ON ELECTROWEAK PRECISION TESTS*,**

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We reemphasize the importance of discriminating fermion-loop and bosonic electroweak corrections in the analysis of electroweak precision data. Most recent data are indeed precise enough to require corrections beyond (trivial) fermion loops. An analysis of these data in terms of the observables $\Delta x \equiv \varepsilon_{N1} - \varepsilon_{N2}$, $\Delta y \equiv -\varepsilon_{N2}$ and $\varepsilon \equiv -\varepsilon_{N3}$ identifies the required additional corrections as vertex corrections at the $W^{\pm} f\bar{f}'$ and $Z^0 f\bar{f}$ vertices. Standard-model values for these corrections are consistent with the experimental data.

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This is an updated version of my talk on the significance of the electroweak precision data as delivered at Szczyrk in September 1993. The theoretical results were obtained by the collaboration of the authors of [1] as an expansion and refinement of our previous work [2, 3] on this subject. The experimental data to be analysed are the most recent ones, reported in March 1994 [4].

As stressed a long time ago [5] by Gounaris and myself, in the analysis of electroweak precision tests, it is essential to clearly discriminate between two sources of electroweak one-loop corrections, *fermion-loop* (vacuum polarization) corrections to γ, $W^{\pm}$ and Z° propagation on the one hand, and *bosonic vacuum polarization and vertex corrections* on the other hand. The reason for the importance of such a discrimination is obvious. The properties of the (light) *fermions* are *empirically well-known* and the mentioned fermion-loop corrections can accordingly be calculated precisely and uniquely upon introducing the mass of the top quark, m_t, as a free parameter. In contrast, the

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additional bosonic corrections contain trilinear and quadrilinear couplings among the vector bosons and to the Higgs scalar which are *empirically entirely unknown.* The difference between the fermion-loop calculations and the full one-loop standard model results thus sets the scale [5] for the accuracy to be aimed at with respect to genuine quantitative experimental tests of the electroweak theory beyond fermion-loops.

In Figs 1–3, we show the three projections of the three-dimensional 68% C.L. volume defined by the data in $(M_{W_\pm}/M_Z, \bar{s}_W^2, \Gamma_l)$-space in comparison with various theoretical results. The data represent the most recent results from the four LEP collaborations, from SLD and from CDF/UA2 [4], $M_Z = 91.1899 \pm 0.0044$ GeV, $M_{W^\pm}/M_Z = 0.8814 \pm 0.0021$, $\Gamma_l = 83.98 \pm 0.18$ MeV, $\bar{s}_W^2$ (all asymmetries LEP) $= 0.23223 \pm 0.00050$, and $\bar{s}_W^2$ (all asymmetries LEP + SLD) $= 0.23158 \pm 0.00045$.

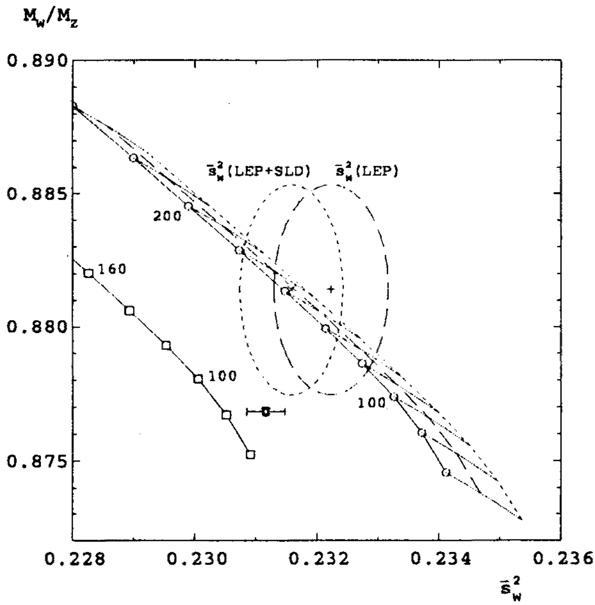

Fig. 1. The experimental data on $(M_W/M_Z, \bar{s}_W^2, \Gamma_l)$ compared with theory.

