CURRENT ENSEMBLE MODEL AND INTERMITTENCY*

B. Ziaja

Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Cracow, Poland
e-mail: beataz@ztc386a.if.uj.edu.pl

(Received March 18, 1994)

Following the idea that very short range correlations in momentum space characteristic for intermittency reflect the power law distribution of space-time region of hadron emission we investigate the origin of the power law dependence. We use the current ensemble model of pion production to describe the boson sources and show the relation between the space-time distribution of the boson sources and intermittency exponents obtained from multiplicities in momentum space.

PACS numbers: 13.85. Ni

1. Introduction

Recently, several experiments [1–3] found that the phenomenon of intermittency [4, 5] is dominated by very short range correlations between momenta of identical hadrons. As it is well known, the HBT correlations [6] reflect the size and shape of the space-time region from which the observed identical particles are emitted [7]. Remembering that intermittency is equivalent to a power law dependence of the hadronic correlation functions (multiplicity distributions), one may conclude that a power law dependence must also be present in the distribution of space-time shapes and sizes of the region of hadron emission [8]. The first analysis of this problem was done in Ref. [9] using the HBT formalism for the purely incoherent hadron source. The considered description of hadron emission was simplified: totally incoherent production from the space region was assumed and therefore the single particle distribution could not be adequately described. Nevertheless,

* Work partly supported by the KBN grants: PB 200929101 and PB 200919101.

this model could explain observed intermittency exponents increasing with rank of the multiparticle distributions.

In the present paper we investigate the relation between space-time structure of the source and multiplicity distributions in a more sophisticated model which allows to correct description of single particle distribution. We assume that pions are produced from a collection of coherent sources. This model was described in detail by Gyulassy, Kauffmann and Wilson in Ref. [10] and we will follow closely their treatment which is summarized in Section 2. The general current ensemble formalism, which was used in Ref. [10] is presented in this section. In the Section 3 we explain briefly how to measure intermittency parameters. In the Section 4 we incorporate the scaling dependence of the production region to the model. At first we discuss the simple power law source distribution following [9] and prove that it does give observable intermittent result in this model. Later we show how to generalize obtained results for any source distribution showing the scaling behaviour.

Our discussion is to prove that in a well working physical model of boson production like the current ensemble model, multiparticle distributions in momentum space can be strongly influenced by the scaling behaviour of space-time structure of the bosonic source.

2. Classical current formalism

2.1. Pion fields

In this section we would like to recall the main ideas of the current ensemble model formulated in [10]. To obtain final pion state produced by a classical current source we should solve field equation for the scalar pion field $\Phi(x)$ with the source current operator $J(x)$:

$$(\partial_\mu \partial^\mu + m_\pi^2)\Phi(x) = J(x)\,. \tag{1}$$

In principle, source current $J(x)$ is an operator coupled to the pion fields and treating it as a complex space-time function is only an approximation. Because we will not specify the conditions of production process it could be difficult to define when this approximation could be used. We assume simply that *we are allowed in our model* to replace the pion current operator $J(x)$ by its expectation value [10].

Solution of (1) gives the coherent final multipion state :

$$|\Phi\rangle = e^{-n/2}\exp\left(i\int d^3kJ(k)a^\dagger(k)\right)|0\rangle\,, \qquad \langle\Phi|\Phi\rangle = 1\,, \tag{2}$$

where $a(\boldsymbol{k})$, $a^{\dagger}(\boldsymbol{k})$ are creation and annihilation operators, n is the average pion multiplicity and $J(\boldsymbol{k})$ is the on-mass-shell Fourier transform of $J(x)$ [7, 10]. Pion density matrix ρ_π constructed from (2) is:

$$\rho = |\Phi\rangle\langle\Phi| \,. \tag{3}$$

Multipion-inclusive distributions $P_m(\boldsymbol{k}_1, \boldsymbol{k}_2 \ldots, \boldsymbol{k}_m)$ are defined by the formula:

$$\begin{aligned} P_m(\boldsymbol{k}_1, \boldsymbol{k}_2 \ldots, \boldsymbol{k}_m) &= \frac{1}{\sigma_\pi} \frac{d^{3m}\sigma(\pi\pi\ldots\pi)}{d^3k_1 d^3k_2 \ldots d^3_m} \\ &= \mathrm{Tr}\left[\rho_\pi a^{\dagger}(\boldsymbol{k}_1)\ldots a^{\dagger}(\boldsymbol{k}_m) a(\boldsymbol{k}_m)\ldots a(\boldsymbol{k}_1)\right] \,. \end{aligned} \tag{4}$$

