THE FERMION NUMBER VIOLATION IN THE SUPERSYMMETRIC STANDARD MODEL*

R. Mańka

Department of Astrophysics and Cosmology
University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland
internet: manka.phys.us.edu.pl

(Received May 27, 1994)

We show that the supersymmetry breaking scale (R-parity), the global U(1) fermion symmetry scale and the electroweak symmetry breaking scale are strictly connected to each other.

PACS numbers: 98.80. Cq, 12.15. Cc

1. Introduction

The idea that the supersymmetry is broken for not too large scale $M_Z < M_{\rm SUSY} < 1$ TeV is promising from the theoretical, experimental and astrophysical point of view. It gives the natural way to cancel the radiative correction in the perturbative approach. The quadrative divergences are still to cancel when the supersymmetry is softly broken. Supersymmetry predicts the rich structure of the new particles which could be observed if the supersymmetry breaking scale $M_{\rm SUSY}$ is not too high. Half of them are boson fields which can condensate breaking for example the global symmetries. Such a case could happen if the minimal supersymmetric standard model (MSSM) [1] is extended by including the right neutrinos and an additional fermion field. The aim of this paper is to construct such a model in which the lepton number is broken in the result of the sneutrinos condensation ($\tilde{\nu}_e$). This condensation breaks also the R-parity. The lepton symmetry breaking for the $\sim$ TeV scale has also an interesting astrophysical meaning [7].

---

* Presented at the XVII International School of Theoretical Physics"Standard Model & Beyond '93", Szczyrk, Poland, September 19–27, 1993.

2. The model with the right sneutrino condensation

Let us consider the simplest supersymmetric version [1] of the standard model where the scalar Higgs sector is built from three fields $\{H, \overline{H}, \Phi\}$. They are in the $\mathrm{SU_L}(2)$ dublet representation

$$H = \left\{ \begin{matrix} H^- \\ H^0 \end{matrix} \right\}, \quad \overline{H} = \left\{ \begin{matrix} H^0 \\ H^+ \end{matrix} \right\}, \tag{1}$$

and one singlet representation Φ. The left handed leptons are in

$$(\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}) \sim L_f = \left\{ \begin{matrix} \nu_e \\ \ \end{matrix} \right\}_{\mathrm{L}}, \quad \left\{ \begin{matrix} \nu_\mu \\ \mu \end{matrix} \right\}_{\mathrm{L}}, \quad \left\{ \begin{matrix} \nu_\tau \\ \tau \end{matrix} \right\}_{\mathrm{L}}; \tag{2}$$

representation (f is the family index). In this model, however, our attention will be focused on the neutrinos in this model. For simplicity we limit ourselves to the first family. Neutrinos are present in our model in the form of two supersymmetric multiplets $\{\nu_{\mathrm{L}}, \psi_{\nu_{\mathrm{L}}}\}$ and $\{\nu^c_{\mathrm{R}}, \psi_{\nu^c_{\mathrm{R}}}\}$ belonging to the $\mathrm{SU_L}(2)\times\mathrm{U_B}(1)$ representations $(\mathbf{2}, -{}^1/_2)$ with the weak isospin $T_3 = {}^1/_2$, the lepton charge $Q_F = 1$ for ν_{L} and with only the lepton charge $Q_F = -1$ for ν^c_{R}. Spinors $\psi_{\nu_{\mathrm{L}}}, \psi_{\nu^c_{\mathrm{R}}}$ eventually will form the Dirac or Majorana fermion fields while the sneutrinos $\nu_{\mathrm{L}}, \nu^c_{\mathrm{R}}$ should gain high masses. But as bosons they could also condensate breaking the lepton number conservation law. The minimal supersymmetric standard model is extended by the additional isosinglet scalar field S carrying the lepton number $Q_F = -1$. These singlets can also arise in superstring models. The Lagrange function may be written as the sum

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{ext}} \tag{3}$$

of the ordinary Glashow–Salam–Weinberg model with

$$\begin{aligned} \mathcal{L}_{\mathrm{GSW}} &= -\tfrac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} + D_\mu H^+ D^\mu H + D_\mu \overline{H}^+ D^\mu \overline{H} \\ &+ D_\mu \nu^+_{\mathrm{L}} D^\mu \nu_{\mathrm{L}} + \partial_\mu \Phi^+ \partial^\mu \Phi - V(H, \overline{H}, \Phi) + i\bar{e}_{\mathrm{R}}\gamma^\mu D_\mu L + h\bar{e}_{\mathrm{R}} H L \end{aligned} \tag{4}$$

with

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + ig\varepsilon_{abc} W^a_\mu W^c_\nu, \tag{5}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{6}$$

$$D_\mu = \partial_\mu - \frac{i}{2} g W^a_\mu \sigma^a - \frac{i}{2} g' Y B_\mu. \tag{7}$$

