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RELATIONISM OF QUANTUM PHYSICS

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This paper develops the hypothesis of *quantum relationism*. Quantum relationism is compared and contrasted with the Cartesian *eventism*, which is the ontology behind the conventional local quantum field theory. In more technical terms the paper deals with a relativistic description of bound quantal systems which, in Author's opinion, provide an ideal testing ground for his hypothesis.

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PREFACE

The aim of this paper is to present a geometrical hypothesis of absolute relational space R_3 that settles the wave–corpuscular duality of quantum physics on the level of first geometry of physics. In this way, directly unobservable relations y between two hypothetical constituents of an elementary *micro-world* $\mathcal{M} = A_1 + A_2$ will precede observable events X, *i.e.* the four-points of spacetime of measurement. Thus, the presented paper is a lecture on the hypothesis of *quantum relationism.* The reader must forgive its length, but several fundamental concepts have to be revised first, in order to get a proper perspective onto them. Such a perspective is required by the very abandoning of the Cartesian eventism of external x-space.

The mathematical basis of R_3 geometry is presented in Sections 10–13. It is shown that mathematics of classical field theory discloses two kinds of geometrical shapes. This branching of geometrical extensions acquires a physical meaning with the quantum momentum–localization duality or the p–x duality ($\hbar \neq 0$) combined with true Minkowskian spacetime L_4 of measurement ($1/c \neq 0$). The point is that eventism of Galilean spacetime G_4 coexists with nonrelativistic (NR) relational space R_3^G.

Two experimental facts make the hypothesis of relationism R_3 possible and — what is even more essential — result in measurable effects which transgress the borders of the present eventism. The first fact is the quantum p–x duality and the second fact consists in the privileged position of the energy–momentum p language of measurement of micro-processes expressed by the S matrix theory parameterized by the Mandelstam p variables and resulting in elimination of spacetime localization of the micro-process under description.

The hypothesis of relationism rules out the one-body problem of eventism in favour of the elementary nature of two-body problem. It is shown that relations y convert into events X of spacetime, provided however, that in the corresponding subspace of the configuration space of the micro-object under description the object itself interacts with a heavy measuring device. Thus symmetry L of events X, much like the events themselves, ceases to be given *a priori* contrarily to what is usually taken for granted in today's physics.

Everybody who insists on the *a priori* character of the symmetry L should remember that physicists opposed the special theory of relativity (STR) because they thougth that symmetry G of Galilean spacetime G_4 should be regarded as if it were given *a priori*. Hence Albert Einstein's motto:
" ... Concepts which have been proved to be useful in ordering things easily acquire such an authority over us that we forget their human origin and accept them as invariable. Then they become "necessities of thought", "given *a priori*", *etc.* The path of scientific progress is then, by such errors, barred for long time. It is therefore no useless game if we are practicizing to analyze current notions and to point out on what conditions their justification and usefulness depend, how they have grown especially from the data of experience. In this way their exaggerated authority is broken. They are removed, if they cannot properly legitimate themselves, corrected, if their correspondence to the given things was too negligently established, replaced by others, if a new system can be developed that we prefer for good reasons. ..."

1. Introduction

The fundamental conflict between the locality of Minkowskian spacetime L_4 of STR and the spacetime nonlocality of quantum physics inclines one to abandon eventism according to which directly observable events X should be regarded as unanalyzable elements of physics. The hypothesis of relational space will precede eventism, because it transfers the momentum–localization ($p-x$) duality of quantum physics onto the first level of physical geometry. Events X and their Lorentz–Poincaré symmetry L will stand on the footing of limited *relations* conditioned, however, by a suitable physical *situation* of the system $\mathcal{M}$ under description. In particular, such a *situation* will accompany any measuring process performed with the aid of heavy (classical) measuring devices.

Note that the p–x duality results in a discretization of internal-energy levels of composite bound systems $\mathcal{M}_n$, which solves the fundamental problems of stability and extension of $\mathcal{M}_n$'s. The same discretization makes the full isolation of *micro-worlds* $\mathcal{M}_n$ from external world to be a realistic state of $\mathcal{M}$'s. In consequence, a composite system $\mathcal{M}$ may remain *hidden* from observation (measurement) and hence, its first metrical relations may break symmetry L of measurement. Of course this requires abandoning the concept of Cartesian metrical x-space (eventism) as a "holder" of all physical extensions.

The p–x duality of relational geometry justifies one to speak of the privileged position of the momentum p language (over the x language) of measurement of micro-processes in the asymptotic zone of kinematics. Such view is taken for granted in the S matrix theory, parameterized by the Mandelstam p invariants, which, in principle, eliminates the spacetime localization of the micro-process under description.

Mathematical basis of the hypothesis of relational space R_3 consists in the bifurcation of geometrical shapes into two different kinds: the event shapes and the relational shapes (*cf.* Sections 11–14). However, before entering mathematics some problems must be clearly formulated which are connected — first of all — with the principle of relativity, *i.e.* with symmetry of Minkowskian spacetime L_4 and its NR limit ($c \to \infty$) given by symmetry G of Galilean spacetime G_4. This limit exhibits the fundamental singularity of symmetry G which consists in the separability of the internal degrees of freedom of isolated systems from the external ones. From the point of view of relational space R_3 this singularity of symmetry G results from *the coexistence* of eventism G_4 with the relationism of 3-space R_3^G, the latter being the NR limit ($c \to \infty$) of relational space R_3. This would explain the tremendous success of NR quantum mechanics and, at the same time, it works in favour of the hypothesis of relational space R_3 accounting for finite universal constant $\hbar/c$ which, however, must go beyond the borders of eventism L_4.

In consequence, 3-space R_3 extends the NR separability of internal (absolute) degrees of freedom of composite micro-structures $\mathcal{M}$ from external (relative) ones. In Section 17 and in Appendix B *the Lorentz limit* of relationism is analyzed. From this analysis follows that symmetry L of measurement ceases to be the one given *a priori* by the Cartesian x-spacetime continuum (eventism), but it represents the limiting case of symmetry R of relationism. Thus, symmetry L introduces an essential dichotomy of measured characteristics into internal-absolute and external-relative ones which enables one to distinguish between the properties of $\mathcal{M}$ itself and those connected with the measuring tools. In general, however, we cannot abstract from the realization of mathematical reference frames S by real reference bodies $\bar{S}$ which characterizes eventism L_4. This will be illustrated by the dilatation effect of the life-times of unstable particles (quantum clocks) which, at the same time, provides us with an R_3 effect connected with the decay mode of composite system (particle) $\mathcal{M}_n$.

The breaking of dilatation symmetry of physics which accounts for finite universal constants $\hbar$, c and M discussed in Section 21 works strongly in favour of the relational origin of metrical physics and the p–x duality of the first continuum of physics that breaks the Thales similarity inherent in flat Cartesian x-space. Relational origin of metrical physics represents philosophy much more akin to this of Leibniz than that of Descartes and Newton. According to the latter, an external space (spacetime) precedes any physical reality and the existence of any physical reality coincides with its existence on the background of spacetime of directly observable but unanalyzable events X.

The author is fully aware that a lot of many important questions remain still open. Nevertheless, in his opinion, the quantum predictions concerning the behaviour of a system $\mathcal{M}$ are rather incompatible with almost all intuitions (the classical eventism included) and for this reason our concept of spacetime must be thoroughly revised.

2. Eventism and the quantum p–x duality

The concepts of external space–and–time continua as preceding any physical reality take their origin in our everyday macro-experience. The globality of these concepts takes for granted — more or less tacitly — that all physical objects $\mathcal{M}$ are intercommunicated because they automatically acquire a coordination in terms of observable events X, *i.e.* four-points of pre-existing spacetime. Therefore, we shall use the term *eventism* for the hypothesis of the external spacetime given *a priori*, although the very word eventism has been coined after Minkowski's geometrization of STR [1]. The point is that this geometrization disclosed a real opposition between *the eventism* and *objectism* [2] concealed by the singularity of Galilean eventism G_4, although the latter is also based on directly observable events X.

From the viewpoint of an instrumentalist, eventism disregards both the physical aspect of measuring process and the physical reality of messengers propagating information about the structure and localization of the observed (measured) object $\mathcal{M}$. In other words, everything that physically exists is automatically *visible* by outer world and commensurable — with any accuracy which might be desired — with *good* measuring rods and clocks with optionally selected units (*e.g.* those of meter and second). Now, let us confront this visibility of $\mathcal{M}$ with the fundamental concept of its isolation as the state of isolation of $\mathcal{M}$ remains at the foundation both of classical and quantum physics. *Visibility* of $\mathcal{M}$ combined with its isolation calls for existence of specific carriers of information which shall be analyzed below.

Still in the framework of classical physics (theory with $\hbar = 0$), two fundamental facts stand in favour of relational nature of *practical (physical) geometry* [3]. The first is the Newtonian action–at–a–distance between at least two objects and the second concerns the relativized simultaneity relation of two events $X_{1,2}$. It is STR which has shown that the propagation of information by light signal with velocity equal to the universal constant c enters the very symmetry of eventism L_4. Simultaneity relation between two events $X_{1,2}$ relativized to the space scaffold of a (real) reference body $\bar{S}$ favours the objectism and, at the same time, the relational origin of the spacetime metrics. In spite of that, classical physics ($\hbar = 0$) is condemned to eventism in which the realization of mathematical reference frames S parameterizing an empty L_4 is done with the help of real reference frames $\bar{S}$. This realization, however, becomes irrelevant as the classical framework admits the existence of *classical carrier of information* (CCINF).

By CCINF we mean a physical "half-being" characterized by the following three properties: (1) at any instant t CCINF is perfectly localized with regard to all physical objects; (2) it carries a negligible (zero) amount of energy and momentum; (3) it interacts — at least indirectly — with any form of physical reality. In consequence, CCINF's determine, with unlimited accuracy, the structure and localization of any object in spacetime without perturbing object's state which remains the same as the one existing before the observation. Indeed, the vanishing energy–momentum transfer between the object and CCINF's makes that the measured object suffers no recoil and, therefore, a classical theory ($\hbar = 0$) admitting existence of CCINF's must be an eventistic one.

After distinguishing between *quantum-potential* existence of $\mathcal{M}$ symbolized by its state ψ and *actualization* characterized by irreversible [4] and registrable track [5], we can conclude that, owing to CCINF's, classical theory reconciles the total actualization of all its entities with the state of their full isolation. Thus, classical physics (framework) deals with *one-level* actualized reality, whereas quantum physics discloses a *two-level* physics with realities on the quantum-potential and classical-actual levels. Note, that actualizations result in a $0-1$ alternative of any property of $\mathcal{M}$, which may be either existent or nonexistent, whereas quantum propensity (potentiality) inclines one to speak of the *fractional–potential–existence* of $\mathcal{M}$ or of some of its properties on the second quantum level of reality. CCINF's reduce this to one-level actual existence which will be discussed further on.

Now, let us show that the classical frameworks ($\hbar = 0$), both relativistic and nonrelativistic (NR) one, admit the existence of CCINF's. In the NR theory they are realized by action–at–a–distance or by very light and small particles of Newtonian mechanics with sharply defined spacetime trajectories. In the classical relativistic theory CCINF's cannot propagate with velocities greater than that of light but they are realizable by sharply localized electromagnetic pulses of classical Maxwell equations which admit negligible amounts of pulse energy and momentum. In the everyday praxis, the real light imitates CCINF's, but the real *quantum* light ($\hbar \neq 0$) cannot approximate CCINF's with an arbitrarily high accuracy.

Similarly to STR which eliminates signals moving with velocities greater than that of light, the quantum p–x duality eliminates CCINF's from the realm of physical world. Indeed, the uncertainty relations

$$\Delta X_j \, \Delta P_k \geq \hbar \delta_{jk}, \quad (j, k = 1, 2, 3) \tag{2.1}$$

which follow from the p–x duality prove that the attributes (1) and (2) of CCINF's can be realized by no physical object. Of course, from the statement that the existence of CCINF's results in eventism does not follow the opposite statement abolishing eventism; nevertheless, the lack of CCINF's shakes the very philosophy of Cartesian x-localization (eventism). Indeed, any observation (measurement) respecting uncertainty relations belongs to physics, hence the separation of the measured object from the measuring tools ceases to be given *a priori*, like it has been in the classical eventistic physics. In particular, direct x-measurements of the localization or structure of a micro-object $\mathcal{M}$ must be connected with an uncontrollable recoil of $\mathcal{M}$, leading to uncontrollable destruction of the $\mathcal{M}$ structure.

Let us remember that the p–x duality establishes (in general) a one–to–one correspondence between the p and x representations of a Hilbert vector $|G\rangle$, as

$$\begin{aligned} \langle x|G\rangle &= \int dp \, U(x,p)\langle p|G\rangle \,, \\ \langle p|G\rangle &= \int dx \, U^*(p,x)\langle x|G\rangle \,, \end{aligned} \tag{2.2}$$

where $U(x,p)$ is the element of the unitary operator $\widehat{U}$ which establishes the one–to–one correspondence between the x and p representations of Hilbert vector $|G\rangle$. Note, that in Dirac's abstract formulation of the state G of $\mathcal{M}$, the physical meaning of the x and p variables remains undetermined. However, in the L_4-eventism, the quantum p–x duality means that $p's$ denote four-momenta of the corresponding degrees of freedom of $\mathcal{M}$ represented in some reference frame S. Let us remark that in the case of, for instance, hydrogen atom neither electron measures proton nor does proton measure electron. Hence — as far as the measurement is concerned — the identification of mutual position x and mutual energy–momentum p with four-vectors of L_4 geometry follows uniquely from the hypothesis of eventism.

However, 4-symmetry of eventism L_4 has explicitly disclosed the experimentally privileged position of the asymptotic language p of relativistic kinematics, as pointed out many years ago by Landau and Peierls [6]. The relativistic S matrix theory still enhances this privileged position of language p by parameterizing matrix elements S_{fi} in terms of Mandelstam L-invariant variables s_J determined by asymptotic four-momenta P_A of free particles A of the initial and final asymptotic states of the collision process under description. One has

$$S_{fi} = \delta^{(4)}(P_i - P_f)T_{fi}(s_1, \ldots, s_K), \tag{2.3}$$

where P_i and P_f denote four-momenta of the whole fully isolated system inside which the quantum-potential collision process takes place. This parametrization of the L-absolute matrix elements S_{fi} with the aid of the L-absolute Mandelstam variables s_J suggests that behind this absolute parameterization stands an L-absolute internal geometry of micro-worlds $\mathcal{M}$ that would *a priori* guarantee the absolute nature of the described process.

The privileged position of the p language means that we can — in principle — first measure exactly the S matrix elements (cross-sections) and then evaluate from (2.2) the x-representations of the corresponding structures. Thus the p–x duality expressed by (2.2) witnesses the coexistence of the p and x aspects of a structure, which is determined (exactly) in the p language, rather than Bohr's *complementarity* [7]. The latter, as based upon Bohr's *incertitude relations*, questions the possibility of reduction of the experimental error of any quantitative characteristic, questioning in this way the very idea of quantitative physics.

Sharp values of four-momenta P_A, resulting in sharp values of s_J variables, make that the collision process described by S_{fi} from (2.3) becomes entirely unlocalized in the spacetime of measurement. However, this paradox may be surmounted by wave-packets which — if sufficiently spread out — lead to the same cross-sections as those evaluated with the help of stationary plane waves with sharply defined P_A's of the A particles entirely unlocalized in the spacetime [8]. In this way, Lorentz 4-symmetry L of the classical principle of relativity required by all repeatable observables becomes reconciled with the quantum nonlocality inherent in the very p language of Mandelstam variables.

Of course, one has to agree with Bohr [7] that L symmetry imposed by heavy, classical devices is an *a priori* condition of any physical theory, no matter how far the concepts of first theory may be remote from intuitions based on classical macro-physics. The aim of this paper is to show that in spite of this unquestionable *a priori*, the quantum p–x duality admits abandoning eventism L_4 and its symmetry L on the quantum-potential level of existence of $\mathcal{M}$, without violating symmetry L of all repeatable observables based on adequate ensembles of actualized data.

In his famous paper, written with Podolsky and Rosen, Einstein (EPR) [9] shows that quantum indeterminism of actualizations is hardly reconcilable with (classical) eventism of quantum physics. Therefore Einstein–realist who, much alike to all his adversaries, accepted eventism, did not agree with nonlocal quantum theory as a complete theory of micro-physics. In this situation, the tremendous success of NR quantum mechanics and relativistic perturbation theory (quantum electrodynamics, in the first place) constitutes a great challenge to our eventistic quantum physics. Moreover, the Bell inequalities [10], violated in perfect agreement with quantum-nonlocal predictions [11], show that Einstein's *classical reality* [9] based on eventism has collapsed. This has inclined Clauser and Shimony to form the conclusion that: "... either one must totally abandon the realistic philosophy of most working scientists, or dramatically revise our concept of spacetime. ..." [12]. The hypothesis of relationism presented below follows the second leg of this alternative.

3. Balances of practical (physical) geometries

We shall call — after Einstein [3] — the Euclidean spaces $E_n (n = 1, 2, 3)$, similarly as the spacetime geometries L_4 and /or G_4, *practical geometries* as they reflect behaviour of *good* measuring rods and clocks. In Bridgman's terminology [13], practical geometries carry *the physical text* of mathematical symbols of physical theory. Without such physical text no confrontation of theory with experiment is possible, as any quantitative theory without physical text underlying theory's symbols is reduced to pure mathematics. Note that Cartesian synthesis of geometry with algebra was, in fact, based on such (at that time) self-evident text of geometry.

Let us point out that in the Cartesian synthesis of algebra with E_n spaces, the orthogonal reference frames S_n, with common (chosen optionally) unit on all axes of all equivalent reference frames S_n, play a distinguished role. Besides the well defined instruction that can be given to each individual observer on the manner he should proceed in order to construct his reference body $\bar{S}_n$ so that his S_n be identical with $\bar{S}'_n$ of another observer (they have to exchange the unit length), the orthogonal reference frames S_n (bodies $\bar{S}_n$) disclose the fundamental symmetry of E_n space. The point is that if $X_j (j = 1, \ldots, n)$ denote the representation of the point X in some S_n and X'_j do the same in S'_n, then X'_j and X_j are connected by $n(n+1)/2$-parameter orthogonal group of transformations

$$X'_j = O^{(n)}_{jk} X_k + A^{(n)}_j , \quad O^{(n)^T} = O^{(n)^{-1}} , \tag{3.1}$$

where $O^{(n)}$ is a $n \times n$ orthogonal matrix and $A_j^{(n)}$ represents the translation symmetry subgroup. Of course, the relative position $x = X_2 - X_1$ representations, where $X_{1,2}$ are two arbitrary points of E_n, are connected by homogeneous rotation group

$$x'_j = O_{jk}^{(n)} x_k \,. \tag{3.2}$$

The distinguished position of orthogonal parametrization of the points X of E_n does not exclude point-transformations to, for instance, spherical (or other) coordination ξ_j of E_N

$$\xi_j = \xi_j(X_k)\,, \quad X_j = X_j(\xi_k)\,, \tag{3.3}$$

but the physical text of initial symbols is imposed by the orthogonal (equivalent) reference frames $S_n(X_j)$. The fundamental reason for the distinguished character of the $S_n(X_j)$ reference frames consists in the facts that: (1) the coordinates X_j of S_n have the same physical text as X'_j of S'_n, X''_j of S''_n, etc. (Using the vocabulary of principle of relativity we shall speak of different *equivalent observers* in different reference frames (bodies) S_n, S'_n, S''_n having at their disposal analogous measuring devices); (2) from the algebraic point of view, the distinguished role of $S_n(X_j)$'s means that the interval r between two points $X_{1,2}$ is $O^{(n)}$-form-invariant, as

$$r'^2 = x'_j x'_j = x_j x_j = r^2 \,. \tag{3.4}$$

Moreover, as it is well known, the opposite statement is also true, namely if quadratic form $x_j x_j$ is to be form-invariant under point transformations (3.3) we have

$$\xi_j = X'_j \,, \tag{3.5}$$

which means that point transformations (3.3) are restricted to those given by (3.1). In other words, only X_j-parametrizations do reflect, algebraically, the symmetry $O^{(n)}$ of Euclidean spaces E_n.

Of course, one may always introduce, in flat spaces E_n, an X_j parametrization when the metrics of E_n is given by finite quadratic form (3.4). However, this aspect of flat spaces ignores the double aspect of transformation (3.1) corresponding with — using the Wigner terminology [14] — the *passive* and *active* interpretations of point transformations like those from (3.1), strictly connected with the existence of $O^{(n)}$-form-invariant extensions $G(r^2)$ embedded in E_n and depending on *the two-point* $x = X_2 - X_1$. This double aspect of suitable point-transformations will be crucial in understanding the principles of relativity in L_4 and in G_4 discussed further on.

In our case of Euclidean spaces E_n, the passive and active interpretations of symmetry group (3.1) means the following: According to the passive interpretation of point-transformation (3.1) we have to deal with a single geometrical object, like a point X of E_n and hence, with a single extension $f(X_j)$, spanned on X's and represented in some fixed reference frame S_n. Simultaneously, we have to deal with an infinite set of representations of the same (single) object represented in all equivalent reference frames S_n. However, the same mathematics (algebra) contained in (3.1) carries quite a different, active interpretation of symmetry (3.1). In the geometrical (physical) language, this means that we are fixing an arbitrary reference frame S_n, while n coordinates X'_J of the left member of (3.1) are the coordinates of another point X' in E_n, measured in the same reference frame S_n as the coordinates X_j of the right member of (3.1). Thus X' is the picture of X under point-transformation (3.1). Physically, this means that we are dealing with a single *observer* in S_n and an infinite set of physical (geometrical) objects, each of them being the picture of another one under a suitable point-transformation (3.1).

The geometrization of physics (so far restricted to static Euclidean spaces E_n) resorts to the passive interpretation of symmetry (3.1) only. Indeed, the question concerns various (possible) and, at the same time, equivalent *positions* of an *observer* as represented by different (existing at observer's disposal) reference frames S_n. Thus if $f(X_j)$ represents a *point-shape* in S_n (as spanned on the points X), the same *point-shape* f takes on another algebraic expression in S'_n (*cf.* Section 5). This *sameness* of f embedded in E_n is guaranteed by the $O^{(n)}$ covariant structure of laws and boundary conditions which determine $f(X_j)$ in any reference frame S_n. However, the $O^{(n)}$ covariant form of laws which determine f's does not involve an $O^{(n)}$-form-invariant form of f's, and it is this very form which constitutes the clue to the active interpretation of a suitable symmetry, *e.g.* — in our case — that given by (3.1). Let us illustrate this by a simple example.

The Poisson equation of the scalar *point-shape* of potential $\Phi(X_j)$ takes the $O^{(n)}$ covariant form

$$\Delta\Phi(X_j) = -\rho(X_j)\,, \tag{3.6}$$

where $\rho(X_j)$ is also a scalar point-shape, representing (in some S_n) a given source ρ of Φ. (For further purposes, we shall also refer to *the point-shapes* as to *event shapes.*) Of course, the algebraic form of external event or point shape $\rho(X_j)$ is not $O^{(n)}$-form-invariant and hence, equation (3.6) represented in another reference frame takes the form

$$\Delta\Phi(X_j) = -\rho\left(O^{(n)}_{kj}(X_k - A_k)\right)\,, \tag{3.7}$$

which is algebraically different from that of (3.6).

A different situation occurs for a homogeneous equation with $\rho = 0$, when

$$\Delta \Phi(X_j) = 0\,, \tag{3.8}$$

as this equation becomes $O^{(n)}$-form-invariant. Consequently, if $\Phi_0(X_j)$ is some particular solution of (3.8), then this solution, together with the picture of $\Phi_0(X_j)$ under transformation (3.1), *i.e.*

$$\Phi(X_j) = \Phi_0 \left(O^{(n)}_{kj}(X_k - A_k) \right)\,, \tag{3.9}$$

represent the $n(n+1)/2$-parameter family of solutions of (3.8). In other words, the $O^{(n)}$-form-invariance of (3.8) means that $O^{(n)}$ is the internal symmetry group of (3.8). In such a case, only some boundary conditions imposed onto the solutions Φ's may distinguish between different reference frames which parametrize S_n.

Now, let us proceed with 4-spacetimes (G_4 and L_4), *i.e.* let us turn our attention to eventism, although the fourth dimension (that of time) was originally *added* to the E_3-space in order to account for the phenomenon of *change* as exemplified by motion of a corpuscular matter. Metrical time, measured by *good* clocks, creates a new *physical text* of *practical geometry* of space–and–time strictly connected with the first Newtonian principle (objectism) based on the state of isolation of the physical system $\mathcal{M}$ under description. The 4-space–and–time has been introduced to define quantitatively the *potential reality* of forces as the entities which are responsible for accelerated motion of $\mathcal{M}$ as a whole in an inertial reference frame S, in which any isolated object $\mathcal{M}$ moves without acceleration. Thus, the inertial reference frames S (we leave out the subscripts) reflect the inertial nature of the global spacetime structure strictly connected with the time dimension.

For the same reasons as those mentioned when discussing E_n-spaces, the events X (points of spacetime) represented in S frames will be parametrized by the same Cartesian coordinates X_j of the E_3-spaces of S's and the time variable t indicated by *good* (and synchronised) clocks at rest in the given reference frame S

$$X = (X_j, X_0); \quad X_0 = ct \text{ in } L_4\,, \quad X_0 = t \text{ in } G_4\,. \tag{3.10}$$

It is worth emphasizing that S_4 acquires the nature of inertial reference frame S if one assumes that S_4 is the rest frame of an infinitely heavy object $\mathcal{M}$ ($M \to \infty$). Indeed, finite forces cannot accelerate $\mathcal{M}$ of an infinite inertia.

Now, let us introduce an important notion of *balance* of practical geometries: E_n, G_4 and L_4 which characterizes the relationship between their mathematical and empirical aspects. We begin with Euclidean geometries E_n $(n = 1,2,3)$ whose foundations resort to the existence of (practically) rigid rods which obey the Pythagoras law. Thus, from the empirical world we borrow the rigid rods and introduce an absolute property — the rods' lengths (measured in some optional units). Hence, the debt of theory amounts to -1. However, after assuming that the practical space-geometry is a Euclidean one with symmetry $O^{(n)}$ given by (3.1), the $O^{(n)}$-form-invariant interval $r = (x_j x_j)^{(1/2)}$ parametrizes properly absolute extensions of real objects (rigid bodies). In consequence, mathematics of E_n geometries returns the debt $(+1)$ which makes it justified to say that *the balance* of practical E_n geometries is equalized: $-1 + 1 = 0$.

Let us consider, in an analogous way, the practical spacetime geometries G_4 and L_4, beginning with G_4. As it follows from the consideration presented above, each of the reference frames S deals with an E_3 space "of its own" with the equalized balance but the metrical time borrows from the empirical world *good* clocks (-1). However, symmetry G of G_4 results in G-invariant intervals Δt of $E_1(t)$ parametrizing absolute intervals of real *good* clocks. Thus, the total balance of G_4 eventism will be characterized by the equality $-2 + 2 = 0$ which means that the balance of G_4 geometry is equalized. Logically, one may expect that the G_4 eventism makes room for a (non-relativistic) *closed* theory capable of reproducing the structures of rods and clocks as its particular solutions.

Minkowskian spacetime L_4 starts with the same negative balance -2 as G_4, but eventism L_4 has to deal with only one L-form-invariant four-interval $(+1)$

$$x'^2 = \boldsymbol{x}'^2 - x_0'^2 = \boldsymbol{x}^2 - x_0^2 = x^2\,, \quad (x_0 = c\Delta t)\,. \tag{3.11}$$

According to the above we should end up with a negative balance: $-2+1 = -1$. In classical physics $(\hbar = 0)$ which employs CCINF's (and hence is condemned to eventism), no other L-invariant spacetime-interval exists *a priori*, but the L-invariance of the velocity of light suggests the possibility of following argumentation in favour of equalized balance of L_4 geometry.

Let us consider a light signal and let the equality

$$r = c\Delta t \tag{3.12}$$

define the measure of space-interval r which reduces such an interval to a time-interval Δt (provided that c is known). (N.B. this is the way we define today the metre.) Such a reduction of the measure of r to the measure of Δt excludes possibility of independent measurements of the velocity of light, but it entails, on the other hand, that we borrow from empirical world only one measure — that of time-interval. This would lead to the initial balance

-1 but the L-invariant four-interval (3.11) equalizes this negative balance: $-1 + 1 = 0$, similarly as in E_n spaces with definite metrics.

Such an operational way of arguing in favour of equalized balance of Minkowskian L_4 spacetime geometry $(-1 + 1 = 0)$ makes an essential use of the classical framework $(\hbar = 0)$ and CCINF's. Here, the measurement of space-interval r with the aid of a light signal can be performed with arbitrary (in principle) accuracy, as the classical light signal (CCINF) causes no uncontrollable recoil of $\mathcal{M}$ which carries the space extension. Consequently, in quantum physics without CCINF's, there is no place for the above (classical) equalization of the balance of L_4 $(-1+1=0)$. In order to avoid having an uncontrollable localization (δX) and uncontrollable velocity $(\delta V \simeq \delta P/M)$ for $\mathcal{M}$, *i.e.* in order to have

$$\delta X \to 0 \quad \text{and} \quad \delta V \to 0 \quad \left(V = \frac{|\boldsymbol{P}|}{M}\right), \tag{3.13}$$

without violating the uncertainty relation

$$(\delta X) \simeq \frac{\hbar}{\delta P} \simeq \frac{\hbar}{M}\frac{1}{\delta V}, \tag{3.14}$$

we must assume $\mathcal{M}$ to be infinitely heavy, which in turn entails the rest frame of $\mathcal{M}$ to be an inertial one as

$$M \to \infty . \tag{3.15}$$

So, in quantum physics, we are left with two qualitatively different properties of space- and time-extensions which proves that the L_4 eventism deals with a negative balance. This supports strongly the opinion formulated by von Weizsaecker: "Spacetime is not the background but a surface aspect of reality" [15]. In the perspective of *balance* of practical (physical geometries) this would mean that the L_4 geometry must be based on some (relational) geometry which would go deeper into the metrical nature of physics and whose balance would be equalized *a priori*.

The most spectacular manifestation of the negative balance of Minkowskian spacetime L_4 takes the form of *no interaction theorems*. In classical relativistic mechanics, subject to canonical p–x symmetry, from the L-form-invariant equations of motion of an isolated composite system $\mathcal{M}$ follows a free motion of each of the system constituents, so these equations admit relativistic kinematics only [16, 17]. The same extreme restrictiveness of symmetry L concerns also field theories, where — as clearly shown by Feynman [18] — the event locality of symmetry L remains in conflict with the internal dynamics responsible for stability of an extended system $\mathcal{M}$. The same *no interaction theorem* is also strongly suggested by the axiomatic S matrix theory [19, 20] based on symmetry L and resulting in $S = 1$.

The *no interaction theorems* support strongly the opinion formulated above according to which the negative balance of geometry L_4 excludes possibility of existence of any *closed* theory based on eventism L_4, *i.e.* any theory which would respect symmetry L and would provide us with the structure of *good* rods as its particular solutions. In this context, it might be worth remembering that "young Einstein", when putting forward the hypothesis of STR, always strongly emphasized that the notion of space- and time-intervals has no other physical sense apart from that of real rods and clocks. At the same time, he maintained that so far we are forced to borrow from the empirical word the space- and time-extensions of rods and clocks, respectively, as the theory is still far from being capable to furnish the structures of rods and clocks as its particular solutions [3]. After Minkowski's geometrization of STR, Einstein's opinion on this subject has remained dominated by eventism (opposed to the previous objectism [2]) most extremely perceived in his GTR.

It is interesting to inspect more closely the reason of fundamental difference between symmetry groups L and G, the latter group being free of any *no interaction theorems*. Let us start with eventism G_4 which admits an orthogonal group of transformations much larger than symmetry G by admitting time-dependent, orthogonal matrix $\widehat{O}$ and space translations A_j (also time-dependent)

$$X'_j = O_{jk}(t)X_k + A_j(t)\,, \quad t' = t + A_0 \quad \left(\widehat{O}^T = \widehat{O}^{-1}\right)\,. \tag{3.16}$$

The point is that both $r = |\boldsymbol{X}_2 - \boldsymbol{X}_1|_{\Delta t=0}$ and the time-interval Δt between arbitrary events $X_{1,2}$ are the two invariants of (3.16). In order to introduce the physical text of inertiality of spacetime we must explicitly resort to the Newtonian principles, similarly as we do it when determining *pseudo-forces* and/or when determining the equations of motion of an NR rigid body.

On the other hand, the universal constant c that enters symmetry L amalgamates space with time as it accounts for (relativistic) Maxwell equations. Consequently, the equations characteristics as parametrized in our Cartesian coordinates X_j and $X_0 = ct$ (carrying the same text when referred to different reference frames) require form-invariance of four-interval x^2, *i.e.* $x'^2 = x^2$. However, contrarily to symmetry G, the form-invariance of the quadratic form $x^2 = \boldsymbol{x}^2 - x_0^2$ determines symmetry L of events X and restricts automatically the reference frames S_4 to inertial reference frames S. Indeed, if $X = (X_j; X_0)$ were to parametrize an object $\mathcal{M}$ in some reference frame $S_4 \neq S$, *e.g.* an object rotating in S with a finite angular velocity Ω, from the very infinity of Cartesian reference frame S_4 would follow that

$\mathcal{M}$ at rest in S_4 acquires a superluminal velocity in some inertial frame S, as $r\Omega > c$ if r is large enough. Thus, we must have $\Omega = 0$ and hence, $S_4 = S$.

Thus, the L_4 eventism is intimately connected with the inertial nature of spacetime which, in the case of the G_4 eventism with its equalized balance, must explicitly resort to the Newtonian laws of motion.

4. Singularity of eventism G_4 and NR relational spacetime I_4^G

The representations in different inertial reference frames S of the L_4 and G_4 spacetimes of an event X are connected by

$$X' = \hat{L}X + A\,, \qquad \text{(L)} \qquad X' = \hat{G}X + A\,, \qquad \text{(G)} \tag{4.1}$$

where $\hat{L}$ and $\hat{G}$ denote the 4×4 matrices of homogeneous 6-parameter Lorentz and Galilean symmetry groups, respectively, and A is the 4×1 matrix of the spacetime symmetry. Neither L_4 nor G_4 are Cartesian products of space $[E_3(\boldsymbol{X})]$ and time $[E_1(t)]$ continua, but from the absoluteness of the Newtonian time follows that G is a semi-group. Thus (4.1 G) rewritten in terms of the space and time variables takes the form

$$X'_j = O_{jk}(X_k - V_k t) + A_j\,, \quad t' = t + A_0 \quad (j, k = 1, 2, 3) \tag{4.2}$$

while the representations of $\boldsymbol{x} = \boldsymbol{X}_2 - \boldsymbol{X}_1$ are connected by

$$x'_j = O_{jk}(x_k - V_k \Delta t)\,, \quad \Delta t' = \Delta t\,. \tag{4.3}$$

As it follows from (4.3), the space-interval R between two events depends on the reference frame S, as

$$R'^2 = \boldsymbol{x}'^2 = \boldsymbol{x}^2 - 2(\boldsymbol{V}\boldsymbol{x})\Delta t + \boldsymbol{V}^2(\Delta t)^2 \neq \boldsymbol{x}^2 = R^2\,. \tag{4.4}$$

However, the G-form-invariant constraint $\Delta t = 0$ imposed onto two events $X_{1,2}$ and contained in the very one-time non relativistic equations of motions makes $r \equiv R|_{\Delta t=0}$ a G-absolute equality. Consequently,

$$r' \equiv R'\big|_{\Delta t=0} = R|_{\Delta t=0} \equiv r \quad \text{and} \quad \Delta t' = \Delta t \tag{4.5}$$

are two G-form-invariant intervals of the spacetime G_4 making the balance of G_4 geometry equalized: $-2 + 2 = 0$.