2.2. Pion source as a current ensemble

Now we are ready to consider the following description of pion emission: pions are produced in some space-time centres $x_1, \ldots, x_N$, distributed with probability density $\rho(x)$. These centres can be for example the effect of N separate collisions "producing" pions [10]. In this picture, the total pion source $J(x)$ would thus be a sum of N different currents $J_i(x)$:

$$J(x) = \sum_{i=1}^{N} J_i(x) \,. \tag{5}$$

We shall consider the situation when the $J_i(x)$ depend only on distance from the individual collision site x_i. It means that if the collision centered in $x = 0$ is parametrized by $j_\pi(x)$ we get:

$$J(x) = \sum_{i=1}^{N} j_\pi(x - x_i) \tag{6}$$

and on-mass-shell Fourier transform is given by:

$$J(\boldsymbol{k}) = j_\pi(\boldsymbol{k}) \sum_{i=1}^{N} \exp\left(i\omega_k t_i - i\boldsymbol{k}\boldsymbol{x}_i\right) \,, \quad \omega_k = \sqrt{\boldsymbol{k}^2 + m_\pi^2} \,. \tag{7}$$

2.3. Multipion distributions for current ensemble

The density matrix ρ_π averaged over the number of sources N and positions x_i of the pion sources is :

$$\rho_\pi = \sum_N P(N) \int d^4x_1 \dots d^4x_N \rho(x_1)\dots\rho(x_N)|\Phi\rangle\langle\Phi| , \qquad (8)$$

where $P(N)$ denotes the probability to find exactly N pion sources and $\rho(x)$ is the probability density to find the source placed at point x. Density $\rho(x)$ is normalized to 1. Using the formulae (2), (4) and (8) one can obtain the inclusive multipion distribution $P_m(k_1,\dots,k_m)$ in the form:

$$P_m(k_1,\dots,k_m) = \sum_N P(N) \int d^4x_1 \dots d^4x_N \rho(x_1)\dots\rho(x_N)|J(k_1)|^2\dots|J(k_m)|^2 . \qquad (9)$$

Substituting $J(k)$ in (9) by (7) one obtains:

$$P_m(k_1,\dots,k_m) = |j_\pi(k_1)|^2\dots|j_\pi(k_m)|^2 \sum_N P(N) \int d^4x_1\dots d^4x_N \rho(x_1)\dots\rho(x_N)$$
$$\times \sum_{i_1=1}^{N} \dots \sum_{i_{2m}=1}^{N} \exp\left(ik_1(x_{i_1}-x_{i_2}\right)\dots\exp\left(ik_m(x_{i_{2m-1}}-x_{i_{2m}}\right) . \qquad (10)$$

One should remember that in the scalar product of 4- dimensional vectors k_i and x_j the first component of k_j is equal to ω_{k_j} as a result of on-mass-shell Fourier transform in (7). The evaluation of (10) is complicated by the combinatorial problem of how many of the N^{2m} terms in (10) have a given number of the indices $(i_1, i_2,\dots,i_{2m})$ equal to one another. The formulae for single- and double pion distribution are:

$$P_1(k_1) = |j_\pi(k_1)|^2 \left(\langle N\rangle + \langle N(N-1)\rangle|\rho(\omega_k,k)|^2\right) , \qquad (11)$$

