UNRESOLVED QUESTIONS IN WEAK-LOCALIZATION IN SEMICONDUCTORS*†

C. JĘDRZEJEK, W. SZOTT**

Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland

AND

W.P. KIRK

NanoFAB, Department of Physics, Texas A&M University
College Station, Texas 77843-4242, USA

(Received May 10, 1994)

A review of weak localization theory is presented in application to electron transport in semiconductors. While the theory qualitatively works well going into more microscopic level reveals difficulties. The important questions concern relaxing contact potential assumption by using weekly screened potential, effect of impurity correlations, and taking into account spatial distribution of impurities and wave function modulation. The origin of weak-antilocalization in strictly two-dimensional semiconductor systems is analyzed. There are two possible sources of this phenomenon: (1) crystalline: inversion asymmetry term and interface or Rashba term, and (2) resulting from the presence of many subbands. The first mechanism does not seem to agree with experimentally extracted dispersion relations for quasi two-dimensional systems. The second mechanism cannot explain a secondary maximum for magnetoresistance observed in some systems. In addition to need to study interaction effect, further progress can be achieved by using semiclassical theory with the correlated noise.

PACS numbers: 72.15. Rn, 72.10. Bg, 71.70. Ej

---

* Presented at the VI Symposium on Statistical Physics, Zakopane, Poland, September 20–29, 1993.

† This material is based in part on work supported by the US-Poland Sklodowska-Curie Fund grant MEN/NSF/92-116.

** Present address: Institute of Gas and Oil Research, Krosno, Poland.

1. Introduction

The physics is disordered electronic systems has been a very active area for the past 15 years. In the weak disorder limit the properties of a single electron interacting with random impurities are basically understood and quantitative comparison with experiment is successful if theory stays at the phenomenological level [1–4].The theory of electron-electron interaction in the presence of impurities is in a worse shape for semiconductors [5]. It has been found in 1984 [5] (and since then there has not been too much progress) that (1) τ_{in} was 10 times larger than indicated by theory, (2) $1-F$, where F is the factor that reduces static screened interaction, was 5 times the expected value, (3) temperature-sensitive negative magnetoresistance in parallel magnetic field was unexplained. The reason may be the use of contact potential assumption for electron impurity approximation. The electron-electron interaction will not be discussed here, although because weak localization (WL) and electron-electron interaction appear together in many cases they cannot be separated. There is parallel progress in WL in the hopping regime [6].

The purpose of this review is to address assumptions made in applying the standard WL to transport in semiconductor systems. The transport in such systems is a many respects different from transport in disordered metals. In particular we will raise the following questions: *(i)* contact potential assumption, *(ii)* effect of impurity correlations, *(iii)* spatial distribution of impurities and wave function modulation in superlattices *(iv)* origin of weak-antilocalization in strictly two-dimensional semiconductor systems.

The question we concentrate on is whether upon relaxing some standard approximation the physics of localization is changed.

2. Standard theory of weak localization

Weak localization effect is caused by quantum interference. Classically probability P for electron to get from point A to B with two possible paths (Fig. 1(a)) equals $P = P_1 + P_2$. Quantum mechanically one has to add amplitudes first and square the result

$$P = |A_1 + A_2| = A_1^* A_1 + A_2^* A_2 + 2\,\mathrm{Re}\, A_1^* A_2 \,. \tag{2.1}$$

With A_1 equal A_2 quantum probability is twice the classical one due to interference. Among the paths connecting points A and B there are ones that form loops as shown in Fig. 1(b). Inside the loop an electron starting from point 0 (Fig. 1(c)) can execute forward trajectories and its time-reversed counterparts. Such pairs of trajectories interfere constructively increasing probability of locating an electron at point 0 in absence of time-reversed

terms. In describing the quantum interference as a diffusive process three lengths play the role: elastic scattering length l, phase breaking (inelastic scattering) length l_{ph} and magnetic length. Phase breaking length radius is visualized in Fig. 1(c) as a radius of a circle enclosing a typical trajectory which has to be much larger than the elastic scattering length in order that the diffusive process occurs. The size of magnetic length is related to a radius of area equivalent to the one encircling a trajectory for which the change of phase due to perpendicular magnetic field is of order 1.

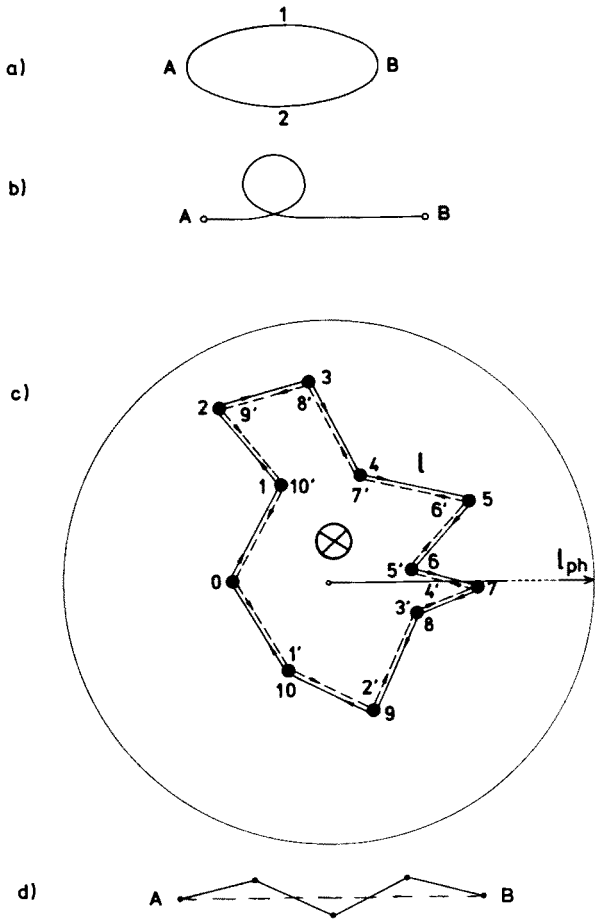

Fig. 1. Schematic representation of paths entering calculation of electron transport (a) interference in the presence of two paths, (b) looped trajectory (c) interference in the presence of many scatterers — l is the mean-free path, l_{ph} is the phase-breaking length, (d) an important trajectory in the hopping regime

The description of weak localization for disordered metals is to be contrasted to interference effects for systems in the hopping regime for which the important trajectories lie inside narrow tubes connecting points A and B due to the fact the transport follows through the overlap of exponentially decaying wave functions, Fig. 1(d) [6].

2.1. Quasiclassical theory of weak localization

In this part we follow the excellent paper by Chakravarty and Schmid [7].

The starting point is the Feynman's expression for the propagation amplitude

$$K(\boldsymbol{r}_f, \boldsymbol{r}_i; t_f, t_i) = \int d[\boldsymbol{r}_t] \exp\left(\frac{i}{\hbar} S[\boldsymbol{r}_t]\right) \tag{2.2}$$

with the action given by

$$\begin{aligned} S[\boldsymbol{r}_t] &= \int_{t_i}^{t_f} dt L(\dot{\boldsymbol{r}}_t, \boldsymbol{r}_t)\,, \\ L &= L_0 + L_1\,, \\ L_0 &= \frac{1}{2} m \dot{\boldsymbol{r}}_t^2 - V_R(\boldsymbol{r}_t)\,. \end{aligned} \tag{2.3}$$

Here V_R is random potential representing disorder, L_0 is dominant part of the Lagrangian, and L_1 — electromagnetic interaction is small but is important because it breaks symmetry with respect to time reversal.

In the quasiclassical limit the path integral $\Rightarrow$ sum of all paths with stationary action

$$K(\boldsymbol{r}_f, \boldsymbol{r}_i; t_f, t_i) = \sum_{\{\boldsymbol{r}_t^{\mathrm{cl}}\}} A[\boldsymbol{r}_t^{\mathrm{cl}}] \exp\left(\frac{i}{\hbar} S[\boldsymbol{r}_t^{\mathrm{cl}}]\right)\,, \tag{2.4}$$

where A represents Gaussian fluctuations around the classical path.