The theoretical results shown in Figs 1–3 are as follows,

(i) the $\alpha(M_Z^2)$ tree-level prediction (denoted by a star) based on $\alpha(M_Z^2) = 1/128.87 \pm 0.12$ [6] which takes into account the change in α from $\alpha(0)$ to $\alpha(M_Z^2)$ due to lepton and quark loops,

(ii) the full fermion-loop prediction, which takes into account the full contribution of all leptons and quarks to the γ, $W^\pm$ and Z° propagators, the mass m_t being varied in steps of 20 GeV (and indicated by squares),

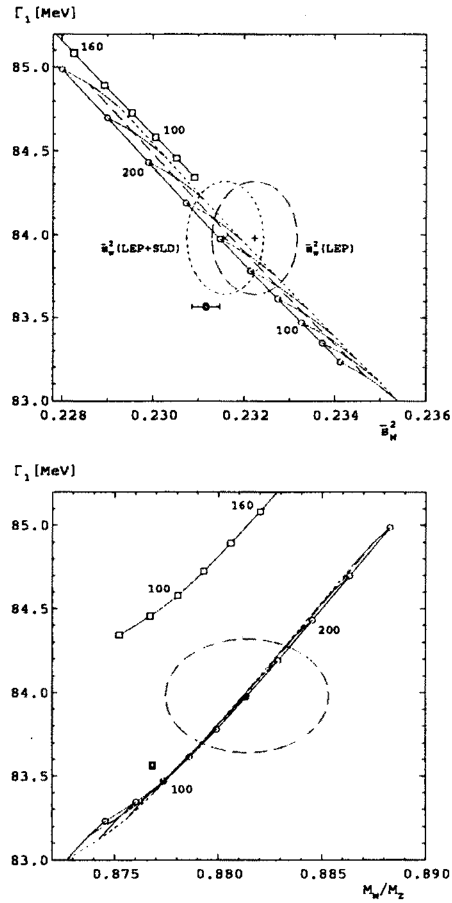

Fig. 2,3. The experimental data on $(M_W/M_Z, \bar{s}^2_W, \Gamma_l)$ compared with theory.

(iii) the full standard $SU(2)_L \times U(1)$ one-loop predictions for Higgs masses of $m_H = 100$ GeV (solid line), 300 GeV (long-dashed line) and 1000 GeV (short-dashed line), 20 GeV steps indicated by circles.

From Figs 1–3, we conclude that the present high-precision data deviate from the $\alpha(M_Z^2)$ tree-level prediction and from the full fermion-loop results. The data are accurate enough to require additional contributions beyond fermion loops, and such contributions are indeed provided by the standard bosonic corrections. A top mass of $m_t \simeq 160$ GeV is required for consistency between experiment and standard theory.

The results in Figs 1–3 can be illuminated by an analysis in terms of the parameters $\Delta x, \Delta y$ and ε which within the framework of an effective electroweak Lagrangian [1, 2] specify possible sources of SU(2) violation. The parameters $\Delta x, \Delta y$ and ε can be deduced from the experimental data on $M_{W^\pm}/M_Z, \bar{s}_W^2$ and Γ_l and compared with standard one-loop results.

The parameter Δx quantifies global SU(2) violation via

$$M_{W^\pm}^2 \equiv (1+\Delta x) M_{W^0}^2 \,, \tag{1}$$

while Δy and ε quantify $SU(2)_L$ violation in vector-boson couplings to fermions, namely

$$g_{W^\pm}^2(0) \equiv 4\sqrt{2} G_\mu M_{W^\pm}^2 (1+\Delta y) g_{W^0}^2(M_Z^2) \,, \tag{2}$$

and via mixing

$$\mathcal{L}_{\rm mix} \equiv (e(M_Z^2)/g_{W^0}(M_Z^2))(1-\varepsilon) A_{\mu\nu} W_3^{\mu\nu} \equiv \lambda A_{\mu\nu} W_3^{\mu\nu} \,. \tag{3}$$

Accordingly, the charged-current Lagrangian for vector-boson-fermion interactions is given by

$$\mathcal{L}_C = -\frac{1}{2} W^{+\mu\nu} W_{\mu\nu}^- + \frac{g_W^\pm}{\sqrt{2}} \left(j_\mu^+ W^{+\mu} + \text{h.c.}\right) + M_W^{\pm\,2} W_\mu^+ W^{-\mu} \,, \tag{4}$$

while the neutral-current Lagrangian becomes

$$\begin{aligned} \mathcal{L}_N = &-\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} \frac{M_{W^0}{}^2}{1-\lambda^2} Z_\mu Z^\mu - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} \\ &- e j_{em}^\mu A_\mu - \frac{g_{W^0}}{\sqrt{1-\lambda^2}} \left(j^3 - \lambda \frac{e}{g_{W^0}} j_{em} \right)^\mu Z_\mu \,. \end{aligned} \tag{5}$$

Our parameters, introduced by quantifying different sources of SU(2) violation, are related to the parameters of Altarelli *et al.* [7] via

$$\begin{aligned} \Delta x &= \varepsilon_{N1} - \varepsilon_{N2} \,, \\ \Delta y &= -\varepsilon_{N2} \,, \\ \varepsilon &= -\varepsilon_{N3} \,. \end{aligned} \tag{6}$$

