SOME CONSEQUENCES OF POSSIBLE THERMODYNAMIC–TYPE ASPECT OF PHYSICAL TIME*

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To the memory of Jerzy Rayski

Recently, the familiar analogy between the time evolution of an isolated system and the thermal equilibrium of a system with a thermostat led us to the hypothesis of a "temporal equilibrium" of any so called isolated system with the physical space-time. Then, small virtual deviations from such an equilibrium imply a nonunitary extension of the quantum state equation and, in consequence, a tiny unitarity defect of the S matrix. Another consequence of our conjecture is the possibility that the hypothetic Big Bang of the universe determined the absolute zero of cosmic time, an analogue of the infinite value of absolute temperature.

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As is commonly known, the XX-century physics discovered the geometrical aspect of physical time, unifying it with physical space in one 3+1-dimensional (generally Riemannian) space-time. However, the familiar analogy between the time evolution of an isolated system in the quantum theory and the thermal equilibrium of a system with a thermostat [1] may be a signal that there is possibly also a thermodynamic-type aspect of the physical time [2]. In particular, such an aspect may manifest itself in a tiny unitarity defect of the S matrix for scattering processes [3]. The smallness of this effect and, on the other hand, the not-high-enough precision in measurements of forward scattering amplitudes [4] are perhaps the reasons that

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no deviations from the unitarity of S matrix have been observed yet. As we point out in the present note, another occasion for the manifestation of the thermodynamic-type aspect of physical time was possibly the hypothetic Big Bang of the universe, determining the absolute zero of cosmic time, being an analogue of the infinite value of absolute temperature.

Let us first comment on our argument leading to the possible unitarity defect of S matrix [3]. Its starting point is the formal analogy $kT \leftrightarrow -i\hbar t^{-1}$ or, in consequence, $k(T - T_0) \leftrightarrow -i\hbar(t^{-1} - t_0^{-1})$ between absolute temperature T and time t. If taken at its face value, it suggests that (in the case of a homogeneous matter medium) a conductivity equation of the form

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\varphi(\vec{r}, t_0) = 0\,, \tag{1}$$

should hold to propagate the *inverse-time field*

$$\varphi(\vec{r}, t_0) \equiv t^{-1}(\vec{r}, t_0) - t_0^{-1} = -\frac{t(\vec{r}, t_0) - t_0}{t(\vec{r}, t_0)t_0}\,, \tag{2}$$

(in analogy with the familiar propagation of temperature field $T - T_0$). Here, $t(\vec{r}, t_0)$ is the *time field* running at any space point $\vec{r}$, $t(\vec{r}_0, t_0) = t_0$ denotes its running value at a particular space point $\vec{r}_0$ and $\lambda_\Gamma > 0$ plays the role of a length-dimensional conductivity constant. Of course, the difference between T and t is that the temperature field T may be experimentally fixed at any space point $\vec{r}$, while the time field t always runs at any $\vec{r}$.

In particular, when $t \equiv t_0$ (*i.e.*, $\varphi \equiv 0$) in a Minkowski frame of reference, one can speak of *temporal equilibrium* of the system with the physical space-time playing, therefore, the role of a *chronostat* (in analogy with the thermal equilibrium of the system with a thermostat, characterized by $T \equiv T_0$). In general, when $t \not\equiv t_0$ (*i.e.*, $\varphi \not\equiv 0$), deviations of the time field t from its temporal-equilibrium run t_0 propagate (in a homogeneous matter medium) through the conductivity equation (1).

From the viewpoint of phenomenological thermodynamics the new conductivity equation (1) suggests that the first law of thermodynamics [5] should be now extended to the form

$$dU = \delta W + \delta Q - i\delta\Gamma\,, \tag{3}$$

including an *imaginary* term $-i\,\delta\Gamma$. Here, $\delta\Gamma$ is the infinitesimal amount of a new thermodynamic-type quantity Γ — call it *energy width* — transferred to the system from its surroundings including the physical space-time (or being identical with the space-time if a so called isolated system is considered). The imaginary quantity $-i\Gamma$ is an analogue of heat Q when $-i\hbar t^{-1}$ takes over the role of kT. In the case of temporal equilibrium $t \equiv t_0$, the thermodynamic internal energy U becomes real. In this case, quantum states evolve (in the Schrödinger picture) according to the conventional quantum state equation [6]

$$i\hbar \frac{d\Psi(t_0)}{dt_0} = H\Psi(t_0)\,, \tag{4}$$

where H is a Hermitian operator of total energy (for a so called isolated system).