Conditional probability for an electron on Fermi surface is

$$\begin{aligned} W(\boldsymbol{r}_f, \boldsymbol{r}_i; t_0) = & \frac{1}{2g(E_F)} \sum_{\{\boldsymbol{r}_t^{\mathrm{cl}}\}} \sum_{\{\boldsymbol{s}_t^{\mathrm{cl}}\}} A[\boldsymbol{r}_t^{\mathrm{cl}}] A^*[\boldsymbol{s}_t^{\mathrm{cl}}] \\ & \times \exp\{\tfrac{i}{\hbar}(S[\boldsymbol{r}_t^{\mathrm{cl}}] - S[\boldsymbol{s}_t^{\mathrm{cl}}])\} \\ & \times \{\delta(\varepsilon[\boldsymbol{r}_t^{\mathrm{cl}}] - \varepsilon_F) + \delta(\varepsilon[\boldsymbol{s}_t^{\mathrm{cl}}] - \varepsilon_F)\}\,. \end{aligned} \tag{2.5}$$

Due to irregularities of V_R, it is assumed that no correlations between phases $S[r_t^{cl}]/\hbar$ and $S[r_t^{cl}]/\hbar$ exist on two different paths

$$W(r_f, r_i; t_0) = \frac{1}{g(E_F)} \sum_{\{r_t^{cl}\}} |A[r_t^{cl}]|^2 \delta(\varepsilon[r_t^{cl}] - \varepsilon_F) . \tag{2.6}$$

Conditional (quasi)probability of pairs of time-reversed paths takes the form

$$\begin{aligned} \tilde{W}(r_f, r_i; t_0) = & \frac{1}{g(E_F)} \sum_{\{r_t^{cl}\}} |A[r_t^{cl}]|^2 \delta(\varepsilon[r_t^{cl}] - \varepsilon_F) \\ & \times \exp(i\phi[r_t]) , \end{aligned} \tag{2.7}$$

with

$$\frac{dr_{-t}^{cl}}{dt} = -\dot{r}_{-t}^{cl} , \tag{2.8}$$

$$\begin{aligned} \hbar\phi[r_t] &= S[r_t^{cl}] - S[r_{-t}^{cl}] \\ &= \int_{\frac{-t_0}{2}}^{\frac{t_0}{2}} dt\{L(\dot{r}_t, r_t) - L(-\dot{r}_{-t}, r_{-t})\} \\ &= \int_{\frac{-t_0}{2}}^{\frac{t_0}{2}} dt\{L_1(\dot{r}_t, r_t) - L_1(-\dot{r}_t, r_t)\} . \end{aligned} \tag{2.9}$$

In calculation of probability of classical paths summation over paths and averaging over V_R can be combined in one step. Selecting an interval of length t_0, a path $\boldsymbol{R}_t$ and a neighborhood $[\boldsymbol{R}_t] \cdots [\boldsymbol{R}_t] + d[\boldsymbol{R}_t]$ in the function space, probability of classical paths for a realization of the potential is defined

$$P_{t_0}[\boldsymbol{R}_t] = \frac{1}{g(E_F)} \left\langle \sum_{\{r_t^{cl}\}} |A[r_t^{cl}]|^2 \delta(\varepsilon[r_t^{cl}] - \varepsilon_F) \delta([r_t^{cl}] - \boldsymbol{R}_t) \right\rangle . \tag{2.10}$$

Classical probability density

$$W_{t_0} = \int d[\boldsymbol{R}_t] P_{t_0}[\boldsymbol{R}_t] , \tag{2.11}$$

is related to a probability $W_{t_0} d^d R_t$ for a classical path to return to a volume $d^d R_t$ near a starting point. The phase dependence can be included as

$$\tilde{W}_{t_0} = W_{t_0} \langle\langle \exp(i\phi[r_t]) \rangle\rangle , \tag{2.12}$$

where

$$\langle\langle Y[R_t] \rangle\rangle = W_{t_0}^{-1} \int d[R_t] P_{t_0}[R_t] Y[R_t] . \tag{2.13}$$

$W(r_f, r_i; t_0)$ can readily be calculated for a Boltzmannian motion in which an electron undergoes sharp changes of direction upon collision with impurities. Such approach is often used in weak localization problems which involve velocity at boundaries [8, 9].

In general, it is much easier to use a Brownian motion, in which when $k_F l \gg 1$ the result obtained from Boltzmann equation agrees with the one from diffusion equation. Then the probability for a realization of a path is given by the Wiener measure

$$P_{t_0}[R_t] = \exp\Big(- \frac{1}{4D} \int_{t_i}^{t_f} dt \dot{R}_t^2 \Big) , \tag{2.14}$$

where

$$\tilde{W}(r_f, r_i; t_f, t_i) = \int d[R_t] \exp\Big(- \int_{t_i}^{t_f} dt F(\dot{R}_t, R_t) \Big) , \tag{2.15}$$

$$F(\dot{R}_t, R_t) = \frac{1}{4D} \dot{R}_t^2 - \frac{i}{\hbar} (L_1(\dot{R}_t, R_t) - L_1(-\dot{R}_t, R_t)) . \tag{2.16}$$

In the absence of time-reversal terms the standard diffusion equation in obtained

$$\Big[\frac{\partial}{\partial t_f} - D \frac{\partial^2}{\partial r_f^2} \Big] \tilde{W}(r_f, r_i; t_f, t_i) = \delta(t_f - t_i) \delta(r_f - r_i) , \tag{2.17}$$

with the solution

$$\tilde{W}(r_f, r_i; t_0) = (4\pi D t_0)^{-d/2} \exp\left(- \frac{r_0^2}{4 D t_0} \right) . \tag{2.18}$$

As a consequence the probability of return is

$$\tilde{W}(r_f, r_f; t_0) = (4\pi D t_0)^{-d/2} . \tag{2.19}$$

Taking into account damping due to phase breaking collisions

$$\tilde{W}_{t_0} \Rightarrow \tilde{W}_{t_0} \exp\left(-\frac{t_0}{\tau_{\rm ph}}\right) . \tag{2.20}$$

Correction to conductivity is obtained by invoking that conductivity and diffusion constant are related

$$\sigma = 2e^2 g(E_F) D . \tag{2.21}$$

Diffusion constant can be obtained from the time-integrated velocity-velocity correlation function and as a result

$$\Delta\sigma = -\frac{e^2}{2\pi^2\hbar} \int_{\tau} dt_0 \tilde{W}_{t_0} . \tag{2.22}$$

This leads to the following result

$$\Delta\sigma = -\frac{e^2}{2\pi^2\hbar} \begin{cases} (\frac{3}{\pi})^{\frac{1}{2}} l^{-1} - (D\tau_{\rm ph})^{-\frac{1}{2}} & \text{for } d = 3 \\ \ln(\frac{\tau_{\rm ph}}{\tau}) & \text{for } d = 2 \\ (2\pi)(D\tau_{\rm ph})^{\frac{1}{2}} & \text{for } d = 1 . \end{cases} \tag{2.23}$$

If size of the sample is smaller than the phase breaking length it takes over the role of the characteristic length.

