BUNCHING PARAMETER AND INTERMITTENCY IN HIGH-ENERGY COLLISIONS

S.V. Chekanov, V.I. Kuvshinov

Institute of Physics, Academy of Sciences of Belarus
F. Scaryna Av. 70, 220072 Minsk, Belarus

(Received April 27, 1994)

We introduce the parameter of bunching for analysis of intermittent structure of multihadron production at high-energy collisions following analogy with photon counting in quantum optics. It is shown the existence of a power-law singularity for second-order bunching parameter in small phase-space intervals for monofractal structure of multiplicity distribution and the similar form of high-order parameters for multifractality. The approximation of the high-order bunching parameters by the second-order provides good description of anomalous fractal dimensions for some experimental data with multifractal behaviour.

PACS numbers: 13.85. Hd, 45.50. Dv

1. Introduction

The idea of applying stochastic methods developed for study photon counting statistics of quantum optics to particle production processes was used for a long time [1, 2]. At present, a systematic and careful investigation of multihadron production with the application of methods of quantum optics is very useful because there is a large analogy between these fields of physics. For instance, the interpretation of multiplicity distributions in terms of hadronic field states by analogy to photon counting [3, 4] and squeezed gluon states study [5] seem to be important directions for theoretical research. The problem of damping of the statistical noise and concept of factorial moment analysis to study multihadron production [6] were well-known in quantum optics [7]. Correlators in terms of the moments [8] have analogous form in quantum optics [9].

The purpose of the present paper is to extend some methods of continuous quantum measurement in quantum optics to high-energy physics. We introduce the bunching parameter for analysis of fractal structure of multihadron production. A nontrivial behaviour of it for small phase-space interval for intermittent structure of multiplicity distribution is obtained.

2. The bunching parameter

The bunching parameter $\check{\eta}_n(\delta t)$ of order n in theory of continuous quantum measurement can be expressed for one-mode photon field in terms of the probability $P_n(\delta t)$ to have n photons in time interval δt [10] in the form

$$\check{\eta}_n(\delta t) = \frac{n}{n-1} \frac{P_n(\delta t) P_{n-2}(\delta t)}{(P_{n-1}(\delta t))^2}, \qquad n > 1 . \qquad (1)$$

This parameter determines how does the probability to detect n photons in δt change relatively to the probability to detect $n-1$ photons in the same time interval. If source of light is completely coherent then $\check{\eta}_n(\delta t) = 1$. The corresponding multiplicity distribution is the Poisson one. A radiation field is said to be statistical antibunched in order n if $\check{\eta}_n(\delta t) < 1$ and when $\check{\eta}_n(\delta t) > 1$, then it is said to be bunched of photons in δt. For a wide class of states the bunching parameter is independent of time interval [10].

By analogy with (1), let us consider the bunching parameters (BPs) $\eta_n(\Delta)$ for the multiplicity distributions of secondary particles produced in high-energy interaction

$$\eta_n(\Delta) = \frac{n}{n-1} \frac{P_n(\Delta) P_{n-2}(\Delta)}{(P_{n-1}(\Delta))^2}, \qquad n > 1, \qquad (2)$$

where $P_n(\Delta)$ is the probability to have n particles in Lorentz invariant phase-space interval Δ, for example in rapidity, azimuthal angle and transverse momentum or in just one of these alone.

There is a large class of distributions which has the BPs independent of Δ. Let us find this class. By applying formula (2), any multiplicity distribution can be expressed as

$$P_n(\Delta) = P_0(\Delta) \frac{\lambda^n(\Delta)}{n!} \prod_{m=2}^{n} (\eta_m(\Delta))^{n+1-m}, \qquad n > 1, \qquad (3)$$

where we define $\lambda(\Delta) = P_1(\Delta)/P_0(\Delta)$. If $\eta_s(\Delta)$ is independent of Δ, one gets the general form of the generation function for such distribution

$$G(z, \Delta) \equiv \sum_{n=0}^{\infty} z^n P_n(\Delta) = G(z=0, \Delta) Q\left(z\lambda(\Delta)\right), \qquad (4)$$

where $G(z=0,\Delta)=P_0(\Delta)$, the $Q(z\lambda(\Delta))$ is some analytic function of auxiliary variable z multiplied by function $\lambda(\Delta)$ under condition $Q(\lambda(\Delta))=1/G(z=0,\Delta)$. It is easy to see that the condition (4) is fulfilled for such well-known distributions as Poisson, binomial, and geometric ones. The case of negative binomial distribution will be discussed later.