In general, however, the evolution of quantum states *deviates* from their evolution in the temporal equilibrium $t \equiv t_0$ (or $\varphi \equiv 0$). Then, it is natural to consider for the state equation (in the Schrödinger picture) the form

$$i\hbar \frac{d\Psi(t_0)}{dt_0} = [H - i\mathbf{1}\Gamma(t_0)]\,\Psi(t_0)\,, \tag{5}$$

where $\mathbf{1}$ stands for the unit operator and $\Gamma(t_0)$ is the total energy width (for a so called isolated system).

We proposed that

$$\Gamma(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r}\,\frac{1}{c}\,j^0(\vec{r}, t_0)\varphi(\vec{r}, t_0)\,, \tag{6}$$

where $g_\Gamma > 0$ is a small dimensionless coupling constant and

$$j^\mu(\vec{r}, t_0) \equiv \langle\Psi(t_0)|\; J^\mu(\vec{r})\; |\Psi(t_0)\rangle_{\rm av}\,, \tag{7}$$

denotes the spin-averaged expectation value of the operator of total particle 4-current $J^\mu(\vec{r})$ (in the Schrödinger picture, identical with the Heisenberg and interaction pictures at $t_0 = 0$). Note that in the temporal equilibrium where $\varphi \equiv 0$ we get $\Gamma(t_0) \equiv 0$.

We also proposed that in the nonrelativistic approximation the inverse-time field φ satisfies an inhomogeneous conductivity equation of the form

$$\left(\Delta - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t_0}\right)\varphi = 4\pi g_\Gamma \lambda_\Gamma \left(\frac{1}{c}\frac{\partial j^0}{\partial t_0} + {\rm div}\vec{j}\right)\exp\frac{-c\,t_0}{2\lambda_\Gamma}\,. \tag{8}$$

Such an equation can be approximately derived from a relativistic equation for $\chi = \varphi\exp(ct_0/2\lambda_\Gamma)$ having a tachyonic character.

Due to Eqs (5) and (7) the 4-divergence in the source term of Eq. (8) can be written as

$$\begin{aligned}\frac{1}{c}\frac{\partial j^0}{\partial t_0} + {\rm div}\vec{j} &= \langle\Psi(t_0)|\;\frac{i}{\hbar}[P_\mu\,,\,J^\mu(\vec{r})] - \frac{2}{\hbar c}\Gamma(t_0)J^0(\vec{r})\;|\Psi(t_0)\rangle_{\rm av}\\ &= \langle\Psi(0)|\;\frac{i}{\hbar}\,[P_\mu\,,\,J^\mu_H(\vec{r}, t_0)] - \frac{2}{\hbar c}\Gamma(t_0)J^0_H(\vec{r}, t_0)\;|\Psi(0)\rangle_{\rm av}\\ &\quad\times\exp\left[-\frac{2}{\hbar}\int_0^{t_0} dt_0'\,\Gamma(t_0')\right]\,,\end{aligned} \tag{9}$$

where $P_\mu = (c^{-1}H, \vec{P})$ is the operator of total 4-momentum (in the Schrödinger as well as Heisenberg picture). In particular, if $\Psi(t_0)$ satisfying Eq. (5) is an eigenstate of all P_μ, the 4-divergence (9) vanishes and so no inverse-time field φ can be created by the matter through Eq. (8) (the system persists in the temporal equilibrium, consistently with $\Gamma(t_0) \equiv 0$).