In a presence of magnetic flux time-reversal term due to

$$L_1 = e\dot{\boldsymbol{r}}_t \cdot \boldsymbol{A}(\boldsymbol{r}_t^{\rm cl}) \tag{2.24}$$

gives the phase proportional to the enclosed flux

$$\phi[\boldsymbol{r}_t] = \frac{2e}{\hbar} \int_{-t_0/2}^{t_0/2} dt\, \dot{\boldsymbol{r}}_t \cdot \boldsymbol{A}(\boldsymbol{r}_t^{\rm cl}) = 2\pi \frac{\Psi(\boldsymbol{r}_t^{\rm cl})}{\Psi_0} , \tag{2.25}$$

where $\Psi_0 = h/2e$ is the quantum of flux for two electrons. This is closely connected with the Aharonov–Bohm effect demonstrated by Shavrin and Shavrin (see, Ref. [10]) in doubly connected geometry

$$\begin{aligned} \tilde{W}_{t_0} &= \sum_{n=-\infty}^{+\infty} (4\pi D t_0)^{-d/2} \\ &\quad \times \exp\left(-\frac{n^2 L^2}{4Dt_0} + 2\pi i n \frac{\Psi_L}{\Psi_0} - \frac{t_0}{\tau_{\rm ph}}\right) \\ &\approx -\cos\left(2\pi \frac{\Psi_L}{\Psi_0}\right) . \end{aligned} \tag{2.26}$$

Phase coherence in presence of magnetic flux is obtained from the diffusion equation in which momentum is substituted by a canonical momentum. Such a modified diffusion equation reads

$$\left[\frac{\partial}{\partial t_f} - D\left(-i\frac{\partial}{\partial \boldsymbol{r}_f} - \frac{2e}{\hbar}\boldsymbol{A}(\boldsymbol{r}_t)\right)^2 + \frac{1}{\tau_{\mathrm{ph}}}\right]\tilde{W} = \delta(t_f - t_i)\delta(\boldsymbol{r}_f - \boldsymbol{r}_i). \quad (2.27)$$

Choosing the gauge $A = (0, Bx, 0)$ one obtains

$$\left[\frac{\partial}{\partial t_f} - D\frac{\partial^2}{\partial x_f^2} - D\left(-i\frac{\partial}{\partial y_f} - \frac{2eB}{\hbar}x_f\right)^2 + \frac{1}{\tau_{\mathrm{ph}}}\right]\tilde{W} = \delta(t_f - t_i)\delta(x_f - x_i)\delta(y_f - y_i). \quad (2.28)$$

The solution is in terms of $\Psi_n(x)$ normalized eigenfunctions of the operator $D(\frac{\partial^2}{\partial x^2} + (\frac{2eB}{\hbar})^2)$ with eigenvalues $\omega_n = (\frac{2eBD}{\hbar})(2n+1)$

$$\tilde{W} = \sum_n \int \frac{dk}{2\pi}\Psi_n\left(x_f - \frac{\hbar k}{2eB}\right)\Psi_n\left(x_i - \frac{\hbar k}{2eB}\right) \times \exp\left[ik(y_f - y_i) - \omega_n t_0 - \frac{t_0}{\tau_{\mathrm{ph}}}\right]. \quad (2.29)$$

For $\boldsymbol{r}_f = \boldsymbol{r}_i$

$$\tilde{W} = \frac{2eB}{h}\sum_n \exp\left(-(\tfrac{2eBD}{\hbar})(2n+1)t_0 - \frac{t_0}{\tau_{\mathrm{ph}}}\right). \quad (2.30)$$

This leads to the following correction to magnetoconductivity

$$\begin{aligned}\Delta\sigma &= -\frac{e^2}{2\pi^2\hbar}\int_0^\infty dt_0 \tilde{W}_{t_0}\left[\exp\left(-\frac{t_0}{\tau_{\mathrm{ph}}}\right) - \exp\left(\frac{t_0}{\tau}\right)\right] \\ &= -\frac{e^2}{2\pi^2\hbar}\sum_n\left\{\left[n + \frac{1}{2} + \frac{\hbar}{4eBD\tau_{\mathrm{ph}}}\right]^{-1} - \left[n + \frac{1}{2} + \frac{\hbar}{4eBD\tau}\right]^{-1}\right\} \\ &= -\frac{e^2}{2\pi^2\hbar}\left\{\Psi\left(\frac{1}{2} + \frac{\tau_B}{\tau}\right) - \Psi\left(\frac{1}{2} + \frac{\tau_B}{\tau_{\mathrm{ph}}}\right)\right\}, \end{aligned} \quad (2.31)$$

where $\tau_B = \hbar/4eDB$ and Ψ is *Digamma* function. $\Delta\sigma$ increases with increasing magnetic field.

Analyzing the derivation scheme the type of noise occurs in two places. Firsts in Eq. (2.7) where it is assumed that due to irregularities of V_R, it is assumed that no correlations between phases $S[r_t^{cl}]/\hbar$ and $S[r_t^{cl}]/\hbar$ exist on two different paths. Second, when the averaging over noise is made. The choice of Gaussian white noise is motivated by the facts that it exactly corresponds to the Green's function technique in which only nonintersecting impurity interactions are taken into account. This suggests that using coloured noise may be closer to reality [11]. Unfortunately, Feynman paths for correlated noise do not lead to analytic result [12]. Likewise, we are not aware of any analytic result for the probability of return. The use of correlated noise is even more pressing for systems with correlated impurities such as spin glasses or semiconductor systems with the 2-D electron density approaching 10^{12} cm^{-2} [13, 14].The presence of traps can be simulated by diffusion in random media

$$\left[\frac{\partial}{\partial t_f} - D\frac{\partial^2}{\partial r_f^2}\right]\tilde{W}(r_f, r_i; t_f, t_i) - \lambda V(r_f)\tilde{W}(r_f, r_i; t_f, t_i) = \delta(t_f - t_i)\delta(r_f - r_i)\,, \quad (2.32)$$

where λ is a constant and $V(r_f)$ is time-independent δ-function correlated Gaussian potential. This equation was studied by Tao [15] in 1-dimension and found to be diffusive for times small compared to $\tau = (16D/9\lambda^4)^{1/3}$ and completely dominated by random medium for times large compared to τ when probability of $\tilde{W}(0; t_f, t_i)$ grows with time exponentially. In view of the fact that traps also dephase electrons this interesting theoretical possibility is not applicable. Large classes of dynamical processes in disordered media differ from simple diffusion. Anomalous diffusion is characterized by different from $\gamma = 1$ exponent in dependence of the mean-squared displacement [16–20]

$$< r^2 > \sim t^{\gamma}\,. \quad (2.33)$$

This can be achieved by either using

$$\left[\frac{\partial}{\partial t_f} - \frac{\partial}{\partial r_f}k(r,t)\frac{\partial}{\partial r_f}\right]\tilde{W}(r_f, r_i; t_f, t_i) = 0\,, \quad (2.34)$$

or

$$\left[\frac{\partial^m}{\partial t_f^m} - D\frac{\partial^2}{\partial r_f^2}\right]\tilde{W}(r_f, r_i; t_f, t_i) = 0\,. \quad (2.35)$$

There exists an analogy of weak localization in dynamic systems manifested by diffusive regime in *e.g.* kicked rotor [21, 22] or periodically driven anharmonic oscillator [23]. When time-reversal symmetry is preserved quantum

survival probability is twice its classical counterpart [23]. These authors reported that introducing 10% of random noise in the field destroyed all interference effects. It would be interesting to study the counterpart of this effect in electronic transport. The diffusive nature of the motion exists for kicked rotor even though as shown by Atland [24] the kicks are not δ-function correlated. This resilience of diffusive nature of dynamics is also corroborated by field-theoretic results [25, 26].

2.2. Quantum theory of weak localization

In this part we will concentrate on momentum dependent scattering. For introduction to the subject the reader is referred to the classic textbook by Mahan [27] and the review by Bergmann [3]. Here we start from the general form impurity averaged double scattering probability W illustrated in Fig. 2

$$W = n_{\mathrm{imp}}\langle V_i V_j\rangle_{i=j} = \frac{n_{\mathrm{imp}}V^2}{(2\pi)^d}\delta(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_3-\boldsymbol{p}_4)g(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3,\boldsymbol{p}_4)\,, \quad (2.36)$$

where n_{imp} is the impurity density.

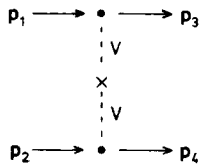

Fig. 2. Schematic representation of the impurity averaged double scattering probability.