On the other hand, the frame-dependent value of R discloses a dilemma of eventism (here of eventism G_4) connected with the spacetime vicinity of two spacetime regions which is the basis of experimental actualizations. Suppose two events $X_{1,2}$ to have, in some reference frame, the same space localization ($\boldsymbol{x} = 0$), but to occur at two different instants $t_{1,2}$, so $\Delta t = t_2 - t_1 > 0$. From (4.4) it can be seen that in the reference frame S' which moves in S with a velocity $V = |\boldsymbol{V}|$ the space distance between the two events $X_{1,2}$ is equal to $R' = V\Delta t$. However, the velocity V of S' with respect to S can be, in G_4, arbitrarily large and hence,

$$R' = V\Delta t \underset{V\to\infty}{\longrightarrow} \infty \,, \tag{4.6}$$

even if Δt is arbitrarily small. Thus, in eventism G_4, the vicinity of two spacetime regions loses the G-absolute meaning unless we impose, from the very beginning, the constraint $\Delta t = 0$ when $r' = r$. No experiment can realize this mathematical accuracy, although the one-time NR theory is free of this dilemma.

The reasons why the above dilemma does not concern practical (experimental) physics are quite different than those in theoretical physics, where the condition $\Delta t = 0$ is imposed by the one-time laws of motion. In practical physics, we tacitly assume that, in spite of Galilean principle of relativity admitting arbitrarily large velocities V, the mutual velocities between corpuscular objects (characterized by some velocity v) are negligible (zero) as compared with the velocity of signals which are the carriers of the information on the space localization of the corpuscular objects. Since information is carried, first of all, by light with the velocity c, this tacit assumption is equivalent to the condition that v is much smaller than c. Indeed, if $l = v\Delta t$ is to characterize the change of space configuration of the observed system during a time-interval $\Delta t = r/c$, where r is the distance between the observer and the observed object, than l should be much smaller than r, hence

$$\frac{l}{r} = \frac{v}{c} \ll 1 \,. \tag{4.7}$$

The finite velocity of light c was discovered by Roemer who observed irregularities of one of the Jupiter's moons due to the varying distance r between the Earth and Jupiter.

In principle, the strong inequality (4.7) that really dominates the world of macro-physics neither abolishes the dilemma of eventism, nor is canceled by the mathematical limit $c \to \infty$ characterizing the NR physics when symmetry L of L_4 converts into symmetry G of G_4. Indeed, even with c tending to infinity, the velocity v may tend to infinity as well, preserving the v/c ratio finite, although less than one. From the point of view of the Galilean principle of relativity (symmetry G), strong inequality (4.7) realized by macro-objects is "accidental" and it was this "accident" which made possible the conceptualization of the NR physics.

From now on — in order to refrain from introducing new symbols — let x_j denote, in G_4, the relative space components of two simultaneous events occurring in G_4, in agreement with the usual notation of NR mechanics. However, let x denote, as before, the relative four-coordinate in G_4 and L_4: $x = (x_j; x_0)$, where $x_0 = \Delta t$ in G_4 and $x_0 = c\Delta t$ in L_4. With this notation equation (4.3) transforms into

$$x'_j = O^G_{jk}\, x_k \quad (\Delta t' = \Delta t = 0)\,. \tag{4.8}$$

The superscript G in the orthogonal matrix is meant to point out that the space-rotation symmetry $\widehat{O}$ is induced by symmetry $\widehat{G}$ of eventism G_4. Accordingly to (4.8), $\boldsymbol{x}'^2 = \boldsymbol{x}^2$, hence the only dependence of x_j's on S results via rotation group $\widehat{O}^G$.

In classical physics, with CCINF's and condemned to eventism, the rotation symmetry of internal variables such as x_j, must be coupled to the rotation symmetry of the whole $E_3(\boldsymbol{X})$ space of some reference frame S, hence $\widehat{O} = \widehat{O}^G$. However, together with the quantum p–x duality, it is possible to presume that a micro-system $\mathcal{M}$, fully isolated from the external world of measurement, constitutes a self-dependent *micro-world* $\mathcal{M}$. In NR theory ($1/c = 0$) this hypothesis is realized by introducing the Euclidean *relational space* R^G_3 whose rotation symmetry $\widehat{O}^R$ is *a priori* independent of the rotation symmetry $\widehat{O}^G$. Consequently, let y_j denote the relational coordinates of a point in the $R^G_3(\boldsymbol{y})$-space whose isotropy results in the 3-parameter rotation symmetry

$$y'_j = O^R_{jk}\, y_k\,, \quad x'_j = O^G_{jk}\, x_k\,, \quad \boldsymbol{y}^2 = \boldsymbol{x}^2\,. \tag{4.9}$$

The lengths of $\boldsymbol{x}$ and $\boldsymbol{y}$ remain the same, but

$$\widehat{O}^R \neq \widehat{O}^G\,. \tag{4.10}$$

According to relationism, the quantum p–x duality expressed by

$$[\widehat{y}_j, \widehat{q}_k] = i\hbar\delta_{jk} \tag{4.11}$$

becomes anchored in the p and x aspects of the relational space R^G_3, *i.e.* in $R^G_3(\boldsymbol{q})$ and $R^G_3(\boldsymbol{y})$. Fourier transforms establish correspondence between $\langle \boldsymbol{q}|F\rangle$ and $\langle \boldsymbol{y}|F\rangle$, *i.e.* between the p and x representations of a Hilbert vector $|F\rangle$ in $R^G_3(\boldsymbol{q})$ and $R^G_3(\boldsymbol{y})$, respectively. Fourier transforms make the relational space R^G_3 an infinite and indivisible whole in its p and x aspects. Duality p–x of the R^G_3 space makes it essentially different from Cartesian x-space $E_3(\boldsymbol{X})$ of classical physics. Of course, Cartesian x-space $E_3(\boldsymbol{X})$ admits Fourier transforms of $\langle \boldsymbol{X}|F\rangle$, but physics without $\hbar$ ($\hbar = 0$) excludes the quantum p–x duality, so there is no room left for the duality of the first background of any classical extension. Therefore, one is justified to speak of R_3^G as of *a geometrical meta-object* connected with a mechanical configuration sub-space rather than with classical spacetime G_4 of pre-existing events X.

However, in spite of the signalized difference between R_3^G and $E_3(X)$, and although $\widehat{O}^R \neq \widehat{O}^G$, R_3^G and G_4 are indeed connected by a mathematical isomorphy which is responsible for the tremendous success of NR quantum physics based on classical eventism G_4. We shall see that this isomorphy between the hypothetical R_3-relational space and eventism L_4 disappears in physics which presumes finiteness of the universal constant $\hbar/c$. Then the distinction between relationism and eventism becomes physically relevant.

From the point of view of the relational space R_3 introduced further on and accounting for the finiteness of the universal constant $\hbar/c$, which in turn leads to abandoning of eventism L_4, the singularity of NR eventism G_4 consists in its coexistence with relational space R_3^G. In consequence, the structure of a system $\mathcal{M}$ composed of N constituents: $\mathcal{M} = A_1 + A_2 + \ldots + A_N$, finds the room in the $3(N-1)$-dimensional configuration space induced by R_3^G: $\overbrace{R_3^G \otimes \ldots \otimes R_3^G}^{N-1}$ whose dual p–x nature reflects the wave–corpuscular duality of the quantum $\mathcal{M}$'s. The coexistence of R_3^G with G_4 would be responsible for the enormous success of NR quantum mechanics which maintains *the classical* spacetime G_4 of actualizations (measurements) as the background of *quantum* relational structures on their potential level characterized by ψ function. In true physics which employs finite $\hbar/c$ constant, the relational structure of $\mathcal{M}$ on its quantum-potential level will be *hidden* [21] in $\overbrace{R_3^G \otimes \ldots \otimes R_3^G}^{N-1}$-space.

Let us emphasize that the elementary (in relationism) two-body system $\mathcal{M} = A_1 + A_2$ introduces two hypothetical constituents $A_{1,2}$, but that does not entail existence of each of them separately in the external spacetime of measurement as it takes place in eventism. The point is that, according to relationism, the spacetime continuum of observable events X will stand on the footing of a limiting case of relationism conditioned by suitable physical situation of the system under consideration. Let us remember that the identification of physical existence of some entity with its existence in the external spacetime of measurement is inherent in eventism and hence, in classical physics ($\hbar = 0$) with CCINF's. The quantum p–x duality makes room for the hypothesis of relational space of directly unobservable points-relations which constitute foundations of metrical physics and make possible to analyze directly observable events X.

In order to emphasize the configurational origin of relationism with its quantum p–x duality, let us stress — after Heisenberg [5] — that the wave–corpuscular nature of matter reveals itself in each (canonical) degree of freedom of the configuration space of $\mathcal{M} = A_1 + \ldots + A_N$. These waves in configurational spaces are then of quite a different nature than the classical waves in spacetime. Within the eventism (G_4 or/and L_4) the wave of matter may appear in spacetime for a one-body problem only.

Let us attach, in G_4, to an isolated micro-world $\mathcal{M}$ (viewed as a whole) a G-absolute internal-time continuum $\mathcal{T}^G(\tau)$ which coincides, up to an arbitrary (*a priori*) translation constant, with the Newtonian time continuum $T(t)$, *i.e.*

$$\tau = t + C\,, \quad \Delta\tau = \Delta t\,. \tag{4.12}$$

This exhibits also the singularity of symmetry G or, in other words, the equalized balance of geometry G_4. The 3-parameter rotation symmetry $\widehat{O}^R$ of R_3^G together with the 1-parameter translation symmetry of internal-time τ constitute the 4-parameter symmetry group R of the *a priori* G-absolute *internal* spacetime I_4^G. Thus I_4^G is a Cartesian product of R_3^G and $\mathcal{T}^G$

$$I_4^G = R_3^G \otimes \mathcal{T}^G\,. \tag{4.13}$$

5. Relational shapes *versus* event shapes and NR mechanics

Full contents of the principle of relativity call for the passive and active interpretations of symmetries (4.1) of L_4 and G_4. These two interpretations of symmetries $\widehat{L}$ and $\widehat{G}$, as well as of the orthogonal symmetry (3.1) of E_n, are connected with the fact that the spaces: L_4, G_4 and E_n, if parametrized by our Cartesian coordinates, deal with x-form-invariant intervals under the transformation groups: L, G and $\widehat{O}$, respectively. In particular, L covariant equations of motion, and thus consistent with STR which requires only the passive interpretation of symmetry L, become L-form invariant if they describe a fully isolated system $\mathcal{M}$ in the whole L_4 spacetime. Had the symmetry L not been the internal symmetry of equations of motion, the analytic form of these equations expressed by X-variables would have distinguished between different (equivalent) reference frames S in conflict with the principle of relativity.

In quantum-relativistic physics, the Bethe–Salpeter equation [22] describing two-body systems is an example of L-form-invariant equation. Free Maxwell and free Dirac equations are other examples of L-form-invariant equations, similarly to equation (3.8) which is $\widehat{O}$-form invariant. However, the L covariant Maxwell equations with external four-current density $j(X)$

break the internal symmetry $\widehat{L}$; the breaking results from the presence of external field $j(X)$. Let us examine this in a more detailed manner which will enable us to introduce some classification of shapes embedded in spacetime.

It is sufficient to confine our considerations to scalar representation of $\widehat{L}$ and $\widehat{G}$ groups given by a scalar property f spanned on spacetime continuum. Consequently, in some (arbitrary) reference frame S, the property f takes the form of a mathematical function (distribution) $f(X)$ and, therefore, let us call f an event shape. Now, let $'f(X)$ denote *the same* event shape f when represented in S', hence

$$'f(X') = f(X), \quad X' = \begin{pmatrix} \widehat{L} \\ \widehat{G} \end{pmatrix} X + A. \tag{5.1}$$

Thus $'f$ parametrized by the same mathematical variables X as f was in S takes the form

$$'f(X) = f\left\{ \begin{pmatrix} \widehat{L}^{-1} \\ \widehat{G}^{-1} \end{pmatrix} (X - A) \right\}. \tag{5.2}$$

If four (mathematical) variables X are interpreted geometrically as variables which parametrize event X in a fixed reference frame S, then (5.2) illustrates the active role of symmetries L and G. Now, if the X variables (*non-primed variables)* in the right member of (5.2) parametrize events in S', while the X variables in the left member of (5.2) parametrize events in S, then the symmetries $\widehat{L}$ and $\widehat{G}$ exhibit their passive role. Equation (5.2) shows that no event shape can represent an L-form-invariant shape, except from $f(X)$ =constant in the whole infinite spacetime. Thus

$$'f(X) \neq f(X) \quad \text{if} \quad f(X) \neq \text{constant}. \tag{5.3}$$

Following the same principle, let us consider another scalar property g which now is spanned on relative coordinates: $x = X_2 - X_1$. Logically, we shall label g a *two-event shape*, and the sameness of g represented in different reference frames and parametrized by four mathematical variables $x = (x_j; x_0)$ results in

$$'g(x) = g\begin{pmatrix} \widehat{L}^{-1} \\ \widehat{G}^{-1} \end{pmatrix} x\,. \tag{5.4}$$

Again symmetries $\widehat{L}$ and $\widehat{G}$ can be interpreted passively and actively and, in general, $'g(x) \neq g(x)$.

However, two-event shapes disclose the fundamental property of flat spaces, such as E_n, G_4 and L_4, namely that these spaces deal (in our Cartesian parametrization) with form-invariant metrics. Let us remember that in the case of E_n and L_4 geometries the requirement of form-invariance of interval $r^2 = x_j x_j$ in E_n and the form-invariance of four-interval x^2 in L_4 result in symmetries (3.1) and (4.1 L), respectively. In consequence, there is a sub-class of two-event shapes in L_4 and in G_4 — denoted henceforth by capital letter G — which are form-invariant under the corresponding $\widehat{L}$ and $\widehat{G}$ transformation groups. In L_4 we deal with

$$'G(x^2) = G(x^2)\,, \tag{5.5}$$

while in G_4 we deal with two types of G-form-invariant two-event shapes, namely

$$\begin{aligned} 'G(\Delta t) &= G(\Delta t)\,, & (i)\\ 'G(x) &= F(x^2)\delta^{(1)}(\Delta t)\,. & (ii) \end{aligned} \tag{5.6}$$

We shall denote by $\mathcal{L}_4(x)$ and $\mathcal{G}_4(x)$ the four-space spanned on relative four-coordinates $x = X_2 - X_1$, depending on whether the corresponding spacetime is L_4 or G_4, respectively. Since $\mathcal{L}_4(x)$ will induce the configuration spaces $\mathcal{L}_4 \otimes \mathcal{L}_4 \otimes \ldots$ of composite micro-objects and micro-processes, the p–x duality of these objects and processes imparts the p and x aspects to the 4-space $\mathcal{L}_4$ itself, hence we deal with $\mathcal{L}_4(p)$ and $\mathcal{L}_4(q)$, respectively. Infinity of 4-space $\mathcal{L}_4$, together with its indivisibility connected with the unitary transformation structure (Fourier transforms) establishing the correspondence between the p and x representations of *quantum* extensions make $\mathcal{L}_4$ — besides R_3^G — another *geometrical meta-object.*

The L-form-invariance of form factor $G(x^2)$ of particle $\mathcal{M}$ embedded in $\mathcal{L}_4(x)$ or, interchangeably, L-form-invariance of $\tilde{G}(p^2)$ embedded in $\mathcal{L}_4(p)$, carries two different physical texts resulting from the passive and active interpretation of symmetry L. The same concerns the G-form-invariance of form factor $G(x)$ from (5.6). Passive interpretation means that G takes the same analytic form if parametrized by invariant intervals of spacetime, independently of the lab-system in which G is being measured. Active interpretation means that the external motion of $\mathcal{M}$ as a whole in a fixed lab-system does not affect the analytic structure of G. Thus the form-invariance of form factors (5.5) (in L_4) and (5.6) (in G_4) manifests the separation of the internal and external degrees of freedom of $\mathcal{M}$. In L_4, the L-form-invariance of $G(x^2)$ means that the structure of $\mathcal{M}$ suffers from no *relativistic distortions* [23], the best known among which is the Lorentz contraction, a distortion of a purely kinematic origin [24]. The non-separability of the internal degrees of freedom of $\mathcal{M}$ from the external ones, inherent in the L_4 eventism, will be discussed in Section 7 and, as we may see, it stands in fundamental conflict with L-form-invariant form factor $G(x^2)$ (or, interchangeably, $G(p^2)$). The equalized balance of geometry G_4 makes this conflict vanish in G_4, because of the coexistence of R_3^G with G_4 which entails the separation of the internal degrees of freedom of $\mathcal{M}$ from its external ones.

Besides the L- and G-form invariant two-event shapes (5.5) and (5.6), respectively, the eventism itself results in the L- and G-form invariance of the four-dimensional Dirac function

$$'\delta^{(4)}(x) = \delta^{(4)}\left(\begin{matrix}\hat{L}^{-1} \\ \hat{G}^{-1}\end{matrix}\, x\right) = \delta^{(4)}(x)\,. \tag{5.7}$$

This is obvious, as $\delta^{(4)}(x)$ accounts for the L- (G-)absoluteness of the coincidence of two events when $x = X_2 - X_1 = 0$. In consequence, $\delta^{(4)}(x)$ is the universal L-(G-)form-invariant form factor of all point–particles. Owing to the equalized balance of geometry G_4 and confronting (5.6 *ii*) with (5.7) we see that the space-locality factor $\delta^{(3)}(\boldsymbol{x})$ of $\delta^{(4)}(x)$ can be continuously approximated by a space nonlocal and G-form-invariant form factor $F(r)$,

$$F(r) \rightarrow \frac{\delta^{(1)}(r)}{2\pi r^2} = \delta^{(3)}(\boldsymbol{y})\,, \quad (r = |\boldsymbol{y}|)\,. \tag{5.8}$$

The negative balance of geometry L_4 excludes any L-form invariant form factor which would account for the space nonlocality (in a similar way as (5.8) does in G_4). This results immediately in a dilemma of eventism L_4 analogous to that of eventism G_4 discussed in Section 4. Indeed, besides the spacetime-local form factor $\delta^{(4)}(x)$, any other L-form-invariant form factor takes the form $G(x^2)$ and occupies the whole $\mathcal{L}_4(x)$-space remaining constant on the unbounded *Minkowski's spheres* $x^2 =$ constant. This shows why $\mathcal{L}_4(x)$ is an infinite and indivisible whole, called *the geometrical meta-object.* The discontinuity between $G(x^2)$ and $\delta^{(4)}(x)$ shows that it is impossible to express L-absolutely the vicinity of two bounded spacetime regions. This fact will be essential when we will be discussing the adiabatic hypothesis in Section 9. The same dilemma of eventism L_4 explains why relativistic (spacetime-local) field theory is restricted to point-particles only [25] with the universal L-form-invariant form factor $\delta^{(4)}(x)$.

Now let us show how NR mechanics exhibits the singularity of symmetry G which — as we know — consists in the coexistence of G_4 and R_3^G and hence, in the separability of the internal degrees of freedom of an isolated system $\mathcal{M}$ from its external ones. It is sufficient to consider the elementary composite two-body system $\mathcal{M} = A_1 + A_2$ starting with the (equal-time) 6-dimensional configuration space $E_3(\boldsymbol{X}_1) \otimes E_3(\boldsymbol{X}_2)$ based on an arbitrary reference frame S parametrizing G_4. The Newtonian, G-absolute, time parametrizes absolute evolution of all possible states of $\mathcal{M}$, both scattering or bound ones, generated by Hamiltonian H^G. This Hamiltonian can be taken in its simplest form

$$H^G = \frac{\boldsymbol{P}_1^2}{2m_1} + \frac{\boldsymbol{P}_2^2}{2m_2} + V\left((\boldsymbol{X}_2 - \boldsymbol{X}_1)^2\right). \tag{5.9}$$

Here, the dependence of H^G on S enters via the kinetic energy operator only, as the action–at–a–distance given by potential $V\left((\boldsymbol{X}_2 - \boldsymbol{X}_1)^2 = \boldsymbol{y}^2\right)$ is a G-absolute relational shape embedded in R_3^G.

The well known canonical transformation

$$\left.\begin{array}{ll} \boldsymbol{X} = a^G \boldsymbol{X}_1 + (1-a^G)\boldsymbol{X}_2\,, & \boldsymbol{x} = \boldsymbol{X}_2 - \boldsymbol{X}_1 \\ \boldsymbol{P} = \boldsymbol{P}_1 + \boldsymbol{P}_2\,, & \boldsymbol{p} = a^G \boldsymbol{P}_2 - (1-a^G)\boldsymbol{P}_1 \end{array}\right\} \tag{5.10}$$

with *the weight* a^G given *a priori* by masses of the constituents

$$a^G = \frac{m_1}{m}\,, \qquad 1 - a^G = \frac{m_2}{m}\,, \qquad m = m_1 + m_2\,, \tag{5.11}$$

guarantees (*a priori*) the G-absoluteness of internal canonical variables $\boldsymbol{x}, \boldsymbol{p}$

$$\begin{aligned} \boldsymbol{p}' &= a^G P_2' - (1-a^G)P_1' = a^G(\boldsymbol{P}_2 + m_2\boldsymbol{V}) + (1-a^G)(\boldsymbol{P}_1 + m_1\boldsymbol{V}) \\ &= a^G\boldsymbol{P}_2 - (1-a^G)\boldsymbol{P}_1 = \boldsymbol{p}\,, \\ \boldsymbol{x}' &= \boldsymbol{X}_2' - \boldsymbol{X}_1' = \boldsymbol{X}_2 + \boldsymbol{V}t + \boldsymbol{A} - \boldsymbol{X}_1 - \boldsymbol{V}t - \boldsymbol{A} = \boldsymbol{X}_2 - \boldsymbol{X}_1 = \boldsymbol{x}\,, \end{aligned} \tag{5.12}$$

and changes the parametrization of the configuration space according to

$$E_3(\boldsymbol{X}_1) \otimes E_3(\boldsymbol{X}_2) = E_3(\boldsymbol{X}) \otimes E_3(\boldsymbol{x})\,. \tag{5.13}$$

According to (5.12), $\boldsymbol{x}^2 = \boldsymbol{y}^2$ and $\boldsymbol{p}^2 = \boldsymbol{q}^2$, and H^G becomes separated (in the new variables) into external ($\boldsymbol{X}$) and internal ($\boldsymbol{x}$) variables, as

$$\left.\begin{array}{ll} H^G & = \frac{\boldsymbol{P}^2}{2m} + h^G(\boldsymbol{x}^2 = \boldsymbol{y}^2, \boldsymbol{p}^2 = \boldsymbol{q}^2)\,, \\ h^G & = \frac{\boldsymbol{q}^2}{2\mu} + V(\boldsymbol{y}^2)\,, \qquad \mu = m_1 m_2/m\,. \end{array}\right\} \tag{5.14}$$

The relationship between the *a priori* G-absolute characteristics of $\mathcal{M}$ obtained from h^G and the corresponding measurement data obtained in some lab-system in G_4 is given by (5.10). Keeping in mind that $O^R \neq O^G$, the connections between the space orientations of $\mathcal{M}$ in R_3^G and in the space E_3 of lab-system S are established *a priori*. If the isolated system $\mathcal{M} = A_1 + A_2$ is in a scattering state, this connection is established by means of the asymptotic momenta of A_1 and A_2 in the asymptotic zone of measurement. If $\mathcal{M}$ is in a bound state, the orientation of internal structure of the (composite) particle $\mathcal{M}$ in R_3^G, with regard to the external space of lab-system, requires some polarization effects such as those in the Stern–Gerlach experiment.

The coexistence of R_3^G with G_4 manifests itself in (5.13) or/and in the separation of the external (free) Hamiltonian $\boldsymbol{P}^2/2m$ from the internal G-absolute Hamiltonian h^G as shown in (5.14). The same separation concerns any isolated N-body system parametrized from its inside by $3(N-1)$ relational coordinates $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1}$ of the configuration space $R_3^G(\boldsymbol{y}_1) \otimes \ldots \otimes R_3^G(\boldsymbol{y}_{N-1})$. In the Schrödinger equation, the $\boldsymbol{p}$–$\boldsymbol{x}$ duality is realized by relational momenta $\boldsymbol{q}_1, \ldots, \boldsymbol{q}_{N-1}$ conjugate with $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1}$ and equal to $\widehat{\boldsymbol{q}}_J = -i\hbar\partial/\partial \boldsymbol{y}_J$. The Schrödinger equation then takes the form

$$i\hbar \frac{\partial \psi^G(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1}; \tau)}{\partial \tau} = \widehat{h}^G\, \psi^G(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1}; \tau)\,, \tag{5.15}$$

which results, for stationary states, in an eigen-problem of h^G

$$\widehat{h}^G\, \psi_n^G(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1}) = w_n^G\, \psi_n^G(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1})\,. \tag{5.16}$$

The remaining 3 degrees of freedom $\boldsymbol{X}$which describe $\mathcal{M}$ as a whole, *i.e.* as a single particle in G_4, are determined from the Schrödinger equation

$$i\hbar \frac{\partial \Psi^G(\boldsymbol{X}, t)}{\partial t} = \frac{\widehat{\boldsymbol{P}}^2}{2m}\, \Psi^G(\boldsymbol{X}, t) \tag{5.17}$$

with $\widehat{\boldsymbol{P}} = -i\hbar\partial/\partial \boldsymbol{X}$.

The coexistence of R_3^G with G_4 makes the *internal* equation (5.15) and the *external* equation (5.17) to be quite different from each other. In true physics with finite $\hbar/c$ constant, when L_4 is the spacetime of measurement, the hypothesis of (L-absolute) relational space R_3 must abandon the L_4 eventism and the solutions of the corresponding *internal* and *external* Schrödinger equations will be subject to a hierarchy: first one has to establish the state of $\mathcal{M}$ in R_3 and then only solve the external equation. Briefly speaking, this hierarchy results from the energy–mass relation which, as it will be seen, has far-reaching geometrical consequences.

If $\mathcal{M}$ as a whole is in the eigenstate of total momentum $\widehat{\boldsymbol{P}}$ and, simultaneously, in the eigenstate Ψ_n^G of $\widehat{h}^G$ as defined in (5.16), it follows from (5.14) that the total energy E_n^G of $\mathcal{M}$ is equal to

$$E_n^G = \frac{\boldsymbol{P}^2}{2m} + w_n^G\,. \tag{5.18}$$

The dependence of E_n^G on external reference frames S enters via kinetic energy $\boldsymbol{P}^2/2m$ only, while w_n^G is G-absolute *a priori.* In consequence, the equality

$$E_n^{G*} = w_n^G \tag{5.19}$$

in the rest frame S^* of $\mathcal{M}$, in which $\boldsymbol{P}^* = 0$, takes place *a posteriori* and hence, it does not distinguish the reference frame S^* in a way which would conflict with the Galilean principle of relativity.

6. Relative time variable Δt as a degree of freedom

Low-energy physics and, in particular, low–energy transport phenomena in micro-physics which are very well described by the NR Schrödinger equation, are based on classical spacetime background of heavy, classical measuring device. In spite of this background, the *quantum motion* is hardly reconcilable with the physical text of space and time continua found by measuring rods and classical clocks [26, 27]. The same concerns the electron motion inside loosely bound systems like atoms [28, 29]. This suggests that, on the elementary level of micro-worlds $\mathcal{M}$, the metrical relations existing in $\mathcal{M}$'s are originated by relationism accounting — by its very nature — for the p–x duality rather than by the macro-eventism of classical physics. One of the problems which arises together with the opposition *relationism versus eventism* is connected with the role of relative time variable in N-body states ($N \geq 2$). Let us confine ourselves to the simplest two-body system $\mathcal{M} = A_1 + A_2$ when the problem concerns a single relative time variable $\Delta t = t_2 - t_1$.

In classical physics, after determining the trajectories $\boldsymbol{X}_{1,2}(t)$ from the one-time NR equations of motion ($\Delta t = 0$), the classical spacetime background does not forbid to speak *a posteriori* of each of these trajectories at different instants $t_1 < t_2$ with $\Delta t \neq 0$

$$\boldsymbol{X}_1 = \boldsymbol{X}_1(t_1)\,, \quad \boldsymbol{X}_2 = \boldsymbol{X}_2(t_2)\,, \quad \Delta t = t_2 - t_1 \neq 0\,. \tag{6.1}$$

It is instructive to analyze how does this problem look like in two-body quantum systems starting with the most reliable case of two free particles (kinematics) in spacetime L_4.

Let A_1 and A_2 be in the eigenstates of their momenta $\widehat{\boldsymbol{P}}_{1,2}$ with corresponding eigenvalues $\overset{o}{\boldsymbol{P}}_{1,2}$ and let $\overset{o}{P}_{1,2}$ denote their four-momenta when $A_{1,2}$ are on their mass-shells; hence, $\overset{o}{P}_{1,2}$ are subject to two constraints

$$\overset{o}{P}{}^2_{1,2} = -m_{1,2}^2\, c^2, \text{ hence } \overset{o}{E}_{1,2} = c\left(m_{1,2}^2 + \widehat{\boldsymbol{P}}{}^2_{1,2}\right)^{1/2}, \tag{6.2}$$

where $\overset{o}{E}_{1,2}$ denote the energies of $A_{1,2}$ in some arbitrary reference frame S parametrizing L_4. The corresponding eigenstates of $\widehat{P}_{1,2}$ take the well known form of plane waves

$$\overset{o}{\Psi} = A\exp(i\overset{o}{\Phi}), \quad \overset{o}{\Phi} = \frac{\overset{o}{P}_1 X_1 + \overset{o}{P}_2 X_2}{\hbar}, \tag{6.3}$$

which embed $\overset{o}{\Psi}$ in the 8-dimensional configuration space $L_4(X_1) \otimes L_4(X_2)$. Thus one may determine $\overset{o}{\Psi}$ for an arbitrary value of *the* $\Delta t = t_2 - t_1$ *degree of freedom* as required by 4-dimensional continuum L_4. However, as it has been pointed out by Dirac [30], the two constraints (6.2) show, in the p language of measurement, that four-dimensionality of L_4 is not a *simple* extension of the 3-symmetry of NR physics into the 4-symmetry of STR, as this extension calls for some additional constraints eliminating the relativistic redundancy of degrees of freedom.

Now, following the NR procedure expressed by (5.10), we intend to change the parametrization $X_{1,2}$ of 8-dimensional configuration space of $\mathcal{M} = A_1 + A_2$ in a way which would separate the internal variables from the external ones. Among the new variables there must be the total (external) four-momentum $P = P_1 + P_2$ and the (internal) relative four-coordinate $x = X_2 - X_1$. As the p–x duality must concern the old variables as well as the new ones, the change of variables must preserve the unitarity of transformations between the corresponding x- and p-representations of all states of $\mathcal{M}$. The necessary and sufficient condition for this is the following identity of the expression for Φ in the old and new x and p variables. Omitting the superscript o, this identity may be written in the form

$$\hbar\Phi = P_1 X_1 + P_2 X_2 \equiv PX + px\,. \tag{6.4}$$

Under these assumptions we end up with

$$\left.\begin{aligned} X &= aX_1 + (1-a)X_2\,, & x &= X_2 - X_1\,, \\ P &= P_1 + P_2\,, & p &= aP_2 - (1-a)P_1\,, \end{aligned}\right\} \tag{6.5}$$

with the arbitrary (so far) *weight* a as a free parameter. In consequence, the elements of the unitary operator $\widehat{U}$ between the x and p representations in the old and new variables are of the form

$$\begin{aligned} U &= (2\pi\hbar)^{-4}\exp\left\{\frac{i}{\hbar}(P_1X_1 + P_2X_2)\right\} \\ &\equiv (2\pi\hbar)^{-4}\exp\left\{\frac{i}{\hbar}(PX + px)\right\}\,. \end{aligned} \tag{6.6}$$

Hence, the state (6.3) in the new parametrization takes the form

$$\overset{o}{\Psi} = A \exp\left[\frac{i}{\hbar}\left(\overset{o}{P}_1 X_1 + \overset{o}{P}_1 X_2\right)\right] \equiv A \exp\left[\frac{i}{\hbar}(\overset{o}{P} X + \overset{o}{p} x)\right], \tag{6.7}$$

independently of the value of a.

Although the internal phase $\overset{o}{\phi}$ of $\overset{o}{\Psi}$ with

$$\hbar \overset{o}{\phi} = \overset{o}{p} x, \tag{6.8}$$

remains L-invariant independently of the value of a, similarly as do x^2 and $\overset{o}{p}{}^2$, the four-length $\overset{o}{p}{}^2$ depends on a. Let us confront this with the G-invariance of $\overset{o}{p}{}^2 = \overset{o}{q}{}^2$ from (5.12) due to the NR weight $a^G = m_1/m$, which is crucial for the separability (in G_4) of the internal degrees of freedom of $\mathcal{M} = A_1 + A_2$ from the external ones as well as for the abstraction of relational space R_3. The relativistic energy–mass relation makes that the weight a cannot be given *a priori* (as in the NR case), because it depends on the internal state of $\mathcal{M}$ which decides about the value of the mass M of $\mathcal{M}$. This very fact imposes *the hierarchic* description of the whole state of $\mathcal{M}$; such a description is excluded by eventism L_4 and calls for the hypothesis of relational space R_3 extending R_3^G to physics of finite $\hbar/c$. We shall explain this in the following sections, after introducing the R_3 space, but now let us present the following argument from which some fixed value of a will result, confining ourselves to the *trivial* case under discussion when $\mathcal{M}$ is composed of free constituents on their mass-shells as in (6.2).

The point is that if the variables x and p are to be internal variables of an isolated micro-world $\mathcal{M}$ separated from its external variables X and P then, in the case of free constituents $A_{1,2}$ of $\mathcal{M}$, $\mathcal{M}$'s absolute mass M should be parametrized by L-invariant four-length of p as this four-length is the only internal, L-invariant quantity which we have at our disposal. Thus

$$M = M(p^2) = \frac{W(p^2)}{c^2}, \tag{6.9}$$

where W is the internal energy of $\mathcal{M}$. However, using (6.5) and taking into account the mass-shell constraints (6.2) one obtains

$$P_0^* = Mc = (m_1^2c^2 + \boldsymbol{p}^{*2})^{1/2} + (m_2^2c^2 + \boldsymbol{p}^{*2})^{1/2} = \frac{W}{c}, \tag{6.10}$$

where $\boldsymbol{p}^*$ is the space component of p in the rest-frame S^* of $\mathcal{M}$ in which $\boldsymbol{P}^* = 0$.

Equality (6.10) conflicts with the requirement (6.9) because p^2 coincides with $\boldsymbol{p}^{*2}$ provided, however, that $p_0^* = 0$ which, rewritten in a manifestly L covariant form, means that

$$Pp = 0 \quad (p_0^* = 0). \tag{6.11}$$

Thus the external four-momentum P imposes a constraint onto p which conflicts with its internal self-dependent character required by (6.9). The constraint (6.11) proves that 4-symmetry L excludes the separation of the internal degrees of freedom of isolated systems $\mathcal{M}$ from the external ones already on the elementary level of relativistic kinematics. The fact that 4-symmetry L_4 conflicts with the separability of internal dynamics of $\mathcal{M}$ was first disclosed by Dirac *et al.* [31] and discussed after by Foldy [32] in connection with the *semi-relativistic* equation of motion of two-body system put forward by Eddington [33]. Within the L_4 eventism, Eddington's semi-relativistic equation starts with the CM system S^* which — from the point of view of symmetry L as the one given *a priori* by eventism L_4 — postulates the separability of internal dynamics of $\mathcal{M}$ conflicting with symmetry L. Let us remember that (5.14), which follows from (singular) symmetry G, means the separability of the internal (absolute) coordination of $\mathcal{M}$ from the external (G-relative) one. Thus, the *a priori* G–absolute internal Hamiltonian $\widehat{h}^G$ realizes the idea of Eddington of an absolute equation of motion without distinguishing CM system S^* and hence, without violating the (Galilean) principle of relativity.

Thus the two constraints of scattering states (free states) of $\mathcal{M}$, given by (6.2) in the initial parametrization of $\mathcal{M} = A_1 + A_2$, convert, in final parametrization, into two constraints (6.11) and

$$P^2 = -M^2c^2 \tag{6.12}$$

which determines the L-invariant mass M from the outside of $\mathcal{M}$ as given by the external four-momentum P. Now, let us show that the constraint (6.11) and the equalities (6.5) determine the weight a.

