STRONG INTERACTION CORRECTIONS TO ATOMIC PARITY VIOLATION*

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We show that if vector current conservation is consistently implemented, an interesting subclass of graphs relevant for the strong interaction corrections to atomic parity violation has a vanishing small effect on the determination of $\sin^2\theta_W$.

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Although the Z boson has a mass which is huge on atomic length scales, weak neutral currents still have a measurable effect on atomic phenomena. As was first shown by Bouchiat & Bouchiat, [1] virtual Z exchange between the orbital electrons and an atomic nucleus induces a parity violating contribution, $\mathcal{L}^{\text{eff}}_{PV}$, to the atomic potential which is given to a good approximation by

$$\frac{2G_F}{\sqrt{2}}\sum_q\left(g_q^{\rm L}+g_q^{\rm R}\right)\bar{q}\gamma_\mu q\left(g_e^{\rm L}-g_e^{\rm R}\right)\bar{e}\gamma^\mu\gamma^5 e\,, \tag{1}$$

where the sum is over all valence quarks in the nucleus. The sensitivity of present day experiments in conjunction with refinements on the theoretical side, due to the inclusion of the effects of electro-weak radiative corrections, [2] have permitted a high precision determination of the Z-Nucleus coupling Q_W, and thereby of $\sin^2\theta_W$ [3]. More recent work [4] has shown that $\sin^2\theta_W$ determined in this manner is, for certain nuclei, only weakly dependent on the top mass, and therefore of particular interest for precision tests of the Standard Model. In order to completely pin down the systematic uncertainties in the determination of $\sin^2\theta_W$ it is necessary to consider

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the hitherto neglected strong interaction corrections. At the relevant length scales the effects of QCD corrections cannot be described through quark and gluon loops, but rather through meson loops. However, typical meson--nucleon couplings (*e.g.* $g_{\pi NN}$) are *large* compared with electro-weak couplings, indeed large enough to cast serious doubt on the validity of a purely perturbative determination of $\sin^2 \theta_W$. In what follows, we will show that current conservation arguments can be used to reliably estimate the size of these corrections, large coupling constants notwithstanding.

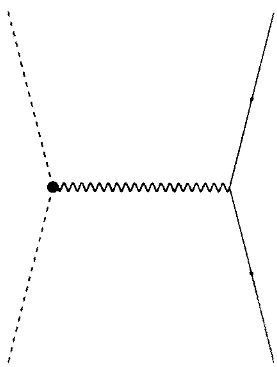

Fig. 1. The three level graph. Dotted lines represent the nucleus, the wavy line the Z propagator.

There are three categories of graphs relevant for the corrections we are interested in. The only graph in which meson-nucleon couplings do not play a role is shown in Fig. 2. A precise calculation of this graph requires knowledge of the VV and AA correlation functions beyond perturbation theory. However, the presence of a second virtual Z propagator leads to an additional suppression of G_F compared with the Born level diagram (Fig. 1), making this contribution too small to be interesting. A similar situation arises in the consideration of radiative corrections to the lifetime of the muon, where the insertions of hadronic vacuum polarization bubbles into the W propagator are generally neglected as they are numerically insignificant [5].

It is not easy to dismiss the graphs shown in Fig. 3. However, it would be grossly misleading to perform the loop integral without taking into account the running of the meson-nucleon-nucleon form factors to arbitrarily large space-like momenta. Since this cannot be done from first principles, the only possibility is to work within the framework of the specific model, which we would like to avoid. Hence, we will not discuss this potentially interesting contribution in any detail, but merely point out that the effect of such graphs can probably be accounted for by the nuclear form factors.

We from now, focus mainly on the graphs in Fig. 4 which as we will

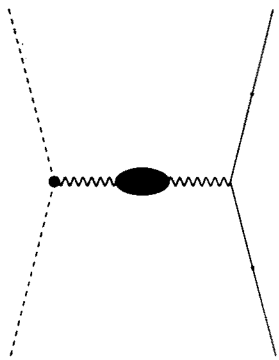

Fig. 2. The blob represents the hadronic self-energy of the Z. Dotted lines represent the nucleus, the wavy line the Z propagator.

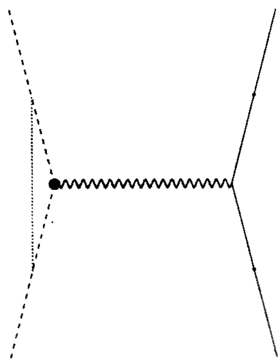

Fig. 3. Mesonic vertex correction to the Z *nucleus-nucleus* coupling. The short dotted line represents any meson with a non-vanishing coupling to the Nucleus, the wavy line the Z propagator. There are similar graphs containing Z-*meson* couplings.

show can be reliably estimated. Before doing so, it is crucial to recall that although the nucleus couples to the Z via both axial and vector form factors, the axial form factor (in the non-relativistic limit we are interested in) gets a contribution only from a nucleon whose spin is not paired while the vector form factor gets contributions from all nucleons. Hence, for a massive nucleus the vector contribution to Q_W is the dominant one, which is implicit in Eq. (1). For this reason, we may restrict ourselves to the effects of vector meson exchange in Fig. 4. Furthermore, we assume all contributions from strange mesons are Zweig rule suppressed. We are effectively dealing only with the first family of fermions. We will next try to estimate the coupling constant of the Z to various vector mesons built out of u and d quarks.