The potential V in the supersymmetric phase is fully defined by the superpotential

$$\mathcal{W} = \mathcal{W}_{\mathrm{SM}} + \mathcal{W}_{\mathrm{ext}}, \tag{8}$$

where the superpotential

$$\mathcal{W}_{SM} = h_e \varepsilon_{i,j} \overline{H}^i L^j e^c_{\rm R} + \sqrt{\lambda}\left(\varepsilon_{i,j}\overline{H}^i H^j \Phi - \tfrac{1}{3}\Phi^3\right), \tag{9}$$

is responsible for the leptons interaction inside the standard model. R parity is a discrete symmetry and if it is an exact symmetry, all particles of the standard model are R-parity even while their superpartners are R-parity odd. The relation between R parity, the total lepton number L, baryon number B and spin S is as follows

$$R_p = (-1)^{3B+L+2S}. \tag{10}$$

R parity can be broken in two ways either explicitly or spontaneously. In this paper the lepton and R symmetry violating term is postulated in the following form

$$\mathcal{W}_{ext} = h_\nu \varepsilon_{i,j} H^i L^j \nu^c_{\rm R} + \sqrt{\lambda} S \nu^c_{\rm R} \Phi. \tag{11}$$

The supersymmetry breaking potential may be written as

$$\delta V = m^2 \Phi^+ \Phi - \lambda u(\varepsilon_{i,j}\overline{H}^i H^j + S\nu^c_{\rm R})\Phi - \kappa u(\Phi^3 + \Phi^{3^+}). \tag{12}$$

The total potential is the sum of two parts

$$V_s = V_0 + \delta V_1, \tag{13}$$

the first one comes from the superpotential $\mathcal{W}$

$$V_0 = \sum_i \left|\frac{\partial \mathcal{W}}{\partial \Phi^i}\right|^2, \tag{14}$$

where $\Phi^i = \{H, \overline{H}, \nu_{\rm L}, \nu^c_{\rm R}, S, \Phi\}$. Let us suppose that the spontaneous lepton number breaking occurs in the result of the boson condensation. This means that H, $\overline{H}$, S and $\nu^c_{\rm R}$ fields get nonvanishing expectation values

$$\langle H \rangle = \frac{1}{\sqrt{2}}\{0, v\}, \qquad \langle \overline{H} \rangle = \frac{1}{\sqrt{2}}\{v, 0\},$$
$$\langle \Phi \rangle = \frac{1}{\sqrt{2}}x, \quad \langle S \rangle = \frac{1}{\sqrt{2}}y, \quad \langle \nu^c_{\rm R} \rangle = \frac{1}{\sqrt{2}}y \tag{15}$$

and most of bosonic and fermionic fields get masses. This superpotential determines the fermion mass matrix

$$\mathcal{M}_{ij} = \frac{\partial^2 \mathcal{W}}{\partial \Phi^i \partial \Phi^j}. \tag{16}$$

The part of this matrix concerning the $\Phi^i = \{H, \bar{H}, \nu_L, \nu_R^c, S, \Phi\}$ fields has the following form

$$\mathcal{M}'_{ij} = \begin{pmatrix} 0 & m_D & \frac{h_\nu}{\sqrt{2}} y & 0 & 0 & 0 \\ m_D & 0 & 0 & 0 & \sqrt{\frac{\lambda}{2}} x & \sqrt{\frac{\lambda}{2}} y \\ \frac{h_\nu}{\sqrt{2}} y & 0 & 0 & \sqrt{\frac{\lambda}{2}} x & 0 & \sqrt{\frac{\lambda}{2}} v \\ 0 & 0 & \sqrt{\frac{\lambda}{2}} x & 0 & 0 & \sqrt{\frac{\lambda}{2}} v \\ 0 & \sqrt{\frac{\lambda}{2}} x & 0 & 0 & 0 & \sqrt{\frac{\lambda}{2}} y \\ 0 & \sqrt{\frac{\lambda}{2}} y & \sqrt{\frac{\lambda}{2}} v & \sqrt{\frac{\lambda}{2}} v & \sqrt{\frac{\lambda}{2}} y & -\sqrt{2\lambda} x \end{pmatrix},$$