From (6.5) we obtain that

$$\begin{gathered} p^2(a) = (aP_2 - (1-a)P_1)^2 = \boldsymbol{p}^{*2} - (aM_2 - (1-a)M_1)^2c^2, \\ M_{1,2} = (m_{1,2}^2 + \boldsymbol{p}^{*2})^{1/2}, \qquad M = M_1 + M_2 \geq m = m_1 + m_2 . \end{gathered} \tag{6.13}$$

The constraint (6.11) results in $p^2 = \boldsymbol{p}^{*2}$ which leads to

$$a(M) = \frac{M_1}{M} = \frac{1}{2}\left[1 + \frac{m_1^2 - m_2^2}{M^2}\right] \underset{c\to\infty}{\longrightarrow} a^G = \frac{m_1}{m} \tag{6.14}$$

and — as it was to be expected — results in the dependence of the weight a on M, *i.e.* on internal state of $\mathcal{M}$. For scattering states, when $M \geq m$, the very notion of *weight* and hence, the dependence of the centre of mass coordinate X [*cf.* (6.5)] on internal state of $\mathcal{M}$ is of secondary importance. However, the same analytic dependence of a on M remains valid for bound states when $M < m$ (*cf.* Section 18). Then the dependence of a on M_n acquires physical interest, much like the *hierarchic* description of the state of M_n connected strictly with the hypothesis of L-absolute relational space R_3.

The space-like relative four-momentum p for $a = a(M)$ results in

$$\hbar\phi = px = \boldsymbol{p}^*\boldsymbol{x}^* \tag{6.15}$$

which means that in the rest-frame S^* of $\mathcal{M}$ internal phase ϕ of internal state of $\mathcal{M}$ is independent of the Δt-*degree of freedom.* Let us review this conclusion in the context of the NR framework ($1/c = 0$) which deals with an *a priori* given weight $a^G = m_1/m$ resulting — in accordance with (5.12) — in the G-invariant internal phase ϕ^G

$$\hbar\phi^{G'} = \boldsymbol{p}'\,\boldsymbol{x}' = \boldsymbol{p}\,\boldsymbol{x} = \hbar\phi^G, \tag{6.16}$$

free of the time component $(p_0x_0)/\hbar$ in all reference frames S.

However, the L-invariant phase ϕ from (6.15) is equal to

$$\hbar\phi = \boldsymbol{p}\,\boldsymbol{x} - p_0x_0\,, \tag{6.17}$$

hence, one should expect the time component of ϕ to vanish identically and the space component to convert into the G-absolute one from (6.16) in the NR limit ($c \to \infty$). Here a delicate point of the NR limit is revealed which will be discussed in a more detailed way in Section 8. The point is that if we put — from the very beginning — $a = a^G$ in the transformation (6.5), in order to guarantee that in the NR limit ($c \to \infty$) the space component $\boldsymbol{p}$ of four-momentum p becomes the G-absolute relative vector $\boldsymbol{p}$ from (5.12), then the time component of ϕ equal to $p_0x_0/\hbar$ does not vanish (in this limit) in S^* of the G_4-spacetime. Thus the dependence of a on M becomes an essential one if one endeavours to obtain the equality (6.15).

In the NR approximation the time components of $P_{1,2}$ take the form

$$(c\,P_{1,2})_0 = m_{1,2}c^2 + \frac{\boldsymbol{P}_{1,2}^2}{2m_{1,2}}\,, \tag{6.18}$$

and hence, from (6.5) and for $a = a^G$, the time component of four-momentum p is equal to

$$cp_0 = \frac{m_1}{m}\left(m_2c^2 + \frac{\boldsymbol{P}_2^2}{2m_2}\right) - \frac{m_2}{m}\left(m_1c^2 + \frac{\boldsymbol{P}_1^2}{2m_1}\right)$$
$$= \frac{1}{2m}\left[\frac{m_1}{m_2}\boldsymbol{P}_2^2 - \frac{m_2}{m_1}\boldsymbol{P}_1^2\right]. \tag{6.19}$$

Thus, the (tending to infinity) rest energies $m_{1,2}c^2$ cancel out; nevertheless, in the rest-frame S^* of $\mathcal{M}$ in which $\boldsymbol{P}_2^* = -\boldsymbol{P}_1^* = \boldsymbol{p}^*$ and in the NR limit with $\boldsymbol{p}^*$ converting into the G-absolute relative momentum $\boldsymbol{p}$, we obtain

$$\lim_{c\to\infty}(p_0^* x_0^*) = \frac{1}{2m}\left(\frac{m_1}{m_2} - \frac{m_2}{m_1}\right)\boldsymbol{p}^2 \Delta t \neq 0. \tag{6.20}$$

As it can be seen from (6.19), p_0 itself vanishes in the NR limit

$$\lim_{c\to\infty} p_0 = 0. \tag{6.21}$$

However, since $x_0 = c\Delta t$, the quantity given in (6.20) remains finite.

In G_4 we deal with the total phase Φ^G of two-body plane wave Ψ equal to

$$\hbar\Phi^G = \boldsymbol{P}\boldsymbol{X} - E^G t + \boldsymbol{p}\boldsymbol{x}. \tag{6.22}$$

This means that from the point of view of 4-symmetry L we must impose onto Φ^G the simultaneity constraint $\Delta t = 0$ resulting from the one-time NR equations of motion. Otherwise we should add to Φ^G the phase (6.20) different from zero, unless $m_1 = m_2$.

7. Separability of scattering and bound states and symmetry L

In scattering states each of the constituents of $\mathcal{M} = A_1 + A_2$ reaches separately the asymptotic zone of relativistic kinematics of measurement and this makes the two constraints (6.2) or (6.11) to be imposed on four-momenta of $\mathcal{M}$. Let us remember that constraint (6.11) excludes separability of internal and external states of $\mathcal{M}$ and, moreover, from this constraint follows that four-momentum p has only 3 degrees of freedom. However, symmetry L recognized by eventism L_4 as the one given *a priori* results in nonseparability of the internal states of $\mathcal{M}$ from the external ones, independently of whether $\mathcal{M}$ is in a scattering state or in a bound one.

According to eventism L_4, equations of motion of a fully isolated system $\mathcal{M} = A_1 + A_2$ must be L-form–invariant, *i.e.* symmetry L must be their internal symmetry group, as otherwise they would be able to distinguish a reference frame, similarly as the semi-relativistic equations do [33]. The Bethe–Salpeter equation is an example of L-form–invariant equation. The spacetime translation invariance of such equations, included in symmetry L, incites one to look for solutions of such equations in the form

$$\Psi^L = A \exp\left[\frac{i}{\hbar}(\overset{o}{P} X)\right] \psi^L(x; \overset{o}{P}), \tag{7.1}$$

where the superscript L insists on the symmetry L of initial laws based on eventism L_4. In order to show that ψ^L may not be separated into internal and external states of $\mathcal{M}$ it is sufficient to consider the subclass of the most symmetric solutions $\psi^L(x; \overset{o}{P})$ which depend on the three L–invariant variables: x^2, $\overset{o}{P}x$, and $\overset{o}{P}{}^2 = -\overset{o}{M}{}^2 c^2$ which can be constructed *a priori* from two independent four-vectors x and $\overset{o}{P}$. Of course, basing on the p–x duality one could introduce instead of x the relative four-momentum p. The $\overset{o}{P}{}^2$ variable is irrelevant in the separability problem, so the x- or (interchangeably) p-representations of $|\psi^L\rangle$ take the form

$$\psi^L = \psi^L(x^2, \overset{o}{P} x) \quad \text{or} \quad \tilde{\psi}^L = \tilde{\psi}^L(p^2, \overset{o}{P} p). \tag{7.2}$$

The state (7.2) could be recognized as separated from the external one parametrized by $\overset{o}{P}$ if ψ^L would depend solely on x^2 (p^2), *i.e.*

$$\psi^L = \psi^L(x^2) \quad \text{or} \quad \tilde{\psi}^L = \tilde{\psi}^L(p^2). \tag{7.3}$$

However, expression (7.3) cannot represent any state in L_4 which would fulfill an L-form invariant two-body equation of motion, because, first of all, $\psi^L(x^2)$ remains constant on Minkowski's spheres v^2 = constant. In consequence, the dependence of ψ^L on the $\overset{o}{P} x$ variable proves that relativistic wave functions [34, 35] of scattering states of $\mathcal{M}$, as well as those of bound ones, cannot be separated from the variables which parametrize external motion of $\mathcal{M}$.

The general conclusion is that no form factor $G^L(x)$ of $\mathcal{M}$ obtained from a theory based on eventism L_4 can be separated from the variables that characterize external motion of $\mathcal{M}$ as a whole; this motion results in *relativistic distortions* of $G^L(x)$.

The separation of the internal degrees of freedom of a micro-world $\mathcal{M}$ from the external ones will be substantial for the hypothesis of relational space R_3 realized automatically (apart from $O^R \neq O^G$) by eventism G_4. Therefore within the eventism L_4, mathematical formulation of this separability is an artificial one and it is preceded by physical text of internal and external coordination of $\mathcal{M}$, alien, in principle, to eventism which regards spacetime as a pre-existent (hence external) background of all extensional degrees of freedom. For the sake of simplicity, we shall confine ourselves to the elementary two-body problem and scalar form factors G of $\mathcal{M}$. Thus x (or, interchangeably, p) is recognized as the internal four-coordinate of $\mathcal{M} = A_1 + A_2$ and C denotes all parameters which determine the analytic form of form factors represented in some reference frame S. Consequently, one gets $G = G(x; C)$; however, the set of parameters C must be divided into two classes of entirely independent parameters: the internal ones C_i and external ones C_e. The geometrical nature of internal parameters C_i belongs to the internal 4-space $\mathcal{L}_4$ containing its p and x aspects, while external parameters C_e belong to the external spacetime L_4 of measurement. The four-momentum $\overset{o}{P}$ of $\mathcal{M}$ (*cf.* (7.1) or (7.2)) represents the external parameters.

Let the Lorentz transformation L_e act uniquely on the external variables and parameters, leaving unchanged the analytic form of representation of G in the internal (geometrical) meta-object $\mathcal{L}_4$. Geometrically, this means that we begin with G represented in some (arbitrary) reference frame S and, in some other S', the external properties of G are — in spite of being *observed in* S' — kept the same (*i.e.*, rewritten L covariantly), whereas the internal properties of G, also observed in S', become transformed under the L^{-1} transformation and hence, they do not remain *the same* from the point of view of eventism L_4. This *active* treatment of the internal properties by L_e transformation, combined with the *passive* treatment of the external properties (or *vice versa*) may be regarded as a mathematical operation conflicting with the universal spacetime background of eventism L_4.

However, one has to remember that the very partition of degrees of freedom into the internal (x) and external (X) ones goes beyond the eventism, as such a partition must be preceded by existence of some reality $\mathcal{M}$ characteristic for *objectism* [2]. Without $\mathcal{M}$ such a partition would be deprived of any sense, whereas the eventism means that empty metrical continuum L_4 precedes any reality $\mathcal{M}$.

In spite of this non-eventistic aspect of the very problem of separability, we shall assume existence of a system $\mathcal{M} = A_1 + A_2$ and treat x and X as system's internal and external degrees of freedom, respectively. Then, transformation L_e enables one to distinguish between the analytical structures of form factors $G(x)$ separated (SE) and non-separated (NS) from the external characteristics of $\mathcal{M}$, according to

$$L_e\{G_{\mathrm{SE}}(x)\} = G_{\mathrm{SE}}(x)\,, \quad (i)$$
$$L_e\{G_{\mathrm{NS}}(x)\} \neq G_{\mathrm{NS}}(x)\,. \quad (ii) \qquad (7.4)$$

According to (7.4), the fundamental consequence of eventism L_4 can be rewritten in the form

$$G^L(x) = G_{\mathrm{NS}}(x)\,. \tag{7.5}$$

On the other hand, the L-form–invariant form factors $G(x^2)$ from (5.5) depend uniquely on the *a priori* L-form invariant four-interval x^2 and they represent SE structures $G_{\mathrm{SE}}(x)$

$$G(x^2) = G_{\mathrm{SE}}(x)\,. \tag{7.6}$$

In order to prove (7.5), it is sufficient to consider the two-event shape $g(x) = (\overset{o}{P}\,x)$ and to apply the L_e transformation to it

$$L_e[g(x)] = (\,[L_e\,\overset{o}{P}]\,x) \neq (\overset{o}{P}\,x) = g(x)\,. \tag{7.7}$$

In consequence, relativistic form factors $G^L(x)$ of $\mathcal{M}$ represent, as well as the relativistic wave functions from (7.2), G_{NS} shapes.

The L covariant expression of simultaneity of two events $X_{1,2}$ in some reference frame S in L_4, calls for an external, time-like *direction field* n $(n^2 = -1)$

$$n_{|S} = (0,0,0;1)\,. \tag{7.8}$$

As $x = X_2 - X_1$, the L covariant expression of simultaneity of $X_{1,2}$ takes the form

$$g(x) = (nx) = 0\,. \tag{7.9}$$

Thus, if x were to denote internal degrees of freedom of $\mathcal{M} = A_1 + A_2$ then, similarly as in (7.7),

$$L_e(nx) = (\,[Ln]\,x) = 0 \tag{7.10}$$

would express the simultaneity of $X_{1,2}$ in some reference frame S' different from S in which relation (7.8) holds. However, apart from $\mathcal{M}$, the relation $x = X_2 - X_1$ is determined in an empty L_4 and its representations in different S's are subject to (homogeneous) symmetry L.

The double aspect of x which serves as the internal coordinates of G (objectism) being at the same time the relative four-coordinate in empty spacetime L_4 (eventism), discloses (in spite of mathematical isomorphy) the fundamental difference between *the geometrical meta-object* $\mathcal{L}_4(x)$, parametrizing composite system $\mathcal{M}$ in its configuration subspace $\mathcal{L}_4(x)$ (or, interchangeably, in $\mathcal{L}_4(p)$), and the classical (Cartesian) eventism $L_4(X)$. This mathematical isomorphy of two meanings of x makes the 4-space $\mathcal{L}_4$ a *mediator* between *the quantum* relationism of micro-structures and micro-processes and *the classical* measurement in the Cartesian spacetime L_4.

Employing criterion (7.4) of separability–nonseparability of form factors G, with x representing the internal degrees of freedom of $\mathcal{M}$ (objectism), we are in position to indicate the essential difference between the bound states of $\mathcal{M}$ and the scattering ones. Such a difference is alien to singular eventism G_4 as it is intimately connected with 4-symmetry L. The point is that in the scattering states of $\mathcal{M} = A_1 + A_2$, $\mathcal{M}$'s internal four-momentum $\overset{o}{p}$ as well, as the external four-momentum $\overset{o}{P}$, represent external parameters C_e of $\mathcal{M}$, because of the constraint (6.11): $\overset{o}{P}\overset{o}{p} = 0$ which couples $\overset{o}{P}$ to $\overset{o}{p}$. However, the mass-shell constraints (6.11) follow from two independent measurements of four-momenta $\overset{o}{P}_{1,2}$ of $A_{1,2}$ which — independently of each other — both reach the asymptotic zone of measurement. If $\mathcal{M}$ is in a bound state ψ_n with (L-absolute) mass $M_n < m = m_1 + m_2$, the entity $\mathcal{M}_n$ represents a single particle hence, for the same reasons as before, there is only one constraint (6.12) imposed onto P_n, namely

$$-P_n^2 = M_n^2 c^2 = \frac{W_n^2}{c^2} < m^2 c^2 . \tag{7.11}$$

In consequence, constraint (6.11) drops out and Fermi four-momenta p conjugate with x gain the 4-parameter freedom, much like the relative four-coordinate x which parametrizes the x representation of form factors G of bound structures $\mathcal{M}$. The 4-parameter freedom of p makes room for L-form–invariant form factor $\tilde{G}(p^2)$ of $\mathcal{M}$ (or, interchangeably, $G(x^2)$) which belong to G_{SE} form factors (*cf.* (7.6)). After introducing relational space R_3, we shall show — *cf.* Section 18 — that 4-freedom of p results in undetermined four-momenta $P_{1,2}$ of the constituents of bound structures of $\mathcal{M}$.

Now we can even better appreciate the singularity of eventism G_4. Indeed, without explicitly distinguishing between eventism (G_4) and objectism, the separability of total Hamiltonian H^G (as in (5.14)) makes that NR quantum mechanics provides us with the G-form–invariant form factors $F(\boldsymbol{x}^2 = \boldsymbol{y}^2)$ (or, interchangeably, with $\tilde{F}(\boldsymbol{p}^2 = \boldsymbol{q}^2)$) which, as such, are separated from the external characteristics of $\mathcal{M}$ as a whole. This is due to the equalized balance of G_4 geometry or, in other words, to the coexistence (apart from $O^R \neq O^G$) of eventism G_4 with relationism of the R_3^G space.

8. NR limit ($c \to \infty$) of geometry L_4

The discontinuity which exists between the one L-form invariant four-interval x^2 of geometry L_4 and the two G-form invariant space and time intervals $r = |\boldsymbol{x}| = |\boldsymbol{y}|$ and $\Delta\tau = \Delta t$ of geometry G_4 implies that G_4 (regarded as the limit of L_4) must conceal an additional assumption besides the condition $c \to \infty$. Having in mind the experimentally privileged position of the p language over the x one, let us start to analyze the NR limit with considering the limits of three kinds of four-momenta p for $c \to \infty$, namely when p is: time-like, isotropic and space-like.

For "dimensional" reasons, let us attach to a time-like p an auxiliary (fictitious) particle with mass m. Then m, together with the universal constant c, provide us with the required dimension of p, namely

$$p^2 = \boldsymbol{p}^2 - p_0^2 = -m^2c^2 < 0\,. \tag{8.1}$$

Thus, a purely dimensional analysis shows that in the NR limit ($c \to \infty$) there is no room for an NR counterpart of time-like four-momentum, as p^2 tends to infinity. The energy–mass relation disappears and we are left with two notions of mass and energy which are essentially different from each other in G_4. A more detailed analysis of the NR limit of time-like p is given at the end of this section.

For isotropic four-momenta when $p^2 = 0$, the representation of p in any reference frame S in L_4 takes the form

$$p = (\boldsymbol{p}; p_0 = \pm|\boldsymbol{p}|)\,, \quad e = |p_0|c\,, \tag{8.2}$$

where e is the energy of a carrier which propagates with the velocity of light c. If the Minkowskian spacetime L_4, with its light cone structure, is to be converted into the Galilean space G_4 (without light cones) a constraint has to be imposed — namely, the time components p_0 of all p's must tend to zero with $c \to \infty$, because only then all isotropic four-momenta p vanish: $p = 0$. Thus, besides the (mathematical) limit $c \to \infty$ we must assume an independent constraint

$$e < K\,c^{1-\epsilon} \Longrightarrow |p_0| = \frac{e}{c} < K\,c^{-\epsilon} \underset{c\to\infty}{\longrightarrow} 0\,. \tag{8.3}$$

Here K is an arbitrarily large, but finite, constant and ϵ is an arbitrarily small, positive number. Of course, a particular signal carries a finite amount of e, say $\overset{o}{e}$, hence $\overset{o}{e}/c = |\overset{o}{p}_0| \underset{c\to\infty}{\longrightarrow} 0$. However, the conversion of L_4 into G_4 concerns two (actualized) infinite *geometrical objects*. Saying that the limit $c \to \infty$ results in the $L_4 \to G_4$ conversion, one tacitly assumes that in some lab-system $\bar{S}$ we deal with some particular signals carrying given, finite values of $\overset{o}{e}$.

A similar situation concerns space-like four-momenta p which — as the only ones — have their NR counterparts. From (6.21) we know that the G symmetry of NR mechanics requires the time components p_0 of all space-like four-momenta p to vanish when $c \to \infty$, similarly as in (8.3). Indeed, in this limit, the L-absolute four-length-squares of all space-like four-momenta p convert into the G-absolute length-squares of relative (and relational) momenta $\boldsymbol{p}(\boldsymbol{q})$, as

$$\lim_{c\to\infty}(\boldsymbol{p};p_0) = (\boldsymbol{p};0) \Longrightarrow \lim_{c\to\infty} p^2 = \boldsymbol{p}^2 = \boldsymbol{q}^2 \,. \tag{8.4}$$

Again, the $\overset{o}{p}_0$ component of a particular four-momentum $\overset{o}{p}$ does vanish with $c \to \infty$, because of finite value of $\overset{o}{e} = c|\overset{o}{\boldsymbol{p}}|$. However, condition (8.4) must concern all four-momenta p at once and this requires the assumption (8.3) which does not uniquely follow from the limit $c \to \infty$.

It is remarkable that a similar situation concerns *the most popular* demonstration of the $L_4 \to G_4$ conversion, because in the limit $c \to \infty$ the Lorentz transformation (4.1 L) converts into the Galilean one (4.1 G). In order to show where is the point, it is sufficient to consider the special, homogeneous Lorentz transformation of one space variable X and time t which takes the well-known form

$$X' = \Gamma(X - Vt)\,, \qquad t' = \Gamma\left(t - \frac{VX}{c^2}\right)\,. \tag{8.5}$$

One may say that, in the limit $c \to \infty$, transformation (8.5) converts into the corresponding (special) Galilean transformation

$$X' = X - Vt\,, \qquad t' = t\,. \tag{8.6}$$

Since $V < c$, for $c \to \infty$ the term VX/c^2 must tend to zero and the Lorentz factor Γ must tend to unity; the transition from (8.5) to (8.6) is then justified if we assume, together with $c \to \infty$, that

$$\left|\frac{V}{c}\right| < Ac^{-\epsilon}\,, \qquad (i)$$

$$|X| < Bc^{1-\epsilon}\,, \qquad (ii) \tag{8.7}$$

where A, B are some, arbitrarily large but finite, constants and ϵ is an arbitrary small, positive number, similarly as in (8.3). Of course, for a particular event $\overset{o}{X}$ and for a particular value of velocity V ($|V| = \overset{o}{V} < c$), inequalities (8.7) are fulfilled automatically. However, the question concerns again all events X at once and all velocities V ($|V| < c$) admitted by symmetry L, because we deal with actualized infinities of *geometrical objects* L_4 and G_4. If so, inequalities (8.7) must be added to the limit $c \to \infty$ in order to convert L_4 into G_4. These additional assumptions — (8.3) and/or (8.7) — are responsible for the discontinuity connected with the NR limit of L_4 and consisting in transition from geometry L_4 of negative balance to geometry G_4 of equalized balance.

This discontinuity reveals itself in the transition from one kind of L-form invariant two-event shapes (5.5) to two kinds of G-form invariant two-event shapes (5.6). The first kind of G-form invariant shapes $G(\Delta t)$ from (5.6 *i*) may be, however, obtained immediately under an additional assumption corresponding to that from (8.7 *ii*), namely that

$$|\boldsymbol{x}| < B\, c^{1-\epsilon}\,, \tag{8.8}$$

as, under this assumption, $s^2 = -x^2/c^2 = (\Delta t)^2 - \boldsymbol{x}^2/c^2 \underset{c\to\infty}{\Longrightarrow} (\Delta t)^2$ and

$$\lim_{c\to\infty} G(s^2) = G(\Delta t)\,, \tag{8.9}$$

which coincides with (5.6 *i*).

In order to obtain the second kind of G-form invariant two-event shapes from (5.6 *ii*) we must resort explicitly to the quantum p–x duality and the p representation of L-form invariant shape G

$$G(x^2) = (2\pi\hbar)^{-4} \int d^3p \int\limits_{-\infty}^{\infty} dp_0\, \tilde{G}(\boldsymbol{p}^2 - p_0^2) \exp\left[\frac{i}{\hbar}(\boldsymbol{p}\boldsymbol{x} - p_0 x_0)\right]. \tag{8.10}$$

Instead of *the relativistic variables* $x_0 = c\Delta t$ and $p_0 = e/c$ which — with $c \to \infty$ — tend to infinity and zero, respectively, we introduce *the NR variables* Δt and e, the latter with the dimension of energy and hence,

$$\Delta t = x_0/c\,, \qquad e = cp_0 \quad\Longrightarrow\quad p_0 x_0 = e\Delta t\,. \tag{8.11}$$

From the previous discussion we know that the NR limit of four-momenta p admits space-like four-momenta only if e fulfills inequality (8.3). This condition will be automatically fulfilled in the limit $c \to \infty$ if $G(x^2)$ defined in (8.10) is interpreted as equal to

$$c\, G(x^2) \to (2\pi\hbar)^{-4} \lim_{c\to\infty} \int d^3p \int\limits_{-Kc^{1-\epsilon}}^{Kc^{1-\epsilon}} de\, \tilde{G}\left(\boldsymbol{p}^2 - \frac{e^2}{c^2}\right) \exp\left[\frac{i}{\hbar}(\boldsymbol{p}\boldsymbol{x} - e\Delta t)\right]. \tag{8.12}$$

Taking into account the limiting values of the limits of the integral over e (*cf.* (8.3)) we obtain

$$\tilde{G}\left(\boldsymbol{p}^2 - \frac{e^2}{c^2}\right) \underset{c\to\infty}{\Longrightarrow} \tilde{G}(\boldsymbol{p}^2) \equiv \tilde{F}(\boldsymbol{p}^2) \tag{8.13}$$

and — since $\pm Kc^{1-\epsilon}$ tends to $\pm\infty$ with $c \to \infty$ — we obtain

$$\lim_{c\to\infty} cG(\boldsymbol{x}^2) = (2\pi\hbar)^{-3} \int d^3p\, \tilde{F}(\boldsymbol{p}^2) \exp\left[\frac{i}{\hbar}(\boldsymbol{px})\right] \delta^{(1)}(\Delta t)$$
$$= F(\boldsymbol{x}^2)\delta^{(1)}(\Delta t) = F(\boldsymbol{y}^2)\delta^{(1)}(\Delta t)\,. \tag{8.14}$$

The Dirac $\delta^{(1)}(\Delta t)$ function reproduces the locality of the Newtonian time.

From (8.10) and (8.14) we get the following identities in $\mathcal{L}_4$ and $\mathcal{G}_4$ 4-spaces, respectively

$$\left.\begin{aligned} &\int_{-\infty}^{\infty} dx_0\; G(\boldsymbol{x}^2 - x_0^2) \equiv F(\boldsymbol{x}^2) && \text{(L)} \\ \text{and}\quad &\int_{-\infty}^{\infty} d(\Delta t)\; G(\boldsymbol{x}^2)\delta^{(1)}(\Delta t) \equiv F(\boldsymbol{x}^2 = \boldsymbol{y}^2)\,. && \text{(G)} \end{aligned}\right\} \tag{8.15}$$

Now let us analyze in a more detailed manner the NR limit of a time-like four-momentum P, where $E = cP_0$ denotes the total energy of an isolated system $\mathcal{M}$ with invariant mass $M = (-P^2/c^2)^{1/2}$ and

$$E = c(M^2c^2 + \boldsymbol{P}^2)^{1/2} = Mc^2(1 + \frac{\boldsymbol{P}^2}{M^2c^2})^{1/2}. \tag{8.16}$$

Here, the NR approximation in the p language of the quantum p–x duality imposes also some constraint as it makes use of the expansion of E into power series of dimension-less variable $\boldsymbol{P}^2/M^2c^2$. The condition for convergence of this series imposes the inequality

$$\frac{\boldsymbol{P}^2}{M^2c^2} < 1 \tag{8.17}$$

which, in the velocity language v, means that

$$\frac{\boldsymbol{V}^2}{c^2} < \tfrac{1}{2}\,. \tag{8.18}$$

In the one-body problem (system $\mathcal{M}$) of eventism L_4, constraint (8.18) is inconsistent with symmetry L, because velocity $\boldsymbol{V}$ is a relative quantity and hence, (8.18) distinguishes some reference frames S of $\mathcal{M}$ conflicting thus with the principle of relativity. It follows that constraint (8.18) works in favour of the relational rather than eventistic origin of the p language, with $\boldsymbol{P}$ referred to some real lab-system $\bar{S}$ which makes $\boldsymbol{P}^2$ an L-absolute quantity.

For fixed value of $\boldsymbol{P}^2 = \overset{o}{\boldsymbol{P}}{}^2$, the left member of (8.17) (and hence, of (8.18)) tends to zero when $c \to \infty$. In general, however, the NR limit imposes the inequality

$$|\boldsymbol{P}| < Ac^{1-\epsilon} \underset{c\to\infty}{\longrightarrow} \infty\,, \tag{8.19}$$

which, for c large enough guarantees the inequality (8.17). This exhibits, once again, an essential difference existing between the NR framework $(1/c \to 0)$ and the NR approximation dealing with a finite universal constant.

Now let us assume $\mathcal{M}$ to be composed of N particles A_J, each of them having an absolute mass m_J and let us decompose the L-absolute internal energy $W = Mc^2$ of $\mathcal{M}$ into two L-absolute components

$$W = mc^2 + w\,, \quad M = m + \frac{w}{c^2} = \frac{W}{c^2}\,, \tag{8.20}$$

with

$$m = \sum_{J=1}^{N} m_J = \lim_{c\to\infty} M\,. \tag{8.20a}$$

Since the strong inequality

$$\left|\frac{w}{mc^2}\right| \ll 1 \tag{8.21}$$

characterizes loosely bound systems, one can say that — in the NR limit $(c \to \infty)$ and with finite $|w|$ — all systems are *infinitely* loosely bound as the left member of (8.21) tends to zero.

Equation (8.16) takes now the form

$$E = mc^2\left[\left(1 + \frac{w}{mc^2}\right)^2 + \frac{\boldsymbol{P}^2}{m^2c^2}\right]^{1/2} \tag{8.22}$$

and, under assumption (8.17), E may be expanded into a convergent power series of $\boldsymbol{P}^2/M^2c^2$ which, together with (8.22), results in

$$E = mc^2 + w + \frac{\boldsymbol{P}^2}{2m} + 0\left(\frac{1}{c^2}\right)\,, \tag{8.23}$$

where $0\left(1/c^2\right)$ vanishes with $c \to \infty$. After subtracting the term $mc^2 \underset{c\to\infty}{\longrightarrow} \infty$ from E, we obtain

$$\lim_{c\to\infty}(E - mc^2) = w^G + \frac{\boldsymbol{P}^2}{2m} = E^G. \tag{8.24}$$

Here, E^G coincides with the total NR energy from (5.18) and

$$w^G = \lim_{c \to \infty} w \tag{8.25}$$

denotes the G-absolute internal energy of $\mathcal{M}$.

Singularity of the NR framework ($1/c = 0$) based on geometry G_4 manifests itself by the fact that the total energy E^G is a sum of two terms, each of them having different properties under the G-transformation: the G-absolute internal energy w_G and the G-relative external kinetic energy $\boldsymbol{P}^2/2m$ of $\mathcal{M}$ as a whole. Let us remember that w^G's are — similarly as the eigenvalues of the internal, G-absolute Hamiltonian h^G embedded in R_3^G — G-absolute *a priori*, *i.e.* without resorting to any fixed reference frame S parametrizing the external spacetime of measurement — here G_4.

9. Adiabatic hypothesis of field theory and eventism L_4

According to Heisenberg's philosophy of his S matrix, a fully isolated micro-process splits into three stages which were largely discussed by Fock [36] and by my master Prof. Weyssenhoff.

In the initial (I) and final (III) stage of a micro-collision process involving two micro-worlds $\mathcal{M}$ and $\mathcal{M}'$ an *observer* has at his disposal classical macro-devices with their localizations and structures embedded in spacetime of the asymptotic zone of the collision process. During these two stages the initial $|i\rangle$ (I) and final $|f\rangle$ (III) state of the micro-system $\mathcal{M} + \mathcal{M}'$ are prepared and detected, respectively. During the stage II the quantum-potential collision process takes place. This process is out of any (spacetime) control and the S matrix should take account of it.

In consequence, the matrix elements $S_{fi} = \langle f|S|i\rangle$ provide us with repeatable observables which could detect the directly non-observable internal collision process occurring on the quantum-potential level of reality. Note, that the very philosophy of the S matrix follows the *objectism* [2] by assuming existence of realities $\mathcal{M}$ and $\mathcal{M}'$. Owing to the quantum p–x duality, the event-nonlocal language p of asymptotic kinematics makes possible reconciliation of the 4-symmetry L of measurement with the quantum nonlocality and its EPR-like correlations which are hardly reconcilable with eventism L_4. The L-absolute, Mandelstam p variables s_J which parametrize S_{fi} elements (*cf.* (2.3)) suggest that quantum structures are based on (absolute) relational space R_3 that extends the absolute relational space R_3^G to physics of finite $\hbar/c$. *The hidden nature* of *quantum* process occurring during the stage II admits the hypothesis of such a hidden continuum of relations which — as it will be shown — precedes events of the spacetime L_4 of measurement.

However, eventism L_4 excludes any L-absolute space R_3 and hence the enormous success of quantum electrodynamics remains a great puzzle, even still greater if one remembers how restrictive is the locality of eventism L_4 to any dynamical theory [16–19]. Our present aim is to show that this success is due to a geometrical aspect of the adiabatic hypothesis that rests at the foundations of the relativistic perturbation theory and which is responsible for the fact that the perturbation theory goes implicitly beyond eventism L_4.

The locality of eventism L_4 causes that quantum-relativistic theories resort to local fields, *i.e.* fields spanned on events X and being subject to *second quantization.* In the case of fully isolated systems, we deal with nonlinear equations of motion whose internal symmetry must coincide with the symmetry L expressing the principle of relativity. In other words, equations of motion must be L-form invariant and hence $x^2 = \boldsymbol{x}^2 - x_0^2$ is the only interval of L_4 consistent with internal symmetry L. In this situation, the internal language of theory cannot express the space separation ($|\boldsymbol{x}| \to \infty$) of interacting subsystems $\mathcal{M}$ and $\mathcal{M}'$ of an isolated system $\mathcal{M} + \mathcal{M}'$ responsible for the asymptotic zone of kinematics of the stages I and III of the *quantum* collision process. As we can see, this dilemma is strictly connected with the negative balance of geometry L_4 and it vanishes in G_4 (of equalized balance) in which the G-form invariant space-interval $r = |\boldsymbol{x}| = |\boldsymbol{y}|$ exists and provides an absolute measure of the space separation between $\mathcal{M}$ and $\mathcal{M}'$.

Let us start with a definite, although arbitrary, reference frame S in L_4 and let us assume that all coupling constants responsible for nonlinear (dynamical) terms of the theory vanish in the asymptotic past ($t \to -\infty$) and absolute future ($t \to \infty$) of the reference frame S. This breaking of the time-translation invariance of the theory by the adiabatic hypothesis results also in breaking of theory's internal L symmetry and involves the scattering states of $\mathcal{M} + \mathcal{M}'$. Consequently, the subsystems $\mathcal{M}$ and $\mathcal{M}'$ determine some mean mutual velocity $\langle v \rangle > 0$ and hence, the space-interval R in S is equal to

$$R = \langle v \rangle |t|, \quad \langle v \rangle > 0\,. \tag{9.1}$$

This interval — independently of the particular value of a positive $\langle v \rangle$ — provides the measure of space separation of $\mathcal{M}$ and $\mathcal{M}'$ in S.

Note, that the adiabatic hypothesis resorts to objectism by introducing existence of physical objects $\mathcal{M}$ and $\mathcal{M}'$. The essential point is that the limit $|t| \to \infty$ of the adiabatic hypothesis leads to

$$R \underset{|t| \to \infty}{\longrightarrow} \infty \tag{9.2}$$

which acquires an L-absolute meaning. In other words, infinite space separation of $\mathcal{M}$ and $\mathcal{M}'$ occurs in all reference frames S which parametrize L_4.

In consequence, the perturbation theory based on the adiabatic hypothesis reproduces the relativistic kinematics of the kinematic zone of measurement of the stages I and III and — by virtue of the L-absolute meaning of the limit (9.2) — results in a manifestly L-invariant structure of the S matrix. This fact is taken for granted in the S matrix theory.