$$\begin{aligned} P_2(k_1,k_2) = {} & |j_\pi(k_1)|^2|j_\pi(k_2)|^2 \\ & \times \Big(\langle N^2\rangle + \langle N(N-1)\rangle\left[|\rho(k_1-k_2)|^2 + |\rho(k_1+k_2)|^2\right] \\ & + \langle N^2(N-1)\rangle\left[|\rho(k_1)|^2 + |\rho(k_2)|^2\right] \\ & + \langle N(N-1)(N-2)(N-3)\rangle|\rho(k_1)|^2|\rho(k_2)|^2 \\ & + \langle N(N-1)(N-2)\rangle\Big[\rho(k_1-k_2)\rho^*(k_1)\rho(k_2) \\ & + \rho(k_1+k_2)\rho^*(k_1)\rho^*(k_2) + \text{c.c.}\Big]\Big) . \end{aligned} \qquad (12)$$

One should notice that for $N = 1$ we recover the coherent field results. Produced particles are then uncorrelated. Hence we exclude the case $N = 1$ from our further investigation.

3. Intermittency exponents

3.1. General definition

Intermittency is equivalent to the power law dependence of the hadronic multiparticle distributions in momentum space. This dependence is described by the intermittency exponents. We would like to discuss exact results for intermittency exponents we can get in the usual way by integrating pion multiplicities $P_m(k_1, \ldots, k_m)$ in the finite region of size δ. We obtain a kind of series in δ:

$$K_m(\delta) = \int_0^\delta P_M(k_1, \ldots, k_m) d^3k_1 \ldots d^3K_m = \sum_j a_j (L\delta)^{-\alpha_j}\,, \qquad (13)$$

where α_j are intermittency exponents, a_j weights and the length L is introduced for dimensional reasons. Experimentally we do not observe such a kind of series in $(L\delta)$. We observe only the term dominating in (13). *So, for our purposes we define the intermittency exponent f_m to be equal to α_j taken from the term dominating in (13).* We should notice it need not to be the term with max/min(α_j) because of weights a_j.The existing data show intermittency exponents increasing with rank of the multiparticle distributions.

3.2. Leading terms for current ensemble multiplicities

Now we will discuss leading terms in (13) for pion multiplicities in current ensemble formalism (10). At first we consider cases (11) and (12). After integrating (11) and (12) in the region $(0, \delta)$ we get respectively:

$$K_1(\delta) = \langle N \rangle c_{11}(\delta) + \langle N(N-1) \rangle c_{12}(\delta)\,, \qquad (14)$$

$$\begin{aligned} K_2(\delta = {} & \langle N^2 \rangle c_{21}(\delta) + \langle N(N-1) \rangle c_{22}(\delta) + \langle N^2(N-1) \rangle c_{23}(\delta) \\ & + \langle N(N-1)(N-2)(N-3) \rangle c_{24}(\delta) + \langle N(N-1)(N-2) \rangle c_{25}(\delta)\,, \end{aligned} \qquad (15)$$

where c_{ij} are results of integrating (11) and (12): they depend on δ but *they are independent of N*. One can notice that for large N term with c_{12} in (14) and term with c_{24} in (15) will dominate. For small N one should take into account also the contributions of other c_{ij} *i.e.* relations among c_{ij} and then carefully investigate which term is leading. "Large" and "small" N mean here that averages we consider in (14), (15) are respectively large and small. It can be observed *i.e.* for "spike" number distribution $P(N)$ with large/small $\langle N\rangle$.

It could be difficult to write down explicitly all terms in Eq. (13). Fortunately, for our purposes it is enough to calculate only the ranks of weights from evaluated formula (10). Some weights are presented below:

term $\left(|\rho(\boldsymbol{k}_1)|^2|\rho(\boldsymbol{k}_2)|^2\dots|\rho(\boldsymbol{k}_m)|^2\right)$

with weight $\langle N(N-1)(N-2)\dots(N-2m+1)\rangle\,,$ (16)

where $N>2m-1$

term $\left(|\rho(\boldsymbol{k}_i)|^2\right)$ with weight $\langle N(N-1)N^{m-1}\rangle\,,$ where $m>1$ etc. (17)