where $m_D = \frac{1}{\sqrt{2}} h_\nu v$. On the tree level the vacuum is determined by the minimum of the potential

$$\begin{aligned} V(v,x,y) =& \tfrac{1}{2}\lambda(y^2+v^2)x^2 + \tfrac{1}{4}\lambda(v^2+y^2-x^2)^2 \\ & - \tfrac{1}{2\sqrt{2}}\lambda u(v^2+y^2)x + \tfrac{1}{2}m^2x^2 - \tfrac{1}{2}u\kappa x^3 \,. \end{aligned} \tag{17}$$

In fact it depends only on one parameter $z = \sqrt{v^2+y^2}$. This means that we have the flat direction for the potential $V(v,x,y)$. One parameter, for example v must appear in the result of the phase transition induced by the radiative correction (the Coleman–Weinberg potential [5, 6]). Keeping only the contributions associated with the gauge bosons W, Z and the quark t the radiative corrections give

$$V_r(v) = \sum_{i=W,Z,t,\ldots} \frac{n_i}{64\pi^2} m_i^4(v) \left\{ \ln \frac{m_i^4(v)}{Q^2} - \tfrac{3}{2} \right\}, \tag{18}$$

where for example

$$m_W^2 = \tfrac{1}{4}g^2v^2\,, \tag{19}$$
$$m_Z^2 = \tfrac{1}{4}(g^2+g'^2)v^2\,, \tag{20}$$
$$m_t^2 = h_q^2v^2, \ldots . \tag{21}$$

n_i depends on the number of degrees of freedom and the particles statistics

$$n_W = 6, \quad n_Z = 3, \quad n_t = -12, \ldots . \tag{22}$$

Q is the renormalization scale. Let us notice that $V_r(0) = 0$. At the extremum point v_0, where $\partial V_r(v)/\partial v = 0$, the potential $V_r(v_0)$ should also be equal to zero. In other case it will produce the nonvanishing cosmological constant in the broken phase ($v = v_0$). This estimates the renormalization scale Q. The total potential

$$V(v,x,y) = V_r(v) + V_s(z,x) \tag{23}$$

with $z = \sqrt{v^2 + y^2}$ will have the extremum point at

$$\frac{\partial V}{\partial v} = \frac{\partial V_r}{\partial v} + \frac{\partial V_s}{\partial z}\frac{\partial z}{\partial v} = 0\,, \tag{24}$$

$$\frac{\partial V}{\partial y} = \frac{\partial V_s}{\partial z}\frac{\partial z}{\partial y} = 0\,, \tag{25}$$

$$\frac{\partial V}{\partial x} = \frac{\partial V_s}{\partial x} = 0\,, \tag{26}$$

and will be determined by the condition

$$\frac{\partial V_r}{\partial v} = 0\,, \quad \frac{\partial V_s}{\partial x} = 0\,, \quad \frac{\partial V_s}{\partial z} = 0\,. \tag{27}$$

The first extremum condition gives

$$v = v_0\,. \tag{28}$$

v_0 is determined by the electroweak symmetry breaking scale $v_0 \sim 250$ GeV. The nontrivial minimum V_s with respect to z gives

$$z^2 = \frac{1}{\sqrt{2}}ux\,. \tag{29}$$

Using (29) the $V(v,x,y)$ (17) potential may be rewritten in the following form

$$V(x) = \tfrac{1}{2}ax^2 - \tfrac{1}{6}bx^3 + \tfrac{1}{24}cx^4\,, \tag{30}$$

with

$$a = m^2 - \tfrac{1}{4}\lambda u^2\,, \tag{31}$$

$$b = 3\kappa u\,, \tag{32}$$

$$c = 6\lambda\,. \tag{33}$$

The potential $V(x)$ depends only on parameters which break supersymmetry. In a result the potential $V(x)$ can be parameterized by one dimensional constant u. The solution with $x = 0$ corresponds to the supersymmetric phase, whereas the solution with $x \neq 0$ to the phase with broken supersymmetry. The parameter v_0 is fixed by the W-mass ($v_0 \sim 250$ GeV). To have the vanishing cosmological constant in the broken phase for small B and ($v_0 \neq 0$) we should choose