The L-absoluteness of limit (9.2), contrasted with S-dependence of finite R, explains the success of the S matrix theory based on the perturbation theory and, simultaneously, the failure of relativistic Moeller matrices [37] expressed by integrals taken over finite time intervals. Moeller matrices assume the NR (and also classical) philosophy according to which states of any system $\mathcal{M}$ (in all its degrees of freedom) evolve with the continuously increasing parameter t of an external reference frame S in L_4. However, the very relativization of the time dimension by eventism L_4 questions that philosophy. Note, that the 4-symmetry L results in 4-dimensional x and p variables which, as such, lose the nature of dynamical variables [6, 38]. The same conclusion may be reached on the basis of all *no interaction theorems* which, in their very formulation, favour Landau's opinion that a decent relativistic S-matrix theory must abandon local equations of motion in L_4.

Adiabatic hypothesis which — accordingly to (9.2) — equalizes implicitly the balance of geometry L_4 may explain the essential difficulties of the perturbation theory in accounting for bound states of $\mathcal{M}$ [39]. Indeed, bound states of $\mathcal{M}$ offer no $\langle v\rangle$-parameters, hence

$$\langle v\rangle = R = 0 \tag{9.3}$$

and the adiabatic hypothesis loses its physical reason consisting in the space separation of $\mathcal{M}$ and $\mathcal{M}'$. From this follows that all dynamical bound structures of $\mathcal{M}$ become decoupled when $t \to -\infty$ and they remain decoupled for ever.

In spite of the success of the adiabatic hypothesis in accounting for scattering states, the relativistic perturbation theory *remembers* the locality of L_4 and fields admitting point-like particles only [25]. The universal form factor of these particles $\delta^{(4)}(x)$ vanishes everywhere but at the point $x = 0$ of 4-space $\mathcal{L}_4(x)$. Let us remember that any other L-form invariant form factor $G(x^2)$ of $\mathcal{M}$ admitted by the phenomenological perturbation theory occupies the whole *geometrical meta-object* $\mathcal{L}_4(x)$ (and, interchangeably, $\mathcal{L}_4(p)$. The spacetime globality of the form factor $G(x^2)$ of $\mathcal{M}$ which enters the integrand of the corresponding integral for the S-matrix element S_{fi} shows clearly that S_{fi} may not be deduced from Moeller matrices. This supports again the Landau's opinion about a decent theory of the S matrix [38].

Although *the orthodox* perturbation theory is restricted to point-particles, it discloses the relational nature of internal dynamics which goes beyond eventism L_4. This manifests itself in the L-form invariance of the propagators $U(x^2)$ which occupy, much like form factors $G(x^2)$, the whole 4-space $\mathcal{L}_4(x)$ and, being such propagators, they are separated from degrees of freedom external to those of the 4-space $\mathcal{L}_4$. Thus, accordingly to (7.4), $G(x^2)$ and $U(x^2)$ belong to the $G_{SE}(x)$ class of two-event shapes. This separability of interaction $U(x^2)$, due to its spacetime nonlocality, stands in opposition to the L_4 eventistic nonseparability of internal dynamics resulting as a consequence of "wrong" *semi-relativistic* equations. The coexistence of R_3^G with G_4 makes the corresponding G-form invariant form factors $F(\boldsymbol{x}^2 = \boldsymbol{y}^2)$ to be consistent with the time-local NR Schrödinger equation. Simultaneously, the G-form invariance of theses form factors means that they do belong to the class of separable two-event shapes G_{SE} in G_4.

The globality of 4-dimensional *geometrical meta-objects* $\mathcal{L}_4(x)$ which, in micro-physics, takes origin in the p–x duality, reminds one to some extent of the concept of *block universe* [40]. For future purposes it is interesting to point out the reason why macro-block-universe was brought into being. Evidently, the coexistence of two physical entities must have an absolute meaning imposed by the very measuring process in which we deal with measured objects and measuring apparatus. Simultaneous existence of both has lost the absolute meaning with the relativized time of eventism L_4. The doubtful (in macro-physics) concept of block universe of all absolutely coexisting events X would be a solution to a really fundamental problem of the meaning of physical coexistence of different physical things. Einstein was fully aware of this problem, which was also discussed later by Weyl [41] and Goedel [42].

It is also worth emphasizing that the perturbation theory has introduced the configuration spaces $\mathcal{L}_4 \otimes \mathcal{L}_4 \otimes \ldots$ based on 4-space $\mathcal{L}_4$ These spaces make room for L-absolute structures reconciling quantum nonlocality with symmetry L of S-matrix elements parametrized and measured in the privileged p language of relativistic kinematics. Thus, configuration spaces of perturbation theory make that the theory becomes similar to the NR mechanics with its configuration spaces $R_3^G \otimes R_3^G \otimes \ldots$ rather than to the original field theory with fields spanned on the universal spacetime background. Indeed, as we have seen from (8.14), 4-space $\mathcal{L}_4(x)$ of L-form invariant shapes $G(x^2)$ becomes contracted in the limit $c \to \infty$ to the 3-space $R_3^G(\boldsymbol{y})$ of the G-form invariant shapes $F(\boldsymbol{y}^2)$ because of the $\delta^{(1)}(\Delta t)$ factor which reflects the locality of Newtonian time.

Finally, let us emphasize that 4-space $\mathcal{L}_4(x)$, in spite of the fact that it gives room for spacetime nonlocal two-event shapes $G(x^2) \subset G_{SE}(x)$ (excluded by the locality of eventism L_4), does not realize the philosophy of relationism. Indeed, $x = X_2 - X_1$ — so x is secondary to events $X_{1,2}$, while relationism would have this order reversed. Events must represent limiting relations, where the corresponding limit would be conditioned by suitable *physical situation*, created — for instance — by heavy measuring devices. In other words, directly observable events X must be analyzable in terms of *more elementary*, directly unobservable relations y.

In the Galilean spacetime G_4, coexisting with the G-absolute relational space R_3^G, the problem of priority of relationism over eventism is physically empty, while the absolute simultaneity of Newtonian time solves the problem concerning the absolute coexistence of different physical things. Together with this, the opposition *objectism–eventism* becomes physically empty too, although one should remember that physics has started with *natural* objectism of corpuscular matter in space, while the dimension of time was added to space in order to account for the category of *change* reflected in the motion of corpuscles in space. The problem of priority of objectism over eventism and/or of relationism over eventism and vice versa becomes of physical importance in the (true) spacetime L_4 of measurement, because the eventism L_4 excludes its coexistence with any L-absolute 3-space R_3 which would extend R_3^G to physics of finite universal constant $\hbar/c$.

10. Interpretation of form factor G

Let us review a fundamental difficulty of eventism L_4 which is connected with interpretation of the form factor G. We shall illustrate this problem by the example of an *ultra-relativistic* elastic electron–proton collision which discloses the proton structure G. We start with a general remark concerning the reason of the privileged position of the p language with respect to the x one, keeping in mind that asymptotic four-momenta P_J of particles A_J which participate in an elementary collision process can be measured with — in principle — arbitrarily high precision without affecting the quantum collision process of the stage II. This fact, as we know it well, has its consequences in the S matrix theory parametrized by sharply defined Mandelstam variables s_J (*cf.* (2.3)).

On the other hand, direct x measurements of micro-structures must consist in registering suitable x-coincidences which, according to the p–x duality, result in uncontrollable disturbances of the state of measured object due to uncontrollable amounts of the energy–momentum transfer between the measured object and the measuring one. The privileged position of the p language consists in controllable energy–momentum transfers as well as in the fact that the energy–momentum conservation laws work on the quantum-potential level of each individual micro-process. In consequence, the p–x duality translates the measured p-extensions into their x-representations which proves the completeness of the quantum description of micro-objects together with their *p–x nature*. Of course, according to the same p–x duality, determination of a repeatable observable (*e.g.* a structure) requires a suitable statistics of individual micro-events which actualize quantum propensity (potentiality) which is being carried by each individual system $\mathcal{M}$.

The presented briefly ontologization of the quantum-potential level of micro-system $\mathcal{M}$ conflicts with the Bohr *complementarity principle* which reduces quantum physics to its epistemological aspect only. According to Bohr's philosophy, a physical reality is attached to observable actualizations only if the obtained knowledge about the x and p aspects of the reality is subject to *the incertitude relations.* Thus, the p and x aspects of micro-objects are encumbered with unavoidable errors δx and δp, with $\delta x \delta p \geq \hbar$, and both aspects are necessary in order to get a full knowledge of a micro-system $\mathcal{M}$, similarly as in the *actualized* classical physics with CCINF's. Bohr's defense of the completeness of quantum mechanics resorts to the experimental *possibilities* restricted by incertitude relations which cannot be surmounted by quantum predictions.

This purely epistemological philosophy of quantum physics questions the very possibility of exact (in principle) measurement of some property of micro-world $\mathcal{M}$, reflected implicitly in the popular opinion that — according to quantum physics — the very measurement affects the measured object. If this were a rule without exceptions it would exclude the very quantitative experimental micro-physics. Indeed, all quantitative data would reflect some undefined states of the measured and measuring objects. However, in spite of the ontologization of the quantum-potential level of micro-system $\mathcal{M}$ with its p–x duality and in spite of the experimentally privileged position of the p language, the geometrical structure of p–x duality based on eventism L_4 is encumbered with the dilemma of how to interpret properly the form factors G of $\mathcal{M}$. We are going to discuss this dilemma with an example of the elastic electron–proton collision and, next, we will show that this dilemma can be eliminated by the hypothesis of L-absolute relational space R_3.

The Rosenbluth cross-section [43] for elastic electron–proton collision, obtained from the phenomenological perturbation theory [44], deals with L-form invariant form factors which are accessible to experimental determination. After some theoretical work we are left with a single form factor G, whose p-representation, determined experimentally, is well approximated by the so-called *dipole fit* [45]

$$\tilde{G}(p^2 = \tilde{t}) = \left(1 + \frac{p^2}{0.71}\right)^{-2}, \quad p^2 = \tilde{t} = (P_i - P_f)^2. \tag{10.1}$$

Here, P_i and P_f denote the initial and final asymptotic four-momentum of proton (or that of electron) given in the (GeV/c) units, respectively. The dilemma starts with the fact that the same four-momenta p (conjugate with the relative four-coordinate x) which parametrize $\tilde{G}(p^2)$ measure simultaneously the recoil of proton which carries the measured form factor G, as $p^2 = \tilde{t}$. Let us consider the two possible geometrical natures of G that result from the (phenomenological) perturbation theory on one side and, on the other side, from the corresponding L-invariant equations of motion which should determine the analytic structure of $G^L(x)$. According to the first approach $G = G(x^2)$ (or, interchangeably, $\tilde{G}(p^2)$) and it belongs to the class of separable two-event shapes $G_{\rm SE}(x)$ so it suffers no relativistic distortions which might be due to the recoil of proton. In the standard interpretation of (10.1) [44] one resorts to the extra symmetry of elastic collisions which says that — in the zero-momentum reference frame S^* of the colliding particles — there is no energy transfer between them and therefore all four-momentum transfers p in S^* take the form $p^* = (\boldsymbol{p}; 0)$. Thus $\tilde{t} = \boldsymbol{p}^{*2}$ and one may define a static and spherically symmetric x-shape in S^* which is called the charge density distribution of proton and is equal to

$$\rho(r^*) = e(2\pi\hbar)^{-3} \int d^3p^* \; \tilde{G}(\boldsymbol{p}^{*2} = \tilde{t}) \exp\left[\frac{i}{\hbar}(\boldsymbol{p}^*\boldsymbol{x}^*)\right] \quad (r^* = |\boldsymbol{x}|)\,, \quad (10.2)$$

where e is the elementary charge of proton and

$$\int d^3x^* \; \rho(r^*) = e\,.$$

This interpretation of ρ cannot be justified within the eventism L_4. Firstly, because $\rho(r^*)$, similarly as $G(x^2)$, represents a two-event shape while charge distribution in L_4 must be given by an event shape. Secondly, it is quite obscure why the proton structure should be spherically symmetric in the S^* frame which depends on the relative motion of electron and proton. Of course, the third possibility — that of a direct interpretation of $G(x^2)$ as the "usual" spacetime shape of proton — is out of question, because $G(x^2)$ is a distribution and it remains constant on Minkowski's spheres $x^2 = \text{const.}$ which occupy the whole $\mathcal{L}_4(x)$.

Now, let us consider the second possibility when $G = G^L(x)$ belongs to the second class of non-separable shapes $G_{\rm NS}(x)$, because its analytic form depends on the proton four-momenta $P_{i,f}$ which do not belong to the configuration subspace $\mathcal{L}_4$. This leads — as we already know it — to relativistic distortions of the proton structure which are inherent in the very determination of the structure, as $\tilde{t} = (P_i - P_f)^2$. Thus, defining *the proper shape* of proton as equal to

$$G^L(x) = G^L(x; P_i = P_f) \qquad (i)$$

or

$$\tilde{G}^L(p) = \tilde{G}^L(p; P_i = P_f) \qquad (ii) \qquad (10.3)$$

(as there exists a proton rest-frame in which $P_i = P_f = (\mathbf{0}; M_p c)$ we see that the measured form factor (10.1) does not coincide with equation (10.3) in which $P_i = P_f$ and $\tilde{t}$ disappears. In consequence, the proper shape of proton is not accessible to experiment.

The conclusion may be formed as follows: in spite of the fact that the p language *is* privileged, in both cases — namely for

$$G = G(x^2) \subset G_{\mathrm{SE}}(x) \qquad (i)$$

and

$$G = G^L(x) \subset G_{\mathrm{NS}}(x) \qquad (ii) \qquad (10.4)$$

— the geometry of eventism L_4 results in an inconsistent picture of what can be perceived as a stable extension of $\mathcal{M}$ embedded in L_4. The problem is not an academic one for high-energy collisions (like the one discussed above) in which the dipole fit may be tested up to the values of the $\tilde{t}/(M_p^2 c^2)$ variable equal to about 30. In such situations the recoils of protons are *ultra-relativistic* and for $G = G^L$ the relativistic distortions of the form factor will be significant.

11. Two kinds of geometrical shapes

The quantum p–x duality is formally based on mathematics of the Fourier analysis which is much older than quantum physics. However, from the physical point of view (represented especially by quantum mechanics) there is a fundamental difference between the mathematical — say k–x — duality of the Fourier analysis and the p–x duality, the latter being strictly connected with the dimensional Planck constant $\hbar$. In particular, Fourier analysis results in the *uncertainty relation*

$$\delta x \, \delta k \geq 1 \qquad (11.1)$$

known in the classical field theory which has little in common with the true $(p - x)$-uncertainty relations of quantum physics. A synthesis of mathematical symmetry of Fourier analysis with reality of micro-physics has been developed by Einstein and de Broglie who have put forward the relationships between the wave-frequency language of four-vector $(\boldsymbol{k}; \omega/c)$ and the four-momentum $(\boldsymbol{p}; E/c)$ of an individual quantum (*atom*) in the form

$$\boldsymbol{p} = \hbar \boldsymbol{k} \quad \text{and} \quad E = \hbar\omega\,, \tag{11.2}$$

where $\boldsymbol{k}$ denotes the wave vector. From now on we can speak of the wave-corpuscular $(p - x)$ duality of an individual object $\mathcal{M}$ and equation (11.1) multiplied by $\hbar$ transforms into Heisenberg's uncertainty relation

$$\delta x\, \delta p \geq \hbar\,. \tag{11.3}$$

Note, that owing to the dimensionality of $\hbar$, micro-extensions measured in momentum–energy units may acquire — via the p–x duality — their corresponding x-representations measured in metres and seconds.

Our intention is to show that the mathematical treatment of classical extensions within the k–x duality provides one with two different types of space extensions. This difference reveals itself in the analytic forms of these space extensions, when *the same* extensions are represented in Euclidean spaces E_n of different dimensions n.

We begin with an n-dimensional Euclidean space E_n parametrized by an orthogonal Cartesian reference frame S_n where $\boldsymbol{X} = X_j^{(n)}$, $(j = 1, 2, \ldots, n)$ represents a point in E_n. In order to simplify the notation, we omit the superscript n in the vector notation of X, as the context shall indicate clearly the dimension of E_n. Let us consider a simple problem of determination of the event shape (point shape) $u^{(n)}(\boldsymbol{X})$, $(n = 1, 2, 3)$ of electrostatic potential, extended over the whole space E_n and free of any boundary conditions in finite region of E_n, determined by a given point shape of the charge density distribution $\rho^{(n)}(\boldsymbol{X})$. The translation and rotation symmetries of the space E_n make the corresponding Green's functions to be two-event (two-point) shapes, $O^{(n)}$-form invariant, so one has

$$G^{(n)}(\boldsymbol{x}) = G^{(n)}\left(x_1^{(n)^2} + \ldots + x_n^{(n)^2}\right), \quad \boldsymbol{x} = \boldsymbol{X}_2 - \boldsymbol{X}_1\,. \tag{11.4}$$

Let us emphasize an essential difference which exists between point shapes parametrized by $\boldsymbol{X}$ subject to translations and two-point shapes parametrized by $\boldsymbol{x}$ without translation subgroup which makes a two-point shape unlocalized in E_n. A more philosophical remark will be also instructive for further considerations, namely that our concept of dimensionality of E_n spaces $(n = 1, 2, 3)$ takes its origin in the point shapes of our everyday experience. It is therefore justified to say that the E_n spaces give room to point shapes or that the very point shapes spaces generate E_n's. From this point of view, Euclidean space of the points $x = X_2 - X_1$ subject to rotation symmetry only, is distinctly different from that of the points X whose representations $\boldsymbol{X}$ are sensitive to translation symmetry of $E_n(X)$. In the case of two-point shapes, our imagination resorts automatically to imagining point shapes $f(\boldsymbol{X})$ or points $\boldsymbol{X}$ alone.

The Green's functions of the discussed problem take the following analytic form in the x and k representations

$$
\begin{aligned}
G^{(1)}(\boldsymbol{x}) &= -\frac{1}{2}\left|x_1^{(1)}\right| , \\
G^{(2)}(\boldsymbol{x}) &= -\frac{1}{4\pi}\ln\left(x_1^{(2)^2} + x_2^{(2)^2}\right) , \qquad (i) \\
G^{(3)}(\boldsymbol{x}) &= +\frac{1}{4\pi}\left(x_1^{(3)^2} + x_2^{(3)^2} + x_3^{(3)^2}\right)^{-1/2}
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{G}^{(1)}(\boldsymbol{k}) &= \left(k_1^{(1)^2}\right)^{-1} , \\
\tilde{G}^{(2)}(\boldsymbol{k}) &= \left(k_1^{(2)^2} + k_2^{(2)^2}\right)^{-1} , \qquad (ii) \\
\tilde{G}^{(3)}(\boldsymbol{k}) &= \left(k_1^{(3)^2} + k_2^{(3)^2} + k_3^{(3)^2}\right)^{-1} .
\end{aligned} \tag{11.5}
$$

Unlike the point shapes $u^{(n)}(\boldsymbol{X})$ and $\rho^{(n)}(\boldsymbol{X})$ which represent the corresponding properties attached to points X in the E_n spaces, the Green's functions of the same (linear) differential equation account, in E_n's of different dimensions n, for the same relation given by the two-point shapes $G^{(n)}(\boldsymbol{x})$ $(n = 1, 2, 3)$ from (11.5 i) which — being such shapes — are not localized in $E_n(\boldsymbol{X})$.

The example of Green's functions inclines one to distinguish between two-point shapes and other shapes which we shall label *relational shapes.* This distinction is a consequence of the two different meanings of the *sameness* of two-point shapes and relational shapes in the spaces of different dimensions n.

Definition:

> $G^{(n)}$ represents a *relational shape* if its k-representations in Euclidean spaces of different dimensions n are all determined by an n-independent function $\tilde{G}(k^2)$ of non-negative argument $k^2 \in [0, \infty)$. Thus the relational shapes determine their *sameness* when represented in Euclidean spaces of different dimensions n by the relation

$$\tilde{G}^{(n)}(\boldsymbol{k}) = \tilde{G}\left(k^2 = k_1^{(n)^2} + \ldots + k_n^{(n)^2}\right). \tag{11.6}$$

In the example of Green's functions from (11.5), $\tilde{G}(k^2)$ takes the form

$$\tilde{G}(k^2) = \frac{1}{k^2}. \tag{11.7}$$

As seen from (11.5 *i*), *the sameness* of a relational property G in the x-representation takes, in E_n spaces of different n's, quite different analytic forms which describe two-point shapes $G^{(n)}(\boldsymbol{x})$.

Much like the point shapes $f(\boldsymbol{X})$ originate the Cartesian spaces E_n, let the relational shapes determine *the relational spaces* R_n which are also Euclidean. The globality and indivisibility of relational spaces R_n, strictly connected with the k–x duality, justifies calling R_n's *the geometrical meta-objects* and distinguishing their p and x aspects as $R_n(\boldsymbol{k})$ and $R_n(\boldsymbol{x})$ spaces, respectively. From this double p–x-aspect of a relational space R_n follows that the x-representations of the same relational shapes $G^{(n)}(\boldsymbol{x})$ take the form

$$\begin{aligned} G^{(n)}(\boldsymbol{x}) &= (2\pi)^{-n} \int d^n k\, \tilde{G}^{(n)}(\boldsymbol{k}) e^{i(\boldsymbol{kx})} \\ &= G^{(n)}\left(x_1^{(n)^2} + \ldots + x_n^{(n)^2}\right). \end{aligned} \tag{11.8}$$

Let *the equality with dot* express *the sameness* of a relational property G represented in R_{n+1} and R_n

$$\tilde{G}^{(n+1)}(\boldsymbol{k}) \doteq \tilde{G}^{(n)}(\boldsymbol{k}), \quad G^{(n+1)}(\boldsymbol{x}) \doteq G^{(n)}(\boldsymbol{x}), \tag{11.9}$$

with

$$\tilde{G}^{(1)}(\boldsymbol{k}) \equiv \tilde{G}(k^2).$$

Of course, the question arises of the translation of relational shapes between spaces of different dimensions in the case when the relational shapes transgress the class of rotation invariant shapes. This question will be discussed after introduction of the (physical) hypothesis of relational space R_3.

As the integrals (11.8) must be well-defined, the k–x duality of relational properties G restricts their class. For example, the Euclidean metrics of relational spaces R_n (in their x and k aspects) represents a relational property G, but it must be introduced indirectly, because if given directly it has the form

$$\tilde{G}^{(n)}(\boldsymbol{k}) = k_1^{(n)^2} + \ldots + k_n^{(n)^2} \tag{11.10}$$

in $R_n(\boldsymbol{k})$ and

$$G_x^{(n)}(\boldsymbol{x}) = x_1^{(n)^2} + \ldots + x_n^{(n)^2} \tag{11.11}$$

in $R_n(\boldsymbol{x})$. However, the integrals which determine $G_k^{(n)}(\boldsymbol{x})$ and/or $\tilde{G}_x^{(n)}(\boldsymbol{k})$ are strongly divergent and the metrics of R_n go beyond the acceptable relational properties G.

In order to emphasize the difference between the relational shapes $G^{(n)}(\boldsymbol{x})$ and the two-point shapes in the standard Cartesian x-space $E_n(\boldsymbol{X})$, let us consider an also $O^{(n)}$-form invariant two-point shape $G^{(n)}(x_1^{(n)2} + \ldots + x_n^{(n)2})$ whose analytic form in the space E_n of a definite dimension n can coincide with the relational shape. We already know what does *the sameness* of a relational shape $G^{(n)}$ mean (when $G^{(n)}$ is translated from R_n to R_{n+1}); this sameness is represented by an equality with dot. Suppose now that $G^{(n)}\left(x_1^{(n)2} + \ldots + x_n^{(n)2}\right)$ is a standard two-point shape and that we want to express *the same* two-point shape in the E_{n+1} space. We add to n axes of S_n the $(n+1)$-th 0-$x_{n+1}^{(n+1)}$-axis of the reference frame S_{n+1} (which parametrizes E_{n+1}). The new axis is perpendicular to all n axes of S_n. The fact that $G^{(n)}\left(x_1^{(n)2} + \ldots + x_n^{(n)2}\right)$ has no extension in the 0-$x_{n+1}^{(n+1)}$-dimension of E_{n+1} space makes that *the same* two-point shape $G^{(n+1)}$, when represented in the subclass of reference frames S_{n+1} in E_{n+1}, has the form

$$G^{(n+1)}(\boldsymbol{x}) = G^{(n)}\left(x_1^{(n+1)^2} + \ldots + x_n^{(n+1)^2}\right)\delta^{(1)}\left(x_{n+1}^{(n+1)}\right). \tag{11.12}$$

We have assumed $G^{(n)}(\boldsymbol{x})d^n x$ to be dimensionless quantities, because this corresponds to the most interesting case in considerations which follow, but the essential point is that *the same* two-point shape breaks the rotation symmetry $O^{(n+1)}$ of E_{n+1} when represented in E_{n+1}. As it can be seen from (11.12), *the same* two-point shape in E_{n+1} introduces a geometrical direction $\boldsymbol{u}^{(n+1)}$ parallel to the 0-$x_{n+1}^{(n+1)}$-axis of our subclass of reference frames S_{n+1} in which $G^{(n+1)}(\boldsymbol{x})$ takes the form (11.12).

Thus, in contrast with relational shapes, *the sameness* of two-point shapes in E_n and E_{n+1} makes the space E_n to be relativized to the space E_{n+1} as a geometrical direction $\boldsymbol{u}_{n+1}$ must be introduced. *The sameness* of relational shapes in the relational spaces R_n and R_{n+1} does not introduce any geometrical direction in R_{n+1}. Therefore, relational spaces which give room to relational shapes remain self-dependent, *i.e.* the space R_n is not relativized to the R_{n+1} one. Although, within the Euclidean spaces E_n, any distinction between two-point shapes and relational shapes is immaterial and any differentiation of shapes into two-point ones and relational ones looks "artificially". We shall see, from Section 13 onward, that the situation changes radically when *practical geometry* of pseudo-Euclidean Minkowskian spacetime L_4 comes into the play.

12. Some properties of relational spaces R_n

In order to make a clear distinction between Cartesian (E_n) and relational spaces (R_n), let $\boldsymbol{x} \to \boldsymbol{y}$ and $\boldsymbol{k} \to \boldsymbol{q}$ denote henceforth the points of relational space R_n in its x and p aspects, respectively. Together with this change of notation, we shall also introduce the Planck constant so the classical k–x duality will convert into the *true* quantum p–x duality represented by $\boldsymbol{y}$ and $\boldsymbol{q}$ coordinates.

The self-dependence of relational spaces R_n, due to the fact that relational shapes $G^{(n)}(\boldsymbol{y}^2)$ in express R_n the same relational property G (under different analytic forms of its representations in R_n of different dimensions n), results in the following fundamental property. A relational shape $G^{(n)}(\boldsymbol{y}^2)$ in R_n represents obviously an explicitly $O^{(k)}$-form invariant function for $k \leq n$, because in this case $O^{(k)}$ is a subgroup of the rotation group $O^{(n)}$. However — and here is the essential point — $G^{(n)}(\boldsymbol{y}^2)$ remains, although implicitly, also $O^{(k)}$-form invariant for $k > n$, in particular for $k = n+1$. Indeed, the same relational property G in R_{n+1} takes the form $G^{(n+1)}(\boldsymbol{y}^2)$ ($\doteq G^{(n)}(\boldsymbol{y}^2)$) which is explicitly $O^{(n+1)}$-form invariant.

Before formulating some important identities, let us point out that the directions of axes of a reference frame S_n parametrizing some space, do not introduce any geometrical direction $\boldsymbol{u}^{(n)}$. Thus, if we impose, in $R_{n+1}(\boldsymbol{q})$ and onto relational shape $G^{(n+1)}(\boldsymbol{q})$, the constraint $q_{n+1}^{(n+1)} = 0$ we do not introduce into R_{n+1} any geometrical direction like $\boldsymbol{u}^{(n+1)}$. According to the definition (11.6) we get the following identity

$$\tilde{G}^{(n+1)}\left(q_1^{(n+1)^2} + \ldots + q_n^{(n+1)^2} = q^2\right)$$
$$\equiv \tilde{G}^{(n)}\left(q_1^{(n)^2} + \ldots + q_n^{(n)^2} = q^2\right). \qquad (12.1)$$

According to the p–x duality, the same identity takes the form in x representation

$$\int_{-\infty}^{+\infty} dy_{n+1}^{(n+1)}\, G^{(n+1)}\left(y_1^{(n+1)^2} + \ldots + y_{n+1}^{(n+1)^2}\right) \equiv G^{(n)}\left(y_1^{(n)^2} + \ldots + y_n^{(n)^2}\right), \tag{12.2}$$

where, similarly as in $R_{n+1}(q)$, the integration over the $y_{n+1}^{(n+1)}$ variable introduces no geometrical direction in $R_{n+1}(\boldsymbol{y})$. Of course, identities (12.1) and (12.2) connect *the same* Green's functions from (11.5).

Relational spaces R_n with their dual p–x aspects have been introduced with the help of relational properties G which reveal their quantitative aspects in the corresponding functions $\tilde{G}(q^2)$, *i.e.* when exposed in the p language. Consequently, the relationship between analytic representations of *the same* property G in relational spaces R_n of different dimensions n given by *the equality with dot* is restricted — so far — to $O^{(n)}$-form invariant functions in the R_n spaces. In accordance with the physical concept of configuration space, a relational space R_n of definite dimension n induces the corresponding configuration spaces $R_n(\boldsymbol{y}_1) \otimes R_n(\boldsymbol{y}_2) \otimes \ldots$ in which the relational shapes $G^{(n)}(\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots)$ are embedded. The extension of the translation $G^{(n)}$ into the language of R_m spaces with $m \neq n$ follows the rule presented for the elementary configuration space $R_n(\boldsymbol{y})$ provided that $G^{(n)}$ is also $O^{(n)}$-form invariant.

However, the question arises of the translation of relational shapes from an R_n to an R_m space in the case when $G^{(n)}(\boldsymbol{y})$ breaks the $O^{(n)}$ symmetry. Seeking some help in physical intuition, let us suppose that $R_n(\boldsymbol{y})$ gives room to the parametrization of the y degrees of freedom of some system $\mathcal{M}$ which are entirely separated from the other degrees of freedom of the system. Let us suppose, moreover, that no geometrical directions are given inside the relational space R_n. Such a situation corresponds to the one in which we have to deal with the $\boldsymbol{y}$ degrees of freedom of a micro-world $\mathcal{M}$ in a bound state of $\mathcal{M}$ composed of spin-less constituents. In such a case neither directions of (relational) momenta $\overset{o}{\boldsymbol{q}}$ nor directions $\boldsymbol{n}$ (onto which spins could be projected) are given and the only "building stuff" of relational shapes in $R_n(\boldsymbol{y})$ (here $n = 3$) is provided by the coordinates y_j of the points $\boldsymbol{y}$ of $R_n(\boldsymbol{y})$ or, interchangeably, by the coordinates q_j of the points $\boldsymbol{q}$ of $R_n(\boldsymbol{q})$. The geometrical meta-object R_n restricts then the class of relational shapes to tensor fields of the form

$$T^{(n)}_{s_1\ldots s_K}(y) = G^{(n)}(\boldsymbol{y}^2)\, y^{(n)}_{s_1}\ldots y^{(n)}_{s_K} \qquad (s_J = 1, \ldots, n) \tag{12.3}$$

or, interchangeably, to analogous tensors in the $q_j^{(n)}$ variables if $T^{(n)}$ is represented in the p language. Of course, tensors $T^{(n)}$ do break the $O^{(n)}$-form invariance and the question arises of the translation of these tensors from the R_n space to an R_m space, the latter having the dimension $m \neq n$. Again, *the equality with dot* must be used

$$T^{(n)}(y) \doteq T^{(m)}(y)\,. \tag{12.4}$$

This question will be solved in Section 14 in connection with the physically essential problem of the relationship which exists between the geometrical objects embedded in 4-space $\mathcal{L}_4(p)$ accessible to measurement and those embedded in the *hidden* L-absolute relational space R_3.

13. Hypothesis of relational space R_3

We return to physics and, following the philosophy of relationism, we take for granted that metrical relations of micro-systems $\mathcal{M}$ are initiated inside the very systems and not in the external spacetime of measurement as in the present physics based on eventism. This philosophy stands in agreement with von Weizsaecker's opinion [15] that: "Spacetime is not the background but a surface aspect of reality". Note that the main idea of relationism, namely that according to which physics takes its origin inside micro-worlds $\mathcal{M}$ is consistent with the two most successful theories: NR quantum mechanics and relativistic perturbation theory. Although both theories start with eventism, their success is due — in the first place — to the separability of the external degrees of freedom of an isolated system $\mathcal{M}$ from the external ones. This is a necessary condition if the hypothesis of relationism is to be put forward. In NR quantum mechanics, apart from $O^R \neq O^G$, the (G-absolute) relational space R_3^G coexists with eventism G_4, while the relativistic perturbation theory introduces implicitly the geometrical meta-object of 4-space $\mathcal{L}_4$ which goes beyond the locality of classical eventism L_4 by realizing the mentioned separability of degrees of freedom. Thus, the two theories, without explicitly renouncing eventism, make a step towards relationism.

According to relationism, an elementary micro-world $\mathcal{M}$ must be composed of two hypothetical components. From this follows that the one-body problem of eventism must become a limiting case of the two-body problem. At the same time, an event X must become a limiting notion of *the more elementary* relation y. Now if one assumes, in order to deal with metrical extensions of $\mathcal{M} = A_1 + A_2$, the existence of two hypothetical constituents

A_1 and A_2, there is no reason whatever for declaring that each of these constituents pre-exists separately in metrical continuum (spacetime) of measurement. Such implication would follow from eventism which is obligatory for classical physics only (in which $\hbar = 0$) with its CCINF's.

The following assumptions stand at the basis of the hypothesis of (L-absolute) internal space–and–time I_4 which extends I_4^G (*cf.* (4.13)) to physics of finite universal constant $\hbar/c$: (1) the quantum p–x duality which accounts for atomism and for wave–corpuscular duality of matter; (2) symmetry L of relativistic kinematics of the asymptotic zone of measurement. The third assumption, which is connected with the separability of the internal and external degrees of freedom of a micro-world $\mathcal{M}$, brings into being the relational space R_3 and it may be formulated as follows: (3) the hypothesis of relationism recognizes that micro-physical symmetries follow the analogy which exists between the Euclidean relational space R_4 and the pseudo-Euclidean 4-space $\mathcal{L}_4$.

Let us remember that absolute relational properties G given by $O^{(n)}$-form invariant relational shapes $G^{(4)}(y^2)$ determine *the same* absolute properties G embedded — by definition — in relational space R_3 and given by

$$\tilde{G}^{(3)}(q^2) \doteq \tilde{G}^{(4)}(q^2)\,, \quad G^{(3)}(y^2) \doteq G^{(4)}(y^2)\,. \tag{13.1}$$

Moreover, and here is the point, relational shapes $G^{(3)}$ which are $O^{(3)}$-form invariant explicitly are also $O^{(4)}$-form invariant implicitly. In other words, the 6-parameter rotation symmetry $O^{(4)}$ of 4-space R_4 keeps the analytic form of relational shapes $G^{(3)}(y^2)$ (and, interchangeably, $\tilde{G}^{(n)}(q^2)$) unchanged. This is due to the fact that R_3 is not relativized to R_4, because *the sameness* of relational shapes in R_3 and R_4 does not introduce any geometrical direction $u^{(4)}$. In the case of Cartesian point shapes in E_3 extended to E_4 — *cf.* (11.12) — *the sameness* of the corresponding point shapes makes E_3 relativized to E_4 by introducing the direction $u^{(4)}$ in E_3.

According to assumption (3), the hypothesis of relationism replaces R_4 with $\mathcal{L}_4$. However, the indefinite metrics of $\mathcal{L}_4$ and the definite one of R_3 restricts the 4-space $\mathcal{L}_4(p)$ to the region of its space-like four-momenta p. Thus we begin with absolute relational properties G expressed in $\mathcal{L}_4(p)$ by L-form invariant functions $\tilde{G}(p^2)$ which, in the L-absolute relational space $R_3(q)$, take the form

$$\tilde{F}(q^2) = \tilde{G}(p^2 = q^2 \geq 0)\,. \tag{13.2}$$

We say that F, embedded in R_3, expresses the same relational property of the system $\mathcal{M}$ from its inside as G does from its outside in $\mathcal{L}_4$ and accessible to measurement in the privileged language p, *i.e.* in $\mathcal{L}_4(p)$. The

p–x duality of geometrical meta-objects R_3 and $\mathcal{L}_4$ determines the corresponding x-representations of relational shapes F and G as being equal to

$$\left.\begin{aligned} F(\boldsymbol{y}^2) &= (2\pi\hbar)^{-3}\int d^3q\,\tilde{F}(\boldsymbol{q}^2)\mathrm{e}^{i/\hbar(\boldsymbol{q}\boldsymbol{y})}, \quad (i) \\ G(x^2) &= (2\pi\hbar)^{-4}\int d^4p\,\tilde{G}(p^2)\mathrm{e}^{i/\hbar(px)}. \quad (ii) \end{aligned}\right\} \tag{13.3}$$

Note that the p–x duality restricts — in principle — the class of relational properties G to those for which the integrals (13.3) are well-defined.