The first step in this process is to make an isospin decomposition of the vector piece of hadronic weak neutral current. Since Z couplings are

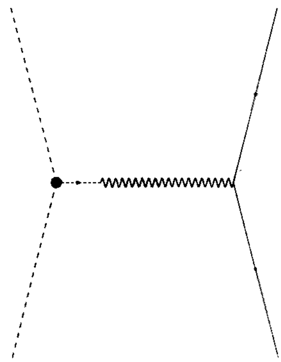

Fig. 4. Vector meson correction to the Z-nucleus-nucleus coupling. Dotted lines represent the nucleus, the wavy line the Z propagator.

flavour diagonal, only iso-scalar (I_0^μ) and the third component of isospin (I_3^μ) pieces can occur. It is tempting to treat the isoscalar current as an interpolating field for isoscalar vector mesons such as the ω and I_3^μ as an interpolating field for vector meson such as the ρ, up to overall constants. However, if we require that the electron in Fig. 4 is non-relativistic (which is often the case) then the vector meson is far off-shell, and form factor effects must also be taken into account. This amounts to assuming that the correct normalization factor relating currents and fields is momentum dependent.

In order to estimate the functional form of the relevant form factor, it is convenient to repeat the same isospin decomposition for the electromagnetic current. Once again, we only get contributions from the third component of isospin and isoscalar currents and once again, we are confronted with the problem of from factor dependence, which is now constrained by the requirement of U(1) gauge-invariance. The usual vector meson dominance prescription of introducing vertices of the form $\sim A_\mu(\rho^\mu, \omega^\mu)$ using their leptonic decay widths to fix the overall normalization violates gauge-invariance and is otherwise phenomenologically unacceptable whenever the mesons are off-shell [6]. As shown in [6], all these problems can be avoided by requiring an explicitly gauge-invariant ρ-A coupling which must be of the form $\rho_{\mu\nu}F^{\mu\nu}$. In momentum space, this generates a vertex of the form $A^\mu(q^2 g_{\mu\nu} - q^\mu q^\nu)\rho^\nu$, with a similar ω-A vertex. For an on-shell vector meson, it is straightforward to relate this prescription to the conventional one. When the internal vector meson couples only to conserved currents, then the $q_\mu q_\nu$ term drops out, leaving behind an interaction term of the form $A^\mu\rho_\mu q^2$. What we have done in effect is to use the requirement of U(1) gauge invariance to constrain the momentum space dependence of the ρ-photon form factor. This form factor behaviour is not a consequence of the fact that one is considering an electromagnetic coupling, but rather arises from the process of generating vector-meson fields by "integrating out" the energetic degrees of freedom in the $I_z = 3$ component of the isospin current constructed from quarks. Of course, the precise momentum-dependence of the form-factor is much more complicated, but we now have all the information we need in order to estimate corrections to $\sin^2\theta_W$ as determined from atomic parity violation experiments.

We will now attempt to write down the amplitude for the Feynman graph shown in Fig. 4. Assuming the ρ couples to a conserved nuclear current (which is actually a common ingredient of many phenomenological models *e.g.* [7]) one can drop the $q^\mu q^\nu$ term in the ρ and Z propagators. Since we are interested only in parity violating contributions, we retain only the axial part of electron current which couples to the Z. With these assumptions we may write the amplitude as (up to overall constants)

$$\tilde{\lambda}(q^2)\frac{q^2}{M_Z^2(q^2 - m_\rho^2)}\mathcal{N}_\mu\left(g_e^{\mathrm{L}} - g_e^{\mathrm{R}}\right)\bar{e}\gamma^\mu\gamma^5 e\,,$$

where $\mathcal{N}^\mu$ denotes the vector current to which the ρ couples and $\tilde{\lambda}(q^2)$ contains the residual momentum dependence of the form factor which we were unable to determine by current conservation arguments alone. This formula bears a striking resemblance to the Born level formula, except for the factor

$$\tilde{\lambda}(q^2)\frac{q^2}{q^2 - m_\rho^2}$$

which for a non-relativistic electron ($q^2 \to m_e^2$) is vanishingly small even if the ρ Nucleus coupling which we have absorbed into $\mathcal{N}^\mu$ is $\mathcal{O}$ (Number of Nucleons)$\times g_{\rho NN}$. We have thus demonstrated that the Z-Nucleus coupling is infinitesimally rescaled. However, we have implicitly assumed that $\tilde{\lambda}(q^2)$ does not have a pole as $q^2 \to 0$.

To justify this assumption and as a cross-check on our reasoning, we observe that on very general grounds, the Z-ρ mixing must be proportional to the hadronic VV correlation function, which as a consequence of vector current conservation is proportional to $(q^2 g^{\mu\nu} - q^\mu q^\nu)$, which is exactly the form we had before. If the overall (q^2 dependent) constant of proportionality indeed had a pole at small q^2, then the renormalized photon propagator, which is sensitive to a very similar correlation function, would not have a simple pole at $q^2 = 0$. Such a situation could arise for example if there would be a Goldstone pole in the vector channel, in which case the Higgs mechanism would force a nonzero photon mass. This justifies our claim that $\tilde{\lambda}(q^2)$ must be well behaved for $q^2 \to 0$. Similar arguments hold for other vector mesons, ρ', ω, *etc.*

This completes our demonstration that the class of graphs we have been considering gives a vanishing small ($\mathcal{O}(10^{-6})$) contribution to the renormalization of Q_W. It is important to note that we have relied *only* on vector current conservation and U(1) gauge invariance to arrive at this result, which is thus probably quite model independent.

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