$$m = \sqrt{\frac{\lambda}{4}}u\sqrt{1+2\left(\frac{\kappa}{\lambda}\right)^2}\,; \tag{34}$$

then

$$x_0 = \frac{\kappa u}{\lambda}. \tag{35}$$

The field Φ gains the mass

$$m_\Phi = \frac{1}{\sqrt{2\lambda}}\kappa u\,. \tag{36}$$

For example, for $\kappa = 10$, $\lambda = 0.3$ and for $v_0 = 250$ GeV we have $u = 51.5$ GeV, $x_0 = 1716.7$ GeV, $m = 665$ GeV, and $m_\Phi = 664.7$ GeV. The equation

$$z = \sqrt{v^2 + y^2} \tag{37}$$

joins together the spontaneous electroweak symmetry breaking scale ($v \sim v_0$), the lepton symmetry breaking scale ($y = y_0$) and the supersymmetry breaking scale ($z \neq 0$). The electroweak breaking is determined by the W-mass ($v_0 \sim 250$ GeV). It is easy to notice that when the lepton number breaking takes place, the scale of the supersymmetry breaking is greater then v_0. The values of this scale will increase with y (the lepton symmetry breaking scale). As a result, due to the see-saw mechanism of

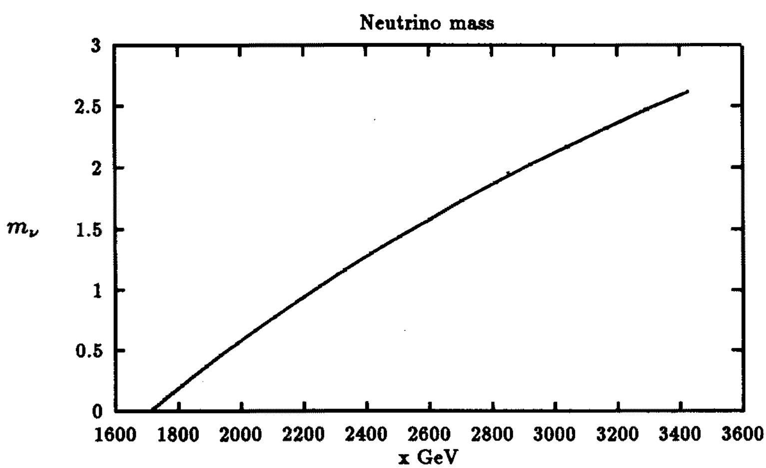

Fig. 1. The light neutrino mass (in 10^{-12} GeV) as a function of the lepton symmetry scale (in GeV).

the neutrino mass generation we shall have the lepton symmetry breaking scale y dependent the light Majorana neutrino mass (Fig. 1). The heavy Majorana neutrino mass meanwhile is of the 1 TeV order. It is interesting that when the lepton symmetry is correct ($y = 0$), we have the massless left Majorana neutrino. In this case, however, the supersymmetry is broken exactly on the electroweak symmetry breaking scale ($v \sim 250$ GeV). The high symmetry phase ($z = 0$) implies that all symmetries, including supersymmetry, the electroweak $SU_L(2) \times U_Y(1)$ symmetry and the global U(1) fermion symmetry, are restored.

3. Discussion

In was shown that the supersymmetry breaking scale (R-parity), the global U(1) fermion symmetry scale and the electroweak symmetry breaking scale are strictly connected to each other.

REFERENCES

[1] H.P. Nilles, *Phys. Rep.* **110**, 1 (1984); U. Ellwanger, M.R. de Traubenberg, *Natural Range of Higgs masses in supersymmetry*, HD-THEP-91-21, CERN-TH.6144/91.

[2] B. Holdom, *Phys. Rev.* **D36**, 1000 (1982).

[3] A.D. Dolgov, O.Yu. Markin, *Prog. Theor. Phys.* **85**, 1091 (1991).

[4] V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985).

[5] Lewis H. Ryder, *Quantum Field Theory*, Cambridge University Press, Cambridge 1987.

[6] P. Langacker, *Phys. Rep.* **72**, 185 (1981).

[7] R.N. Mohapatra, P.B. Pal, *Massive Neutrinos in Physics and Astrophysics*, World Scientific,1990.