# NUCLEAR RADIUS*,**

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The new isospin dependent formula for the nuclear charge distribution radius

$$R = 1.25\left(1 - 0.2\frac{N-Z}{A}\right)A^{1/3}$$

is proposed. Its parameters are found by the fit of the theoretical mean square radii of even-even nuclei to all the available experimental data on the isotopic shift of the charge radius.

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The last decade developments in the laser spectroscopy techniques has given new data concerning the mean square radius of charge distribution in a nucleus. These data evidently show that a nucleus composed of $N$ neutrons and $Z$ protons has a radius depending not only on the mass number $A = N + Z$ but also on the relative neutron excess $I = (N - Z)/A$ and deformation caused by shell effects and long range nuclear forces. The neutron excess in a nucleus plays an important role in the charge radius isotopic dependence.

The traditional liquid drop formula for the nuclear radius

$$R^{\rm old} = 1.2\,A^{1/3}\,, \tag{1}$$

fails in reproduction of the experimental mean square radii (msr) isotopic shifts. The nuclear charge radius grows with the neutron number slower,

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and it can be approximated by the following formula:

$$R^{\mathrm{new}} = r_0 \left(1 - \alpha \frac{N-Z}{A}\right) A^{1/3}. \tag{2}$$

We have found the parameters $r_0$, $\alpha$ by the fit of the theoretical msr values to all the known experimental isotopic shifts of even-even isotopes.

The macroscopic msr estimate is given by the following integral performed over the volume of the deformed nucleus $V(\varepsilon^0, \varepsilon_4^0)$, where $(\varepsilon^0, \varepsilon_4^0)$ are the Nilsson quadrupole and hexadecapole equilibrium deformation parameters [1, 2]

$$\langle r^2 \rangle = \int\limits_{V(\varepsilon^0,\varepsilon_4^0)} r^2 \rho(\vec{r}) d\tau . \tag{3}$$

For the sake of simplicity we assumed here the uniform charge density distribution:

$$\rho(\vec{r}) = \begin{cases} \frac{Z}{\frac{4}{3}\pi (R^{\mathrm{new}})^3} & \text{for} \quad r \leq R(\vartheta, \varphi) \\ 0 & \text{for} \quad r > R(\vartheta, \varphi) \end{cases} . \tag{4}$$

The radius of deformed nucleus $R(\vartheta, \varphi)$ is found from the volume conservation condition

$$\int\limits_{V(\varepsilon^0,\varepsilon_4^0)} d\tau = \tfrac{4}{3}\pi (R^{\mathrm{new}})^3 . \tag{5}$$

The integral (3) for msr is equal

$$\langle r^2 \rangle = \tfrac{3}{5} Z (R^{\mathrm{new}})^2 \left(\frac{\overset{\circ}{\omega}_0}{\omega_0}\right)^5 \left(1 + \tfrac{1}{3}\varepsilon^0\right)^{-1} \left(1 - \tfrac{2}{3}\varepsilon^0\right)^{-1/2}$$
$$\times \left[ \left( \frac{1}{1 + \frac{1}{3}\varepsilon^0} + \frac{1}{1 - \frac{2}{3}\varepsilon^0} \right) \int\limits_0^1 \frac{t^2 dt}{[1 - \frac{2}{3}\varepsilon^0 P_2(t) + 2\varepsilon_4^0 P_4(t)]^{5/2}} \right.$$
$$\left. + \frac{1}{1 + \frac{1}{3}\varepsilon^0} \int\limits_0^1 \frac{dt}{[1 - \frac{2}{3}\varepsilon^0 P_2(t) + 2\varepsilon_4^0 P_4(t)]^{5/2}} \right] , \tag{6}$$

where $P_L$ are the Legendre polynomials. The volume conservation condition (5) gives the equation for the harmonic oscillator frequencies

$$\frac{\overset{\circ}{\omega}}{\omega_0} = \left(1 + \tfrac{1}{3}\varepsilon^0\right)^{1/3} \left(1 - \tfrac{2}{3}\varepsilon^0\right)^{1/6} \left[ \int\limits_0^1 \frac{dt}{[1 - \frac{2}{3}\varepsilon^0 P_2(t) + 2\varepsilon_4^0 P_4(t)]^{3/2}} \right]^{-1/3} . \tag{7}$$