4. Scaling behaviour of the distribution $\rho(x)$

In this section we show that the scaling of distribution of collision sites $\rho(x)$ (introduced in (8)) *can* produce scaling of momentum distribution P_m. Let us assume that distribution $\rho(x)$ of pion production centres can be factorized into the space part $\rho_{\rm s}$ and the time part $\rho_{\rm t}$:

$$\rho(x)=\rho_{\rm t}(t)\rho s(|\boldsymbol{x}|)\,. \tag{18}$$

We further assume that the space part follows a power law:

$$\rho_{\rm s}(\boldsymbol{x})=L^{-\alpha}|\boldsymbol{x}|^{\alpha-3},\quad \alpha<3\,, \tag{19}$$

where the length L was introduced for dimensional reasons. The time part $\rho_{\rm t}$ describes the typical time evolution of the production process (it can be for example Gaussian function of time in the form: $\exp(-t^2/T^2)$ and *has no singularity for any t. We assume also that $\rho_{\rm t}$ is normalized to unity.*

The exact power law behaviour in (19) implies that the $\rho(x)$ in (18) cannot be normalized. Following [9] we can introduce a kind of cut-off to get rid of this problem:

$$\rho_{\rm s}(\boldsymbol{x})=|\boldsymbol{x}|^{\alpha-3}L^{-\alpha}\Theta(L-|\boldsymbol{x}|)\alpha(4\pi)^{-1}\,. \tag{20}$$

In this case L can be interpreted as a size of pion production region. Using the relations for 4-dimensional Fourier transform:

$$\rho(k)=\int d^4x e^{ikx}\rho(x)\,, \tag{21}$$

$$\rho(k=0) = \int d^4x\rho(x), \tag{22}$$

one obtains the distribution in momentum space:

$$\rho(k) = \alpha\rho_t(k_0)(L|\boldsymbol{k}|)^{-\alpha} \int_0^{L|\boldsymbol{k}|} du\, u^{\alpha-2} \sin u\,. \tag{23}$$

One can observe *the power law behaviour in (23) only for* $L|\boldsymbol{k}| \geq 1$. *For* $L|\boldsymbol{k}| \leq 1$ *the singularity will be cut off and* $\rho(\boldsymbol{k})$ *in (23) tends smoothly to 1 for* $L|\boldsymbol{k}| \to 0$. *We notice also that* $\rho(\boldsymbol{k}) \leq 1$ *for any* $\boldsymbol{k}$.

Now we will analyse the multipion distribution $P_m(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_m)$ to get the dominating term there. This term will produce intermittency exponents in (13). Let us consider at first distribution $P_1(\boldsymbol{k}_1)$ obtained from (11) and (14). If N is *large enough* the term with $c_{12}(\delta)$ will be dominating and from (13) we obtain intermittency exponent in the form:

$$f_1 = 2\alpha\,. \tag{24}$$

From the distribution $P_2(\boldsymbol{k}_1, \boldsymbol{k}_2)$ we will get term $c_{24}(\delta)$ dominating in (15) for large N. Hence intermittency exponent is:

$$f_2 = 4\alpha\,. \tag{25}$$

Now we are ready to consider leading terms in $P_m(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_m)$ for any $m \geq 1$. As it was already mentioned in Section 2.2 we will not exactly calculate the weights in (13). We assume N *to be large* and then weights in (13) fulfill the following relation:

$$\text{terms with} \quad (L\delta)^{2k\alpha} \quad \text{behave like} \quad N^{m+k}\,, \text{ where } k = 1, \ldots, m\,, \tag{26}$$

$$\text{terms with} \quad (L\delta)^{(2k-1)\alpha} \quad \text{behave like} \quad N^{m+k-1}\,, \text{ where } k = 2, \ldots, m\,, \tag{27}$$

Hence we get the intermittency exponent in the form :

$$f_m = 2m\alpha\,, \tag{28}$$

because the term (26) with $k = m$ will be dominating in (13). The result we have got above is different from the intermittency exponents obtained in [9]. In [9] the intermittency exponents fulfilled the relation:

$$f_m{}' = m\alpha\,. \tag{29}$$

Nevertheless, because f_m and $f_m{}'$ fulfill the relation:

$$f_m = 2f_m{}', \tag{30}$$

the relative relations among f_m and $f_m{}'$ are the same:

$$\frac{f_k}{f_l} = \frac{f_{k'}}{f_{l'}}. \tag{31}$$

All calculations we have made to get f_m request the assumption N to be large. For small N the formulae are very complicated and it is difficult to see how they could provide the simple power law behaviour as observed in experiment.