Remind that the observed behaviour of the normalized factorial moments (NFM)

$$F_k(\Delta)\equiv\frac{\langle n^{[k]}\rangle}{\langle n\rangle^k}\propto\Delta^{-d_k(k-1)}, \qquad \Delta\to 0 \qquad k\geq 2, \tag{5}$$

is a straightforward manifestation of the nonstatistical intermittent fluctuations in the distributions of secondary particles produced in high-energy interactions [6, 11–12]. In (5) n denotes the number of particles in Δ, $n^{[k]}\equiv n(n-1)\ldots(n-k+1)$, $\langle\ldots\rangle$ is average over all events. The right side of (5) represents by definition the intermittent behaviour characterized by a anomalous fractal dimensions (AFD) d_k depending on rank k of NFM for multifractal behaviour and $d_k=$ const. for monofractality.

Now we shall prove the following statement: a inverse power dependence of the second order BP in phase-space interval Δ and Δ-independence of high-order BPs are necessary and sufficient conditions for monofractal behaviour of AFD. A inverse power Δ-dependence of all BPs is necessary and sufficient for multifractality.

Sketch of proof. By applying (3), for obtaining NFM in terms of BPs, we have

$$F_k(\Delta)=\frac{P_0(\Delta)}{\langle n\rangle^k}\sum_{n=k}^{\infty}\frac{n^{[k]}}{n!}\lambda^n(\Delta)\prod_{m=2}^{n}[\eta_m(\Delta)]^{n+1-m}, \tag{6}$$

$$\langle n\rangle=P_0(\Delta)\lambda(\Delta)+P_0(\Delta)\sum_{n=2}^{\infty}\frac{\lambda^n(\Delta)}{(n-1)!}\prod_{m=2}^{n}[\eta_m(\Delta)]^{n+1-m}. \tag{7}$$

Assuming the approximate proportionality of $\langle n\rangle$ and Δ at small Δ and condition $P_0(\Delta)\to 1$, if $\Delta\to 0$, we have the following approximate expression for small Δ

$$F_k(\Delta)\simeq\prod_{m=2}^{k}[\eta_m(\Delta)]^{k+1-m}. \tag{8}$$

In the case of power-law dependence of NFM (5) with the monofractal behaviour of AFD $d_k=d_2=$ const. we must demand the following properties of BPs

$$\eta_2(\Delta)\propto\Delta^{-d_2}, \qquad 0<d_2<1, \tag{9}$$

$$\eta_s(\Delta)\simeq\text{const.}, \qquad s>2. \tag{10}$$

From expression (8) one can conclude that for multifractality a power-law singularity of the BPs of order $s > 2$ is necessary. Using (8) it is easy to show the sufficient conditions of a inverse power-law behaviour of BPs for both monofractal and multifractal cases.

Now we have possibility to write down the general form of generation function $G^{\mathrm{mon}}(z, \Delta)$ for the multiplicity distribution with monofractal behaviour of AFD

$$G^{\mathrm{mon}}(z, \Delta) = G^{\mathrm{mon}}(z=0, \Delta) \left(1 + \eta_2^{-1}(\Delta) Q^{\mathrm{mon}} \left(\lambda(\Delta)\eta_2(\Delta) z\right)\right) . \quad (11)$$

where $\eta_2(\Delta)$ is defined by (9), Q^{mon} is some analytic function with variable $\lambda(\Delta)\eta_2(\Delta)z$ with the next conditions

$$Q^{\mathrm{mon}} \left(\lambda(\Delta)\eta_2(\Delta) z = 0\right) = 0 , \quad (12)$$

$$Q^{\mathrm{mon}} \left(\lambda(\Delta)\eta_2(\Delta) z = \lambda(\Delta)\eta_2(\Delta)\right) = \eta_2(\Delta) \left(\frac{1}{G^{\mathrm{mon}}(z=0)} - 1\right) . \quad (13)$$

General formal form of generating function for multifractal behaviour one can obtain from (3).

