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DYNAMICS OF SINGLE PARTICLE NUCLEON MOTION*,**

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The dynamics of classical motion of a nucleus is investigated. The Hamiltonian consists of harmonic oscillator potential and spin orbit coupling. It is shown that the classical dynamics of the nucleon motion in such a mean field is very rich. The trajectories live in submanifolds with dimensions ranging from 3 to 6, depending on initial conditions and control parameters. This reflects a transition from ordered to chaotic dynamics. The calculated Poincaré sections, Lyapunov exponents, correlation dimensions and power spectra describe quantitatively this transition.

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1. Introduction

The model discussed here was already described in details in two previous papers [1, 2]. One of our aims was to prove that spin-orbit coupling can be indeed the source of chaos in single particle nucleon motion. Therefore

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we choose a model which is integrable when the spin orbit term is switched off. Quantum Hamiltonian takes form:

$$H = T + V_{\mathrm{HO}} - \kappa\,(\mathbf{l}\cdot\mathbf{s}\,)\,, \tag{1}$$

where V_{HO} is an axially symmetric harmonic oscillator potential (spheroidal deformation). "Classical Hamiltonian" can be constructed with the use of coherent states technique,

$$\mathcal{H} = < CS \mid H \mid CS >\,, \tag{2}$$

where $\mid CS >$ is a coherent state. More precisely, $\mid CS > = \mid \alpha > \mid \beta > \mid \gamma > \mid \zeta >$, where $\mid \alpha >$, $\mid \beta >$, $\mid \gamma >$ – HO coherent states and $\mid \zeta >$ – SU(2) (spin) coherent state. Here $\alpha, \beta, \gamma, \zeta$ are complex variables. There is one to one correspondence $(\alpha, \alpha^*) \Longleftrightarrow (Q, P)$. Then the classical Hamiltonian $\mathcal{H}$ can be expressed in generalized coordinates and momenta spanning 8–dimensional phase space $\mathcal{H} = \mathcal{H}(X, P_x, Y, P_y, Z, P_z, \Phi, P_\Phi)$, where $\{\Phi, \Theta\}$ are angles defining the position of s and $P_\Phi = \frac{1}{2}\cos\Theta = s_z$.

$$\begin{aligned}\mathcal{H} = {} & \tfrac{1}{2}\{\omega_\perp(X^2 + P_x{}^2) + \omega_\perp(Y^2 + P_y{}^2) + \omega_z(Z^2 + P_z{}^2)\} \\ & - \frac{1}{2}\,\kappa\,\omega_o \left\{ \left(\sqrt{\frac{\omega_z}{\omega_\perp}} Y P_z - \sqrt{\frac{\omega_\perp}{\omega_z}} Z P_y\right) \sin\Theta\ \cos\Phi \right. \\ & \left. + \left(\sqrt{\frac{\omega_\perp}{\omega_z}} Z P_x - \sqrt{\frac{\omega_z}{\omega_\perp}} X P_z\right) \sin\Theta\ \sin\Phi - (X P_y - Y P_x)\ \cos\Theta\ \right\}.\end{aligned} \tag{3}$$

Classical equations of motion (EOM) are Hamiltonian equations:

$$\dot{Q}_i = \frac{\partial H}{\partial P_i}\,, \qquad \dot{P}_i = -\frac{\partial H}{\partial Q_i}\,. \tag{4}$$

Alternatively, the same "classical" EOM can be derived from variational principle [2]. EOM are nonlinear and are solved numerically.

2. Poincaré sections and Lyapunov exponents

In previous papers [1, 2], mainly the cases with rather large J_z ($J_z = \frac{9}{2}$) have been investigated. For such trajectories we found only a soft chaos for small deformations and standard nuclear value of spin orbit coupling constant κ. The Poincaré sections showed also that for many initial conditions trajectories can not penetrate the full phase space. Lyapunov exponents for chaotic trajectories are also small, from λ=0.002 for shapes close to spherical to λ=0.02 for deformations corresponding to ground state deformations of deformed nuclei. Trajectories corresponding to large J_z lie close to the equatorial plane and experience only weakly the potential deformation. Therefore searching for more chaotic trajectories, occupying higher dimensional fraction of the phase space one should concentrate rather on the cases with the lowest J_z. In the present paper we investigated cases with $J_z = \frac{1}{2}$. Some of the results are presented below and compared to the earlier results for completness of the discussion. Indeed, the motion of the particle with low angular momentum is much more chaotic. The typical Poincaré sections are composed of random swarm of points. The calculated Lyapunov exponents are also up to one order of magnitude larger, reaching values λ=0.2–0.3.