The results [28] can be actually derived under much less restrictive conditions. To see this, let us first formulate general conditions function $\rho(x)$ must fulfill to be considered as the probability distribution and to show the scaling behaviour:

$$\rho_S(\boldsymbol{x}) \geq 0, \tag{32}$$

$$\int \rho_S(\boldsymbol{x}) d^3\boldsymbol{x} = 1, \tag{33}$$

$$\rho_S(\boldsymbol{k}) \sim \text{const} \cdot (L|k|)^{-\alpha} \text{ in some interval } L|k| \in (a,b) \text{ when } a, b \text{ exist}. \tag{34}$$

One can notice that conditions (32), (33) give the following inequality for 3-dimensional Fourier transform:

$$\rho_S(\boldsymbol{k}) \leq \rho_S(\boldsymbol{k}=0) = 1 \quad \text{for any} \quad \boldsymbol{k}. \tag{35}$$

Now we can easily generalize the results obtained for collision sites distribution $\rho(x)$ defined in (18), (19). For any function $\rho(x)$ which fulfill conditions (32), (33), (34) we get the intermittency exponents behave like (28) increasing with the rank of pion multiplicity for large N. The inequality (35) allows to expect that factors $c_{mi}(\delta)$ in $K_m(\delta)$ are not very large. It means that weights contributions are rather small and hence N do not need to be very large to give intermittency behaviour (28).

5. Conclusions

We have investigated the relation between intermittency and scaling behaviour of space-time collision sites distribution in current ensemble model. Our conclusions can be summarized as follows:

- there is a possibility of intermittency in current ensemble model provided that
  - (a) one assumes power law singularity in distribution of collision sites
  - (b) the number of coherent sources N is large enough. Then intermittency exponents grow with the increasing rank of multiplicities following the Eq. (28).
- for small number of sources N intermittency is generally not observed. There are no well defined leading terms which can give intermittency exponents growing with the rank of moments/ multiplicities as we observe in experiment.

However, we have proved that in a well working physical model of boson production like the current ensemble model, multiparticle distributions in momentum space can be *influenced* by the scaling space-time structure of the bosonic source and give observed *intermittent results. The problem is if the assumption N to be large is realized in nature.* If not, we are forced to conclude that the current ensemble model of intermittent boson production cannot describe intermittency.

I would like to thank Professor A. Bialas for many stimulating discussions and a continuous interest in this work. I appreciate also to Professor K. Fialkowski for his reading the manuscript and helpful comments.

REFERENCES

[1] UA1 Coll., B. Buschbeck *et al.*, CERN-PPE/93-152.
[2] NA22 Coll., M. Atanyan *et al.*, *Z. Phys.* **C54**, 247 (1992).
[3] I. Derado, G. Jancso, N. Schmitz, *Z. Phys.* **C56**, 553 (1992).
[4] A. Bialas, R. Peschanski, *Nucl. Phys.* **B273**, 703 (1986).
[5] A. Bialas, R. Peschanski, *Nucl. Phys.* **B306**, 857 (1988).
[6] R. Hanbury-Brown, R.Q. Twiss, *Nature* 177 (1957).
[7] D.H. Boal, C.-K. Gelbke, B.K. Jennings, *Rev. Mod. Phys.* **62**, 553 (1990).
[8] A. Bialas, *Nucl. Phys.* **A525**, 345c (1991).
[9] A. Bialas, B. Ziaja, *Acta Phys. Pol.* **B24**, 1509 (1993).
[10] M. Gyulassy, S. Kaufmann, L.W. Wilson, *Phys. Rev.* **C20**, 2267 (1979).