3. The BPs for negative binomial distribution

Since a few years, many high energy multiparticle data at various energies have been successfully fitted by the negative binomial distribution (NBD) [13–16] with generation function

$$G^{\mathrm{NBD}}(\delta y, z) = \left(1 + \frac{\langle n(\delta y)\rangle}{k(\delta y)}(1-z)\right)^{-k(\delta y)} , \quad (14)$$

where $\langle n(\delta y)\rangle$ is average multiplicity of final hadrons in restricted rapidity (or pseudorapidity) intervals δy and $k(\delta y)$ is a positive parameter. If the $k(\delta y)$ does not depend on δy, we have no any fractal type of behaviour for NFM of this distribution. Really in this case one can rewrite the generation function in the form (4).

In general case the BPs of NBD are given by the expressions

$$\eta_n^{\mathrm{NBD}}(\delta y) = \frac{k(\delta y) + n - 1}{k(\delta y) + n - 2} , \qquad n = 2, \ldots \quad (15)$$

Let us assume that $k(\delta y) \propto \delta y^{d_2}$. In this case $\eta_2^{\mathrm{NBD}}(\delta y) \propto \delta y^{-d_2}$ and $\eta_s^{\mathrm{NBD}}(\delta y) \simeq$ const., $s > 2$ for small δy. According to Section 2, one gets the monofractal type of behaviour for AFD, *i.e.* $d_n = d_2 =$ const.. Such monofractal behaviour has already been discussed in Ref. [17, 18]. This analysis only illustrate the simplicity of our approach to intermittency in terms of BPs.

4. The Levy-law approximation

In this section we show the possible behaviour of the bunching parameters in rapidity bin for different high-energy collisions.

At the beginning, let us note that for an investigation of intermittency in rapidity bin δy usually [6] one average the factorial moments over all bins of equal width δy normalizing to the overall average number per bin $\overline{\langle n\rangle} \equiv \sum_{m=1}^{M}\langle n_m\rangle/M$, where $\langle n_m\rangle$ is average multiplicity in mth bin, $M = Y/\delta y$, Y is full rapidity interval

$$\bar{F}_k(\delta y) \equiv \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m^{[k]}\rangle}{\overline{\langle n\rangle}^k} \simeq C_k(\delta y)^{-d_k(k-1)}, \tag{16}$$

where C_k are some constants. Similarly, one can introduce the BPs by averaging the probability $P_n^m(\delta y)$ for mth bin over all M bins

$$\bar{\eta}_n(\delta y) = \frac{n}{n-1}\frac{\bar{P}_n(\delta y)\bar{P}_{n-2}(\delta y)}{\left(\bar{P}_{n-1}(\delta y)\right)^2}, \qquad n > 1, \tag{17}$$

where $\bar{P}_n(\delta y) \equiv \frac{1}{M}\sum_{m=1}^{M} P_n^m(\delta y)$.

Following the same procedure as in Section 2, we can see that the approximate expression for NFM (16) in terms of the BPs (17) has the same form for $\delta y \to 0$ as that of Eq. (8), if we substitute $\bar{F}_k(\delta y)$ instead of $F_k(\Delta)$ and $\bar{\eta}_s(\delta y)$ instead of $\eta_s(\Delta)$. Then we have

$$\bar{\eta}_2(\delta y) \simeq \bar{F}_2(\delta y), \qquad \bar{\eta}_s(\delta y) \simeq \frac{\bar{F}_s(\delta y)\bar{F}_{s-2}(\delta y)}{\left(\bar{F}_{s-1}(\delta y)\right)^2}, \tag{18}$$