However, if we start with relational shapes F in R_3 and we want to get the corresponding relational shapes G in $\mathcal{L}_4$, a general problem arises of the extension of $\tilde{F}(\boldsymbol{q}^2)$ over the negative values of $\boldsymbol{q}^2$ which correspond to the time-like four-momenta p. This extension is given directly, in several important problems mentioned in Appendix A, by the very analytic form of $\tilde{F}(\boldsymbol{q}^2)$. Assuming that the problem of extension of $\tilde{F}(\boldsymbol{q}^2)$ over the negative values of $\boldsymbol{q}^2$ can be solved, $\tilde{F}(\boldsymbol{q}^2)$ determines *the same* relational shape $\tilde{G}(p^2)$ in the whole 4-space $\mathcal{L}_4(p)$ and

$$\tilde{G}(p^2) = \tilde{F}(\boldsymbol{q}^2 = p^2 \geq 0)\,. \tag{13.4}$$

Once again we may see the singularity of the semi-group G according to which G_4 coexists with relationism R_3^G (apart from $O^R \neq O^G$). As we know from Section 8, NR physics eliminates the counterparts of time-like four-momenta p and, hence — without explicitly resorting to the hypothesis of relationism — we obtain a one–to–one correspondence between the G-form invariant two-event shapes $G(\boldsymbol{x})$ from (5.6) and the (explicitly) G-form invariant relational shapes $F(\boldsymbol{y}^2)$ embedded in R_3^G. We see also that the NR limits (8.9) and (8.14) of the L-form invariant two-event shapes $G(x^2)$ convert into G-form invariant shapes $G(\boldsymbol{x})$ from (5.6).

The pseudo-Euclidean character of 4-space $\mathcal{L}_4$ does not violate the identities (12.1) and (12.2) which result now from the constraint $p_0 = 0$ and integration over the x_0 variable. The only difference between R_4 (with definite metrics) and $\mathcal{L}_4$ (with indefinite metrics) consists in the fact that the directions which are perpendicular to the hyper-plane $p_0 = 0$ and the direction of the 0-x_0-axis of some reference frame S in $\mathcal{L}_4$ are not arbitrary ones (as in R_4), but must be of the time-like character. Thus, according to the definition (13.2), the constraint $p_0 = 0$ results, in any reference frame S parametrizing L_4 and $\mathcal{L}_4$, in an identity analogous to that from (12.1)

$$\tilde{G}(p^2 = \boldsymbol{p}^2 - p_0^2 = \boldsymbol{p}^2 = \boldsymbol{q}^2 \geq 0) \equiv \tilde{F}(\boldsymbol{q}^2)\,. \tag{13.5}$$

The x-counterpart of (13.5) corresponding to (12.2) takes the form

$$\int_{-\infty}^{+\infty} dx_0 \, G(x^2 = \boldsymbol{x}^2 - x_0^2) = F(\boldsymbol{x}^2 = \boldsymbol{y}^2), \tag{13.6}$$

which is valid in any reference frame S.

Since equality (13.6) is valid in any reference frame S, the identification of the numerical value of the space-interval square $\boldsymbol{x}^2$ with the L-absolute interval square $\boldsymbol{y}^2$ in $R_3(\boldsymbol{y})$ expresses the fact that the proper lengths of unit rods of all reference bodies $\tilde{S}$ are the same. In the NR limit, as can be seen from (8.15 G), the identity $\boldsymbol{x}^2 = \boldsymbol{y}^2$ follows directly from the coexistence of G_4 with R_3^G or, in other words, from the equalized balance of geometry G_4.

Note, that the self-dependence of the R_n spaces, much like that of R_3 and $\mathcal{L}_4$, means that the geometrical directions $\boldsymbol{u}^{(3)}$ in R_3 and $u^{(4)}$ in $\mathcal{L}_4$ are *a priori* independent. This results automatically in an inequality analogous to that from (4.10), with G_4 replaced by L_4

$$O_3^R \neq O_3^L, \tag{13.7}$$

where O_3^L is the space rotation included in the L-symmetry group. In Section 18 we show that the connection between some $\boldsymbol{u}^{(3)}$ and $\boldsymbol{u}^{(4)}$ directions is established *a posteriori* and one should expect that in the case of fully isolated micro-worlds $\mathcal{M}$ this connection will concern the 3-momentum $\overset{o}{\boldsymbol{q}}$ of $R_3(\boldsymbol{q})$ and the four-momentum $\overset{o}{p}$ of $\mathcal{L}_4(p)$ which characterize the scattering states of $\mathcal{M}$.

The separability of the internal degrees of freedom of an $\mathcal{M}$ embedded in the corresponding configuration space $R_3(\boldsymbol{y}_1) \otimes R_3(\boldsymbol{y}_2) \otimes \ldots$ from the external ones in $L_4(X)$, does indeed conflict with eventism L_4. Therefore, a hierarchic description of any state of the composite system $\mathcal{M}$ must be used. First one has to determine the c-number, (L-)absolute characteristics of $\mathcal{M}$ in the corresponding (mechanical) configuration space induced by relational space R_3 and next to translate them (if necessary) into the language of the corresponding configuration space induced by 4-space $\mathcal{L}_4$ whose p aspects are accessible to measurement. Of course, this translation is given by the equalities with dot which relate the corresponding relational shapes in R_3 to the two-event shapes in $\mathcal{L}_4$.

The extension of relational shapes $F(\boldsymbol{y})$ (functions), obtained from *mechanical* equations based on geometry, to *the same* two-event shapes $G(x)$ (distributions) embedded in $\mathcal{L}_4(x)$ shows that the hypothesis of relationism R_3 eliminates the relativistic redundancy of degrees of freedom [30]. At the same time, the relational variables $\boldsymbol{y}$ and $\boldsymbol{q}$ regain the character of dynamical variables of NR physics free of *the relativistic redundancy* of degrees of freedom because of the G-absoluteness of the Newtonian time.

Relational origin of metrical physics and its p–x duality are also strongly supported by non-local quantum EPR-like correlations [9] which break Bell's inequalities [10] in perfect agreement with quantum predictions [11] and in full conflict with Einstein's *classical reality* [9] based on eventism. Thus, two *practical geometries*, namely that of *hidden* relational spaces R_3 and that of spacetime of directly observable events X would disclose the *two-level* nature of physical reality which must be extended over the quantum potentiality of *ψ-states*. We would deal with the quantum-potential level of reality which exhibits quantum propensity of an individual micro-object $\mathcal{M}$ and the classical actualized level of measurement which is being performed by classical macro-devices registering some irreversible tracks [5]. As rightly pointed out by Messiah [46], *the gap* between potential and actual levels of reality does not necessarily mean that the macro-objects avoid the p–x duality. The only point is that the large masses of extremely "involved" macro-objects justify one to ascribe to such an object a relatively sharp localization X and velocity V at each instant t — *cf.* (3.12) — when the wave–aspect of such object ceases to be detectable. Therefore, first the classical mechanics and next the whole classical physics have (tacitly) taken for granted the one-level physics, actualized *a priori* on the background of a pre-existing metrical spacetime of measurement (eventism).

Thus, let us emphasize, as this will be important in considerations which follow, that according to relationism, an infinitely heavy reference body $\bar{S}$ must stand behind reference frames S, if relations referred to $\bar{S}$ are to become isomorphic with events. In classical physics ($\hbar = 0$) with CCINF's we can abstract from reality of reference bodies $\bar{S}$ which remain behind reference frames S, so classical physics is condemned to eventism. Consequently, physical existence of any entity means its actualized existence on the background of spacetime and there is no room left for *quantum-potential* existence which is symbolized by ψ.

14. Translation of relational shapes into two-event shapes

By virtue of definition we know how to translate the $O^{(3)}$-form invariant relational shapes $F(\boldsymbol{y}^2)$ of R_3 into *the same* two-event shapes $G(x^2)$ of $\mathcal{L}_4$ which are L-form invariant. However, this translation, represented by an equality with dot, requires an extension onto $\mathcal{M}$'s composed of N constituents, *i.e.* when $\mathcal{M} = A_1 + \ldots + A_N$ ($N > 2$) and, moreover, when relational shapes describing the structure of $\mathcal{M}$ cease to be $O^{(3)}$-form invariant. In both cases we must distinguish between $\mathcal{M}$'s in bound states and in scattering ones, because in scattering states we deal with fragments of $\mathcal{M}$ which reach, each of them separately, the asymptotic zone of relativistic kinematics subject to the symmetry L of measurement. In this section we shall treat the case of bound structures of spin-less particles, whereas in Section 18 we shall proceed with the problem of scattering states.

Remaining still within the frame of the $O^{(3)}$-form invariant form factors, let us show that the equality with dot of the corresponding form factors F and G, $G \doteq F$, of an $\mathcal{M}$ composed of several constituents obeys the same rule as in the case of the elementary two-body $\mathcal{M}$, $\mathcal{M} = A_1 + A_2$. For this purpose it is sufficient to consider the three-body problem of an $\mathcal{M} = A_1 + A_2 + A_3$ whose structures G are embedded in the corresponding 6-dimensional configuration space $R_3(y_{12}) \otimes R_3(y_{13})$, with A_1 taken (quite arbitrarily) as the origin of the reference frame S_3 which parametrizes $R_3(y)$. The corresponding L-form invariant form factors G embedded in the 8-dimensional configuration space $\mathcal{L}_4(x_{12}) \otimes \mathcal{L}_4(x_{13})$ take the form

$$G = G(x_{12}^2, x_{13}^2, x_{23}^2)\,, \quad x_{23} = x_{13} - x_{12}\,, \tag{14.1}$$

where $x_{jk} = X_k - X_j$. Following the same rule as in (13.2), the L-absolute relational shape F is determined in the p language as

$$\tilde{F}(q_{12}^2, q_{13}^2, q_{23}^2) \equiv \tilde{G}(p_{12}^2 = q_{12}^2, p_{13}^2 = q_{13}^2, p_{23}^2 = q_{23}^2) \tag{14.2}$$

with $q_{jk}^2 \geq 0$.

Much like in (13.2), this projection of 8-dimensional configuration space $\mathcal{L}_4(p_{12})\otimes\mathcal{L}_4(p_{13})$ onto 6-dimensional configuration space $R_3(q_{12})\otimes R_3(q_{13})$, where $q_{23} = q_{13} - q_{12}$ and $p_{23} = p_{13} - p_{12}$, restricts four-momenta p_{12}, p_{13}, p_{23} to space-like ones only. Thus the determination of G by means of F, in the whole configuration space $\mathcal{L}_4(p_{12}) \otimes \mathcal{L}_4(p_{13})$, requires a proper extension of $\tilde{F}(q_{12}, q_{13})$ for the case of negative values of q_{12}^2, q_{13}^2, q_{23}^2. Besides, the p–x duality restricts the class of *relational properties* $F = G$ to those for which the corresponding Fourier integrals are defined. This fact may be of physical importance (*cf.* Appendix A). As these problems have the same nature for $N = 2$ and for $N \geq 3$, it is also sufficient to discuss the question of the equality with dot, which links F and G when they break the symmetries $O^{(3)}$ and L, respectively, in the example of the elementary two-body problem with $\mathcal{M} = A_1 + A_2$.

Similarly as it has been signalized in Section 12 (*cf.* (12.3)) for bound states of $\mathcal{M} = A_1 + A_2$ under discussion, the very geometry of meta-objects in R_3 and in $\mathcal{L}_4$ restricts the building stuff of geometrical objects in $R_3(y)$ and in $\mathcal{L}_4(x)$ to the coordinates y of $R_3(y)$ and to the coordinates $x = (x_j, x_0)$ of the points x of $\mathcal{L}_4(x)$. According to the p–x duality, the same can be formulated — interchangeably — in the p language of the $R_3(q)$ and

$\mathcal{L}_4(p)$ spaces. Thus, in analogy to (12.3), the most general relational shapes in $R_3(y)$ and two-event shapes in $\mathcal{L}_4(x)$ take the form of tensors

$$T_{s_1 \dots s_K}(y) = F(y^2) y_{s_1} \cdots y_{s_K} \quad (s_J = 1,2,3) \tag{14.3}$$

and

$$\Theta_{\sigma_1 \dots \sigma_K}(x) = G(x^2) x_{\sigma_1} \dots x_{\sigma_K} \quad (\sigma_J = 0,1,2,3). \tag{14.4}$$

The determination of the equality with dot between the corresponding relational shapes T and two-event shapes Θ will be based on the already established equality with dot, which links the form invariant shapes in the R_3 and $\mathcal{L}_4$ spaces, and on the identities

$$\left.\begin{aligned} F(y^2)\, y_s &= \tfrac{1}{2} \left(\frac{\partial}{\partial y_s}\right) F^{(1)}(y^2), \quad &(i) \\ G(x^2)\, x_\sigma &= \tfrac{1}{2} \left(\frac{\partial}{\partial x_\sigma}\right) G^{(1)}(x^2). \quad &(ii) \end{aligned}\right\} \tag{14.5}$$

Here $F^{(k)}(z)$ and $G^{(k)}(z)$ denote the k-fold integrals of $F(z) \equiv F^{(0)}(z)$ and $G(z) \equiv G^{(0)}(z)$, respectively, hence

$$\left.\begin{aligned} \frac{d^k}{dz^k} \begin{Bmatrix} F^{(k)}(z) \\ G^{(k)}(z) \end{Bmatrix} &= \begin{Bmatrix} F(z) \\ G(z) \end{Bmatrix} \quad &(i) \\ F^{(k)}(y^2) &\doteq G^{(k)}(x^2). \quad &(ii) \end{aligned}\right\} \tag{14.6}$$

with

Let us remember that $O_3^R \neq O_3^L$ and therefore the orientations of the space axes of reference frame in R_3 and those of reference frame S in L_4 are independent *a priori*. Consequently, partial derivatives of $F(y^2)$ with respect to y_s and those of $G(x^2)$ with respect to x_σ do not introduce any connection between the orientations of R_3 space and E_3 space of some S — a fact which is consistent with (13.7).

Let us start with a four-vector field of tensor T from (14.4) where

$$\Theta_\sigma(x) = G(x^2) x_\sigma. \tag{14.7}$$

The identity

$$\int_{-\infty}^{+\infty} dx_0\, G(\boldsymbol{x}^2 - x_0^2) x_0 = 0 \tag{14.8}$$

makes that the time component $\Theta_0(x)$ of $\Theta_\sigma(x)$ has no residuum in R_3 which leads to

$$\Theta_0(x) = G(\boldsymbol{x}^2 - x_0^2) x_0 \doteq 0. \tag{14.9}$$

From identity (13.6) one gets

$$\Theta_s(x) = G(x^2)x_s \doteq F(y^2)y_s = T_s(y) \quad (s = 1,2,3) \tag{14.10}$$

which, together with (14.9), determine the equality with dot between the four-vector field $\Theta_\sigma(x)$ in $\mathcal{L}_4(x)$ and the vector field $T_s(y)$ in $R_3(y)$.

Following the same rules, the equality with dot between 2-rank tensors $\Theta_{\sigma\lambda}$ and T_{sl} has the form

$$\begin{aligned} \Theta_{sl}(x) &= G(x^2)x_s x_l \doteq F(y^2)y_s y_l = T_{sl}(y)\,, \\ \Theta_{s0}(x) &= \Theta_{0s}(x) = G(x^2)x_0 \doteq 0\,, \\ \Theta_{00}(x) &= G(x^2)x_0^2 \doteq \tfrac{1}{2}F^{(1)}(y^2)\,. \end{aligned} \tag{14.11}$$

As $\Theta_{00}(x)$ is an even function (distribution) of x_0, its R_3-counterpart does not vanish but it is equal to $^1/_2 F^{(1)}(y^2)$. Equivalent equalities with dot could be obtained starting with the p representations of tensors T and Θ.

Exactly in the same way one can establish the equalities with dot between tensors T and Θ of an arbitrary rank; the same concerns structures $\mathcal{M}$ composed of N constituents $N > 2$ when the corresponding tensors are spanned on $3(N-1)$ and $4(N-1)$ variables of the configuration spaces $\overbrace{R_3 \otimes \ldots \otimes R_3}^{N-1}$ and $\overbrace{\mathcal{L}_4 \otimes \ldots \otimes \mathcal{L}_4}^{N-1}$, respectively.

Let us emphasize that in absence of any internal direction $u^{(4)}$ in $\mathcal{L}_4$ which characterizes bound structures of $\mathcal{M}$, the symmetry L of geometrical meta-object $\mathcal{L}_4$ excludes all two-event structures $G(x)$, like *e.g.* $\Theta(x)$ tensors, which, in some reference frame S parametrizing L_4 (hence also $\mathcal{L}_4$) would be independent of the relative time variable $x_0 = c\Delta t$. This very fact makes that the spacetime two-event shapes $G(x)$ are essentially different from an event shape $f(X)$. Indeed, there exist event shapes which become independent of the time variable $X_0 = ct$ in some reference frames S of L_4. However, these Cartesian shapes $f(\boldsymbol{X})$ seen in the perspective of 4-dimensional spacetime L_4 can be perceived as 4-dimensional structures rolled along the time axis of the reference frames S in which f becomes independent of time. Thus, the very translation symmetry of such shapes introduces a geometrical direction $u^{(4)}$ parallel to that of 0-X_0-axis. Two-event shapes $G(x)$ in question are never rolled along the 0-x_0-axis of some reference frame S in $\mathcal{L}_4(x)$, which makes the 4-space $\mathcal{L}_4(x)$ essentially different from the 4-spacetime L_4 with its Cartesian event shapes $f(X)$ which have found "on their own" the picture of the space–and–time that we adopt.

The restricted class of the relational shapes T and corresponding two-event shapes G (or Θ) is strictly connected with the quantum p–x duality giving rise to the dual p and x aspects of geometrical meta-objects R_3

and $\mathcal{L}_4$ which give room to quantum nonlocality. In consequence, relational geometry R_3 and its *Lorentz limit* discussed in Section 16 admit laws of motion which are in position to originate, starting from (hypothetical) point constituents of $\mathcal{M}$, some extended and stable structures (*plena*) which describe the measuring rods and are particular solutions of these laws. These structures, based on the quantum p–x duality, remain at the same time consistent with the symmetry L of measurement. Thus, the R_3 relationism makes room for a *closed* theory that reconstructs measuring rods as the objects which found our metrical spacetime. Let us remember that eventism which follows the Cartesian philosophy of a pre-existing x-space cannot avoid the old paradox of *labyrinth* due to the fundamental opposition which exists between structure-less point particles and continuum. NR quantum mechanics surmounts this paradox with the help of eventism G_4 only because G_4 coexists with relationism R_3^G.

In Section 21 we show also that the p–x duality of the first *practical geometry* R_3 of relations — even the flat one — breaks the Thales similarity of *small* and *large* objects inherent in the Cartesian (flat) spacetime (eventism). In consequence, a declaration that some real object is small and/or large attains an absolute meaning which is alien to eventism.

15. Internal time and internal spacetime I_4 of micro-objects

As we remember, it was time which, when added to 3-space, has disclosed the Galilean symmetry G relativizing 3-space to G_4. In spite of that, the G-absolute 3-space $R_3^G(y)$ and the equally G-absolute Newtonian time make eventism G_4 coexist with internal space–and–time continuum I_4^G. Contrary to G_4, I_4^G is a Cartesian product of $R_3^G(y)$ and $\mathcal{T}^G(\tau)$ (*cf.* (4.13)). Thus I_4^G reminds one of the Aristotelian space and time. Still within the frame of classical physics ($\hbar = 0$), this situation becomes radically changed by STR which restricts the velocities of CCINF's introducing a universal constant c. The CCINF's do maintain the eventism of Cartesian x-space (time), but the very fact that any intercommunication interferes with the symmetry (L) of pre-existing spacetime L_4 shakes the faith in the eventism with time rates being referred to space scaffolds.

Another aspect of classical physics which also favours the objectism of relational origin of metrical physics is connected with additional symmetries of an isolated system $\mathcal{M}$. Indeed, a complete description of an isolated $\mathcal{M}$ resorts to the Hamiltonian and/or Lagrangean formalisms. In consequence, the very formalism responsible for disclosing the fundamental p–x canonical symmetry excludes dissipative systems which become described incompletely. This leads to the well known and still challenging problems of statistical physics connected with the notion of irreversibility. In quantum physics, this problem appears in the irreversibility of the actualization processes of quantum potentiality ψ called *the reduction of the wave packet* [5].

Following the *quantum* relationism R_3 and its p–x duality, we start with an isolated micro-world $\mathcal{M}$ on its elementary quantum-potential level. Thus, an $\mathcal{M}$ composed of N constituents A_J is embedded in the $3(N-1)$-dimensional configuration space $\overbrace{R_3 \otimes \ldots R_3}^{N-1}$ induced by the R_3 space. In accordance with the idea of indivisibility of quantum states, we attach to this configuration space the *a priori* L-absolute internal time τ of the continuum $\mathcal{T}_M(\tau)$. The L-absolute time continuum $\mathcal{T}_M(\tau)$ enhances then the philosophy of relationism according to which a fully isolated system $\mathcal{M}$ creates a self-dependent *micro-world* $\mathcal{M}$. In the NR limit ($c \to \infty$) the internal-time continuum $\mathcal{T}_M$ converts into a G-absolute internal-time continuum $\mathcal{T}^G$. (The subscript M may be omitted because of the G-absoluteness of the Newtonian time.) However, a synchronization of internal times τ_M corresponding to different, independent $\mathcal{M}$'s remains, even in G_4, *a priori* undetermined. We shall show in Sections 22 and 23 that the rates of internal times τ_M, as they depend on $\mathcal{M}$, account for the time dilatation effect on the quantum-potential level of $\mathcal{M}$.

The hierarchic description of the states of $\mathcal{M}$ imposed by relationism R_3 starts with the Schrödinger equation in the $R_3 \otimes \ldots \otimes R_3 \otimes \tau_M$ spacetime and the p–x duality of R_3 is realized in the Schrödinger x-representation by putting

$$\boldsymbol{y}_j = \boldsymbol{y}_j\,, \quad \boldsymbol{q}_j = -i\hbar \frac{\partial}{\partial \boldsymbol{y}_j} \quad (j = 1, \ldots, N-1)\,. \tag{15.1}$$

Let the internal O_3^R-form invariant Hamiltonian $\widehat{h}$ of $\mathcal{M}$ be a sum of internal kinetic energy (operator) $\widehat{W}^{(k)}$ and internal potential V, hence

$$\widehat{h} = \widehat{W}^{(k)}(\hat{\boldsymbol{q}}_1, \ldots, \hat{\boldsymbol{q}}_{N-1}) + V(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1})\,. \tag{15.2}$$

This form of $\widehat{h}$ shows explicitly that $\boldsymbol{y}$ and $\boldsymbol{q}$ are dynamical variables, similarly as in NR mechanics.

The 3-symmetry of relational space R_3 makes that the V interaction describes action–at–a–distance in $R_3 \otimes \ldots \otimes R_3 \otimes \tau_M$, which results in the third Newtonian law, while the L-absolute limits

$$r_{jk} = |\boldsymbol{y}_{jk}| \to \infty \tag{15.3}$$

determine the corresponding asymptotic zone of $\mathcal{M}$ without resorting to the adiabatic hypothesis, *i.e.* treating scattering states and bound states in the same way. Let us remark that, similarly as in NR mechanics, action–at–a–distance does not introduce any additional degrees of freedom into $\mathcal{M}$. This solves the problem of stability of bound structures of $\mathcal{M}$ in physics of finite universal constant $\hbar/c$.

The L-absolute internal kinetic energy operator $W^{(k)}$ must be consistent with the assumed relativistic kinematics of the asymptotic zone of measurement. Therefore, the analytic form of $W^{(k)}$ must coincide with that of the relativistic kinetic energy $E^{(k)}$, represented in the zero-momentum reference frame in which $P^* = 0$. However, contrary to the conclusions which result from the wrong semi-relativistic equations [31–33] based on eventism L_4, the analytic coincidence of $W^{(k)}$ with $E^{(k)*}$ occurs on two different geometrical backgrounds of these two quantities. $W^{(k)}$ is an L-absolute object, embedded in $\overbrace{R_3 \otimes \ldots \otimes R_3}^{N-1}$, whereas $E^{(k)*}$ is embedded in L_4 and, moreover, it must resort to the c-number condition $\boldsymbol{P}^* = 0$. The same concerns the *a priori* L-absolute interaction $V(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{N-1})$ which becomes independent of the reference frame S parametrizing external spacetime L_4 of free motion of $\mathcal{M}$ as a whole.

Finally, in one of different possible parametrizations of the configuration space $\overbrace{R_3 \otimes \ldots \otimes R_3}^{N-1}$, $\widehat{W}^{(k)}$ takes the form

$$\begin{aligned}\widehat{W}^{(k)} =& c\Big[(m_1^2c^2 + \widehat{\boldsymbol{q}}_1^2)^{1/2} + \ldots + (m_{N-1}^2c^2 + \widehat{\boldsymbol{q}}_{N-1}^2)^{1/2} \\ &+ (m_N^2c^2 + (-\widehat{\boldsymbol{q}}_1 - \ldots - \widehat{\boldsymbol{q}}_{N-1})^2)^{1/2}\Big], \end{aligned} \tag{15.4}$$

where

$$\widehat{\boldsymbol{q}}_N \equiv -(\widehat{\boldsymbol{q}}_1 + \widehat{\boldsymbol{q}}_2 + \ldots + \widehat{\boldsymbol{q}}_{N-1}) . \tag{15.5}$$

The point is that the sum of all relational momenta $\widehat{\boldsymbol{q}}_j$ referred to the constituent A_1 vanishes identically being a q-number

$$\widehat{\boldsymbol{q}}_1 + \widehat{\boldsymbol{q}}_2 + \ldots + \widehat{\boldsymbol{q}}_{N-1} + \widehat{\boldsymbol{q}}_N \equiv 0 . \tag{15.6}$$

The hierarchy of description of the state of $\mathcal{M}$, which follows from relationism R_3, makes that $\mathcal{M}$ (viewed as a whole) may be, in L_4, in an external state superposed of different momenta $\boldsymbol{P}$ when the rest-frame S^* does not exist but, nevertheless, this does not affect identity (15.6) embedded in R_3. The L-absolute internal Hamiltonian $\widehat{h}$ acts as a generator of infinitesimal translation of internal time τ_M of $\mathcal{M}$ which leads to the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial \tau_M} = \widehat{h}\,\psi(y_1, \ldots, y_{N-1}; \tau_M) \,. \tag{15.7}$$

Note, that according to (15.6), the numerical coincidence of $\Delta\tau_M$ with the time interval Δt^* of the rest frame S^* of $\mathcal{M}$ (if such a frame does exist) takes place *a posteriori.*

The internal time τ_M (being a c-number parameter) is essentially different from the relational degrees of freedom y_j. In particular, as seen from (15.7), τ_M has the 1-parameter translation symmetry

$$\tau'_M = \tau_M + \tau_{0M} \,. \tag{15.8}$$

If, omitting the subscript M, we confine ourselves to the elementary two-body system, then the internal spacetime of $\mathcal{M} = A_1 + A_2$, *i.e.* spacetime I_4, composed of $R_3(y)$ and $\mathcal{T}(\tau)$, has the 3-parameter rotation symmetry of R_3 and the 1-parameter translation symmetry (15.8). In consequence, I_4 has a 4-parameter symmetry R, the same as R_3^G has, and — much like as in the NR limit — we have

$$I_4 = R_3(y) \otimes \mathcal{T}(\tau) \,. \tag{15.9}$$

For stationary states when

$$\psi = \psi_W(y_1, \ldots, y_{N-1}) \exp\left(-\frac{i}{\hbar} W \tau_M\right) \tag{15.10}$$

equation (15.7) leads to the eigenproblem of $\widehat{h}$

$$\widehat{h}\psi_W = W\psi_W(y_1, \ldots, y_{N-1}) \,, \tag{15.11}$$

where $W = Mc^2$ denotes the *a priori* L-absolute total internal energy W (and mass M) of $\mathcal{M}$ in the eigenstate ψ_W of $\widehat{h}$. Thus, different bound eigenstates of $\widehat{h}$ determine different composite particles $\mathcal{M}_n$, because they have different invariant masses M_n.

In spite of the fact that a far-going analogy exists between equation (15.11) and its NR limit (in which $I_4 \to I_4^G$), let us strongly emphasize that the NR framework ($1/c = 0$) is unable to account for the (perfectly well known and proven) mass defect of composite structures. Indeed, if m_J is the mass of J-th constituent A_J of $\mathcal{M}$, then

$$M_n \underset{c \to \infty}{\longrightarrow} m = \sum_{J=1}^{N} m_J \tag{15.12}$$

which means that all particles $\mathcal{M}_n$ synthetised *mechanically* are of the same mass $M_n = m$ determined by the masses m_J. Thus, the fundamental fact known from nuclear physics, chemistry, *etc.*, *i.e.* existence of a spectrum of masses M_n for each of composite particles $\mathcal{M}_n$ (built of the same constituents and interacting via the same forces), cannot be explained on the ground of the NR theory. As the binding energies ϵ_n of loosely bound systems are determined by the NR Schrödinger equation, we obtain the corresponding mass defects dividing ϵ_n by c^2 — $\Delta M_n = \epsilon_n/c^2$, as it follows from the relativistic energy–mass relation. This very fact shows clearly that NR Schrödinger equation must be recognized as the NR approximation of (15.11) accounting for finite universal constant $\hbar/c$. However, according to relationism one has to resort to the hypothesis of the R_3-space which explicitly goes beyond eventism L_4.

16. Some consequences of relationism R_3

Direct unobservability of the points $(y; \tau)$ of I_4 makes that the I_4 geometry has to deal with two L-absolute intervals

$$r = |y| \quad \text{and} \quad \Delta\tau \,. \tag{16.1}$$

Thus, in opposition to eventism L_4 with negative balance of geometry L_4, the metrical physics based on primordial nature of relations admits a *closed* theory which would reproduce the dynamical structure of measuring rods. According to relationism I_4, the true spacetime L_4 of measurement is not a manifestation of eventism but it represents the limiting case of geometry I_4 conditioned by physical situation created by *classical*, heavy measuring devices. In consequence, mathematical reference frames S parametrizing L_4 cannot abstract from reality of (*infinitely* heavy) reference bodies $\bar{S}$.

The hierarchic description of the state of $\mathcal{M}$, which is inherent in the *two-level* relational physics, starts — as we know it — with determination of the structure of composite particle $\mathcal{M}_n$ and of its all internal L-absolute characteristics like mass M_n and spin embedded in the configuration space $\overbrace{R_3 \otimes \ldots \otimes R_3}^{N-1}$. In the next step only we attach the corresponding L_4 geometry to these c-number characteristics. In particular, we attach to $\mathcal{M}_n$ (as a whole) the overall four-coordinate X_n and the conjugate four-momentum P_n, where

$$P_n^2 = -M_n^2 c^2 \,. \tag{16.2}$$

Suppose that $\psi_{m,n}(y)$ are the bound states, normalized to unity, of the Hermitian operator of internal Hamiltonian $\widehat{h}$ of the elementary two-body system $\mathcal{M} = A_1 + A_2$. Actually, these states describe two different particles $\mathcal{M}_m$ and $\mathcal{M}_n$, as we assume that $M_m \neq M_n$. Thus

$$F_{nm}(\boldsymbol{y}) = \psi_n^*(\boldsymbol{y})\psi_m(\boldsymbol{y}) \tag{16.3}$$

represents a relational shape of the form factor between the internal states ψ_m and ψ_n. Since $W_m \neq W_n$ ($M_{m,n} = W_{m,n}/c^2$), the orthonormality of states $\psi_{mn}(\boldsymbol{y})$ takes the form

$$\int d^3y\, F_{mn}(\boldsymbol{y}) = \delta_{mn}\,. \tag{16.4}$$

As the collisions of $\mathcal{M}$ with another object $\mathcal{M}'$ disclose F in the structure of the corresponding cross-sections (in the privileged p language), we must deal with *the same* relational property F represented in 4-space $\mathcal{L}_4$. Hence we get

$$G_{nm}(x) \doteq F_{nm}(\boldsymbol{y})\,. \tag{16.5}$$

Similarly as in (13.6), $G_{nm}(x)$ leads to the identity

$$\int_{-\infty}^{+\infty} dx_0\, G_{nm}(\boldsymbol{x}, x_0) \equiv F_{nm}(\boldsymbol{x})\,, \tag{16.6}$$

hence the orthonormality condition (16.4) takes a manifestly L-invariant form

$$\int d^4x\, G_{nm}(x) = \delta_{nm}\,. \tag{16.7}$$

Let us remember that the relativistic wave functions deduced on the basis of eventism result, due to the non-separability of internal and external degrees of freedom of $\mathcal{M}$, in relativistic distortions of $G^L_{nm}(x)$ which destroy the orthogonality relation of the initial *proper* wave functions of $\mathcal{M}$.

The separability of internal and external degrees of freedom of $\mathcal{M}$ which follows from relationism solves the dilemma connected with physical interpretation of the proton form factor discussed in Section 10. Let us remember that for elastic scattering the four-momentum transfers p take, in the zero-momentum reference frame S^* of the colliding particles, the form: $p^* = (\boldsymbol{p}^*; 0)$, hence $p_0^* = 0$ and $\tilde{t} = p^2 = \boldsymbol{p}^{*2}$. However, as follows from identities (13.5) and (13.6), the charge density distribution of proton defined in (10.2) coincides with the L-absolute relational shape $\rho(r)$ with $r^* \to r = |\boldsymbol{y}|$. Thus, without distinguishing any reference frame in L_4, $\rho(r)$ represents the L-absolute relational structure of proton (composed of some constituents) which remains hidden *a priori* in R_3. Elastic collisions of proton and point-like particles make this structure to appear on *the surface of measurement* and hence, *the same* form factor represented in $\mathcal{L}_4$ takes the form

$$eG(x^2) \doteq \rho(r) \qquad (r = |y|)\,. \tag{16.8}$$

From the *a posteriori* equality (16.2) follows that the total energy E_n of an isolated particle $\mathcal{M}_n$ being in the eigenstate of total momentum P is equal to

$$E_n = (W_n^2 + c^2 P^2)^{1/2}, \qquad W_n = M_n c^2\,. \tag{16.9}$$

Consequently, one can speak of the rest frame S^* of $\mathcal{M}_n$ in which

$$\begin{aligned} E_n^* &= W_n\,, &\qquad (i)\\ \Delta t^* &= \Delta\tau\,. &\qquad (ii) \end{aligned} \tag{16.10}$$

Note that these c-number equalities do not distinguish S^* in a way which would conflict with Einstein's principle of relativity as they are inherent in the relativistic kinematics.

However, taking into account quantum superposition of states, the determination of the mass M_n as an L-invariant quantity equal to $M_n = (-P_n^2/c^2)^{1/2}$ is more general than that of the rest mass equal to $M_n = E_n^*/c^2$, because in the first case $\mathcal{M}$ can be in a state superposed of different momenta $\boldsymbol{P}$ for which no rest frame S^* exists. This distinction becomes relevant in determining the mean life-times of unstable particles $\mathcal{M}_n$ from the corresponding energy uncertainties — *cf.* Section 24.