From the Lagrangian, one easily obtains

$$\begin{aligned} \bar{s}_W^2 (1-\bar{s}_W^2) &= \frac{\pi\alpha(M_Z{}^2)}{\sqrt{2} G_\mu M_Z{}^2} \frac{y}{x} (1-\varepsilon) \frac{1}{\left(1+\frac{\bar{s}_W^2}{1-\bar{s}_W^2}\varepsilon\right)} \,, \\ \frac{M_W^{\pm\,2}}{M_Z{}^2} &= (1-\bar{s}_W^2)\, x \left(1+\frac{\bar{s}_W^2}{1-\bar{s}_W^2}\varepsilon\right) , \\ \Gamma_l &= \frac{G_\mu M_Z{}^3}{24\pi\sqrt{2}} \left(1+(1-4\bar{s}_W^2)^2\right) \frac{x}{y} \left(1-\frac{3\alpha}{4\pi}\right) . \end{aligned} \tag{7}$$

Expanding in linear order in $\Delta x, \Delta y, \varepsilon$, one obtains [1, 2]

$$\bar{s}_W^2 = s_0^2 \left[1 - \frac{1}{c_0^2 - s_0^2}\varepsilon - \frac{c_0^2}{c_0^2 - s_0^2}(\Delta x - \Delta y)\right],$$

$$\frac{M_W^{\pm}}{M_Z} = c_0 \left[1 + \frac{s_0^2}{c_0^2 - s_0^2}\varepsilon + \frac{c_0^2}{2(c_0^2 - s_0^2)}\Delta x - \frac{s_0^2}{2(c_0^2 - s_0^2)}\Delta y\right],$$

$$\begin{aligned} \Gamma_l = \Gamma_l^{(0)} \Bigg[1 &+ \frac{8s_0^2(1-4s_0^2)}{(c_0^2 - s_0^2)(1+(1-4s_0^2)^2)}\varepsilon \\ &+ \frac{2(c_0^2 - s_0^2 - 4s_0^4)}{(c_0^2 - s_0^2)(1+(1-4s_0^2)^2)}(\Delta x - \Delta y)\Bigg], \end{aligned} \tag{8}$$

where

$$\begin{aligned} s_0^2(1-s_0^2) \equiv c_0^2 s_0^2 &= \frac{\pi\alpha({M_Z}^2)}{\sqrt{2}G_\mu {M_Z}^2}, \\ \Gamma_l^{(0)} &= \frac{\alpha({M_Z}^2)M_Z}{48 s_0^2 c_0^2}\left[1 + (1-4s_0^2)^2\right]\left(1 + \frac{3\alpha}{4\pi}\right). \end{aligned} \tag{9}$$

The inversion of (8),

$$\begin{aligned} \Delta x =& \frac{1}{(c_0^2 - s_0^2)^2 + 4s_0^2}\Bigg[2(1-2s_0^2)\bar{s}_W^2 + \frac{s_0^2(1+(1-4s_0^2)^2)}{2\Gamma_l^{(0)}}\Gamma_l \\ &- 1 - 3s_0^2 + 8s_0^4c_0^2\Bigg] + 2\left(\frac{M_W}{M_Z c_0} - 1\right), \\ \Delta y =& \frac{-2}{(c_0^2 - s_0^2)^2 + 4s_0^4}\Bigg[(c_0^2 - 5s_0^2)\bar{s}_W^2 + \frac{c_0^2(1+(1-4s_0^2)^2)}{4\Gamma_l^{(0)}}\Gamma_l \\ &- \frac{1}{2} + \frac{3}{2}s_0^2 + 4s_0^6\Bigg] + 2\left(\frac{M_W}{M_Z c_0} - 1\right), \\ \varepsilon =& \frac{1}{(c_0^2 - s_0^2)^2 + 4s_0^4}\Bigg[-\frac{c_0^2 - s_0^2 - 4s_0^4}{s_0^2}\bar{s}_W^2 - \frac{c_0^2(1+(1-4s_0^2)^2)}{2\Gamma_l^{(0)}}\Gamma_l \\ &+ 2c_0^2 - 5s_0^2 + 8c_0^2s_0^4\Bigg], \end{aligned} \tag{10}$$

may now be used to deduce the parameters $\Delta x, \Delta y, \varepsilon$ from the experimental data on $M_{W^\pm}/M_Z$, $\bar{s}_W^2$ and Γ_l in order to compare them with the fermion-loop and the full standard-model predictions.

