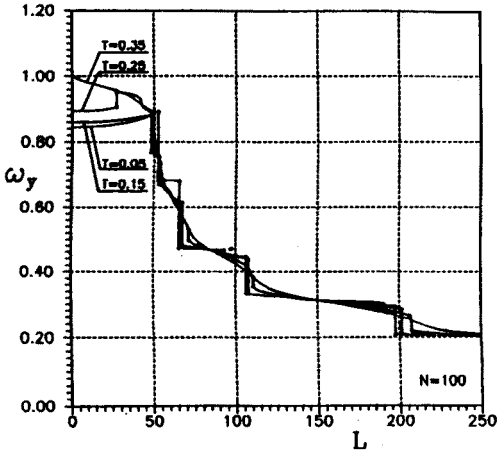
with $\omega_x = \omega_o^3/\omega_y\omega_z$.

Eq. (14) is a system of coupled, highly nonlinear equations in variables ω_y, ω_z, ω, μ and with parameters T, A, L. The analytical solution of (14) is not possible.

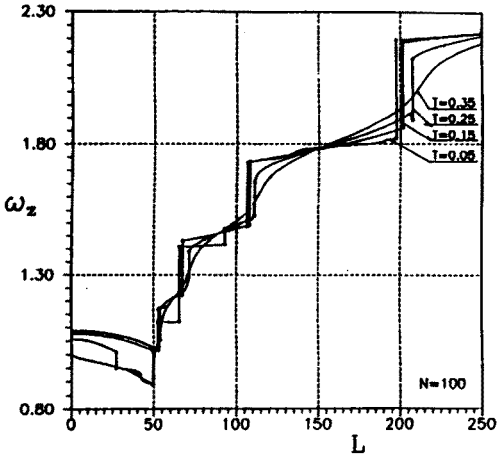
There exists a rich family of solutions of Eq. (14) — as usual for nonlinear problems. The solution of physical significance is, however, that one which minimizes the thermodynamical potential Φ. This solution turns out to be a piece-wise continuous function and the points of its discontinuity correspond with first order phase transitions of the nucleus shape. The step-type changes of the variables ω_x, ω_y, ω_z at these phase transition points correspond to step-type deformations of the nucleus. The minimal solution of Eq. (14) (for several temperatures and for particle number 100) is depicted in Figs 1a, b, c, 2 and 3 (for variables $\omega_x(L)$, $\omega_y(L)$, $\omega_z(L)$, in



(a)



(b)



(c)

Fig. 1. The dependence of ω_x — (a); ω_y — (b); ω_z — (c) *versus* L at several temperatures (particle number $A = 100$). Steps along the curve are due to the shape changes of the system ($\omega_0 = 1$, $\hbar = 1$, $k_B = 1$).

Fig. 1a, b, c, $\mu(L)$ in Fig. 2 and $\omega(L)$ in Fig. 3, respectively). The characteristic steps are very transparent for temperatures $T < 0.5$ (in units of ω_0, *i.e.*, it was put: $\omega_0 = 1$, $\hbar = 1$, $k_B = 1$). The interesting result is that the rigid body angular momentum *versus* the total angular momentum, *i.e.*,

$$L_{\mathrm{rig}}(L) = m\omega(\langle y^2 \rangle + \langle z^2 \rangle), \tag{15}$$

calculated for the equilibrium state (these piece-wise continuous functions

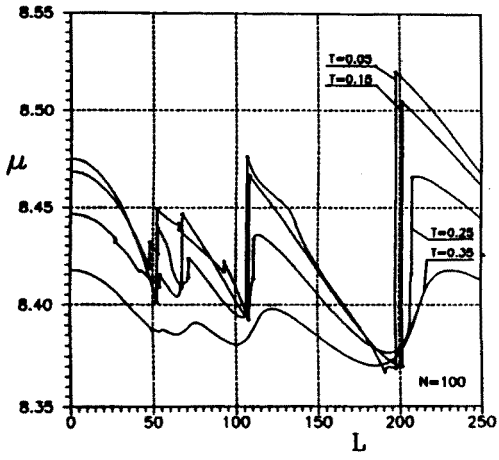


Fig. 2. The dependence of the chemical potential μ *versus* L at several temperatures, particle number $A = 100$ ($\omega_0 = 1$, $\hbar = 1$, $k_B = 1$).

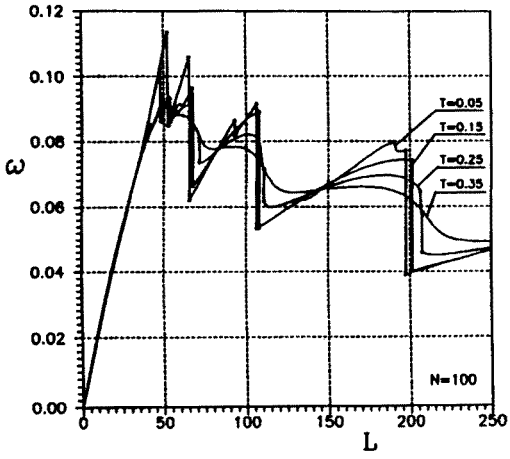


Fig. 3. The cranking angular velocity ω *versus* L at several temperatures, particle number $A = 100$ ($\omega_0 = 1$, $\hbar = 1$, $k_B = 1$).

presented in Figs 1–3) is exactly the straight line:

$$L_{\mathrm{rig}}(L) = L\,. \tag{16}$$

Note that in many other considerations of the cranked harmonic oscillator (*cf. e.g.*, [4], [5]) $L_{\mathrm{rig}}(L) \neq L$ since considered in those papers arbitrary assumed configurations are usually far from the sole equilibrium state.

Summarizing, we have found, at sufficiently low temperatures ($T < 0.5$), several shape phase transitions of nucleus within the cranked oscillator model. We have also proved that, despite the very complicated shape phase

transitions, the moment of inertia of the equilibrium state of a nucleus in the oscillator model is of the rigid body type at any temperature.

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