Another consequence of relationism R_3 concerns the already mentioned problem of direct interaction $V(y^2)$ (relational shape) at–a–distance in I_4 which obeys the third Newtonian principle. Thus, an interaction–at–a–distance in I_4 avoids the wave zone of signalization which is intimately connected with the locality of L_4 geometry and which destroys the stability of relativistic composite systems. The wave zone of relativistic fields introduces its own degrees of freedom and the best-known example of the dilemma of stability of composite systems interacting by means of local relativistic fields is the Bremsstrahlung of classical atoms. On the other hand, *the two-level* physics based on relationism remains consistent with the L symmetry of measurement, because *the same* relational property V expressed in $\mathcal{L}_4$ takes a manifestly L-form invariant two-event shape U. The corresponding equalities with dot take the form (in L_4 and G_4)

$$\begin{aligned} V(y^2) &\doteq U(x^2)\,, &\qquad (\mathrm{L})\\ V(y^2) &\doteq V(x^2)\delta^{(1)}(\Delta t)\,. &\qquad (\mathrm{G}) \end{aligned} \tag{16.11}$$

It must be remembered that $U(x^2)$ (much like $G(x^2)$) although being consistent with symmetry L goes beyond the locality of eventism L_4. Therefore interaction $U(x^2)$ results in quantum correlations of finite space-like four-intervals x^2 ($x^2 > 0$) conflicting with Einstein's *classical reality* [9]. In particular, Feynman propagators of relativistic perturbation theory are examples of such interaction $U(x^2)$. In agreement with general discussion from Section 9, this proves that perturbation theory goes implicitly beyond the eventism L_4 with its locality.

Finally, let us consider the relational velocity $\boldsymbol{v}$ in I_4 which points to the discontinuity of the Lorentz limit of I_4 discussed in greater detail in the next section. In the elementary two-body systems with internal Hamiltonian as from (15.2) for $N = 2$, relational velocity is defined in a standard manner as equal to

$$\boldsymbol{v} = \frac{d\boldsymbol{y}}{d\tau} = \frac{\partial \widehat{h}}{\partial \boldsymbol{q}}. \tag{16.12}$$

The point is that $\boldsymbol{v}$ takes an account of the mutual (relational) motion of both constituents A_1 and A_2 of $\mathcal{M}$ and determines the relation $\boldsymbol{y}$ as, in accordance with (15.2) and (16.12), we obtain

$$\boldsymbol{v} = \boldsymbol{q}\left[\left(m_1^2 + \frac{\boldsymbol{q}^2}{c^2}\right)^{-1/2} + \left(m_2^2 + \frac{\boldsymbol{q}^2}{c^2}\right)^{-1/2}\right]. \tag{16.13}$$

In consequence, if $|\boldsymbol{q}| \to \infty$ velocity $v = |\boldsymbol{v}|$ tends to $2c$ instead of c. Relationism I_4 accounts automatically for *the recoil* of both interacting constituents of $\mathcal{M} = A_1 + A_2$.

As one might expect, if we assume that in the limit of an infinitely heavy A_1 ($m_1 \to \infty$)

$$\lim_{m_1 \to \infty} \frac{\boldsymbol{q}^2}{m_1^2 c^2} = 0, \tag{16.14}$$

which means that A_1 suffers no recoil (like Bohr's *basis* of measuring apparatus [47]), then $\boldsymbol{v}$ from (16.13) converts into

$$\boldsymbol{v} = \frac{\boldsymbol{q}}{\sqrt{m_2^2 + \boldsymbol{q}^2/c^2}} \xrightarrow[|\boldsymbol{q}| \to \infty]{} c. \tag{16.15}$$

Thus, in the limit (16.14) the maximal velocity v falls from $2c$ to c, where the latter value represents the maximal velocity of a signal referred to infinitely heavy reference bodies $\bar{S}$ of geometry L_4. This justifies to regard (16.14) as the condition for *the Lorentz limit of I_4*.

The Lorentz limit of I_4 is then characterized by the discontinuity between $v^{(I)}_{\max} = 2c$ and $v^{(L)}_{\max} = c$ — such a discontinuity is alien to symmetry G and points to different natures of relationism and eventism, resulting from the coexistence of I^G_4 and G_4. This coexistence reveals itself here in the equally infinite values of $2c$ and c of the NR limit ($c \to \infty$) which makes that the discontinuity disappears.

17. Lorentz limit of I_4

The double-faced nature of symmetry L of measurement reveals itself in the following: On the one hand the symmetry L of measurement (asymptotic zone) determines *the absoluteness* of relations y but, on the other hand — if one insists on having a self-consistent hypothesis — the internal space–and–time I_4 must convert into L_4 under some conditions which, in particular, accompany any measuring process. The hypothesis of relationism promotes this double-faced character of symmetry L by introducing 3-space R_3 different from 3-space E_3 of any reference frame S parametrizing spacetime L_4 of measurement.

The double role of classical physics of measurement in quantum theory was strongly emphasized by Landau [48] without, however, renouncing eventism. Actualizations that constitute any measurement are in reality induced by *classical* macro-devices [7] which provide us with *physical text* [13] of quantum predictions.

Let us not forget that symmetry G which amalgamates the 3-space with time, so the G_4 spacetime ceases to be a Cartesian product of 3-space and time of Aristotelian physics, is a consequence of the symmetry of the Newtonian equations of motion. Similarly, symmetry L of Minkowski's spacetime results from the symmetry of the Maxwell equations. Thus, the symmetry of the first physical background is always strictly connected with the symmetry of the basic equations of motion which form the basis for quantitative physics. In the case of I_4 continuum its symmetry R is also an internal symmetry of equation (15.7). So, in the case when the symmetry R of equation (15.7) changes — in a consequence of some particular situation which might build up in I_4 — into a "broader" symmetry L, one may say that relationism I_4 transforms, in an appropriate configuration space, into relationism of L_4 spacetime.

It follows from the preceding considerations that, according to relationism I_4, the existence of a metric *outer world* which constitutes the spacetime of measurement calls for one of the constituents of the micro-world $\mathcal{M} = A_1 + A_2$ to be infinitely heavy (*cf.* (16.14)). Let us emphasize that the limit $m_1 \to \infty$ differs essentially from the limits $\hbar \to 0$ and $c \to \infty$

which are responsible for the old paradigms. Those two limits are of a formal character as both $\hbar$ and c are universal constants. On the other hand, the limit (16.14) depicts a situation of $\mathcal{M} = A_1 + A_2$ which may, or may not, occur, as m_1 is not a universal constant, and it may take on different possible values.

In a region without interaction ($V(\boldsymbol{y}^2) = 0$), the condition (16.14) may be always fulfilled by referring a non–interacting A_2 particle to an A_1 of infinite inertia. The absolute character of R_3 space makes that the asymptotic region where $V(\boldsymbol{y}^2) = 0$ is specified also in an absolute manner. Any modifications which might follow from the dynamics of relationism I_4 can only concern the description of bound states (structures) of finite inertia.

Let us consider a two-body Hamiltonian $\hat{h}$ of the two-body system $\mathcal{M} = A_1 + A_2$ in the zero-momentum reference frame S^*. (The limit (16.14) means that the infinitely heavy A_1 suffers no recoil and becomes a *good* reference body $\bar{S}$.) In order to deal with finite quantities we subtract the term m_1c^2 from Hamiltonian $\hat{h}$ and then take the limit for $m_1 \to \infty$. Taking into account (16.14), we obtain

$$
\begin{aligned}
\widehat{H} &= \lim_{m_1 \to \infty} \left[\hat{h} - m_1 c^2\right] \\
&= \lim_{m_1 \to \infty} c\left[(m_1^2c^2 + \hat{\boldsymbol{q}}^2)^{1/2} + (m_2^2c^2 + \hat{\boldsymbol{q}}^2)^{1/2}\right] + V(\boldsymbol{y}^2) - m_1c^2 \\
&= c\left(m_2^2c^2 + \hat{\boldsymbol{q}}^2\right)^{1/2} + V(\boldsymbol{y}^2)\,.
\end{aligned}
\tag{17.1}
$$

For our purposes it is sufficient to consider the simplest scalar equation which may be obtained by replacing $\hat{h}$ in (15.7) with $\widehat{H}$ from (17.1) and squaring both terms. Equation (15.7) transforms into

$$
\left\{\left[i\hbar\frac{\partial}{\partial\tau} - V(\boldsymbol{y}^2)\right]^2 - m_2^2c^4 - c^2\hat{\boldsymbol{q}}^2\right\}\psi(\boldsymbol{y},\tau) = 0\,. \tag{17.2}
$$

Introducing new variables

$$
\boldsymbol{y} = \boldsymbol{X}_2^*\,, \quad c\tau = X_{20}^*\,, \quad X_2^* = (\boldsymbol{X}_2^*; X_{20}^*)\,, \tag{17.3}
$$

and defining four new functions $U^*(X_2^*)$ of the form

$$
U^*(X_2^*) = \left(0, 0, 0, ; \frac{V(\boldsymbol{y} = \boldsymbol{X}_2^*)}{c}\right), \tag{17.4}
$$

equation (17.2) can be rewritten in an explicitly L covariant form

$$\left\{\left[-i\hbar\frac{\partial}{\partial X_2}-U(X_2)\right]^2+m_2^2c^2\right\}\psi(X_2)=0\,, \tag{17.5}$$

with X_2^* denoting the four-coordinate of X_2 event in the zero-momentum reference frame S^* of the infinitely heavy A_1, and U^* representing four-potential U in S^*. Indeed, equation (17.5) viewed in S^* takes on the very form of equation (17.2).

An infinitely heavy A_1 does nor enter equation (17.5), so it remains *hidden*, but the 4-parameter symmetry R of the points $(\boldsymbol{y}; c\tau)$ of I_4 gains a 3-parameter freedom of translation in R_3 and another, also 3-parameter, freedom of *boost*. In consequence, the $(4+6=10)$-parameter symmetry L of $(\boldsymbol{y}; c\tau)$ makes I_4 isomorphic with L_4. At the same time, the L-absolute relational shape $V(\boldsymbol{y}^2)$ becomes isomorphic with the event shape $U(X)$ embedded in L_4 which — as can be seen from (17.5) — plays the role of an external dynamical field. In spite of that, the L covariant equation (17.5) is not L-form invariant or, in other words, symmetry L is not its internal symmetry group. The same happens to the Maxwell equations with external event shape of current density $j(X)$ as well, as to the Dirac equation with external electromagnetic field $U(X)$ *etc.*, as no event shape (except from $U(X)\equiv$ constant in the whole L) is L-form invariant. Nevertheless, equation (17.5) remains consistent with STR which requires the passive interpretation of symmetry only and this is guaranteed by the L-covariance of (17.5).

Equation (17.5) becomes L-form invariant when $U(X)=0$ which stays in agreement with the hypothesis of R_3 based on relativistic kinematics of the asymptotic zone of measurement. Thus, the relativistic kinematics can also be regarded as a limiting case of the two-body problem elementary in the I_4 relationism. In this case, however, the infinitely heavy term A_1 of initial relation $\boldsymbol{y}$ is *hidden* in the asymptotic zone. This would complete the relational origin of the spacetime of measurement which ceases to reflect the eventism, *i.e.* the pre-existing background of metrical spacetime, but follows the relationism of I_4 continuum.

The absence of an infinitely heavy A_1 in (17.5) results in the notion of an external world of A_2 characterized by its metric and symmetry L which are essential for the theory of measurement. The point is that symmetry L creates a dichotomy of all physical characteristics of the measured object $\mathcal{M}$ dividing them into L-absolute and L-relative ones. Such a dichotomy is induced by (infinitely) heavy measuring devices imposing automatically the Lorentz limit of I_4. As a consequence — which is crucial for measurement itself — we obtain possibility of deciding which observable represents an internal absolute property of $\mathcal{M}$ factored out from physics of measuring tools. For example, the measurement of the L-relative momentum $\boldsymbol{P}$ and energy $E = cP_0$ of a free $\mathcal{M}$ enables one to determine its L-absolute mass $M = (P^2/c^2)^{1/2}$.

On the quantum-potential level of reality, micro-worlds $\mathcal{M}$ are *a priori hidden* from measurement (observation) and, hence subject to symmetry R of I_4 much weaker than symmetry L of L_4. This enlarges the class of dynamical models of $\mathcal{M}$ as compared with the case of the extremely restrictive symmetry L of eventism L_4 [16–19]. As can be seen from (16.11), the L-absolute interaction $V(\boldsymbol{y}^2)$ in I_4 is consistent with symmetry L, while in the Lorentz limit of I_4 it transforms into the event shape $U(X)$ of (17.4). Note that $V(\boldsymbol{y}^2)$ and $U(X)$ are described by functions, whereas $U(x^2)(\doteq V(\boldsymbol{y}^2))$ represents a distribution, *i.e.* a two-event shape in $\mathcal{L}_4(x)$. This is why equation (17.5) is a *good* L covariant equation of the one-body problem in L_4, while the two-body equations of motion encounter, in the same limit $m_1 \to \infty$, fundamental difficulties [49].

Indeed, according to eventism, a full isolation of $\mathcal{M} = A_1 + A_2$ requires the equations of motion of $\mathcal{M}$ to be L-form invariant, because otherwise they would distinguish between different reference frames S. Therefore these equations can only accept an interaction given by L-form invariant two-event shapes $U(x^2)$ of distributions. The Bethe–Salpeter equation, for instance, illustrates the mentioned class of two-body equations of fully isolated $\mathcal{M} = A_1 + A_2$. It deals with the L-form invariant interaction $U(x^2)$ and, at the same time, suffers from relativistic redundancy of degrees of freedom [30] which, in the two-body problem, concerns the relative time variable $x_0 = c\Delta t$. We see then that, if one starts with the L_4 eventism, the limit $m_1 \to \infty$ cannot eliminate the redundant x_0 degree of freedom which enters $U(x^2)$. Consequently, we cannot regain a *good*, L covariant one-body equation like that from (17.5) obtained on the basis of relationism I_4 and free of relativistic redundancy of degrees of freedom of $\mathcal{M}$.

The identification of $\boldsymbol{y}$ with $\boldsymbol{X}^*$ (as in (17.3)) attaches automatically to an infinitely heavy A_1 the classical world–line which has been (arbitrarily) identified with the 0-t^*-axis of the rest frame S^* of A_1. Thus, in the Lorentz limit of one of the constituents of a composite $\mathcal{M}$, the rotation symmetry O^R of R_3 coincides with the rotation symmetry O^L of 3-space E_3^* of reference frame S^*.

The most puzzling consequence of the Lorentz limit of I_4 is that equation (17.5) remains L covariant even if $U(X)$ — unlike as in (17.4) — represents an almost arbitrary event shape. This implies that the Lorentz limit $m_1 \to \infty$ allows implicitly A_1 to become an extended object in L_4, deprived of the space–rotation symmetry in E_3^* as well, as of the time–translation invariance. The same concerns L covariant Maxwell equations with (almost arbitrary) event shape of charge–density current $j(X)$. Owing to that, we can shape the external fields like $U(X)$ or/and $j(X)$ and, in consequence, prepare the spacetime organization of the stages I and III of a micro-collision process. The very process occurs during the stage II during which micro-world $\mathcal{M} + \mathcal{M}'$ is inaccessible to any external action of *an observer*. Such an action would have to resort to external fields and boundary conditions in classical — hence divisible — spacetime L_4, opposed to indivisible and infinite (quantum) geometrical meta-objects R_3 and $\mathcal{L}_4$ in which the stage II of the collision process takes place on its quantum-potential level and being subject to the p–x duality. Note that the freedom of this "shaping" of the external event shapes $U(X)$ or/and $j(X)$ of the stages I and III is strictly connected with the Lorentz limit of I_4 resulting in the success of relativistic dynamics of the one-body problem. In particular, the whole industry of *ultra-relativistic* accelerators proves the correctness of relativistic dynamics of the one-body problem.

The variety of event shapes that can accompany the Lorentz limit of I_4 is a consequence of atomism. Indeed, any increase of the mass m_1 of A_1 would be accompanied by an increase of A_1's extension and complexity, resulting in an increase of the number of A_1's degrees of freedom. Consequently, the majority of energy gaps which separate the stationary quantum states of A_1 must tend to zero and, as pointed out by Landau [50], the complete quantum description of A_1 becomes broken by an arbitrarily small perturbation of A_1. The state of isolation of A_1 becomes fictitious and A_1 acquires the property of *a classical* object (described incompletely [5]) and capable of getting information (actualizations) about reality external with respect to A_1. As a matter fact, this coexistence of physical reality on its two levels: the quantum-potential one and the classical–actual one, is responsible for our physics.

In Appendix B we analyze the Lorentz limit of a 3-body system where two independent Lorentz limits are possible, as two independent relational coordinates y_{12} and y_{13} parametrize the internal configuration space $R_3(y_{12}) \otimes R_3(y_{13})$ of $\mathcal{M} = A_1 + A_2 + A_3$. The Lorentz limit can concern independently the subspaces $R_3(y_{12})$ and $R_3(y_{13})$. This clearly exhibits the fact that our spacetime L_4 of measurement is intimately connected with one-body problem of the Lorentz limit of I_4.

18. Bound and scattering states

In Section 6 a fundamental difference was shown which exists between bound and scattering states of composite systems and which is strictly connected with the 4-symmetry L alien to symmetry G. The point is that for the bound states of $\mathcal{M} = A_1 + A_2$, the relative four-momentum p has 4 degrees of freedom, whereas for the scattering ones, when each of the $\mathcal{M}$ constituents reaches the asymptotic zone, the constraint (6.11) ($Pp = 0$) restricts p to a 3-parameter freedom ($p_0^* = 0$). In consequence, bound structures of $\mathcal{M}$ can be described by L-form invariant form factors $\tilde{G}(p^2)$ (or, interchangeably, $G(x^2)$) entirely separated from the external states of their carriers $\mathcal{M}$. As we know, this fact is crucial for the hypothesis of relational space R_3. Now, after introducing R_3, it is necessary to look at the difference between bound and scattering states from the point of view of relationism.

Let us begin with bound states of $\mathcal{M} = A_1 + A_2$ in R_3, assuming that interaction $V(\boldsymbol{y}^2)$ between A_1 and A_2 tends to zero when $r = |\boldsymbol{y}| \to \infty$

$$V(\boldsymbol{y}^2) \underset{r\to\infty}{\longrightarrow} 0\,, \quad r = |\boldsymbol{y}|\,. \tag{18.1}$$

Unlike in NR mechanics, the energy–mass relation makes the normalization of V an absolute one hence, in the asymptotic zone given by (18.1), the L-absolute mass M_n of a bound state of $\mathcal{M}$, with $M_n < m = m_1 + m_2$, can be written in the form

$$M_n = \left(m_1^2 - \frac{k_n^2}{c^2}\right)^{1/2} + \left(m_2^2 - \frac{k_n^2}{c^2}\right)^{1/2} < m = m_1 + m_2\,. \tag{18.2}$$

Thus, a bound state $\psi_n(\boldsymbol{y})$ takes, in the asymptotic zone, the form

$$\psi_n^{(as)}(\boldsymbol{y}) = N_n(\theta,\phi)\exp\left(-\frac{k_n r}{\hbar}\right)\,, \tag{18.3}$$

where the angles θ and ϕ determine some internal direction in $R_3(\boldsymbol{y})$. Thus, if $\mathcal{M}_n$ is to be a stable particle with a real mass M_n, physics of finite $\hbar/c$ imposes an upper limit onto k_n^2 and, at the same time, a lower limit onto M_n

hence,
$$\left.\begin{aligned} &0 < k_n^2/c^2 \le \min(m_1^2, m_2^2) \\ &|m_1^2 - m_2^2|^{1/2} \le M_n < m = m_1 + m_2\,. \end{aligned}\right\} \tag{18.4}$$

In consequence, a massless particle A_2 ($m_2 = 0$) cannot be bound mechanically to A_1.

By virtue of (18.2), relations $(M_{1,2})_n = (m_{1,2} - k_n^2 c^2)^{1/2}$ may be recognized as *the effective masses* of $A_{1,2}$ in the bound state ψ and, therefore, we assume that the undefined *weight* a from (6.5) becomes dependent —

much like in the case of scattering states (6.14) — of the mass of $\mathcal{M}$ and, hence equal to

$$a = a_n = \frac{M_{1n}}{M_n} = \tfrac{1}{2}\left[1 + \frac{m_1^2 - m_2^2}{M_n^2}\right] = a(M_n)\,. \tag{18.5}$$

Similarly as in (6.14), a_n tends — in the NR limit — to the universal NR weight a^G, as

$$a_n = a(M_n) \underset{c\to\infty}{\longrightarrow} a(m) = \frac{m_1}{m} = a^G. \tag{18.6}$$

By virtue of (6.5) the overall four-coordinate X of $\mathcal{M}_n$, when expressed by four-coordinates $X_{1,2}$ of the constituents $A_{1,2}$, becomes dependent of the state of $\mathcal{M}$ via the weight a_n, as

$$X_n = a_n X_1 + (1 - a_n) X_2\,, \quad P_n^2 = -M_n^2 c^2\,. \tag{18.7}$$

Note that the 4-parameter freedom of Fermi four-momenta p, which parameterize bound states ψ_n in their p-representations embedded in $\mathcal{L}_4(p)$, leaves the four-momenta $(P_{1,2})_n$ of the $A_{1,2}$ constituents undetermined. Indeed, the relations

$$(P_1)_n = a_n P_n - p\,, \quad (P_2)_n = (1 - a_n) P_n - p \tag{18.8}$$

with the 4-freedom of p make that the lengths of $(P_{1,2})_n$ are undetermined, even if p^2 is fixed and equal to $\boldsymbol{q}^2$ ($p^2 = \boldsymbol{q}^2$), as we get

$$\left.\begin{aligned}(P_1^2)_n &= -a_n M_n^2 c^2 + \boldsymbol{q}^2 + 2a_n M_n c p_0^*\,,\\ (P_2^2)_n &= -(1 - a_n)^2 M_n^2 c^2 + \boldsymbol{q}^2 - 2(1 - a_n) M_n c p_0^*\,,\end{aligned}\right\} \tag{18.9}$$

where S^* is the rest frame of $\mathcal{M}_n$. The time component of space-like Fermi momentum p (represented in S^*) plays the role of an arbitrary parameter — it is a consequence of the 4-parameter freedom of p. Note that with p^2 being a fixed value

$$p^2 = \boldsymbol{p}^2 - p_0^2 = \boldsymbol{q}^2 \geq 0 \tag{18.10}$$

the determination of the space components ($\boldsymbol{p}$) of p can be done also up to an arbitrary parameter p_0^*. The quantities which are well-defined in R_3 are the Fermi 3-momenta $\boldsymbol{q}$.

In scattering states of $\mathcal{M}$, the constraint $Pp = 0$ results in $p^* = 0$ which restricts p's to a 3-parameter freedom. Moreover, if we take for the scattering states the weight $a(M)$ from (6.14), the equalities (18.9) transform into two constraints (6.2)

$$P_{1,2}^2 = -m_{1,2}^2 c^2\,, \tag{18.11}$$

as it could be expected. It is remarkable that the indetermination of p_0^* and hence, of $(P_{1,2}^2)_n$ from (18.9), concerns arbitrarily loosely-bound NR states. This shows, once again, the difference between the NR approximation ($1/c \neq 0$) and the NR framework ($1/c = 0$). In the NR framework the very notion of four-momentum disappears and, in the limit $c \to \infty$, equations (18.9) and (18.11) lead to

$$\frac{P_{1,2}^2}{c^2} \underset{c \to \infty}{\longrightarrow} - m_{1,2}^2 \,. \tag{18.12}$$

Let us remember that in the NR limit the p_0 component of a space-like p vanishes identically (*cf.* (8.3)) and $\boldsymbol{p}^2 = \boldsymbol{q}^2$ in agreement with the NR meaning of Fermi momentum of $\mathcal{M}$.

Besides the mathematical limit of the NR framework ($c \to \infty$), the dependence of the weight a_n on the internal state of $\mathcal{M}$ vanishes also in the realistic Lorentz limit ($m_1 \to \infty$) when

$$\frac{M_n}{m_1} \underset{m_1 \to \infty}{\longrightarrow} 1 \quad \text{hence,} \quad \lim_{m_1 \to \infty} a_n = 1 \,. \tag{18.13}$$

The Lorentz limit of I_4 is realized automatically under this form when, for instance, the one-body Dirac equation of electron in the external field of nucleus is regarded as the model of hydrogen-like atoms [51]. Indeed, this makes the Coulomb interaction $V(\boldsymbol{y}^2) = -Ze^2/r$; ($r = |\boldsymbol{y}|$ in $R_3(\boldsymbol{y})$) to become identified with an event shape of external field generated by an infinitely heavy nucleus, as stated in (17.4). From the point of view of the NR framework, in which R_3^G coexists with G_4, this means that the reduced mass μ of electron and nucleus is replaced by the electron mass m_e. Thus, the NR corrections due to $\mu < m_e$ are extremely significant, so the Lorentz limit of the relativistic one-body approximation is quite unjustified.

Let us consider now a scattering state of $\mathcal{M} = A_1 + A_2$. In the asymptotic zone of R_3, where $V = 0$, internal states of $\mathcal{M}$ can be taken in the form of plane waves which are the eigenstates of $\widehat{\boldsymbol{q}}$ belonging to some arbitrary eigenvalue $\overset{o}{\boldsymbol{q}}$. Thus

$$\psi(\boldsymbol{y};\tau) = A \exp\left(\frac{i}{\hbar}(\overset{o}{\boldsymbol{q}}\boldsymbol{y} - \overset{o}{W}\tau)\right), \tag{18.14}$$
$$\overset{o}{W} = \overset{o}{M} c^2 = c\left[(m_1^2c^2 + \overset{o}{\boldsymbol{q}}{}^2)^{1/2} + (m_2^2c^2 + \overset{o}{\boldsymbol{q}}{}^2)^{1/2}\right],$$

and the plane-wave solution of asymptotic equation (15.7) introduces explicitly an internal direction $\overset{o}{\boldsymbol{q}}$ in R_3. As we still have $O^R \neq O^L$, it does not mean that $\overset{o}{\boldsymbol{q}}$ determines a space direction in some reference frame S in

L_4. Relational momentum $\overset{o}{q}$ becomes an extra building stuff for geometrical objects in R_3 which may undergo a translation into the corresponding $\mathcal{L}_4$-geometry objects by means of the equality with dot. In the case of the relational shape (18.14) this concerns the relational shape $F(y) = (\overset{o}{q}y)$ of the internal L-absolute phase ϕ of ψ.

Following the p–x duality of R_3 and $\mathcal{L}_4$ geometries, we attach to 3-momentum $\overset{o}{q}$ a space-like four-momentum $\overset{o}{p}$ of the same length as that of $\overset{o}{q}$ ($\overset{o}{p}{}^2 = \overset{o}{q}{}^2$) and *the same* L-absolute scalar property ϕ represented in R_3 and in $\mathcal{L}_4$ takes now the form

$$\hbar\phi = F(y) = (\overset{o}{q}y) \doteq (\overset{o}{p}\,x) = G(x)\,, \quad \overset{o}{p}{}^2 = \overset{o}{q}{}^2\,. \tag{18.15}$$

Let us emphasize that *the same* phase represented in R_3 and in $\mathcal{L}_4$ ($F(y) \doteq G(x)$) has the same dimensions in both cases, whereas *the same* form factors of bound structures $\mathcal{M}$ are of different dimensions, as $F(y)$ is given by a function, while $G(x) \doteq F(y)$ by a distribution.

Equality with dot (18.15) solves the problem of representation of internal state of a free $\mathcal{M} = A_1 + A_2$ in $\mathcal{L}_4$. There still remains an analogous problem with external state of $\mathcal{M}$ determined by the L-absolute external phase $\Phi = -\overset{o}{W}\tau/\hbar$ of ψ from (18.14) embedded initially in I_4. Note that the translation symmetry of internal time τ (*cf.* (15.8)) belonging to symmetry R of I_4, much like the translation symmetry of external time t belonging to symmetry L of L_4, determine Φ up to an unessential additive constant which can be neglected.

It is worth emphasizing that the translation of Φ into a manifestly L-invariant form reminds one of the way in which de Broglie has introduced the wave of matter Ψ, following the old, Hamilton's analogy between mechanics and optics. De Broglie has adopted Bohr's concept of stationary energy levels and has attached to the energy $\overset{o}{E}$ an (external) phase $\Phi = -\overset{o}{E}\,t/\hbar$. However, in order to satisfy symmetry L of STR, he has identified $\overset{o}{E}$ with the time component of four-momentum $\overset{o}{P} = (\overset{o}{\boldsymbol{P}}\,; \overset{o}{E}/c)$ of the system $\mathcal{M}$ (atom) treated as a whole, regarding $\mathcal{M}$ as a free particle. In consequence, de Broglie has replaced $\Phi = -\overset{o}{E}\,t/\hbar$ by a manifestly L-invariant phase $\Phi = (\overset{o}{P}\,X/\hbar)$ obtaining *the wave of matter* Ψ

$$\begin{aligned}\Psi &= A\exp\left(-\frac{i}{\hbar}(\overset{o}{W}\,\tau)\right) \doteq A\exp\left(\frac{i}{\hbar}(\overset{o}{P}\,X)\right)\\ &= A\exp\left(\frac{i}{\hbar}(\overset{o}{\boldsymbol{P}}\,\boldsymbol{X} - \overset{o}{E}\,t)\right)\end{aligned} \tag{18.16}$$

with

$$\overset{o}{E} = (\overset{o}{W}{}^2 + c^2 \overset{o}{P}{}^2)^{1/2}.$$

Let us not forget that the external phase Φ (in I_4 and L_4) admits, in contradistinction to the internal phase ϕ, an arbitrary additional constant connected with the inhomogeneous symmetry group L of events X and the translation symmetry of internal time in I_4 which belongs to symmetry R of I_4.

Although de Broglie took the phase $\Phi = -\overset{o}{E}t/\hbar$ from Bohr's concept of stationary energy levels of atoms, *i.e.* of many-body systems, his relativized phase $\Phi = \overset{o}{P}X/\hbar$ concerned atom $\mathcal{M}$ as a whole, *i.e.* as a one-body problem $\mathcal{M}$. In consequence, de Broglie's *wave of matter* ψ describes a one-body problem embedded in spacetime L_4, as if ψ were a classical wave embedded — by its very nature — in spacetime and not in configuration spaces of quantum mechanics. As it was always pointed out by Heisenberg [4], with the p–x duality of QM resulting in the wave–corpuscular duality of the whole configuration space of $\mathcal{M}$ composed of many constituents, the waves in configuration spaces have little in common with classical waves in spacetime. A one-body problem represents a singularity when its configuration (external) space coincides with space E_3 of some (arbitrary) reference frame S parametrizing L_4.

Finally, the equality with dot from (18.15) and that of de Broglie (18.16) complete the translation of the L-absolute state ψ from (18.14) embedded in I_4 into a manifestly L-invariant two-body state Ψ

$$\begin{aligned}\psi(\boldsymbol{y};\tau) &= A\exp\left(\frac{i}{\hbar}(\overset{o}{q}\boldsymbol{y} - \overset{o}{W}\tau)\right)\\ &\doteq A\exp\left(\frac{i}{\hbar}(\overset{o}{p}x + \overset{o}{P}X)\right) = \Psi\,.\end{aligned} \tag{18.17}$$

According to (6.5), which is equivalent to identity (6.4), Ψ from (18.17) can be rewritten in the form

$$\begin{aligned}\Psi &= A\exp\left(\frac{i}{\hbar}(\overset{o}{p}x + \overset{o}{P}X)\right)\\ &= A\exp\left(\frac{i}{\hbar}(\overset{o}{P}{}_1X_1 + \overset{o}{P}{}_2X_2)\right)\,.\end{aligned} \tag{18.18}$$

Of course, four-momenta $\overset{o}{P}{}_{1,2}$ and $\overset{o}{P}, \overset{o}{p}$ are subject to the constraints (6.2) and (6.11), (6.12), respectively, because the constituents $A_{1,2}$ of $\mathcal{M} = A_1 + A_2$ reach, each of them separately, the asymptotic zone of relativistic kinematics.

The space components of four-momenta $\overset{o}{P}_{1,2}$ as well, as of $\overset{o}{P}, \overset{o}{p}$ determine space directions in 3-space E_3 of each reference frame S in the L_4-spacetime *a posteriori*, *i.e.* after translating ψ embedded in I_4 into *the same state* Ψ in L_4 ($\psi \doteq \Psi$). The quantities $\overset{o}{q}$ and y determine, *a priori*, L-absolute internal directions in R_3 which have nothing in common with any space direction in E_3's — a fact which is consistent with $O^R \neq O^L$.

The equality with dot (18.17) shows that the hypothesis of relationism I_4 does not introduce any modification in the description of asymptotic scattering states of $\mathcal{M}$ which had been adequately described in L_4. This could have been expected, as each constituent $A_{1,2}$ reaches the asymptotic zone of relativistic kinematics. The I_4-effects become apparent when one starts to analyze the bound states and bound structures of $\mathcal{M}$. The adequacy of the L_4 eventism in describing scattering states is responsible for the fact that the S-matrix theory parametrized by Mandelstam variables remains consistent with the hypothesis of relationism. The essential point is that both the S-matrix theory and relationism R_3 take for granted the objectism of a real system $\mathcal{M}$ rather than the eventism of local field theory.

19. Two mechanisms of creation–annihilation of particles

There are two different mechanisms of creation-annihilation of particles which are both backed by a solid experimental evidence. The first consists in a mechanical synthesis of particles $\mathcal{M}_n$ composed of *more elementary* constituents and it is here that NR quantum mechanics may claim its greatest success. This success of the NR framework is, however, accompanied by the theory's fundamental imperfection, namely the theory does not explain the mass defect of bound particles $\mathcal{M}_n$ known best from chemistry and nuclear physics. In consequence, it is the hypothesis of relationism R_3 that may justify regarding the NR Schrödinger equation as an approximation of the L-absolute Schrödinger equation (15.7) accounting for finite $\hbar/c$ and hence, capable of explaining the mass defect of $\mathcal{M}_n$'s.

The second, *field mechanism* of creation–annihilation processes results from relativistic local field theory which made its appearance for the first time with the Maxwell equations in vacuum. In contradistinction to the NR framework, the propagation (in L_4) of a discontinuity of field, *i.e.* of a signal, is limited by the universal constant c. This leads to creation of a wave-zone of classical fields carrying finite amounts of energy and momentum represented by time-like four-momenta P. In quantum physics, this classical wave-zone of field converts into the field mechanism of creation–annihilation of the corresponding quanta (particles).

As far as the form factor structures of particles (quanta) of local (relativistic) field theory are concerned, let us not forget that these structures are given by the universal form factor $\delta^{(4)}(x)$ in $\mathcal{L}_4(x)$. Its p representation is equal to $\tilde{G}^{(4)}(p) = \text{const} = 1$ in the whole 4-space $\mathcal{L}_4$ admitting space-like and time-like four-momenta p. In consequence, point–particles of local fields admit the field mechanism of creation–annihilation of particles excluded by form factors $\tilde{G}(p)$ vanishing for time-like p which occurs — as a rule — in the NR framework ($1/c = 0$). Thus the field mechanism of creation–annihilation of particles is intimately connected with physics of finite c and of the energy–mass relation.

As far as we remain within mechanical systems based on relational 3-space R_3, the field mechanism of creation–annihilation of particles is similar to that in the NR framework, because the form factors $\tilde{F}(\boldsymbol{q})$ are obtained for $\boldsymbol{q}^2 \geq 0$ and, consequently (*cf.* (13.4)), they determine form factors $\tilde{G}(p^2)$ for space-like four-momenta p only ($p^2 = \boldsymbol{q}^2 \geq 0$). However, unlike as in the case of NR framework, *the sameness* of a relational property F in R_3 and in $\mathcal{L}_4$ given by $G \doteq F$ requires $\tilde{G}(p)$ to be extended over the time-like p's and, in general, the form factor $\tilde{G}(p)$ determined in this way in the whole 4-space $\mathcal{L}_4(p)$ does not vanish for time-like p. This fact, taken together with the phenomenological perturbation theory, enable one to regard both mechanisms of creation–annihilation of particles within the framework of one theory which, however, must resort to *the hierarchic description* of the state of a composite $\mathcal{M}$ undergoing the (quantum) collision.