Only the equilibrium deformations, taken from Ref. [1, 2], were obtained microscopically within the generator coordinate method with local approximation of long range forces and pairing interaction. The Nilsson potential with Seo correction term was taken as the single particle potential.

We have calculated the msr values for 220 even-even nuclei for which their isotopic shifts are known experimentally. The value of $\alpha$ parameter in (2) was established from the experimental slope of the msr isotopic shifts for each element while $r_0$ is evaluated from the experimental gold radius. After the inclusion of the isospin dependence (2) of the spherical nuclear radius the sum of square errors decreased about 10 times. The best agreement of the experimental [4] and macroscopic values has been achieved for the parameters

$$r_0 = 1.25\ \text{fm}, \qquad \alpha = 0.2\,.$$

The parameter $\alpha$ does not change much with increasing number of elements taken into consideration [3], and we do not expect its large change after inclusion of the odd isotopes.

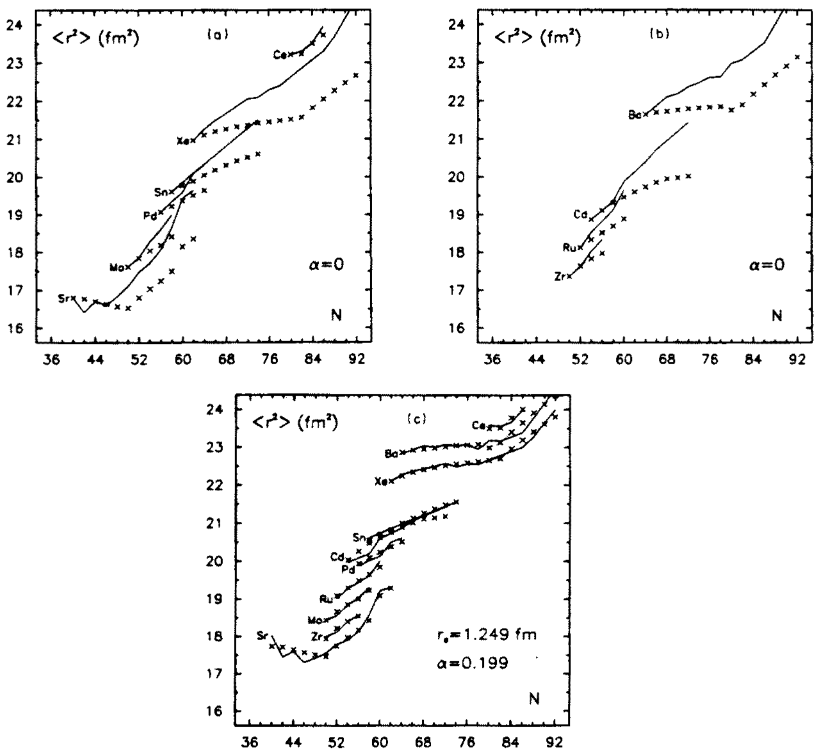


Fig. 1. The charge mean square radii of Sr–Ce even-even isotopes calculated (solid lines) macroscopically with the old (a), (b) and new (c) radius compared with the experimental msr isotopic shifts (crosses).