where $\bar{F}_1(\delta y) = 1$, $s > 2$. Using (16), (18), we obtain the following expression for BPs

$$\bar{\eta}_2(\delta y) \simeq C_2\delta y^{-\beta_2}, \qquad \bar{\eta}_s(\delta y) \simeq \frac{C_s C_{s-2}}{\left(C_{s-1}\right)^2}\delta y^{-\beta_s}, \tag{19}$$

where $C_1 = 1$ and

$$\beta_2 = d_2, \tag{20}$$

$$\beta_s = d_s(s-1) + d_{s-2}(s-3) - 2d_{s-1}(s-2), \qquad s > 2. \tag{21}$$

Note that we can obtain Eq. (21) using the approximation $\langle n_m^{[k]} \rangle \simeq k! P_k^m(\delta y)$ for $\langle \bar{n} \rangle \ll 1$ (the similar analysis of factorial moments in terms of the probabilities for one bin one can find in Ref. [19]).

For analysis of the parameters β_n we shall use the Levy-law approximation for AFD [20] which was introduced to describe random cascading models

$$d_n = \frac{d_2}{2^\mu - 2} \frac{n^\mu - n}{n - 1}, \tag{22}$$

with the Levy index μ. The Levy index is known as the degree of multifractality and allows a natural interpolation between the monofractal case ($\mu = 0$) and multifractality ($\mu > 0$). The case $\mu = 2$ ($d_n = nd_2/2$) corresponds to the log-normal approximation. Substituting (22) into (21), one gets the following expression

$$\beta_n = d_2 E_n, \qquad E_n \equiv \frac{n^\mu + (n-2)^\mu - 2(n-1)^\mu}{2^\mu - 2}. \tag{23}$$

In case of monofractal behaviour of AFD ($\mu = 0$) we have $\beta_s = 0$ for $s > 2$, *i.e.* the high-order BPs are independent of δy. Then the values are completely determined by the coefficients C_n. Note that in the case of multifractality the values of E_n are positive for all n and the BPs increase indefinitely for $\delta y \to 0$. Thus, in the case of multifractal behaviour one can say about strong bunching of particles in δy. Note that for log-normal approximation ($\mu = 2$): $E_n = 1$, $\beta_n = d_2$ and all bunching parameters have the same power-law behaviour $\bar{\eta}_n \propto \delta y^{-d_2}$.

Thus, there are two important cases which correspond to the monofractality and log-normal approximation for multifractality

$$\mu = 0, \quad \bar{\eta}_2(\delta y) \propto \delta y^{-d_2}, \quad \bar{\eta}_s = \text{const.}, \quad \text{for all} \quad s \geq 3, \tag{24}$$

$$\mu = 2, \quad \bar{\eta}_n(\delta y) \propto \delta y^{-d_2}, \quad \text{for all} \quad n \geq 2. \tag{25}$$

In real physical situation the Levy index μ is different for different reactions [12, 20, 21] and, strictly speaking, it is not equal to integer value:

(i) Nucleus-nucleus reaction $S - AgBr$. Levy index is $0 \leq \mu < 0.55$ (in fact it is almost a monofractal system) [20]. The value of E_n is almost zero and the BPs approximately are independent of δy. This behaviour is typical for intermittency at second-order phase transition and thus has been advocated [22] in favour of the formation of a quark-gluon plasma.

(ii) e^+e^-, pA, AA, hh reactions with $\mu \simeq 1.3 - 1.6$ [21] corresponding to the parameter $0 < E_n < 0.7$. In case of μp deep inelastic scattering the Levy index is largest, $\mu \simeq 3.2 - 3.5$ [20] and $1 < E_n < 5$. In these cases we have power-law singularity in behaviour of the BPs.

5. Simple approximation of the high-order BPs

The Levy law approximation allows a simple description of multifractal properties of random cascading models using only one free parameter μ. However, using the interpretation of intermittency via the BPs we can make some approximation of high-order BPs in order to obtain more simple linear expression for AFD maintaining the number of free parameters.