Let us remark that the universal form factor $\delta^{(3)}(\boldsymbol{y})$ describes the point–particles in $R_3(\boldsymbol{y})$, hence its p-representation in $R_3(\boldsymbol{q})$ is given by $\tilde{F}^{(3)}(\boldsymbol{q}) = \text{constant} = 1$ in the whole space $R_3(\boldsymbol{q})$. Consequently, the analytic extension of $\tilde{F}^{(3)}(\boldsymbol{q})$ onto imaginary $\boldsymbol{q}$, with $\boldsymbol{q}^2 = p^2 < 0$, determining *the same* point–particle form factor in the whole 4-space $\mathcal{L}_4$ results in

$$\tilde{F}^{(3)}(\boldsymbol{q}) \doteq \tilde{G}^{(4)}(p) \quad \text{and} \quad \delta^{(3)}(\boldsymbol{y}) \doteq \delta^{(4)}(x)\,. \tag{19.1}$$

We regain thus the L-form invariant form factor $\delta^{(4)}(x)$ of point–particles of relativistic (local) field theory. The energy–mass relation makes that the mechanical synthesis of composite particles $\mathcal{M}_n$ in $R_3 \otimes R_3 \otimes \ldots$ results in a whole spectrum of different particles $\mathcal{M}_n$. Indeed, the same constituents and the same internal forces acting between them may create particles $\mathcal{M}_n$ of different masses M_n and/or of different spins. In the framework of field theory based on eventism L_4, we must attach to each such particle $\mathcal{M}_n$ a separate (local) field operator $\Phi_n(X)$ restricting $\mathcal{M}_n$'s to point particles [25].

As the limitation (by c) of the velocity of relativistic local fields propagation leads to creation of the wave zone with its degrees of freedom, the general problem arises of instability of any composite object $\mathcal{M}$. Unlike as in the case of action–at–a–distance admitted by (coexisting with R_3^G) eventism

G_4 as well as by *hidden* continuum I_4, the spacetime locality of eventism L_4 results in the wave zone carrying out internal energy of $\mathcal{M}$ and, consequently, making $\mathcal{M}$ unstable. This dilemma is best known from the model of atom; the constituents of atom interact by means of *relativistic* (local) electromagnetic forces which results in Bremsstrahlung and hence, in the collapse of the very atom. Note that the same Bremsstrahlung mechanism could be used for the description of multiple production of mesons or other field–quanta [52, 53] in high-energy collision of hadrons, as its origin goes as deep as eventism L_4 itself. Consequently, in order to avoid dilemma of instability of composite structures we must call for the non-local relationism I_4 which precedes events of spacetime of measurement. Then, action–at–a–distance in I_4 is given by relational shape $V(y^2)$ which does not introduce any additional degree of freedom of $\mathcal{M}$ and is given, as *the same* relational property represented in $\mathcal{L}_4(x)$, by two-event shape $U(x^2) \doteq V(y^2)$, consistently with symmetry L of measurement.

In the present, *one-level* physics based on eventism L_4, interaction $U(x^2)$ which, in general, does not vanish on space-like intervals ($x^2 > 0$) looks as *a spooky action–at–a–distance*, employing the very expression Einstein used for describing nonlocal quantum correlations conflicting with his *classical reality* [9] based on eventism.

Let us summarize our present situation which insists on eventism: In high-energy physics, where symmetry L must be respected, creation–annihilation processes are treated in the framework of field mechanism. The price to be paid for this is the particle's loss of any internal structure, because the locality of relativistic fields admits point–particles only. On the other hand, low-energy physics resorts to eventism G_4 ($1/c = 0$) consistent with mechanical synthesis of extended particles $\mathcal{M}$; however, an energy–mass relation does not necessarily exclude field mechanism. Nevertheless, the energy–mass relation concerns both *low*-energy physics and *high*-energy one. Similarly, the discontinuity between the negative balance of geometry L_4 and the equalized balance of geometry G_4 does not follow from the *low*-energy physics but from mathematical limit ($c \to \infty$) which converts the L symmetry into a G one. One usually ignores this fundamental difference between the NR framework (which "neglects" finite c ($1/c = 0$)) and the NR approximation of the characteristics of an individual state of $\mathcal{M}$. The net conclusion is that, within the *one-level* physics based on eventism, no theory exists that would reconcile the two mechanisms.

This *gap* between the NR mechanical synthesis of $\mathcal{M}$ and the relativistic field mechanism of creation of $\mathcal{M}$ is eliminated by *the two-level* relational physics which accounts for finite $\hbar/c$ and which is condemned to abandoning eventism. It is remarkable that both mechanisms of creation–annihilation of particles occur simultaneously in the most *popular* process of emission–absorption of light quanta accompanying deexcitation–excitation of an atom $\mathcal{M}$. Indeed, the very energy–momentum conservation in this process makes that we must resort to relationism R_3. In order to prove this let us consider two internal energy levels of an atom $\mathcal{M}$ in the excited internal state ψ_n of internal energy W_n which returns to the ground state ψ_0 of internal energy $W_0 < W_n$. The two different masses of $\mathcal{M}$: $M_{n,0} = W_{n,0}/c^2$ make that we are dealing with two different particles: $\mathcal{M}_n$ and $\mathcal{M}_0$.

Electromagnetic interaction between the atom electrons and atom nucleus gives rise to a field mechanism of photon production in the deexcitation process of $\mathcal{M}_n$. Photon, which is only virtually present in $\mathcal{M}$, passes on its mass-shell $p^2 = 0$ owing to the interaction of $\mathcal{M}$ with an infinite vacuum. On the other hand, the $\mathcal{M}$ atom is synthetised mechanically of electrons and of a nucleus, hence the deexcitation of $\mathcal{M}_n$ is an example of two-body reaction

$$\mathcal{M}_n \to \mathcal{M}_0 + h\nu_n \tag{19.2}$$

that deals with both mechanisms of creation of the new particles: $\mathcal{M}_0$ and $h\nu_n$.

Let S^* be the rest frame (laboratory system) of excited atom $\mathcal{M}_n$ and let ν_n denote the photon frequency in S^*. From the energy–momentum conservation law one obtains

$$\begin{aligned} \nu_n &= \nu_n^{(B)}\left(1 - \frac{\Delta W_n}{2M_n c^2}\right), \\ \nu_n^{(B)} &= \frac{\Delta W_n}{h}, \quad \Delta W_n = W_n - W_0 \,. \end{aligned} \tag{19.3}$$

Here $\nu_n^{(B)}$ denotes the standard Bohr's frequency, hence (19.3) shows that ν_n is less than $\nu_n^{(B)}$, as in ν_n an account is taken of the recoil of atom $\mathcal{M}_n$. The dimensionless correction term responsible for inequality $\nu_n \neq \nu_n^{(B)}$ is equal to

$$\frac{\Delta W_n}{2M_n c^2} \,. \tag{19.4}$$

In atomic physics this term is very small — of the order of 10^{-9} or less. Such a small term can be neglected if one considers the achievable accuracy of measurement. However, apart from the fact that an analogous correction term in nuclear physics is about 10^6 times greater, a fact of far greater importance is that small numerical corrections may conceal a deep theoretical foundation. The text-book example of *a small effect*, *i.e.* the hypothesis of *ether-wind* negated by the Michelson–Morley experiments, is the best illustration to the above statement.

The correction term from (19.4) vanishes in two cases: The first of them might be called *realistic*; it is when $\mathcal{M}_n$ becomes infinitely heavy realizing the Lorentz limit of I_4. The second has a purely mathematical character; it is when $c \to \infty$ and L_4 converts into G_4. The Lorentz limit is always assumed in the standard radiation theory of atoms [54] by the very fact that atomic wave functions are parametrized by x-coordinates of the lab-system S^*, which ignores atom recoils accompanying radiation processes. The second case — NR limit ($c \to \infty$) — is implicitly assumed by the Bohr theory with the correction (19.4) neglected in the $\nu_n^{(B)}$ in spite of the fact that real atoms are of finite inertia. As a matter of fact, the limit $c \to \infty$ is self-consistent, because it excludes any massless particles like photon. Indeed, the photon momentum is equal to $h\nu_n/c$ and it vanishes in the limit $c \to \infty$. Thus, since $c < \infty$ and the masses M_n of real atoms are finite, the internal L-absolute structure of atoms must be embedded in the corresponding configuration space $R_3 \otimes R_3 \otimes \ldots$ induced by relational space R_3.

20. Symmetry L and NR quantum mechanics

The separation of internal (relational) and external (eventistic) degrees of freedom of an isolated micro-world $\mathcal{M}$ makes that the external energy of $\mathcal{M}$, either *low*, or *high*, is entirely independent of the internal structure of $\mathcal{M}$. This separability, which calls for *two-level* relational physics, explains the success of NR quantum mechanics just because eventism G_4 coexists with R_3^G. In order to exhibit more fully this singularity of NR quantum mechanics, let us start with a composite system $\mathcal{M} = A_1 + \ldots + A_n$ embedded in the L-absolute configuration space $\overbrace{R_3 \otimes R_3 \otimes \ldots}^{N-1}$, taking thus an account of finite universal constant $\hbar/c$.

Let $\widehat{h}$ be the internal Hamiltonian of $\mathcal{M}$ and let us assume that $\mathcal{M}$ is in a loosely-bound state. We may then split $\widehat{h}$ into two L-absolute parts

$$\widehat{h} = mc^2 + \widehat{\bar{h}}\,, \qquad m = \sum_{J=1}^{N} m_J \tag{20.1}$$

and, with the phase of ψ renormalized accordingly to $\bar{\psi} = \psi \times \exp[i/\hbar(mc^2\tau)]$, equations (15.7) and (15.11) take the form

$$i\hbar\frac{\partial\bar{\psi}}{\partial\tau} = \widehat{\bar{h}}\bar{\psi}\,, \qquad \widehat{\bar{h}}\bar{\psi}_n = w_n\bar{\psi}_n\,, \qquad w_n = W_n - mc^2\,. \tag{20.2}$$

For simplicity, let us consider a two-body system $\mathcal{M} = A_1 + A_2$ assuming that it is loosely bound. This means that almost all Fermi relational momenta $\boldsymbol{q}$ parametrizing $\bar{\psi}_n$ in the p-representation satisfy the strong, L-absolute inequality

$$\frac{\boldsymbol{q}^2}{c^2} \ll \min(m_1^2, m_2^2)\,. \tag{20.3}$$

In the NR limit ($c \to \infty$), inequality (20.3) is satisfied perfectly as the left member of (20.3) tends to zero which — from the point of view of physics of finite c — means that in the NR framework we always deal with loosely bound systems.

In true physics — that of finite c — inequality (20.3) determines the NR approximation. As the p-representations of NR bound states ψ_n^G have to deal with $\boldsymbol{q}^2 \in [0, \infty)$, the NR approximation assumes that the fraction of $\boldsymbol{q}$'s which might break strong inequality (20.3) is negligible. Consequently, kinetic energy $\widehat{\bar{W}}^{(k)} = W^{(k)} - mc^2$ which enters $\widehat{\bar{h}}$ will be approximated by a local operator

$$\widehat{\bar{W}}^{(k)} = \left[(m_1^2c^2 + \widehat{\boldsymbol{q}}^2)^{1/2} + (m_2^2c^2 + \widehat{\boldsymbol{q}}^2)^{1/2}\right] - mc^2 = \frac{\widehat{\boldsymbol{q}}^2}{2\mu}, \quad \mu = \frac{m_1 m_2}{m}, \tag{20.4}$$

resulting in an L-absolute internal Hamiltonian $\widehat{\bar{h}}$ of the form

$$\widehat{\bar{h}} = \frac{\boldsymbol{q}^2}{2\mu} + V(\boldsymbol{y}^2) \tag{20.5}$$

and equations (20.2) coincide with the NR Schrödinger equations (5.15) and (5.16) (for $N = 2$).

In the NR approximation ($1/c \neq 0$), equations (20.2) do not follow from the symmetry G of G_4 but rather from the assumption (20.3) which corresponds to the notion of loosely bound structures of $\mathcal{M}$ embedded in the L-absolute relational space R_3. For bound structures F in R_3, the difference between the true spacetime of measurement (L_4) and the wrong one (G_4) reveals itself in the corresponding equalities with dot which take the forms in L_4 and G_4, respectively

$$\begin{aligned} F(\boldsymbol{y}) &\doteq G(\boldsymbol{x})\,, && \text{(L)} \\ F(\boldsymbol{y}) &\doteq F(\boldsymbol{x})\delta^{(1)}(\Delta t)\,. && \text{(G)} \end{aligned} \tag{20.6}$$

At the same time, equation (20.2) with $\widehat{\bar{h}}$ given by (20.5) determines the L-absolute internal energy levels w_n in the NR approximation — let us denote them w_G^n — which approximate well the L-absolute internal energy eigenvalues W_n of $\hat{\bar{h}}$, as

$$W_n \simeq mc^2 + w_n^G\,, \quad M_n \simeq m + \frac{w_n^G}{c^2}\,, \quad w_n^G < 0\,. \tag{20.7}$$

Thus w_n^G represents a characteristic of $\mathcal{M}$ which is simultaneously L-absolute and G-absolute one. However, in opposition to the NR framework ($1/c = 0$), the NR approximation (20.3) of loosely bound structures explains the relativistic mass defect. Note, that the NR approximation of internal structures of atoms and their internal energy levels was used in paper [55] in which some R_3-effects, mentioned in Appendix B, are analyzed.

The success of NR quantum mechanics under the form of the NR approximation of the L-absolute relationism throws new light on *the non-eventistic nature* of quantum-potential motion in low-energy physics mentioned in Section 6. According to relationism I_4, transport phenomena [26, 27], similarly as the NR motion of electrons inside atoms [28, 29], take place on the background of internal space–and–time I_4 and not on the background of Galilean spacetime G. It must be remembered that today G can be used as a helpful mathematical model but the true spacetime of measurement is the space L_4. Thus, as shown in (20.6 L), even in low-energy (NR) physics the spacetime structure of form factors $G(x) \doteq F(y)$ exhibits a spacetime nonlocality which, in general, conflicts with the locality of eventism L_4.

21. Dilatation symmetry

The fundamental property of geometrical meta-objects R_3 and $\mathcal{L}_4$ is that they include the p–x duality. Thus $R_3(y)$ and $R_3(q)$ give room for the x- and p-representations of the same Hilbert vector $|F\rangle$ and, similarly, $\mathcal{L}_4(x)$ and $\mathcal{L}_4(p)$ give room for the x- and p-representations of the same Hilbert vector $|G\rangle$. Note, that the p–x duality of 4-space $\mathcal{L}_4$ does not call explicitly for *a new practical geometry*, because $x = X_2 - X_1$ and events X of Cartesian x-space $\mathcal{L}_4(X)$ span the x-aspect $\mathcal{L}_4(x)$ of $\mathcal{L}_4$. Similarly, the p–x duality of 3-space R_3^G does not need *a new practical geometry*, because, as before: $x = X_2 - X_1$ and (simultaneous) events X of G_4 span 3-space R_3^G in its x-aspect $R_3^G(y)$. The hypothesis of a new practical geometry is required when one intends to extend the G-absolute relational 3-space R_3^G to L-absolute relational 3-space R_3. Only then one has to resort explicitly to the privileged p language and, consequently, to the p–x duality of the first physical (practical) geometry.

Thus the p–x duality of the first metrical background R_3 of (quantitative) physics abandons the Cartesian philosophy of an external x-continuum preceding any physical extensions and imposing onto them a measure of its own. Of course, eventisms G_4 and L_4 follow the Cartesian philosophy thus transferred onto the actual quantum physics. Therefore, the p–x duality of the present quantum theory may be perceived as the symmetry of *quantum laws* which leaves the classical Cartesian spacetime background unaffected. To some extent, such situation reminds of the one which existed before Einstein's STR hypothesis, when the Galilean spacetime G_4 was regarded as if it were given *a priori*. At that time some hypotheses *ad hoc* were needed in order to reconcile the symmetry G of G_4 with the symmetry L of Maxwell equations of motion (*electrodynamical laws*).

Dilatation symmetry D is inherent in the flat Cartesian x-space, hence in the spacetimes L_4 and G_4, provided that the dilatation factor D is common to all four coordinates $(X_j; X_0)$,

$$X \to \bar{X} = DX\,, \quad D \neq 0\,. \tag{21.1}$$

Indeed, symmetries L and G remain D-form invariant. The space aspect of dilatation symmetry D results in the Thales similarity of all *small* and *large* objects embedded in L_4 and G_4. Therefore, according to Boscovich and Euler, the unanalyzable *atoms* of Newtonian atomism must be point–particles, as their *point structure* remains invariant under dilatation symmetry D. However, within the Cartesian eventism of the G_4 as well as of the L_4 spacetime, point–particles cannot form up any extended object which would found the metrical geometry of eventism. The p–x duality of the very first metrical continuum R_3 changes radically this situation. Our present aim is to show that the breaking of the dilatation symmetry of practical (physical) geometry, which enables one to surmount *the labyrinth paradox* of the Cartesian continuum, is strictly connected with the existence of universal constants $\hbar$, c and M. Here M denotes the mass of some stable micro-particles which symbolize atomism.

Although the Bohr radius of atom $r_B = \hbar^2/(m_e e^2)$ makes use of the dynamical coupling constant e^2, it constitutes by itself a universal constant of the dimension of length without indicating any real atom. Thus the assertion that a real object measured in r_B-units is *large* or *small* acquires an absolute meaning expressed by dimensionless numbers larger or smaller than unity. In the classical framework $(\hbar = 0)$ $r_B = 0$ and, according to the Thales similarity, the statement that some real extension is *large* or *small* has a relative meaning only and calls for introducing some real measuring rods which could be recognized as a unit length. In macro-physics this manifests itself in the purely conventional character of the units of *e.g.* metre and second.

Nevertheless, the breaking of the Thales similarity by *planetary atom* and the planetary system can still be regarded as a consequence of different laws determining those structures which are both embedded in the same Cartesian space of spacetime. The question arises whether such opinion may be extrapolated onto micro-physics (as it happens in the case of eventism), with three *super-facts* characterized by the three universal constants: $\hbar$, c and M. The first super-fact is the quantum p–x symmetry discovered with finite $\hbar$, the second concerns the symmetry L of relativistic kinematics ($1/c \neq 0$) and the third accounts for atomism ($1/M \neq 0$).

The point is that these three constants determine natural units of the dimensions of metre and second

$$l_0 = \frac{\hbar}{Mc}, \quad [l_0] = \mathrm{m}, \qquad t_0 = \frac{l_0}{c} = \frac{\hbar}{Mc^2}, \quad [t_0] = \mathrm{sec.}, \tag{21.2}$$

already on the elementary level of kinematics. Therefore, the breaking of the Thales similarity occurring on the most elementary physical level of kinematics suggests strongly that metrical physics cannot be based of the (Cartesian) eventism with its dilatation symmetry D. In other words, the p–x duality ($\hbar \neq 0$) must be inherent in the first physical (practical) geometry, as it is realized by geometrical meta-objects R_3 and $\mathcal{L}_4$. Similarly, the universal constant c cannot be *added* to the Galilean spacetime G_4 just in order to reconcile it with the symmetry L of Maxwell equations.

The self-consistency of relational space R_3 and 4-space $\mathcal{L}_4$ with finite universal constant $\hbar/c$ and, simultaneously, with dilatation symmetry D imposes D-invariance of the L-absolute internal phase ϕ which, in the elementary two-body problem, takes the form

$$\hbar\phi = qy \doteq px, \quad p^2 = q^2. \tag{21.3}$$

Thus, the dilatation symmetry D in R_3 and in $\mathcal{L}_4$ and the D-invariance of ϕ in R_3 and $\mathcal{L}_4$ require the following relations to be fulfilled

$$\left.\begin{array}{lll} y_j \to \bar{y}_j = Dy_j, & q_j \to \bar{q}_j = D^{-1}q_j, & \text{(R)} \\ x \to \bar{x} = Dx, & p \to \bar{p} = D^{-1}p. & \text{(L)} \end{array}\right\} \tag{21.4}$$

There still remains the third super-fact of atomism characterized by finiteness of M. This super-fact must be interpreted in the following way: Atomism provides us with a definite spectrum of masses M_A of micro-particles and should these masses be referred to a single mass M (of one of these particles) then atomism would be characterized by the existence of a definite set of dimensionless numbers

$$\chi_A = \frac{M_A}{M}, \quad A = 1, 2, \ldots. \tag{21.5}$$

In other words, atomism declares that the set of χ_A numbers is absolute and no real physical symmetry can change the values of these numbers.

Before analysing the third super-fact let us recall the ambiguity which arises at interpreting such point transformations as $O^{(n)}$, L and G and also the dilatation symmetry D from (21.4). In order to avoid the ambiguity connected with *the passive* and *the active* interpretation of transformation D and be able to speak of its active interpretation only, we assume that the universal constant $l_0 = \hbar/Mc$ is finite and we introduce dimensionless x and p coordinates. Then the space and time extensions of any real object–process acquire dimensionless characteristics, while the dilatation transformation gets the active interpretation relevant for the physical meaning of terms *large* and *small.* Thus we put

$$z = \frac{x}{l_0} \quad \text{and} \quad u = \frac{p}{Mc}, \tag{21.6}$$

keeping in mind that the universal constant l_0 remains unchanged under dilatation transformation D. We do not introduce any dimensionless coordination of R_3 as the relationship existing between atomism and symmetry D will be viewed in $\mathcal{L}_4$.

It is interesting to point out that the Planck constant does not appear in the dimensionless p language but it enters the x-language only, as $l_0 = \hbar/Mc \neq 0$ provided that $\hbar \neq 0$. This reflects the privileged position of the language p of measuring process and stays in agreement with the Thales similarity of macro-(classical-)measuring devices. The p–x duality enables one to translate the measured p extensions into x extensions measured in metres and seconds of the spacetime of measurement.

The L-absoluteness of the phase ϕ, which accounts also for the first two super-facts ($\hbar/c \neq 0$), remains also D-invariant as, by virtue of (21.6) we get

$$\phi \to \bar{\phi} = \bar{u}\bar{z} = (D^{-1}u)(Dz) = uz = \phi\,. \tag{21.7}$$

Note that the equality $\bar{u} = D^{-1}u$ must concern space-like and time-like four-momenta u, because a difference of two space-like four-momenta can create a time-like four-mentum and vice versa. If u_A is the four-momentum of a free *atom* A with mass M_A, then $u_A^2 = -\chi^2$ and the dilatation symmetry D from (21.6) results in

$$u_A^2 = -\chi^2 \to \bar{u}_A^2 = D^{-2}u_A^2 = -D^{-2}\chi_A^2 \tag{21.8}$$

and

$$\chi_A \to \bar{\chi}_A = \frac{\chi_A}{D}\,. \tag{21.9}$$

The net conclusion is that all three super-facts are in conflict with symmetry D, because the absoluteness of the set of χ_A numbers imposes the constraint

$$D = 1 \tag{21.10}$$

eliminating symmetry D.

Note that the universal constant $l_0 \neq 0$ vanishes in the mathematical limit $\hbar \to 0$ of the classical framework which was to be expected as classical physics (theory) is condemned to eventism with its symmetry D. Also, $l_0 \to 0$ in the NR limit ($c \to \infty$) which is consistent with the coexistence of relationism R_3^G and eventism G_4. The third *realistic* case when l_0 also vanishes corresponds to the Lorentz limit of I_4 when $M \to \infty$. This is consistent with symmetry L of measurement and the Thales similarity of macro–measuring–devices. We may then conclude that in all these three cases $l_0 = 0$ and physics indeed is condemned to the Cartesian philosophy of an external x-space preceding any real entity (eventism). However, the inequality $l_0 \neq 0$ enables one to abandon eventism in the favour of a relational origin of metrical physics.

One has to emphasize that the relational origin of metrical physics based on the objectism of elementary two-body system $\mathcal{M} = A_1 + A_2$ works in favour of the Leibniz philosophy of spacetime [56]. Let us remember that according to this philosophy, the metrical spacetime disappears together with the reality of $\mathcal{M}$. In opposition to such view, Clarke (Newton) argued in favour of pre-existing absolute space and time continua. Since at the time of the argument none of the three super-facts was known — so the equality $l_0 = 0$ was automatically presumed — the Leibniz–Clarke dispute was won by the Clarke–Newton team.

There still remains a realistic limiting case when, unlike as in the case of Lorentz limit of I_4, l_0 tends to infinity because $M \to 0$

$$l_0 = \frac{\hbar}{Mc} \underset{M \to 0}{\longrightarrow} \infty \, . \tag{21.11}$$

Here one regains also the symmetry D of eventism, because an infinite natural unit gives no absolute normalization of real, extended object–process. Thus, it was no accident that the classical eventistic physics could "discover" Maxwell equations. In consequence, the Maxwell equations open the way to both the new symmetry L of eventism L_4 and the quantum wave–corpuscular $(p - x)$ duality of light. Both facts were recognized by Einstein who put forward the hypotheses of STR and of light quanta which could be admitted by the classical Maxwell equations just because of the massless nature of photon, *i.e.* $M_{\text{ph}} = 0$, resulting from (21.11).

In consequence, the spacetime–wave of electromagnetic fields of Maxwell equations accounts for the wave aspect of *corpuscular* photon without explicitly calling for a new notion of the wave function ψ. However, some twenty years later, when de Broglie extended the wave–corpuscular $(p - x)$ duality onto mass–particles with finite Compton wavelengths (in accordance with $l_0 = \hbar/Mc$), he was forced to introduce a new notion of the wave of matter ψ. Again, Einstein was the first to be aware of the fundamental character of de Broglie's hypothesis and the equally fundamental controversy between the classical eventism with its symmetry D and the quantum-potentiality characterized by ψ.

It is not quite true that Einstein did not accept quantum mechanics as a fundamental theory of micro-physics because of its indeterministic nature conflicting with the deterministic nature of the classical field theory regarded by Einstein as a perfect theory. The point is that, following the tradition, Einstein regarded physics as a system reflecting the ontology of nature and not only as the best tool for predicting registrable actualizations. Note that Einstein, much like his all adversaries, believed in eventism giving room to any possible physical entity. In consequence, Einstein–realist could not agree with the positivistic trend recognizing quantum mechanics (based on eventism!) as a complete *tool* which could be used for statistical predictions of all possible observables; Einstein–realist–and–eventist could not accept this purely epistemological philosophy of micro-physics which neglected the micro-physics ontology.

One must agree with Bohr's opinion that classical measuring devices and their *possibilities* constitute always the language of experimental physics, independently of the degree of abstraction of the theory. However, from this inspiring *a priori* of physics does not necessarily follow that a physical reality cannot go beyond *the surface* of directly observable phenomena. For example, NR quantum mechanics determines the form factors $F_{nm}(y)$ of hydrogen atom initially *hidden* in $R_3^G(y)$. These form factors are measurable, but only indirectly, in the p language of the corresponding cross-sections which describe collisions of hydrogen atom with other particles $\mathcal{M}$. In contradistinction to the positivistic philosophy, F_{nm}'s represent registrable realities, although their measurement does not consist in measuring the localization of electron by proton or that of proton by electron. Any measurement must be connected with an irreversible and registrable actualization, whereas an atom represents *a priori* an isolated indivisible micro-world $\mathcal{M}$. It is because of these *impossibilities* of any experiment that there is a place left for the hypothesis of *hidden* relational space R_3 and hence, for extension of *physical reality* onto quantum-potential structures such as form factors F_{nm}.

The point is that the notion of *physical reality* is not *a priori* given. Directly unobservable forces — first of all, forces acting–at–a–distance — did not belong to the physical reality of Democritean and/or Cartesian physics. They became *real* in the Newtonian physics although their observation is always an indirect one and based on the motion of *a visible* particle $\mathcal{M}$. Without *invisible* forces of hypothetical structure the Newtonian physics would not have made its appearance. As a matter of fact, it has been *the invisible action–at–a–distance* which has initiated the concept of an external spacetime as *a holder* of everything that might (physically) exist.

Nowadays, after numerous experiments which attest to the quantum nonlocality of EPR-like correlations and after "welcher Weg" experiments [11], we know that Einstein's concept of *classical reality* [9] based on eventism has collapsed. Thus, in full agreement with the opinion of Clauser and Shimony [12], physical realism " ... must dramatically revise our concept of spacetime". It is author's opinion that *quantum propensity (potentiality)* forces one to extend the notion of physical reality, at the price — this time — of abandoning *classical eventism* in favour of *quantum relationism.*

If one takes into account, together with the most fundamental constants $\hbar, c, M$, the dimensionless dynamical coupling constant $\alpha = e^2/\hbar c = 1/137$ which appears in the Bohr radius $r_B = \hbar^2/(m_e e^2)$, the breaking of the Thales similarity occurs already in: *(i)* the classical–relativistic model ($\hbar = 0, 1/c = 0$) and *(ii)* NR quantum mechanics ($\hbar \neq 0,\ 1/c = 0$), although the natural length $l_0 = \hbar/Mc$ vanishes in both these models. Thus, the two models are condemned to eventism with its dilatation symmetry D, but — at the same time — these very models break this symmetry. Indeed, within these models we deal with finite *units* of length and duration

$$\left.\begin{aligned} l_0' &= \alpha l_0 = \frac{e^2}{Mc^2}\,, \quad t_0' = \frac{e^2}{Mc^3}\,, \quad &(i)\\ l_0'' &= \frac{l_0}{\alpha} = \frac{\hbar^2}{Me^2}\,, \quad t_0'' = 0\,. \quad &(ii) \end{aligned}\right\} \tag{21.12}$$

For $M = m_e$, l_0' coincides with the classical radius of electron r_e, and l_0'' with the Bohr radius of atom.

A finite l_0 attests to the self-inconsistency of classical electrodynamics, as it excludes existence of any charged point-particle *i.e.* of the only *atom* consistent with eventism and its symmetry D. This inconsistency is strictly connected with the one-body problem of eventism and, most of all, with the notion of event shape $f(X)$ of a field. Indeed, a charged point-atom would carry an infinite amount of internal (electromagnetic) energy and, according to the relativistic energy–mass relation ($1/c \neq 0$), an infinite mass. Thus, a finite r_e results from the identification of the electromagnetic mass of electron with its experimental value m_e.

The second model *(ii)* of NR quantum mechanics is a self-consistent one. This is due to the coexistence of classical eventism G_4 and quantum relationism R_3^G. Of course, the model excludes the energy–mass relation and leads to $t' = 0$ which reflects the locality of the Newtonian time or, in other words, infinite velocity of signals admitted by the symmetry G of G_4.

Finally, let us emphasize that the existence of natural units l_0 and t_0 (*cf.* (21.2)) which are responsible for the breaking of dilatation symmetry topped by the hypothesis of relational space R_3 does not mean that *an elementary lattice* must exist. On one hand such a *lattice* would conflict with the *practical* differential manifold while, on the other hand, it would preserve the Cartesian philosophy of x-localization against the fundamental p–x duality which — in author's opinion — stands at the very foundation of metrical physics.

22. Time dilatation effect of classical and quantum clocks

In classical, eventistic physics, the time dilatation effect of a moving clock

$$T = \Gamma T_0 \,, \quad \Gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \,, \tag{22.1}$$

represents a one-body, relative effect. Indeed, the T_0 seconds (proper seconds) of a single clock measured in the clock's reference frame S^* correspond to the T seconds in the reference frame S in which the clock moves with the velocity $\boldsymbol{V}$. Thus the dilatation effect is a relative one and — as we know — realization of a clock is, much like the realization of mathematical reference frames S by reference bodies $\bar{S}$, immaterial within the classical framework ($\hbar = 0$).

However, a measure of time intervals is also supplied by *quantum clocks* represented by unstable micro-particles and metastable states of mechanically synthetised composite particles $\mathcal{M}$. Indeed, micro-worlds are manifestly subject to the p–x duality of the first *practical geometry* of relations and hence, besides direct x-measurement of T, a second kind of indirect measurements opens based on the uncertainty relation between time t and energy E of $\mathcal{M}$. Thus, we shall resort to the standard Gamov's phenomenology and we shall attach to an unstable particle $\mathcal{M}$ a complex value of the L-absolute, internal energy W together with a complex "mass" M

$$W = \overset{o}{W} - i\delta\frac{W}{2} \,, \quad M = \overset{o}{M} - i\delta\frac{M}{2} = \frac{W}{c^2} \,. \tag{22.2}$$

Consequently, the stationary states of $\mathcal{M}$ (if $\mathcal{M}$ is a stable particle $\delta W = \delta M = 0$) convert into metastable states which are subject to the one-exponential (OE) decay law. Conversely, if the uncertainty δW (hence δM)

is well defined, $\mathcal{M}$ undergoes the OE decay with the L-absolute (proper) mean life-time T_0 equal to

$$T_0 = \frac{\hbar}{\delta W} = \frac{\hbar}{\delta M c^2}\,. \tag{22.3}$$

Virtually, the two L-absolute p-characteristics of an unstable particle $\mathcal{M}$, *i.e.* $\overset{o}{M}$ and δM make the description of $\mathcal{M}$ to transgress the limits of relativistic kinematics, as the latter deals with a unique L-invariant length of real four-momentum P. The same conclusion follows also from the canonical representation of ten generators of Lie algebra of Poincaré group L realized by free particles A_J. This representation requires sharply defined real masses m_j of A_J ($\delta m_J = 0$), *i.e.* it requires the particles to be stable. Using a more intuitive approach, one might also say that an unstable particle $\mathcal{M}$ cannot reach the asymptotic zone of relativistic kinematics.

In opposition to eventism L_4, the I_4 relationism admits unstable particles, because geometry I_4 deals with two *a priori* L-absolute p-characteristics: q^2 and W which are the counterparts of the two L-absolute x-characteristics y^2 and $\Delta\tau$. Our intention is to show that symmetry L resulting from (22.1) will result, if viewed in the perspective of relationism I_4 (instead of that of eventism L_4), in a self-consistent description of the dilatation effect (22.1) of $\mathcal{M}$ on the quantum-potential level of existence of $\mathcal{M}$ in I_4. In consequence, an analysis of decay process of $\mathcal{M}$ (quantum clock) must call for an elementary — in I_4 — two-body system $\mathcal{M} + A$ where the reality of the second, stable body A would further coincide with the reality of its rest frame $\bar{S}$ if one assumes A to be infinitely heavy. We shall show that the replacement of a real $\bar{S}$ by a mathematical reference frame S may be justified provided, however, that we have to deal with pure kinematics of stable particles, free of the time characteristics T.

The internal, L-absolute mean life-time T_0 of an unstable $\mathcal{M}$ coincides then with the L-absolute internal time interval $\Delta\tau$ of internal space–and–time I_4 of the two-body micro-world $\mathcal{M} + A$ if the relational momentum q of this system vanishes ($q = 0$) and A is stable. Thus T_0 from (22.3) (based on the p–x duality) represents an internal L-absolute property of an individual micro-object $\mathcal{M} + A$ ($q = 0$) on its quantum-potential level of existence in I_4. Let us emphasize that any experimental determination of T_0 based on the determination of δM must resort to a suitable statistics of actualized decay processes of identical particles $\mathcal{M}$. Moreover, T_0, as a statistically repeatable observable, must respect symmetry L of measurement. The point is that according to relationism, there is no possibility of such replacing of the reference body A ($\bar{S}$) (even an infinitely heavy one) by mathematical reference frames S that would result in the one-body problem of eventism L_4.

Before going to relationism I_4 let us remark that the momentum operator $\widehat{\boldsymbol{P}} = -i\hbar\partial/\partial \boldsymbol{X}$ of $\mathcal{M}$ which realizes the p–x duality is *a priori* independent of the mass of particle $\mathcal{M}$. Hence, no matter whether $\mathcal{M}$ is stable or not, we can speak of the eigenstates of $\widehat{\boldsymbol{P}}$ with definite real eigenvalues $\boldsymbol{P}$ in some arbitrary reference frame S in L_4. Let S be the rest frame S^* of $\mathcal{M}$ in the eigenstate of $\widehat{\boldsymbol{P}}$ with eigenvalue $\boldsymbol{P}^* = 0$. Thus, *a posteriori*, $\Delta\tau$ coincides numerically with the time interval in S^*, *i.e.* $\Delta\tau = \Delta t^*$ and hence, the conditional probability $\Pi(t^*|0)$ $(t^* \geq 0)$ that $\mathcal{M}$ exists in S^* at the instant $t^* \geq 0$, if it existed at $t^* = 0$ is equal to

$$\Pi(t^*|0) = \exp\left(\frac{-t^*}{T_0}\right), \quad t^* \geq 0\,. \tag{22.4}$$

Note that the probabilistic language of quantum physics, in particular that used in the interpretation of $\Pi(t^*|0)$, resorts directly to actualizations (measurements) which are subject to the sharp alternative: *exists — does not exist*, *i.e.* to a *0 – 1* alternative. This agrees well with eventism and, without any doubt, it is crucial for the relationship between theory and experiment; however, it ignores the fact that the quantum propensity ψ precedes the probabilistic language of actualizations. All quantum interference effects follow from the structure of the directly unobservable quantity ψ which, in turn, determines the probability language of measurement. This justifies fully one to speak of neo-realism of *fractional–potential–existence* of an individual micro-object $\mathcal{M}$ in I_4.