Two equations, the state equation (5) (with $\Gamma(t_0)$ as defined in Eq. (6)) and the thermodynamic-type equation (8), form together a coupled nonrelativistic system of equations for $\Psi(t_0)$ and $\varphi(\vec{r}, t_0)$ — call such a theory (nonrelativistic) *chronodynamics*. Strictly speaking, this mixed system is nonlinear (and nonlocal) in $\Psi(t_0)$, slightly violating thereby the superposition principle for the state vector. However, it becomes linear (and local) in the excellent approximation, where $\Psi(t_0)$ in the definition (7) of j^μ is replaced by the zero-order $\Psi^{(0)}(t_0)$ satisfying the conventional, temporal-equilibrium state equation (4) (with $\Psi(t_0)$ denoting now $\Psi^{(0)}(t_0)$, but $\Psi^{(0)}(t_0)$ cannot be here an eigenstate of all P_μ in order to avoid the trivial case of temporal equilibrium). Then, Eq. (6) gives the first order $\Gamma^{(1)}(t_0)$ as

$$\Gamma^{(1)}(t_0) \equiv g_\Gamma \hbar \int d^3\vec{r}\, \frac{1}{c} j^{0(0)}(\vec{r}, t_0)\varphi^{(1)}(\vec{r}, t_0)\,, \tag{10}$$

where the first-order $\varphi^{(1)}(\vec{r}, t_0)$ is evaluated from Eq. (8) with the zero-order source term involving $\Psi^{(0)}(t_0)$ in place of $\Psi(t_0)$.

Hence, the first-order S matrix comes out related to the conventional, temporal-equilibrium S matrix through the formula

$$S^{(1)} = S^{(0)} \exp(-d^{(1)})\,, \tag{11}$$

where the zero-order $S^{(0)}$ is the unitary conventional S matrix and

$$d^{(1)} \equiv \frac{1}{\hbar} \int_{-\infty}^{\infty} dt_0\, \Gamma^{(1)}(t_0) = O(g_\Gamma^2) \tag{12}$$

measures the first-order unitarity defect. The experimental bounds on $d^{(1)}$ in particular processes may be estimated by looking for deviations from the optical theorem in these cases. Note that the formal time interval $-\infty < t_0 < \infty$ in Eq. (12) (as usually for scattering processes) corresponds to a microscopic time interval of effective interaction.

Now, one may proceed a step further with exploiting the analogy $kT \leftrightarrow -i\hbar t^{-1}$ between temperature T and time t. As T is the absolute temperature varying in the interval $0 \leq T < \infty$, one may ask the question whether also the physical time can get (in some circumstances) an absolute meaning in such a sense that $0 \leq t < \infty$, and the limits $t \to 0$ and $t \to \infty$ are analogues of the limits $T \to \infty$ and $T \to 0$, respectively.

Obviously, it would be natural to ascribe the limit $t \to 0$ to the hypothetic Big Bang of the universe, which thus would determine the absolute zero of cosmic time, an analogue of the infinite value of absolute temperature. In this limit, deviations of the universe from an overall temporal equilibrium $\varphi \equiv 0$ would be singular and so not well defined. In contrast, in the limit $t \to \infty$, the universe would approach an overall temporal equilibrium $\varphi \equiv 0$. In fact, under the assumption that the 4-divergence (9) is finite at $t_0 \to \infty$, the inverse-time field φ created by the matter through Eq. (8) tends to zero with $t_0 \to \infty$ (then, $t \to t_0 \to \infty$ due to Eq. (2) with $\varphi \to 0$). Thus, for the "old" universe, the time deviations $t - t_0$ from its overall temporal equilibrium $t \equiv t_0$ (and, of course, any laboratory time intervals $t' - t$) are always negligibly small in comparison with the age of universe given by the spatial average of $t = t(\vec{r}, t_0)$. Then, the universe approaches asymptotically the "temporal death", where time runs uniformly at all space points, $t \equiv t_0$ (or $\varphi \equiv 0$), and $t_0 \to \infty$. A fascinating question arises, how "old" the universe is now (the answer depends on the value of the small coupling constant g_Γ).

As the Reader perhaps noticed, in this paper having a phenomenological character we did not discuss the problem of physical nature of space-time that is certainly connected, to some extent at least, with the theory of gravitation (or supergravitation) and its expected quantization. Analogically, in the phenomenological thermodynamics the physical nature of a thermostat or heat reservoir is not discussed. So, our thermodynamic-type theory considering the physical space-time as a chronostat or energy-width reservoir may be rightly called *phenomenological chronodynamics*.

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