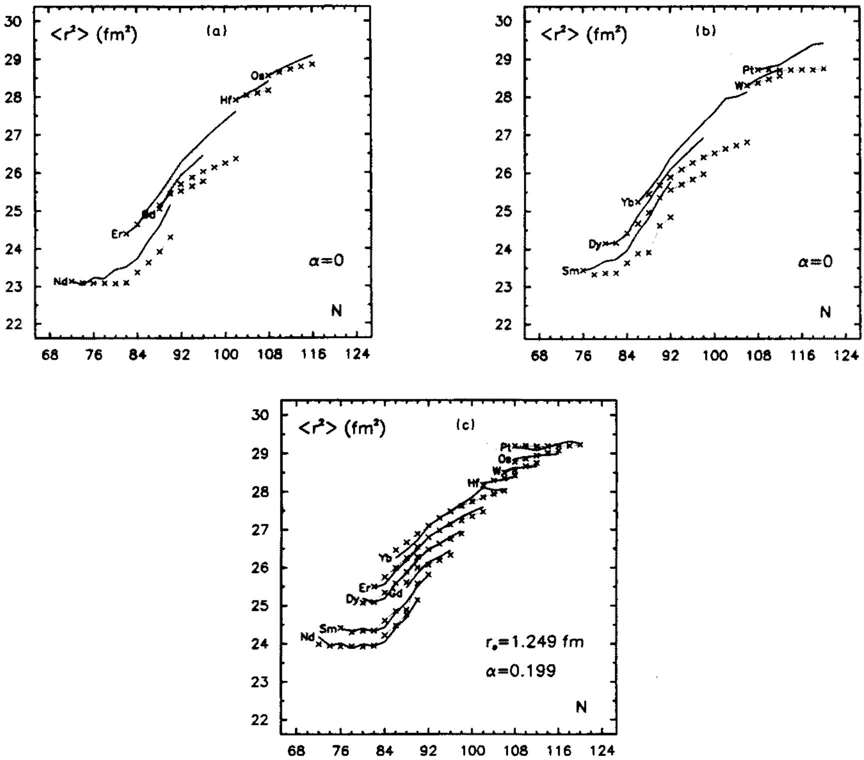


Fig. 2. The charge mean square radii of rare-earth even-even isotopes calculated (solid lines) macroscopically with the old (a), (b) and new (c) radius, compared with the experimental msr isotopic shifts (crosses).

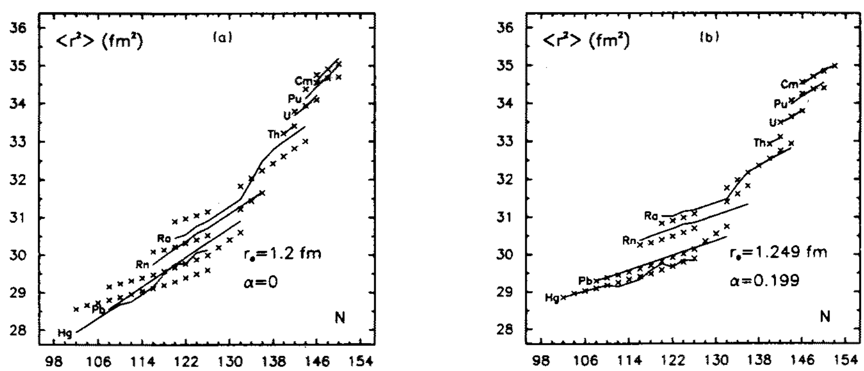


Fig. 3. The charge mean square radii of actinides calculated macroscopically (solid lines) with the old (a) and new (b) radius, compared with the experimental msr isotopic shifts (crosses).

The results on msr macroscopic values are drawn in Figs 1–3. The values obtained with the old formula for radius (1) are so far from the experimental data, that it was even impossible to show them in one picture. That is why the Figs 1(a), (b) and 2 (a), (b) contain the msr for every second element.

It is worthwhile to add that the liquid droplet model gives also similar, but weaker as in Eq. (2), dependence of the nuclear charge radius on the relative neutron excess. This dependence can be seen when one expands the liquid droplet radius [5] into a power series in $(N - Z)/A$.

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