There have been several calculations for important anisotropic cases. Beal-Monod and Forgacs [28] analyzed the correlated binary alloy occupying a D-dimensional hypercubic lattice with the host atoms B of concentration $1-c$ and the concentration of impurities A represented by a small number c. They used the following correlated alloy scattering probability $W^{(ca)}$

$$W^{(ca)}(\boldsymbol{p}-\boldsymbol{p}') = c(1-c)(V_A-V_B)^2\left[1-\frac{c(1-c)\Delta V}{k_BT}\sum_{\boldsymbol{d}}\exp i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{d}\right], \quad (2.37)$$

where $\Delta V = V_{AA}+V_{BB}-2V_{AB}$ with V_{ij} energy of neighbouring pairs of atoms ij, T is the characteristic temperature of ordering, and $\boldsymbol{d}$ is a lattice vector.

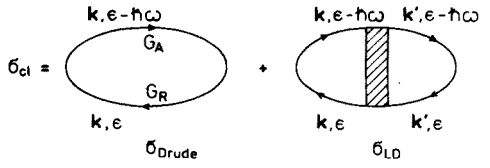

Fig. 3. Diagrams contributing to the classical conductivity.

Beal-Monod, Theumann, and Forgacs [29] considered screened Coulomb potential in 2 dimensions

$$V(r) = \frac{k_F}{2\sqrt{\pi}} \frac{\exp(-\gamma k r)}{r}, \tag{2.38}$$

The application of this potential leeds to the following form of the screened Coulomb impurity averaged scattering probability $W^{(sc)}$

$$W^{(sc)} = n_{\text{imp}} V^2(|\boldsymbol{q} - \boldsymbol{k}'|) \approx \frac{\pi n_{\text{imp}}}{2} \left(\cosh\phi - \frac{\boldsymbol{k}\cdot\boldsymbol{k}'}{k_F^2}\right)^{-1}, \tag{2.39}$$

where $\cosh\phi = (\frac{\gamma^2}{2}) + 1$.

Szott, Jedrzejek, and Kirk [30] constructed a tight-binding model of a superlattice for general impurity profile and wave function modulation. W contained the angle-dependent part g

$$\begin{aligned} g(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4) &= g(p_{1z}, p_{2z}, p_{3z}, p_{4z}) \\ &= a_1 + 2a_2 \sum_{i=1}^{4} \cos p_{iz} a + 2a_3[\cos(p_{1z} + p_{2z})a \\ &+ \cos(p_{1z} - p_{3z})a + \cos(p_{1z} - p_{4z})a]. \end{aligned} \tag{2.40}$$

In contrast to previous cases they obtained momentum dependent scattering lifetime τ

$$\frac{1}{\tau(\boldsymbol{p})} = -2\,\text{Im}\,\Sigma(\boldsymbol{p}, i0^+) = \frac{1}{\tau_0}(1 + b\cos p_z a), \tag{2.41}$$

with

$$\frac{1}{\tau_0} = \frac{b_1^2(0)}{\tau_0^*} = \frac{2\pi g_{||}(\epsilon_F) n_{\text{imp}} V^2 b_1^2(0)}{a}, \tag{2.42}$$

and where Σ is the self-energy, $g_{||}(\epsilon_f)$ is the projected density of states per spin, a is the superlattice period and b, b_1 and coefficients a_i represent various impurity averages defined in Ref. [30].

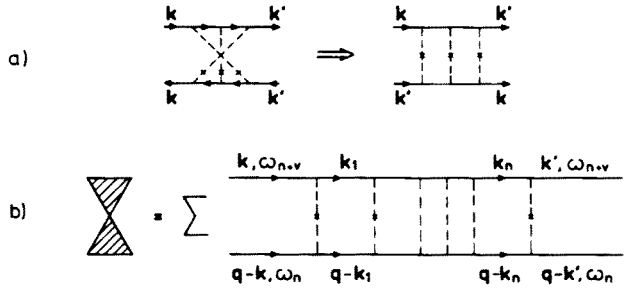

Fig. 4. Propagators contributing to the quantum correction to conductivity: (a) maximally crossed diagrams, (b) representation of the particle-particle diffusion propagator (Cooperon).

The contribution to the static conductivity consists of the classical Drude and ladder diagrams, where the hatched rectangle in Fig. 3 represents a diffuson. The weak-localization effect is related to maximally crossed diagrams shown in Fig. 4(a), with the particle-particle diffusion propagator (Cooperon) shown in Fig. 4(b). It has been demonstrated by Vollhardt and Wölfle [31] and Bhatt, Wölfle, and Ramakrishnan [32] that all diagrams representing WL conductivity σ_{MC} reduce to one diagram with the Cooperon propagator (see Fig. 5).

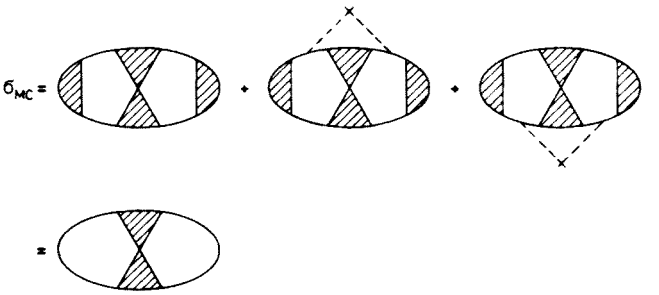

Fig. 5. Diagrams contributing to the quantum correction to conductivity.

Using the Matsubara formalism one obtains static WL conductivity from the diagram in Fig. 5 as a term linear in external frequency

$$\sigma_{ij}(\omega_\ell) = \frac{2e^2}{\beta}\sum_n \int \frac{d^dk}{(2\pi)^d}\frac{d^dq}{(2\pi)^d} v_i(\boldsymbol{k})v_j(\boldsymbol{q}-\boldsymbol{k})G(\boldsymbol{k},\epsilon_n)G(\boldsymbol{k},\epsilon_n+\omega_\ell)$$
$$\times G(\boldsymbol{q}-\boldsymbol{k},\epsilon_n)G(\boldsymbol{q}-\boldsymbol{k},\epsilon_n+\omega_\ell)W(\boldsymbol{k},-\boldsymbol{k},\boldsymbol{q},\epsilon_n,\omega_\ell)\,, \qquad (2.43)$$

where the Matsubara frequencies are $\omega_\ell = 2\pi T(l+\frac{1}{2})$, with T being the temperature. The condition $\epsilon_n(\epsilon_n+\omega_\ell) < 0$ of nonvanishing Cooperon $W(\boldsymbol{k},-\boldsymbol{k},\boldsymbol{q},\epsilon_n,\omega_\ell)$ is assumed.

The easiest way to calculate the Cooperon $W_c(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q},\omega_{n+\nu},\omega_n)$ is to use the recurrence relation [29]

$$W_c^{(i+1)}(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q},\omega_{n+\nu},\omega_n) = \int \frac{d^d p}{(2\pi)^d} G(\boldsymbol{p},\omega_{n+\nu}) G(\boldsymbol{p}-\boldsymbol{q},\omega_n) \times n_{\mathrm{imp}} V^2(|\boldsymbol{q}-\boldsymbol{k}'|) W_c^{(i)}(\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q},\omega_{n+\nu},\omega_n)\,, \tag{2.44}$$

In the standard approximation $V^2(|\boldsymbol{q}-\boldsymbol{k}'|)$ reduces to the constant. Then the right-hand side of Eq. (2.44) does not depend on $\boldsymbol{k}'$ and neither is $W_c^{(i+1)}(\boldsymbol{k},\boldsymbol{k}')$. Thus $W_c^{(i)}(\boldsymbol{k},\boldsymbol{p})$ is independent of $\boldsymbol{p}$ and can be pulled out in front of integral which reduces the integral over $\boldsymbol{p}$ to the product of the two Green's function and leeds to the geometric series for W_c.