The results for $\Delta x, \Delta y, \varepsilon$ thus obtained are displayed in Figs 4–6. The results are striking. According to Fig. 4, the fermion-loop predictions for

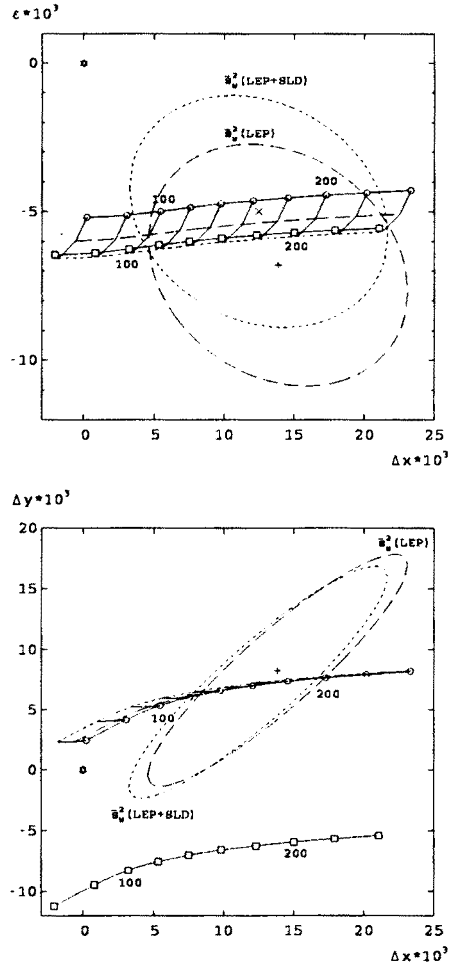

Fig. 4,5. The experimental data on $\Delta x, \Delta y, \varepsilon$ compared with theory.

Δx and ε practically coincide with the complete one loop results, $\Delta x \simeq \Delta x$ (fermion loops), $\varepsilon \simeq \varepsilon$ (fermion loops), *i.e.*, the m_H-dependent standard bosonic vacuum-polarization effects in Δx and ε are of minor importance (and vanishingly small for large values of m_H). In contrast, in Figs 5 and 6, we find a significant non-fermion-loop contribution to Δy, $\Delta y \simeq \Delta y$ (fermion loops) $+\Delta y$ ($W^{\pm}$ -vertex plus box, Z^0 -vertex), which is due to vertex (and box) corrections to the $W^{\pm} f\bar{f}'$ vertex (entering the analysis via the Fermi coupling G_μ extracted from μ decay) in conjunction with $Z^0 f\bar{f}$ vertex corrections. *The differences between fermion-loop and full-one-loop theoretical results in Figs 1–3 accordingly have been traced back to significant genuine electroweak $W^{\pm} f\bar{f}'$ vertex (and box) corrections appearing in*

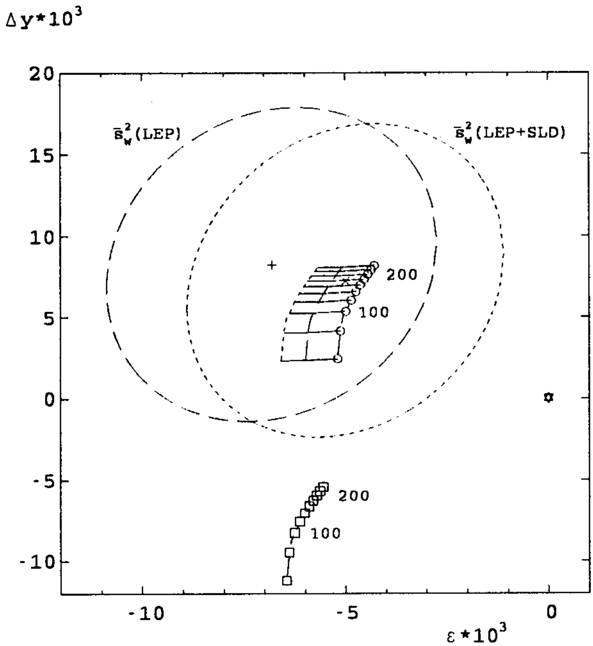

Fig. 6. The experimental data on $\Delta x, \Delta y, \varepsilon$ compared with theory.

conjunction with $Z^0 f\bar{f}$ vertex corrections in the parameter Δy.

It is remarkable that the experimental data have *reached a precision which allows one to isolate loop corrections beyond fermion loops.* More specifically, the data require significant vertex corrections. The magnitude required for the parameters $\Delta x, \Delta x, \varepsilon$ rules out large SU(2)-symmetry violations and is consistent with the one-loop prediction of the standard electroweak theory.

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