3. Trajectory dimension

The calculation of the dimension of a dynamic system embedded in high-dimensional phase space is generally a very difficult task. Here we used the same method as in the previous papers, based on the concept of the correlation dimension, introduced by Grassberger and Procaccia [3]. We applied it with a few technical improvements. According to this method, the number of trajectory points N_i which drop into a phase space sphere of a given radius r_i should scale as

$$N_i \approx {r_i}^D , \tag{5}$$

where D is the dimension of the manifold filled by the trajectory. Then in log-log plot of N_i as a function of r_i the slope represents the dimension. With some care, the method works very effectively and precisely for many systems.

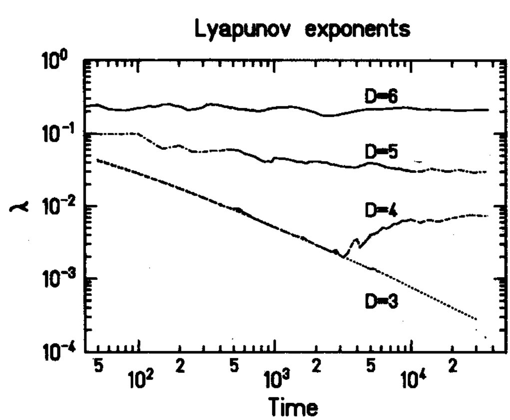

Fig. 1. Lyapunov exponents for trajectories listed in Table I.

Figure 2 contains results on dimension calculation for the trajectory 4 of Table I. The analysed trajectory contained about $2.3 \cdot 10^6$ points (data files occupying over 100 M bytes of the disk space). Only such long trajectory enabled us to determine the dimension with good enough precision. The solid line in the figure stands for slope $a = 6$, dot–dot–dashed line for the slope $a = 5$. The calculated data are displayed by full squares and lie almost exactly on the solid line (the best fit to the data corresponds to the slope $a = 5.94$). As there are 2 constants of motion in our system (E and J_z) the dimension has to be an integer not greater than 6. Therefore we conclude that results presented in Fig. 2 strongly indicate $D = 6$ as the proper value of the dimension. Depending on the initial conditions and control parameters (in this case both deformation ε and spin orbit coupling κ), the trajectory can span 3, 4, 5 and 6 dimensional subsets in 8 dimensional phase space. The examples of such trajectories are given in Table. I. The result $D = 6$ for the last trajectory shows that, in general, there is no room for integrals of motion other than E and J_z.

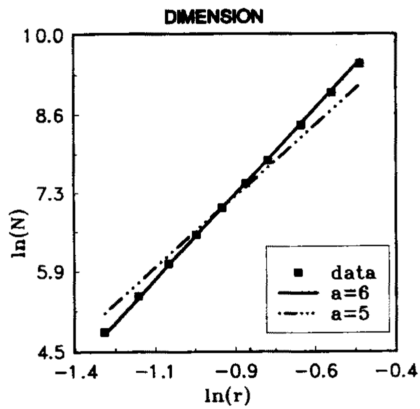

Fig. 2. Dimension of the trajectory 4 of Table I.

TABLE I

Examples of the trajectories with different dimensions

No	ε	κ	J_z	l_{z0}	D	λ_{max}
1	0.02	0.06	$\frac{9}{2}$	4.6	3	0.0
2	0.02	0.06	$\frac{9}{2}$	4.7	4	0.002
3	-0.3	0.6	$\frac{9}{2}$	4.1	5	0.02
4	0.3	0.6	$\frac{1}{2}$	0.1	6	0.2

5. Power spectra

To complete the discussion of the dimensions we calculated also power spectra from the time series of one of the coordinates along trajectory. The results appear to be closely related both to dimension and Lyapunov exponent. Fig. 3 displays the power spectra for four trajectories which are described in Table I. It is striking, how the differences in the dynamics of the particular trajectories are exhibited in the power spectra. The transition from regular trajectories with low dimension to chaotic ones with high dimension is accompanied by increasing power in higher frequencies.

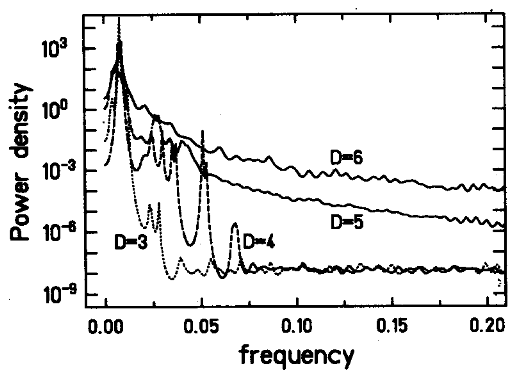

Fig. 3. Power spectra of the trajectories listed in Table I.

REFERENCES

[1] R. Arvieu, P. Rozmej, M. Płoszajczak, in *New Trends in Theoretical and Experimental Nuclear Physics*, World Scientific, Singapore 1992, p. 397.
[2] P. Rozmej, R. Arvieu, *Nucl. Phys.* **A545**, 497c (1992).
[3] P. Grassberger, I. Procaccia, *Phys. Rev. Lett.* **50**, 346 (1983).