For a contact potential the Cooperon reads

$$W_c = \frac{1}{\tau^2}\frac{1}{\omega_n + Dq^2}\,, \tag{2.45}$$

where $D = v_F^2\tau/d$, where d is the dimension.

For a contact potential the Drude conductivity has the form

$$\sigma_{DR} = \begin{cases} \frac{k_F^2 l}{3\pi^2} & \text{for } d=3 \\ \frac{k_F l}{2\pi} & \text{for } d=2 \\ \frac{2l}{\pi} & \text{for } d=1. \end{cases} \tag{2.46}$$

For independent impurities and the contact potential the classical conductivity consists of only the Drude part. For general potentials, such as considered in Refs [28, 29] also the ladder diagram contributes with the effect of replacing the elastic scattering length by transport scattering length.

For the correlated alloy [28] the Drude diagram $\sigma_{DR}^{(ca)}$ remains unchanged compared to the isotropic contact potential. The WL correction to conductivity also has the functional form of the contact potential but with the changed cut-off ($l \to l'$)

$$\Delta\sigma^{(ca)} = -\frac{e^2}{2\pi^2\hbar}\begin{cases} \left(\frac{2}{\pi l'}\right) & \text{for } d=3 \\ \ln\left(\frac{L}{l'}\right) & \text{for } d=2 \\ (2\pi)(L-l') & \text{for } d=1. \end{cases} \tag{2.47}$$

For screened impurities [29] the WL correction to conductivity in 2 dimensions has exactly the functional form of the contact potential, however,

the classical conductivity decreases when the range of potential increases (ϕ decreases)

$$\sigma_{\text{class}} = \frac{k_F}{2\pi n_{\text{imp}}} \frac{\exp\phi + 1}{2}. \tag{2.48}$$

Finally, for the superlattice with arbitrary contact impurity profile the result for conductivity is [30]

$$\sigma_{||,\text{WL}} = -\frac{2e^2}{\pi\hbar} D_{||} \int \frac{d^3q}{(2\pi)^3} \frac{1}{D_{||} q_{||}^2 + D_z q_z^2 + \tau_{\text{ph}}^{-1}} = -\frac{e^2}{\pi\hbar} \frac{1}{\sqrt{D_z \tau_{\text{ph}}}}, \tag{2.49}$$

and

$$\sigma_{z,\text{WL}} = \frac{D_{z,c}}{D_{||}} \sigma_{||,\text{WL}}, \tag{2.50}$$

where $D_z = D_{z,c} + D_{z,h}$ with $D_{z,c}$ is the coherent part of the diffusion coefficient in z-direction and $D_{z,h}$ is the hopping part of the diffusion coefficient in z-direction. Here the standard procedure of introducing dephasing time τ_{ph} was followed. This equation provides the largest modification among systems with momentum dependent scattering probabilities compared to the uniformly distributed impurities interacting with the contact potential.

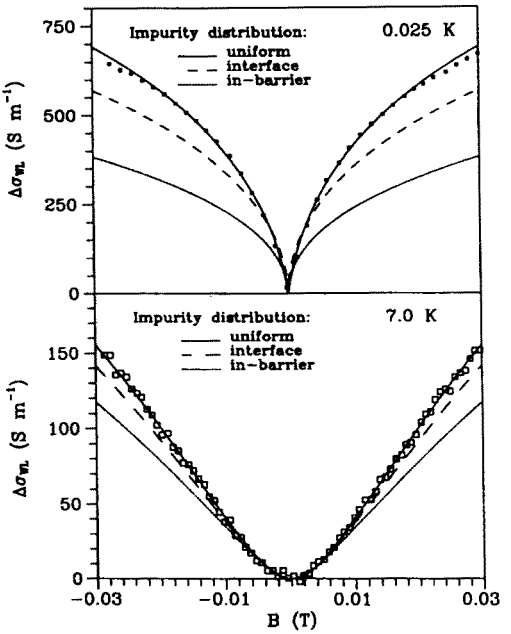

Fig. 6. Weak localization corrections to magnetoconductivity in superlattice SLN — high barrier, narrow miniband, at two different temperatures [33]. Squares represent experimental points. Curves are theoretical results for three different impurity distributions.

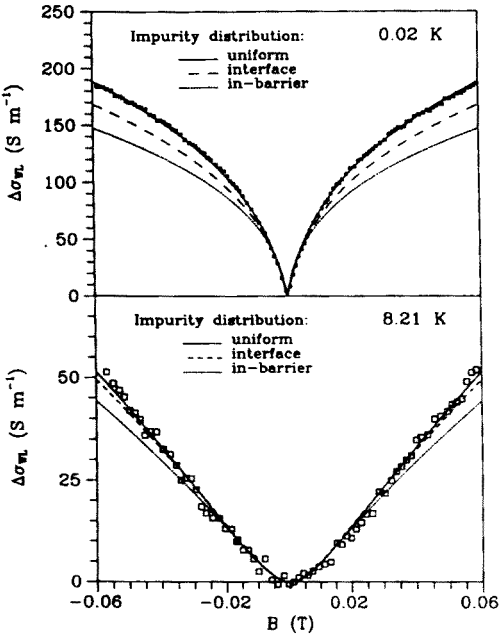

Fig. 7. As in Fig. 6 for superlattice SLW - low barrier, wide miniband. Note the agreement over three decades of temperature up to $\frac{1}{2}B_e$ in magnetic field.

Recently Szott, Jedrzejek, and Kirk [33] reported measurements of quantum corrections to the magnetoconductivity in two superlattice samples of GaAs/$Al_xGa_{1-x}As$ differing in the $Al_xGa_{1-x}As$ alloy composition (and therefore in the miniband width). The sample with a smaller bandwidth displayed a larger correction to magnetoconductivity; however, this result could not be described by a 2-dimensional theory. Using both a systematic and detailed analysis of the transport data they found that weak-localization (WL) rather than electron-electron interaction dominated the quantum transport corrections. An extensive quantitative interpretation of the transport results was done with an advanced weak localization theory for superlattices, incorporating dopant distribution, wave function modulation, and a higher magnetic field range beyond the eikonal approximation [34]. However, this advanced theory still assumed a delta-function impurity scattering potential. The theoretical fits to the data were not compatible with the as-grown impurity profiles, but were best fit with an effective uniform impurity distribution. This is shown in Figs 6 and 7. While silicon dopant migration smeared the impurity profile [35] we think some effect came from the inadequacy of the point scattering assumption commonly adopted in WL theory, which suggests that the long-range potential scattering in WL

effects was operative. The most conclusive results for indicating the need to go beyond the contact potential in describing quantum transport could come from vertical transport measurement in superlattices. Such measurements were so far performed in only 2-terminal geometry [36]. Four terminal measurements present a major fabrication challenge. Even for classical vertical transport it is difficult to ascertain validity of the Bloch picture [37].

3. Weak antilocalization in 2-dimensional systems

Recently there has been a resurgence of interest in weak-antilocalization that as is well-known is due to spin-orbit scattering. In the classic paper by Hikami, Larkin and Nagaoka [38] the Cooperon has been shown to be

$$W_{\alpha\beta\beta\alpha} = \frac{1}{2\pi g(E_F)\tau^2}\Big\{\Big(Dq^2 - i\omega + 2\big(\frac{1}{\tau_{so}^x} + \frac{1}{\tau_{so}^z} + \frac{1}{\tau_s^x}\big)\Big)^{-1} - \frac{1}{2}\Big(Dq^2 - i\omega + 2\big(\frac{2}{\tau_s^x} + \frac{1}{\tau_s^z}\big)\Big)^{-1} + \frac{1}{2}\Big(Dq^2 - i\omega + 2\big(\frac{2}{\tau_{so}^x} + \frac{1}{\tau_s^z}\big)\Big)^{-1}\Big\}. \tag{3.1}$$

Here q is the sum of momenta of the initial and final electron states in the particle-particle propagator, ω is the frequency, $g(\varepsilon_F)$ is the density of states per spin at the Fermi energy, τ is the total scattering, τ_{so}^i and τ_s^i are spin -orbit and magnetic scattering times, respectively in ith direction. Now there are three cases that determine conditions for weak antilocalization in two dimensions (2-D):

1. If there is neither the spin-orbit interaction nor the spin interaction ($\tau_{so} = \infty, \tau_s = \infty$) the last two terms cancel each other, and the first term gives the diffusion pole (localization)
2. If the spin interaction is strong ($\tau_s = finite$) there is no diffusion pole (no localization).
3. If there is no spin interaction and the spin-orbit interaction is strong ($\tau_{so}^x = finite, \tau_s = \infty$) than from the second term the logarithmic term appears with the coefficient $-1/2$ (antilocalization).