Let us make the assumption that high-order BPs can be expressed in terms of second-order BP and a constant $r > 0$ as

$$\bar{\eta}_s(\delta y) = (\bar{\eta}_2(\delta y))^r, \qquad s > 2 \tag{26}$$

with

$$\bar{\eta}_2(\delta y) \propto \delta y^{-d_2}. \tag{27}$$

For the given case, the multiplicity distribution with multifractal behaviour has the following form ($n > 1$)

$$\bar{P}_n(\delta y) = \bar{P}_0(\delta y)\frac{\bar{\lambda}^n(\delta y)}{n!}\left[\bar{\eta}_2(\delta y)\right]^{n-1+\frac{r}{2}(n-n^2+8)(1-\delta_{2n})}, \tag{28}$$

where $\bar{\lambda}(\delta y) = \bar{P}_1(\delta y)/\bar{P}_0(\delta y)$ and $\delta_{22} = 1$, $\delta_{2n} = 0$ for $n \neq 2$. Using (18), (26)–(27), the AFD of such distribution is given by the expression

$$d_n = d_2(1-r) + d_2 r\frac{n}{2}. \tag{29}$$

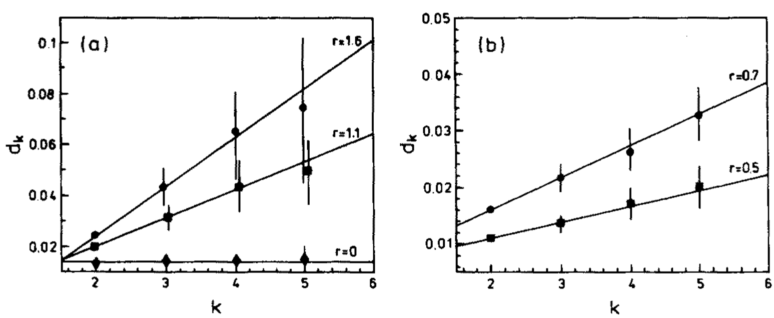

Fig. 1. Some experimental data for AFD. Continuous lines show best fits using the (29). (a) • — Z^0 decay, DELPHI [23], $r = 1.6(8)$, ■ — p–Ag/Br, KLM [24], $r = 1.1(5)$, ♦— S–Ar/Br, KLM [24], $r = 0.0(2)$; (b) • — O + Ag/Br, KLM [24], $r = 0.7(2)$, ■ — $p\bar{p}$, UA1 [25], $r = 0.5(3)$.

The linear approximation of AFD is, in our opinion, very interesting because it allows interpolation between the monofractal case ($r = 0$) and log-normal approximation ($r = 1$) as the Levy-law approximation (22). The results of fits of some experimental data by the expression (29) are reported in Fig. 1(a), (b). This analysis gives good agreement to the experimental data. Thus the approximation of high-order BPs by the second-order is valid for such reactions.

6. Conclusion

We introduced the bunching parameters for analysis of multiparticle production in high-energy physics by analogy with the theory of continuous quantum measurement for one-mode photon fields. It is shown that an inverse power-law singularity of second-order BP leads to monofractal behaviour of AFD at small rapidity interval if high-order BPs are independent of phase-space intervals. We found general form of generation function with monofractality using such dependence of second-order BPs on phase-space intervals. For multifractality an inverse power-law singularity for all order BPs is necessary and sufficient. Using the experimental data, we can conclude that the majority of reactions possess strong bunching of particles in all order for $\delta y \to 0$.

We have shown that some experimental behaviour of AFD can be understood as the simple approximation of high-order BPs in terms of second-order. We believe this to be an important conclusion as it leads to a description of the multifractal multiplicity distribution for the reactions with minimum number of free parameters.

The use of BPs is interesting because it gives a general answer to the problem of finding multiplicity distribution leading to intermittency. This method is also interesting since it may provide a link between theory of continuous quantum measurement and the investigations of multifractal structure of multiplicity distributions in particle collisions at high-energies. It gives also possibility to analyse the intermittency phenomenon in quantum optics.

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