For thin metal films (of thickness less than 100 Å) a coherence length can be larger than a film thickness rendering the quantum transport two-dimensional [3], yet the spin-orbit scattering has a three-dimensional nature and τ_{so}^x = finite. It is not so in strictly 2-D systems (to which one subband semiconductor heterojunction is a good approximation at small electron densities). This comes out by inspecting the scattering matrix element (see, Fig. 8)

$$\langle \mathbf{k}', \beta, n' | V(\mathbf{r}) | \mathbf{k}, \alpha, n \rangle_{so} = i\hbar \sum_l u_{nn'}^{so}(q)\,(\sigma_{\alpha\beta} \times q) \cdot k \exp[-iq \cdot R_l]\,. \tag{3.2}$$

In strictly two dimensions the cross product of p, p' is perpendicular to the 2-D plane and the scalar product with $\sigma_{\alpha\beta}$ involves only the z-component of $\sigma_{\alpha\beta}$.

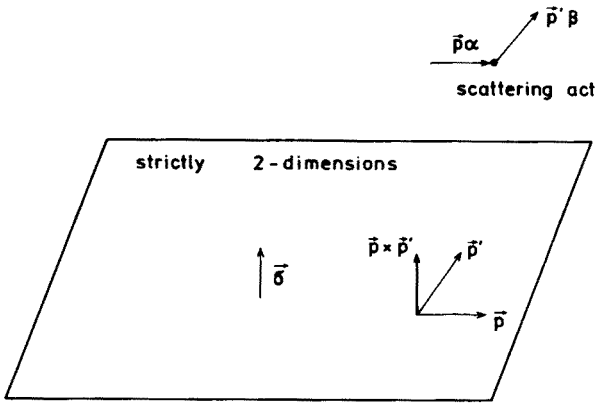

Fig. 8. Geometry of quantities involved a spin-orbit scattering

Yet weak-antilocalization has been seen in systems with no strong spin-orbit scattering of the alloy (impurity) type, such as GaAs/GaAlAs [39–42]. There are two possible explanations of weak antilocalization in 2-D electron gas. First, is a band-structure induced spin-orbit level splitting [39, 43] based on Dyakanov and Perel [44] mechanism. Actually in the context of weak localization this mechanism was first invoked in the semimagnetic semiconductor $Cd_{0.95}Mn_{0.05}Se$:In, however, for the 3-dimensional system by Sawicki *et al.* [45]). Upon neglecting confinement in the z-direction the Hamiltonian that conforms to T_d crystal symmetry has two additional terms of a spin-orbit character: inversion asymmetry term H_I and interface or Rashba term H_R [46]. The total Hamiltonian is:

$$H = H_0 + H_I + H_R, \tag{3.3}$$

where

$$H_0 = \frac{\hbar^2 k^2}{2m^*} + g\mu_B \sigma \cdot B, \tag{3.4}$$

$$H_I = \gamma \sigma \cdot \kappa, \tag{3.5}$$

$$H_R = \eta(\sigma \times k) \cdot \hat{z}. \tag{3.6}$$

Here

$$\hbar k = p + eA. \tag{3.7}$$

The vector κ is given by

$$\kappa_x = \tfrac{1}{2}[k_x(k_y^2 - k_z^2) + (k_y^2 - k_z^2)k_x], \tag{3.8}$$

$$\kappa_y = \tfrac{1}{2}[k_y(k_z^2 - k_x^2) + (k_z^2 - k_x^2)k_y], \tag{3.9}$$

$$\kappa_z = \tfrac{1}{2}[k_z(k_x^2 - k_y^2) + (k_x^2 - k_y^2)k_z]. \tag{3.10}$$

Both these terms give nonzero contributions involving σ_x and σ_y.

On a more fundamental level the band-structure induced spin-orbit level splitting has its origin in the Aharonov-Casher [47] effect for which the time-reversal symmetry breaking term is

$$L_1 = -\boldsymbol{\mu} \cdot \dot{\boldsymbol{r}} \times \boldsymbol{E}(\boldsymbol{r}). \tag{3.11}$$

This leads to modified diffusion equation in presence of crystal field [43]

$$\Big[\frac{\partial}{\partial t_f} - D\Big(-i\frac{\partial}{\partial \boldsymbol{r}_f} - \frac{\mu}{2}(\boldsymbol{\sigma}_a + \boldsymbol{\sigma}_b) \times \boldsymbol{E}(\boldsymbol{r}_t)\Big)^2 + \frac{1}{\tau_{\rm ph}}\Big]\tilde{W} = \delta(t_f - t_i)\delta(\boldsymbol{r}_f - \boldsymbol{r}_i). \tag{3.12}$$

Another possibility proposed *ad hoc* by Taboryski and Lindelof [40] and put into quantitative form by Jedrzejek, Szott and Kirk [48] is the existence of two subbands. If one takes a spin-orbit Hamiltonian for a free electron

$$H_{\rm so} = u_{\rm so}(\boldsymbol{k})(\boldsymbol{\sigma} \times i\boldsymbol{k}) \cdot \frac{\nabla}{i}, \tag{3.13}$$

then the matrix element

$$\langle \Psi(\boldsymbol{r}_\parallel)\zeta_1(z)|H_{\rm so}|\Psi(\boldsymbol{r}_\parallel)\zeta_2(z)\rangle \approx \int_{-\infty}^{\infty} \zeta_1(z)\frac{\partial}{\partial z}\zeta_2(z)dz \neq 0. \tag{3.14}$$

In the simplest version for one subband

$$\int_{-\infty}^{\infty} \zeta_1(z)\frac{\partial}{\partial z}\zeta_1(z)dz = \tfrac{1}{2}\int_{-\infty}^{\infty} \frac{\partial}{\partial z}(\zeta_1(z)\zeta_1(z))dz = 0, \tag{3.15}$$

for localized functions. Note that if in $H_{\rm so}$ there is additional z-dependence [49, 50] the effect can occur even for one-subband.

According to Dresselhaus *et al.* [39] crystal-field induced spin-orbit scattering leads to inversely proportional relation between spin-orbit scattering time and elastic scattering time.

$$\frac{1}{\tau_{\rm so}} = \frac{\langle \Delta E_c^2 \rangle}{4\hbar}\tau_{\rm el}, \tag{3.16}$$

with

$$H_{so} = \eta n^2 , \tag{3.17}$$

where ΔE_c is the spin-orbit energy splitting, η is constant dependent on nature of crystal, and n electron density

Contrary to the point scattering from spin-orbit impurities spin-orbit scattering time and elastic scattering time are proportional, because they come from the scattering on the same centre

$$\frac{1}{\tau_{so}} = 2\pi(g-2)^2 R^2 n \frac{1}{\tau_{el}} , \tag{3.18}$$

where g is the band electron giromagnetic factor, and R the scattering cross section. In Dresselhaus *et al.* experiment [39] the electron density was changed by a gate. Although in Ref. [39] H_{so} has indeed been found proportional to square density of carriers very different (fair to say opposite) dependence was found by Hansen *et al.* [41]. This means that changing the gate voltage causes other changes in a sample in addition to the carrier density. Moreover, the Dresselhaus *et al.* [39] assumption of zero pseudovector κ in the z-direction was criticized on theoretical [51] and experimental [52] grounds.

Even more challenging is to explain the WL results which display secondary peak in the magnetoresistance for systems into nonstandard way of introducing impurities [40], Fig. 9 and [53], Fig. 10.

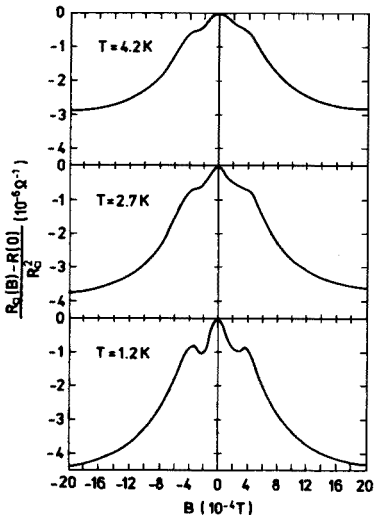

Fig. 9. Magnetoresistivity for a helium-ion implanted sample [40] displaying a satellite peak

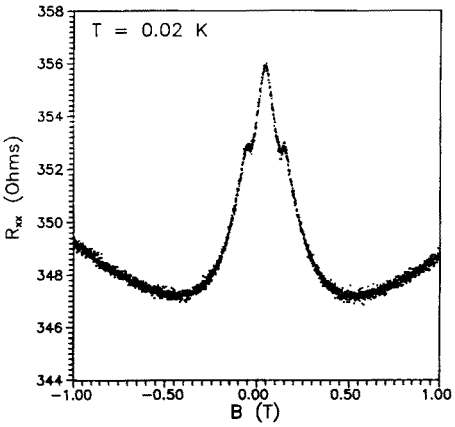

Fig. 10. Magnetoresistivity for a superlattice with magnetic field perpendicular to the superlattice growth direction [53]. A satellite peak does not come from the size effect.

To investigate this case Jedrzejek, Szott and Kirk [48] extended the two-band calculations by Iwabushi [54] avoiding the parallel conduction in two bands as assumed by Taboryski and Lindelof [40]. With the assumption of the phase breaking time common in two bands.

$$\sigma(B)_{\mathrm{WL}} = -\frac{e^2}{2\pi^2\hbar}\left\{\left[\sum_{n=1,2} C_n \Psi\left(\tfrac{1}{2}+\frac{\tau_B}{\tau_{n,\mathrm{I}}}\right)\right] - \Psi\left(\tfrac{1}{2}+\frac{\tau_B}{\tau_{\mathrm{II}}}\right) + \tfrac{1}{2}\Psi\left(\tfrac{1}{2}+\frac{\tau_B}{\tau_{\mathrm{III}}}\right) - \tfrac{1}{2}\Psi\left(\tfrac{1}{2}+\frac{\tau_B}{\tau_{\mathrm{IV}}}\right)\right\}. \quad (3.19)$$

where

$$C_n = \frac{D_n g_n(\varepsilon_F)}{D_1 g_1(\varepsilon_F) + D_2 g_2(\varepsilon_F)}, \quad (3.20)$$

$$\frac{1}{\tau_{n,\mathrm{I}}} = \frac{1}{\tau_{nn,0}} + \frac{1}{\tau_{n\bar{n},0}} + \sum_i \left(\frac{1}{\tau^i_{nn,so}} + \frac{1}{\tau^i_{nn,s}} + \frac{1}{\tau^i_{n\bar{n},so}} + \frac{1}{\tau^i_{n\bar{n},s}}\right),$$

$$\frac{1}{\tau_{\mathrm{II}}} = \frac{1}{\tau_{\mathrm{ph}}} + \sum_{nn'} 2\rho_n(\varepsilon_F)\left(\frac{1}{\tau^z_{nn',so}} + \frac{1}{\tau^x_{nn',so}} + \frac{1}{\tau^x_{nn',s}}\right),$$

$$\frac{1}{\tau_{\mathrm{III}}} = \frac{1}{\tau_{\mathrm{ph}}} + \sum_{nn'} 2\rho_n(\varepsilon_F)\left(\frac{1}{\tau^z_{nn',s}} + \frac{2}{\tau^x_{nn',s}}\right),$$

$$\frac{1}{\tau_{\mathrm{IV}}} = \frac{1}{\tau_{\mathrm{ph}}} + \sum_{nn'} 2\rho_n(\varepsilon_F)\left(\frac{1}{\tau^z_{nn',s}} + \frac{2}{\tau^x_{nn',so}}\right). \quad (3.21)$$

Here $i = x, y, z$,
$n, n' = 1, 2$ and the axial symmetry is assumed.

$$\rho_n(\varepsilon_F) = \frac{g_n(\varepsilon_F)}{g_1(\varepsilon_F) + g_2(\varepsilon_F)}. \tag{3.22}$$

$$D = D_1\rho_1(\varepsilon_F) + D_2\rho_2(\varepsilon_F). \tag{3.23}$$

$$g_2(\varepsilon_F) * \tau^i_{12,\delta} = g_1(\varepsilon_F) * \tau^i_{21,\delta}. \tag{3.24}$$

$$\tau_B = \frac{\hbar}{4eDB}. \tag{3.25}$$

Also

$$\sigma_{\text{clas}} = e^2 \sum_{n=1,2} D_n g_n(\varepsilon_F), \tag{3.26}$$

where

$$D_n = \frac{v_n^2(\varepsilon_F)\tau_n}{2}. \tag{3.27}$$

The formula depends on many parameters, yet for no fit we succeeded in producing the secondary maximum in the magnetoresistance. Even including the spin-scattering [55–57] will not likely explain this effect. There is some hope that semiclassical studies of systems with time-reversal broken symmetry (transitions between Gaussian orthogonal, Gaussian unitary, and Gaussian symplectic ensembles [58]), can shed some light on this problem.

4. Scaling theory of localization and conclusions

The analytic results for weak-localization are important to verify whether one-parameter scaling theory is operative. Weak scattering theory is valid for $(k_F l)^{-1} \ll 1$. For a metal with $L \gg l$, the Ohm's law is fulfilled

$$g(L) = \sigma L^{d-2}. \tag{4.1}$$

In the hopping regime with $L \gg \xi$ we expect

$$g(L) \approx \exp -\frac{L}{\xi}. \tag{4.2}$$

Abrahams *et al.* [59] introduced a scaling function

$$\beta = \frac{d \ln g}{d \ln L}, \tag{4.3}$$

interpolating between these regimes. They postulated one parameter scaling theory-β depends on L only through g independent of dimensionality. For example for small conductance

$$\beta(g) = \ln \frac{g}{g_c}. \tag{4.4}$$

For weak disorder

$$\beta(g) = (d-2) - \frac{a}{g}. \tag{4.5}$$

Whether this is generally true cannot be conclusively stated. Numerical studies of the Anderson model either confirm [60] or indicate against [61] the one-parameter scaling theory. Extensions to weak-localization beyond the contact potential, although do not change the structure of the theory, introduce another scale.

New systems such as these reported by Wang *et al.* [62] and Jiang *et al.* [63] in which the change of magnetic field brings the experimental system (strongly disordered 2-D layer caused by the presence of the gate above) from the insulator to the quantum Hall effect state (quantum Hall insulator [64]) pose even greater change.

To answer the old and new questions progress in the electron-electron effect [65] is needed.

REFERENCES

[1] B.L. Altshuler, A.G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, Vol. 44, Eds. A.L. Efros and M. Pollak, North-Holland, Amsterdam 1985.
[2] P.A. Lee, T.V. Ramakrishnan, *Rev. Mod. Phys.* **B57**, 287 (1985).
[3] G. Bergmann, *Phys. Rep.* **107**, 1 (1984).
[4] H. Fukuyama, in *Electron-Electron Interaction in Disordered Systems,* Vol. 44, Eds. A. L. Efros and M. Pollak, North-Holland, Amststerdam 1985.
[5] B.F. Lin, M.A. Paalanen, A.C. Gossard, D.C. Tsui, *Phys. Rev.* **B29**, 927 (1984).
[6] B.I. Shklovskii, B.Z. Spivak, in *Hopping Transport in Solids,* Eds. M. Pollak and B.I. Shklovskii, Elsevier, 1991, Ch. 9, p. 271.
[7] S. Chakravarty, A. Schmid, *Phys. Rep.* **140**, 193 (1986).
[8] C.W.J. Beenakker, H. van Houten, in *Solid State Physics,* Vol. 44, Eds. H. Ehrenreich and D. Turnbull, Academic Press, 1991.
[9] D.E. Khmelnitskii, M. Yosefin, *Physica* **A200**, 525 (1993).
[10] A.G. Aronov, Yu.V. Shavrin, *Rev. Mod. Phys.* **B59**, 755 (1987).
[11] J. Łuczka, M. Niemiec, E. Piotrowski, *Phys. Lett.* **A167** 475 (1992).
[12] T.G. Venkatesh, L.M. Patnaik, *Phys. Rev.* **E48**, 2402 (1993).

[13] J. Kossut, Z. Wilamowski, T. Dietl, K. Świątek, in *The Physics of Semiconductors*, Proc. 20th Int. Conf., Eds. E. M. Anastassakis, J.D. Joannopoulos, World, Scientific, Singapore 1990, p. 613.
[14] P. Sobkowicz, Z. Wilamowski, J. Kossut, *Acta Phys. Pol.* **A84**, 685 (1993).
[15] R. Tao, *Phys. Rev.* **A43**, 5284 (1991).
[16] X.-J Wang, *Phys. Rev.* **A45**, 8407 (1992).
[17] J. Klafter, A. Blumen, M.F. Shlesinger, *Phys. Rev.* **A35**, 3081 (1987).
[18] J. Klafter, G. Zumofen, M.F. Shlesinger, *Physica* **A200**, 222 (1993).
[19] H. Scher, E W. Montroll, *Phys. Rev.* **B12**, 6 (1975).
[20] A. Aharony, O. Entin-Wohlman, A.B. Harris, *Physica* **A200**, 171 (1993).
[21] T. Dittrich, U. Smilansky, *Nonlinearity*, **4**, 59; 85 (1990).
[22] N. Argaman, Y. Imry, U. Smilansky, *Phys. Rev.* **B47**, 4440 (1993).
[23] N. Ben-Tal, N. Moiseyev, S. Fishman, F. Bensch, H.J. Korsch, *Phys. Rev.* **E47**, 1646 (1993).
[24] A. Altland, *Phys. Rev. Lett.* **71**, 69 (1993).
[25] P. Wölfle, R. N. Bhatt, *Phys. Rev.* **B30**, 3542 (1984).
[26] S. Hikami, E. Brezin, *J. Phys.* (Paris) **46**, 2021 (1985).
[27] G.D. Mahan, *Many-Particle Physics*, Second Edition, Plenum Press, New York 1990.
[28] M.T. Béal-Monod, G. Forgacs, *Phys. Rev.* **B45**, 3971 (1992).
[29] M.T. Béal-Monod, A. Theumann, G. Forgacs, *Phys. Rev.* **B46**, 15726 (1992).
[30] W. Szott, C. Jedrzejek, W.P. Kirk, *Phys. Rev.* **B46**, 15905 (1992).
[31] D. Vollhardt, P. Wölfle, *Phys. Rev.* **B22**, 4666 (1980).
[32] R.N. Bhatt, P. Wölfle, T.V. Ramakrishnan, *Phys. Rev.* **B32**, 569 (1985).
[33] W. Szott, C. Jedrzejek, W.P. Kirk, *Phys. Rev.* **B48**, 8979 (1993).
[34] W. Szott, C. Jedrzejek, W.P. Kirk, *Phys. Rev.* **B45**, 3565 (1992).
[35] L. Pfeiffer, E.F. Schubert, K.W. West, C.W. Magee, *Appl. Phys. Lett.* **58**, 2258 (1991).
[36] M.A. Lee, S. Solin, D.R. Hines, *Phys. Rev.* **B48**, 11921 (1993).
[37] S.-R.E. Yang, S. Das Sarma, *Phys. Rev.* **B37**, 10090 (1988).
[38] S. Hikami, A.I. Larkin, Y. Nagaoka, *Progr. Theor. Phys.* **63**, 707 (1980).
[39] P.D. Dresselhaus, C.M. Papavassiliou, R.G. Wheeler, R.N. Sacks, *Phys. Rev. Lett.* **68**, 106 (1992).
[40] R. Taboryski, P.E. Lindelof, *Semicon. Sci. Technol.* **5**, 933 (1990).
[41] J.E. Hansen, R. Taboryski, P.E. Lindelof, *Phys. Rev.* **B47**, 16040 (1993).
[42] E. Palm, W.P. Kirk, to be published.
[43] H. Mathur, A.D. Stone, *Phys. Rev. Lett.* **68**, 2964 (1992).
[44] M.I. Dyakanov, V.I. Perel, *Zh. Eksp. Teor. Fiz.* **60**, 1954 (1971) [*Sov. Phys. JETP* **33**, 1053 (1971)].
[45] M. Sawicki, T. Dietl, J. Kossut, I. Igalson, T. Wojtowicz, W. Plesiewicz, *Phys. Rev. Lett.* **56**, 508 (1986).
[46] Yu.A. Bychkov, E.I. Rashba, in *The Physics of Semiconductors*, Eds. J.D. Chadi and H.A. Harrison, Springer, New York, 1985), p. 321.
[47] Y. Aharonov, A. Casher, *Phys. Rev. Lett.* **53**, 319 (1992).
[48] C. Jedrzejek, W. Szott, W.P. Kirk, to be published.

[49] R. Sizmann, F. Koch, *Semicon. Sci. Technol.* **5**, S115 (1990).
[50] R. Wollrab, R. Sizmann, F. Koch, J. Ziegler, H. Mayer, *Semicon. Sci. Technol.* **4**, 491 (1989).
[51] P.V. Santos, M. Cardona, *Phys. Rev. Lett.* **72**, 432 (1994).
[52] D. Richards, B. Jusserand, H. Peric, B. Etienne, *Phys. Rev.* **B47**, 16028 (1993).
[53] W. Szott, C. Jedrzejek, W.P. Kirk, to be published.
[54] S. Iwabuchi, Y. Nagaoka, *J. Phys. Soc. Jap.* **58**, 1325 (1989).
[55] S. Maekawa, H. Fukuyama, *J. Phys. Soc. Jap.* **50**, 2516 (1981).
[56] Y. Shapir, Z. Ovadyahu, *Phys. Rev.* **B40**, 12441 (1989).
[57] A. Stern, O. Entin-Wohlman, *Phys. Rev.* **B44**, 10976 (1991).
[58] M. Thala, R. Blumel, U. Smilansky, *Phys. Rev.* **E48**, 1784 (1993).
[59] E. Abrahams, P.W. Anderson, D.C. Licciardello, T.V. Ramakrishnan, *Phys. Rev. Lett.* **42**, 673 (1979).
[60] A. McKinnon, B. Kramer, *Z. Phys.* **B53**, 1 (1983).
[61] R. L. Weaver, *Phys. Rev.* **B49**, 5881 (1994).
[62] T. Wang, K.P. Clark, G.F. Spencer, A.M. Mack, W.P. Kirk, *Phys. Rev. Lett.* **72**, 709 (1994).
[63] H.W. Jiang, C.E. Johnson, K.L. Wang, S.T. Hannahs, *Phys. Rev. Lett.* **71**, 1439 (1993).
[64] S. Kivelson, D.-H. Lee, S.-C. Zhang, *Phys. Rev.* **B46**, 2223 (1992).
[65] G. Strinati, C. Castellani, C. DiCastro, *Phys. Rev.* **B44**, 6078 (1989).