Of course, atomism together with the spacetime globality of ψ (embedded *a priori* in relational space–and–time) must, if testified experimentally, call usually for a decent statistics of actualizations of quantum potentialities ψ. An operationalist would than maintain that ontologization of fractional–potential existence is illegitimate physically, because the language of direct measurement may concern actualizations only. It would be, however, absolutely wrong to accept such an *ascetic* programme of physics restricting its notions to directly measurable ones. If we were to adopt such restriction, we would be forced to eliminate the notion of field of forces $\boldsymbol{f}(\boldsymbol{X}, t)$ of classical physics, because even if we detect some trajectories of $\mathcal{M}$ we never do detect the whole field of forces. Any theoretical knowledge of empirical world must introduce some hypothetical, directly unobservable entities which explain the observable surface of the (never self-depending) empirical world. Of course, the whole model must be self-consistent and must predict all possible experimental facts (actualizations). Only then the ontologization of force $\boldsymbol{f}$ as well as that of ψ are fully justified; they enlarge the world of *physical reality*.

The dependence of Π on time only (*cf.* (22.4)) suggests strongly a relational rather than eventistic origin of this quantity. Indeed, we may attach an analogous probability Π to the same $\mathcal{M}$ described in the reference frame S in which S^* moves with a velocity $\boldsymbol{V}$. This probability has the form

$$\Pi(t|0) = \mathrm{e}^{-t/T}, \quad T = \Gamma T_0 , \quad (t \geq 0) \tag{22.5}$$

which accounts for the dilatation of the mean life-time T of $\mathcal{M}$ in S.

The actualizations which make possible direct measurements of T are not realized automatically (as in the classical framework with CCINF's) but call for an appropriate detector with the time resolution power $1/\delta t$ much larger than $1/T$. According to the p–x duality we have

$$\frac{1}{\delta t} \leq \frac{\delta E}{\hbar} \xrightarrow[\hbar \to 0]{} \infty , \tag{22.6}$$

where δE is the uncertainty of the energy transfer between $\mathcal{M}$ and detector. Thus, direct measurement of T requires

$$\frac{1}{\delta t} \gg \frac{1}{T} \qquad \text{or} \qquad T \gg \delta t . \tag{22.7}$$

As the resolution power $1/\delta t$ tends, in the classical limit ($\hbar \to 0$), to infinity for any, even arbitrarily small, value of δE, the strong inequality (22.7) is satisfied ideally and the direct x-measurement of T is the only possible one. Indeed, in the classical limit, the quantum p–x duality vanishes, hence no room exists for any indirect determination of T; formally, $\lim_{\hbar \to 0}\{T_0 = \hbar/\delta W\} = 0$.

However, strong inequality (22.7) is satisfied even by quantum clocks like μ-meson with very large $T_0 = 2 \times 10^{-6}$ sec. ($cT_0 = 600$m). Thus, the direct x-determination of dilatation effect of the mean life-time T of muon was a spectacular success of STR which regards muon as if it were a classical wandering clock. Indeed, a muon created at a height of about 40 kms ($= 70\ cT_0$), if it had not been subject to the dilatation effect it would have never (practically) reached the Earth surface (the probability of a muon reaching the Earth surface would have amounted to 4×10^{-31}). Thus the presence of muons at the Earth surface is due to the muon's large dilatation factor Γ resulting in $T \gg T_0$. Nevertheless, a quantitative proof of dilatation effect requires a large ensemble of identical unstable particles $\mathcal{M}$ — let us call it $Z(N_0, V)$ — with $N_0 \gg 1$, where all $\mathcal{M}$'s have the same velocity $V = |\boldsymbol{V}|$ in some lab-system $\bar{S}$ and, consequently, they all have the same value of dilatation factor Γ.

Let $T^{(r)}$ ($r = 1, \ldots, N_0$) be the life time of the r-th muon in the ensemble $Z(N_0, V)$. The globality of the mean life-times T of muons manifests itself in the fact that an individual actualized value of the decay instant $T^{(r)}$

of the r-th muon represents no repeatable observable. A repeatable observable would require the whole ensemble $Z(N_0, V)$ to be an infinite one as

$$T = \lim_{N_0 \to \infty} \frac{1}{N_0} \sum_{r=1}^{N_0} T^{(r)}. \tag{22.8}$$

An observable given by quantum-potential predictions must be given by a suitable statistics, because actualizations of individual micro-events are relatively well localized (in spacetime) , whereas quantum-potentiality ψ occupies a very large spacetime region. The point is that — in opposition to eventism which is characterized by an infinite resolution power of space- and time-intervals of all processes — micro-objects and macro-devices have to deal with their own finite resolution powers of space and time intervals. It is here that the origin of indeterministic nature of quantum predictions lies, due to relational rather than eventistic nature of metrical physics.

23. Quantum–relativistic puzzle of indirect measurements of T

For very short-living particles, an indirect determination of their mean life-times is the only way of measurement. Thus, equation (22.3) determines indirectly the proper mean-life time T_0 of $\mathcal{M}$ and, having in mind the dilatation effect and the quantum p–x duality, it is reasonable to expect that the equality

$$T = \frac{\hbar}{\delta E^{(\text{in})}} \tag{23.1}$$

would determine indirectly T in any lab-system $\bar{S}$. By $\delta E^{(\text{in})}$ we mean that part of the uncertainty of energy E of $\mathcal{M}$ which is due solely to the uncertainty of the $\mathcal{M}$'s internal energy W. Assuming that $\delta E^{(\text{in})}/E \ll 1$ and $\delta E^{(\text{ex})}/E \ll 1$, let us put

$$\delta E = \delta E^{(\text{in})} + \delta E^{(\text{ex})}, \tag{23.2}$$

where $\delta E^{(\text{ex})}$ is the uncertainty of E due solely to the fluctuating external motion of $\mathcal{M}$ as a whole (in $\bar{S}$). One should expect that T, as determined by (23.1), should by subject to the dilatation effect: $T = \Gamma T_0$. However, symmetry L of eventism L_4 combined with the quantum p–x duality results in a puzzle which can be solved only if one abandons the L_4 eventism in favour of relationism I_4.

The first aspect of this quantum–relativistic puzzle concerns the difference between the quantum language p and the relativistic velocity language

v. The second aspect consists in an essential difference between the determination of T on the quantum-potential level of description of decay process of $\mathcal{M}$ and the measurement of T which requires a suitable ensemble of actualized decay events which may serve for determining the repeatable observable T. In order to determine T from (23.1) let us consider the decay processes of an unstable (free) particle $\mathcal{M}$ which decays always into the same n stable particles A_J ($J = 1, \ldots, n$) with sharply defined masses m_J. It is the sharpness of all theses masses which attests to the stability of these particles ($\delta m_j = 0$).

We can attach, to each particle and in the asymptotic zone of the decay event, a four-momentum P_J where $P_J^2 = -m_J^2 c^2$ and hence, the cluster of all decay–products A_J gets also a well defined four-momentum P, where

$$P = \sum_{J=1}^{n} P_J\,, \quad P^2 = -M^2 c^2 = -\frac{W^2}{c^2}\,, \tag{23.3}$$

and M denotes the L-invariant mass of the cluster, *i.e.* a mass which is independent of the reference frame S in which P_J's are represented. As the energy–momentum conservation law must be valid on the quantum-potential level of each individual decay–event of $\mathcal{M}$, P and M from (23.3) denote also the four-momentum and L-invariant mass of $\mathcal{M}$, respectively, at the moment of $\mathcal{M}$'s decay. Therefore, even if the masses $M^{(r)}$ of the unstable particle $\mathcal{M}$ fluctuate (r enumerates the decay–events: $r = 1, \ldots$, $N_0 \gg 1$), the energy E of $\mathcal{M}$ in a fixed reference frame S (reference body $\bar{S}$) takes the same analytic form as if $\mathcal{M}$ were stable, *i.e.*

$$E = \sqrt{W^2 + c^2 \boldsymbol{P}^2}\,, \quad W = Mc^2\,. \tag{23.4}$$

After collecting a large sample $Z(N_0)$ ($N_0 \gg 1$) of decay–events of $M^{(r)}$'s, a trustworthy *average* value $\overset{o}{M}$ of masses $M^{(r)}$ and the dispersion δM of $M^{(r)}$'s around $\overset{o}{M}$ determine the mass ($\overset{o}{M}$) of $\mathcal{M}$ and the proper mean life-time T_0 of $\mathcal{M}$, as $T_0 = \hbar/(\delta M c^2)$ — *cf.* (22.3). The L-invariance of masses $M^{(r)}$ makes T_0 independent of momenta $\boldsymbol{P}^{(r)}$ of $M^{(r)}$ in the fixed lab-system $\bar{S}$. Using equality (23.4), we introduce the velocity language v, where

$$\boldsymbol{V} = \frac{\partial E}{\partial \boldsymbol{P}} = \frac{\boldsymbol{P}}{\sqrt{M^2 + \boldsymbol{P}^2 c^2}} \tag{23.5}$$

denotes the velocity of $\mathcal{M}$ in the same reference frame S ($\bar{S}$) in which $\boldsymbol{P}$ and E are represented. In the classical v language, E and P take the form

$$E = \Gamma W\,, \quad \boldsymbol{P} = M\Gamma \boldsymbol{V}\,, \quad \Gamma = \frac{1}{\sqrt{1 - \boldsymbol{V}^2/c^2}} = \sqrt{1 + \frac{\boldsymbol{P}^2}{M^2 c^2}}\,. \tag{23.6}$$

From the v-expression of $\boldsymbol{P}$, $\boldsymbol{P} = M\Gamma\boldsymbol{V}$, follows the equality

$$\delta\boldsymbol{V} = \frac{\delta\boldsymbol{P}}{M\Gamma^3} \qquad (i)$$

or

$$\delta\boldsymbol{P} = M\Gamma^3\delta\boldsymbol{V} \qquad (ii) \tag{23.7}$$

relating the uncertainties $\delta\boldsymbol{P}$ and $\delta\boldsymbol{V}$. It is the very inequality which exhibits an essential difference between the v and p languages. Indeed, let us assume that δP is practically limited by a small, but finite, space interval l in $\bar{S}$, as one has

$$\delta P \simeq \frac{\hbar}{\delta X} \simeq \frac{\hbar}{l} < \infty\,. \tag{23.8}$$

From (23.7 i) and (23.8) follows that the fluctuation (uncertainty) of velocity $\boldsymbol{V}$ tends to zero when $M \to \infty$ which means that — within the accuracy l of the space localization of $\mathcal{M}$ — $\mathcal{M}$ follows a classical trajectory.

According to the quantum symmetry Q which has not been yet confronted with symmetry L, we can assume that, in the same $\bar{S}$, the unstable particle $\mathcal{M}$ is in the eigenstate $\psi_{\boldsymbol{P}}$ of $\widehat{\boldsymbol{P}} = -i\hbar\partial/\partial\boldsymbol{X}$ with an eigenvalue $\boldsymbol{P}$ and hence

$$\delta\boldsymbol{P}^Q = 0\,. \tag{23.9}$$

The superscripts Q and L will denote, from now on, the corresponding symmetries of the languages p and v, respectively. From (23.5) we may see that both languages p and v indicate the same rest frame S^* in which

$$\boldsymbol{P}^* = \boldsymbol{V}^* = 0\,. \tag{23.10}$$

Thus, with $\mathcal{M}$ being at rest in S^* which in turn moves in S with a sharply defined velocity $\boldsymbol{V}$ (dictated by symmetry L), we obtain

$$\delta\boldsymbol{V}^L = 0\,. \tag{23.11}$$

The p- and v-representations of energy E from (23.4) and (23.6) lead to

$$\left.\begin{aligned} \delta E^Q(W,\boldsymbol{P}) &= \frac{1}{\Gamma}\delta W + \boldsymbol{V}\delta\boldsymbol{V}\,, && (\mathrm{Q}) \\ \delta E^L(W,\boldsymbol{V}) &= \Gamma\delta W + M\Gamma^3\boldsymbol{V}\delta\boldsymbol{V}\,. && (\mathrm{L}) \end{aligned}\right\} \tag{23.12}$$

Under constraints (23.9) and (23.11) we get

$$\delta E^{(\mathrm{in})Q} = \frac{1}{\Gamma}\delta W\,, \qquad (\mathrm{Q})$$

$$\delta E^{(\mathrm{in})L} = \Gamma\delta W\,, \qquad (\mathrm{L}) \tag{23.13}$$

which, according to (23.1), result in

$$T^Q = \Gamma T_0\,, \qquad \text{(Q)}$$
$$T^L = \frac{T_0}{\Gamma}\,. \qquad \text{(L)} \tag{23.14}$$

Thus,

$$T^Q \neq T^L \quad (\Gamma > 1) \tag{23.15}$$

tops the quantum–relativistic puzzle.

The classical v language of symmetry L combined with the quantum symmetry Q of the p–x duality in determining T lead immediately to a wrong *contraction effect* of T. On the other hand, T^Q determined on the quantum-potential level of $\mathcal{M}$ coincides with the correct dilatation effect; however, so far we have been ignoring the relationship between the constraint (23.9) and symmetry L. The point is that the constraint $\delta \boldsymbol{P}^Q = 0$ disagrees with symmetry L of eventism L_4, because such a constraint distinguishes the initial reference frame in which $\delta \boldsymbol{P}^Q$ is represented. The reason for this conflict is that the four numbers: $(\boldsymbol{P}; E)$ attached (in S) to an unstable particle $\mathcal{M}$ do not constitute any four-vector as in the case of a stable particle $\mathcal{M}$. In consequence, if $(\boldsymbol{P}; E/c)$ is recognized — in agreement with symmetry L — as the four-momentum of $\mathcal{M}$, then the finite uncertainty $\delta E^{(in)}$ in S results in $\delta \boldsymbol{P}^{Q'} \neq 0$ in an S' moving in S. This shows that the quantum constraint (23.9) in S is not an L-absolute one and we end up with a wrong value of $T^{Q'}$ in S'.

The correct value of T^Q obtained on the quantum-potential level of description of $\mathcal{M}$ inclines one to abandon symmetry L as a symmetry imposed by eventism L_4 and to recognize T^Q as an L-absolute characteristic of the (elementary in I_4) two-body problem $\mathcal{M} + A$. Let us emphasize that the L-symmetry breaking takes place on the quantum-potential level of the two-body system $\mathcal{M} + A$, but the symmetry L cannot be violated on the level of measuring actualizations which determine T, because this would mean a violation of the (classical) principle of relativity. Without restricting the generality of our considerations, we assume A to be a stable and infinitely heavy object. Consequently, numerical value of relational momentum square $\boldsymbol{q}^2$ of $\mathcal{M} + A$ coincides with momentum square of the momentum $\boldsymbol{P}$ of $\mathcal{M}$ in the rest frame S of the infinitely heavy A. Then, instead of the L-absolute constraint (23.9) we come to deal with an L-absolute constraint

$$\delta \boldsymbol{q} = 0\,. \tag{23.16}$$

Thus we deal with internal space–and–time I_4 of $\mathcal{M} + A$ where the uncertainty $\delta \mathcal{W}$ of the total internal energy $\mathcal{W}$ of $\mathcal{M} + A$ is due solely to the uncertainty of the L-absolute mass M of the constituent $\mathcal{M}$ of $\mathcal{M}+A$. One obtains immediately

$$\delta\mathcal{W} = \frac{1}{\Gamma}\delta W\,, \quad \Gamma = \sqrt{1+\frac{q^2}{M^2c^2}} = \sqrt{1+\frac{\boldsymbol{P}^2}{M^2c^2}}\,, \tag{23.17}$$

which results in the L-absolute proper mean life-time of $\mathcal{M}+A$ determined indirectly as equal to

$$T^0 = \frac{\hbar}{\delta\mathcal{W}} = \Gamma T_0 \tag{23.18}$$

and coinciding numerically with T^Q from (23.14 Q).

The instability of the system $\mathcal{M}+A$ is due to the instability of its constituent $\mathcal{M}$ only, while $T_0 = \Delta\tau$ represents the L-absolute mean life-time of $\mathcal{M}+A$ coinciding (numerically) with the time-interval Δt of the rest frame S of A. Thus, in spite of the correct dilatation of T^0: $T^0 = T^Q = \Gamma T_0$, the quantity T^0 ceases to be a relative characteristics of the two-body problem $\mathcal{M}+A$ embedded in I_4. Therefore, on the quantum-potential level of relationism, even if A is infinitely heavy it cannot be replaced by mathematical reference frame S. If, performing a *Gedanken experiment*, we replace A by a similar A' which moves with respect to A, then we obtain $T^{0'} = \Gamma' T_0$ in agreement with the dilatation effect of $\mathcal{M}$ in the rest frame (S') of A' but again, we deal with an L-absolute two-body characteristic of $\mathcal{M}+A'$.

Thus, the L-absolute relationism of internal space–and–time I_4 explains *the kinematic* dilatation effect on the quantum-potential level of system $\mathcal{M}+A$, but it does not mean that an indirect (p) measurement of $T^0 = \Gamma T_0$ is possible. The point is that our two-body systems $\mathcal{M}+A$, $\mathcal{M}+A'$, … are not bound and therefore, the L-symmetry breaking which is connected with the determination of T^0 and which must accompany the indirect measurement of $T^0 = T^Q$ would mean the breaking of L-symmetry on the level of measurement. However, it turns out that *measuring possibilities* admit indirect determination of T_0 only, when the conflict (23.15) vanishes with $\Gamma = 1$. In the next section we shall show that indirect measurement can determine T_0 only and the determined quantity will remain consistent with symmetry Q of the p–x duality as well as with symmetry L of measurement.

24. Indirect measurements of T

It has been said that the quantum-potential predictions given by non-local observables such as $|\psi|^2$ or/and T require a suitable statistics of (locally) actualized micro-events. Therefore, neither single mark left by electron on the screen in the vicinity of a point $\boldsymbol{X}$, nor single instant $T^{(r)}$ of decay–events of $M^{(r)}$ represents a repeatable observable. A repeatable observable requires an adequate statistics. A quite contrary situation happens in the classical theory ($\hbar = 0$) based on the Cartesian x-space free of the p–x duality of relational geometry R_3. Here the localization of an individual particle $\mathcal{M}$ at a point $\boldsymbol{X}$ represents a repeatable observable if we maintain the same dynamics together with the same p and x initial conditions which remain under our control.

Both constraints (23.9) and (23.11) imposed on the state of $\mathcal{M}$ — *i.e.* on the quantum-potential level on its existence — disagree with the symmetry L of measurement which conflicts also with the (classical) principle of relativity. Therefore, even if the relationism I_4 of two-body systems $\mathcal{M} + A$ accounts properly for the dilatation effect of T determined indirectly by relation (23.1), an indirect measurement of T, $T = \Gamma T_0 \;\; (\Gamma > 1)$, will conflict with the symmetry L of measurement. However, it turns out that the symmetry L cannot be threatened by any experiment, because the ensembles $Z(N_0, P)$ and $Z(N_0, V)$ necessary for detecting the conflict (23.15) are unrealistic ones. Here $Z(N_0, P)$ and $Z(N_0, V)$ are large ensembles of $\mathcal{M}$'s having, in some fixed lab-system $\bar{S}$, the same momentum $P = |\boldsymbol{P}|$ and the same velocity $V = |\boldsymbol{V}|$.

Thus, *the experimental possibilities* keep the conflict $T^Q \neq T^L \;\; (\Gamma > 1)$ from being disclosed by an experiment. The limited *experimental possibilities* are inherent in the p–x duality. This makes that in a fixed lab-system $\bar{S}$ we deal with $\mathcal{M}$'s of different momenta and energies and, what is even more important, the states ψ of $\mathcal{M}$'s are superposed of different momenta $\boldsymbol{P}_M$. Thus the four-momenta $P^{(r)}$ of $\mathcal{M}^{(r)}$'s become sharply defined *a posteriori*, whereas the ensembles $Z(N_0, P)$ and $Z(N_0, V)$ assume that these quantities are sharply defined *a priori* and, moreover, that all $\mathcal{M}^{(r)}$'s have the same $P = |\boldsymbol{P}|$ and $V = |\boldsymbol{V}|$ in a fixed lab-system $\bar{S}$. Such ensembles do not exist.

However, according to the L-invariance of masses $M^{(r)}$, as they are equal to: $M^{(r)} = (-{P^{(r)}}^2/c^2)^{1/2}$, the ensemble $Z(N_0)$ is a realistic ensemble which determines the spectrum of masses $M^{(r)}$ and hence, the uncertainty δM of $\mathcal{M}$. This mass uncertainty determines indirectly the proper mean life-time T_0 of $\mathcal{M}$ as stated in (22.3). In this case, the quantum–relativistic conflict disappears, because T_0 coincides numerically with the proper mean life-time T^* of $\mathcal{M}$ as, for $\Gamma = 1$, we have

$$T^* = T_0 = T^{Q*} = T^{L*} \quad (\Gamma = 1)\,. \tag{24.1}$$

Thus *the experimental possibilities* rule out the quantum–relativistic controversy and exclude, at the same time, possibility of indirect (p) measurement of dilatation effect of T.

The presented above controversy between the proper value of $T = T^Q$ determined indirectly on the quantum-potential level of an individual micro-object $\mathcal{M}$ and the impossibility of its detection (requiring an adequate statistics) is strictly connected with the Redhead *locality 5* (LOC_5) [57]. In Appendix C a similar quantum–relativistic puzzle is presented. This new puzzle is connected with the decay mode and mean life-time of the bound system of a meso-atom whose instability is due to the instability of its meson component. It is shown there that such bound states and their decay modes and mean life-times provide us with repeatable observables which can help us in deciding whether it is the spacetime (L_4) of measurement or the relational space–and–time I_4 which constitutes the true background of internal motion of meson (electron) inside atom.

Appendix A

Relationism and confinement of the constituents of $\mathcal{M}$

The more reliable becomes the quark model, the more fundamental becomes one of the model's great questions, namely that of *the confinement* of quarks [58]. Therefore, it would be interesting to indicate a possible reason of the confinement which is based on a geometrical argumentation which results from the hypothesis of relationism. The point is that the coexistence of eventism G_4 and relationism R_3^G makes that any relational shape $F(\boldsymbol{y}^2)$ of R_3^G determines automatically *the same* two-event shape in G_4 equal to $F(\boldsymbol{x}^2)\delta^{(1)}(\Delta t)$. Thus, from the above follows that NR quantum mechanics imposes no restrictions onto the structure of composite particles $\mathcal{M}$. Quite a different situation occurs in physics of finite $\hbar/c$.

The equality with dot which determines *the sameness* of the relational shapes $F(\boldsymbol{y}^2)$ in R_3^G and two-event shapes $G(x^2)$ in L_4 ($\mathcal{L}_4$) must explicitly resort to the privileged position of the p language, hence a problem arises of the existence of $G(x^2)$. The problem concerns the convergence of the corresponding integrals (13.3 *ii*) as well, as the extension of $\tilde{F}(\boldsymbol{q}^2)$ over the negative $\boldsymbol{q}^2$ values, as $\tilde{G}(p^2)$ is to be determined in the whole 4-space $\mathcal{L}_4(p)$. In consequence, the hypothesis of relationism R_3 accounting for finite $\hbar/c$ creates additional constraints which may be responsible for *the confinement.* Let us illustrate this problem by analysing of two classes of relational shapes.

Suppose that $F(\boldsymbol{y}^2)$ represents the relational shape of Yukawa potential corresponding to the exchanged particle $\mathcal{M}$ of mass M, hence

$$F(\boldsymbol{y}^2) = \frac{1}{4\pi r} e^{-Kr} , \quad r = |\boldsymbol{y}| , \quad K = \frac{Mc}{\hbar} . \tag{A.1}$$

In NR physics ($1/c = 0$) and for $M = 0$, $F(y^2)$ coincides with Coulomb interaction in R_3^G (interaction of two unit charges) which, in agreement with the dilatation symmetry D of eventism, is of infinite range. If one of these charges is infinitely heavy ($m_1 \to \infty$), equation (A.1) converts, in the G_4 and L_4 spacetimes, into the event shape of static Coulomb field in the rest frame of the infinitely heavy A_1.

In the p representation $\tilde{F}(q^2)$ takes the form

$$\tilde{F}(q^2) = \frac{1}{M^2c^2 + q^2} \tag{A.2}$$

which determines, without any change of the form, $\tilde{G}(p^2)$ in the whole 4-space $\mathcal{L}_4(p)$

$$\tilde{G}(p^2) = \frac{1}{M^2c^2 + p^2}\,. \tag{A.3}$$

The singularity of $\tilde{G}(p^2)$ on the hyper-surface $p^2 = -M^2c^2$ in $\mathcal{L}_4$ results in the well known ambiguities in x-representations of G in $\mathcal{L}(x)$. These ambiguities can be removed by indicating definite contours of integration (C) in the complex p_0-plane which leads to the Jost functions $G^{(C)}(x)$,

$$G^{(C)}(x) = \Delta^{(C)}(x; K)\,. \tag{A.4}$$

In particular, one of the contours determines the Feynman propagator — $\Delta^{(F)}(x; K)$, *i.e.* the Green's function (distribution) of the inhomogeneous free Klein–Gordon equation of the particle $\mathcal{M}$.

Thus different Green's functions (distributions) and their convolutions occupying the whole 4-space $\mathcal{L}_4(x)$ of the quantum–relativistic perturbation theory provide us with the first class of relational shapes for which *the same* two-event shapes are well defined. This would explain why the quantum–relativistic perturbation theory reflects correctly relationism R_3, without explicitly abandoning eventism L_4. There is, however, a second class of relational shapes $F(y^2)$ in R_3 for which *the same* two-event shapes do not exist, just because of the divergence of the corresponding integrals determining $G(x^2) \doteq F(y^2)$. Let us confine our attention to the relational shape $F(y^2)$ whose analytic form is that of the NR wave function of harmonic oscillator

$$F(y^2) = \frac{1}{(2\pi)^{3/2}}\frac{1}{R^3}\exp\left(\frac{-y^2}{2R^2}\right)\,,$$
$$\tilde{F}(q^2) = \exp\left(-\frac{R^2q^2}{2\hbar^2}\right)\,. \tag{A.5}$$

Direct analytic continuation of $\tilde{F}(q^2)$ onto negative q^2 (like in the previous example) results in

$$\tilde{G}(p^2) = \exp\left(-\frac{R^2p^2}{2\hbar^2}\right), \tag{A.6}$$

however, the time-like four-momenta p ($p^2 < 0$) result in a strong divergence of integral (13.3 *ii*) making $G(x^2) \doteq F(y^2)$ (embedded in $\mathcal{L}_4(x)$) undefined.

This would mean that, in the p language of measurement, we can speak of structures composed of some more elementary *units* (like quarks) which do affect the corresponding S matrix elements, but which can never reach the surface of relativistic kinematics. In other words, this means *the confinement* of such structures. Such a confinement would have a purely *relativistic* origin connected with the four-momentum notion which is alien to eventism G_4. Indeed, the corresponding two-event shape in G_4 is well defined in the whole 4-space $\mathcal{G}_4(x)$ and is equal to

$$G(x) = \frac{1}{(2\pi R^2)^{3/2}} \exp\left(-\frac{x^2}{2R^2}\right) \delta^{(1)}(\Delta T). \tag{A.7}$$

Appendix B

Collisions of composite particles

The hypothesis of relationism R_3 combined with the phenomenological quantum perturbation theory enable one to account for creation and annihilation of particles synthetized from more elementary ones or those obtained via the field mechanism. This is due to the fact that the geometrical meta-objects R_3 and $\mathcal{L}_4$ with their p–x aspects make room for extended particles consistent with the symmetry L of measurement, with dynamical structures determined first in I_4. For the sake of illustration we shall evaluate (in the lowest–order approximation) the S matrix element describing the collision of a scalar and point–particle A_3 with an extended particle $\mathcal{M} = A_1 + A_2$ composed of two hypothetical scalar and also point-like particles A_1 and A_2. Moreover, we assume that A_3 interacts with $\mathcal{M}$ via the A_1 constituent only and, therefore, the evaluated cross–section will detect the structure of the corresponding form factor of $\mathcal{M}$.

Let the initial and final states of $\mathcal{M}$ be the bound eigenstates $\psi_{i,f}(y^2)$ of $\widehat{h}$ with L–absolute masses $M_{i,f}$, respectively ($M_f > M_i$). Spherical symmetry of the states $\psi_{i,f}$ in R_3 means that the composite particles $M_{i,f}$ are also scalar ones. According to (16.3), relationism R_3 provides us with an L-absolute form factor F_{fi} of $\mathcal{M}$ which will enter the corresponding matrix element S_{fi} Consequently, we must know *the same* form factor represented in $\mathcal{L}_4$ which is realized by the equality with dot

$$F_{fi}(y_{12}^2) = \psi_f^*(y_{12}^2)\psi_i(y_{12}^2) \doteq G_{fi}(x_{12}^2)\,. \tag{B.1}$$

Here y_{jk} denotes the relational coordinate of the constituents A_j and A_k in $R_3(y)$, while $x_{jk} = X_k - X_j$ parametrizes the corresponding configuration subspace $\mathcal{L}(x_{jk})$. In our three-body system, $R_3(y_{12}) \otimes R_3(y_{13})$ and $\mathcal{L}_4(x_{12}) \otimes \mathcal{L}_4(x_{13})$ are the corresponding configuration spaces, with A_1 being taken as the origin of the reference frame S_3 parametrizing $R_3(y)$. Of course, one can choose another parametrization of the above configuration spaces provided that *the same* transformation concerns the y and the x variables.

Let $V(y_{13}^2)$ be the relational shape of interaction between A_3 and A_1 which — much like the form factor $F_{fi}(y_{12}^2)$ — becomes measurable in the terms of cross-section and hence, it must also be expressed by *the same* interaction $U(x_{13}^2)$ in $\mathcal{L}_4(x_{13})$, in accordance with

$$V(y_{13}^2) \doteq U(x_{13}^2)\,. \tag{B.2}$$

This interaction can be of quite a different nature from the interaction responsible for the structure of the form factor $F_{fi} \doteq G_{fi}$ of $\mathcal{M}$. The lowest-order Born approximation, in which the matrix element S_{fi} will be evaluated, concerns the interaction V from (B.2). Before putting forward an analytic expressions for S_{fi} let us point out two reasons which make that the relationism R_3 results in some R_3-effects alien to eventism L_4. These effects make the hypothesis of relationism R_3 experimentally testable.

The first reason is the L-form invariance of $G_{fi}(x_{12}^2)$ which becomes thus separated from the external motion of its carrier $\mathcal{M}$. This makes that G_{fi} suffers no relativistic distortions. However, as it is known from elastic electron–proton collisions, any test of *the existence or non-existence* of relativistic distortions requires very large *(relativistic)* momentum transfers — *cf.* Section 10. Note that form factors $\tilde{F}_{fi}(\boldsymbol{q})$ of loosely bound structures found in the NR approximation cannot be used in the case of relativistic Fermi momenta $\boldsymbol{q}$.

There is, however, a second R_3-effect also mentioned in paper [55] which should occur in relatively low-energy collisions of A_3 and $\mathcal{M} = A_1 + A_2$ and which is connected with the weight a. This weight depends on the mass of $\mathcal{M}$ (*cf.* (18.5)). Consequently, inelastic collisions $M_i \to M_f$, with $M_f > M_i$, take place with *a jump* of the centre-of-mass of $\mathcal{M}$ in accordance with

$$X_i = a_i X_1 + b_i X_2 \to a_f X_1 + b_f X_2 = X_f \neq X_i\,, \tag{B.3}$$

with $b_{i,f} = 1 - a_{i,f}$. This results in *the jump-effect* mentioned above. Note that *the jump-effect*, much like the mass defect for bound structures, vanishes in the NR framework ($1/c = 0$), because in the limit $1/c = 0$ we get $a_i = a_f = a^G$. However, in the NR approximation which accounts for finite universal constant $\hbar/c$, the jump effect becomes crucial for the hypothesis of relationism R_3.

Note that our simplifying assumption of A_3 interacting solely with A_1 is a realistic one in (for instance) electron–atom collisions. From experiments we know that the interaction between impinging electron and atom is very well approximated by the sum of two interactions: that of (impinging) electron with atomic electrons and that of electron with atomic nucleus. Then, for $\tilde{t}$ large enough ($\tilde{t} \gg \hbar^2/r_B^2$), one can pick up from the electron–atom cross–section the part which describes the electron–nucleus collision and, at the same time, maintains the atom excited ($M_i \to M_f$) but still unfragmented. The cross-section for this exclusive reaction will be relevant in analysis of the jump effect and the S_{fi} elements (evaluated further on) will concern only this fraction of the electron–atom collision. We shall see that in spite of $\tilde{t} \gg \hbar^2/r_B^2$), in evaluating S_{fi} it will be fully justified to use the internal $\psi_{i,f}$ states of atom evaluated in the NR approximation. Indeed, we shall see that the argument q^2 parametrizing the p representation of the form factor $\tilde{F}_{fi}(q^2) \doteq \tilde{G}_{fi}(p^2 = q^2)$ fulfills the strong inequality (20.3).

In the assumed Born approximation, the initial and final 3–body states $\Psi_{i,f}$ take the form

$$\left.\begin{aligned} \Psi_i &= A_i A_{3i} \exp\left\{\frac{i}{\hbar}\left[P_i(a_i X_1 + b_i X_2) + P_{3i} X_3\right]\right\}\psi_i(y_{12}^2) \\ \Psi_f &= A_f A_{3f} \exp\left\{\frac{i}{\hbar}\left[P_f(a_f X_1 + b_f X_2) + P_{3f} X_3\right]\right\}\psi_f(y_{12}^2) \end{aligned}\right\} \tag{B.4}$$

with

$$P_{i,f}^2 = -M_{i,f}^2 c^2\,, \quad P_{3i}^2 = P_{3f}^2 = -m_3^2 c^2\,.$$

In agreement with the relativistic kinematics of asymptotic zone, these states separate the external (relative) degrees of freedom of A_3 (electron) and $\mathcal{M}$ as a whole (atom) embedded in L_4 from the L–absolute (relational) degrees of freedom parametrizing internal states of an $\mathcal{M}$ *hidden a priori* in R_3.

This is due to the singularity of the NR framework ($1/c = 0$) in which $\psi_{i,f}^G(y_{12}^2)$ embedded in R_3^G are, at the same time, embedded in the external spacetime G_4 of measurement, because R_3^G coexists with G_4. Therefore, NR quantum mechanics can preserve the classical philosophy of equations of motion according to which the states of system $\mathcal{M}$ under description evolve in all system's degrees of freedom with continuously increasing parameter t denoting the absolute Newtonian time. Consequently, in the Born approximation, the S_{fi} element takes the form

$$S^G_{fi} = \langle \Psi^G_f | V(\mathbf{x}^2_{13}) | \Psi^G_i \rangle \tag{B.5}$$

represented by a (9+1)-fold integral over the $\mathbf{X}_{1,2,3}$ coordinates and the single time variable.

The true principle of relativity expressed by symmetry L that must be respected by all repeatable observables — in particular, by cross-section deduced from S_{fi} — requires S_{fi} to be of an L covariant structure. Relationism R_3 will satisfy this requirement provided that: *(i)* the L-absolute interaction $V(\mathbf{y}^2_{13})$ is replaced by *the same* L-form invariant interaction $U(x^2_{13}) \doteq V(\mathbf{y}^2_{13})$, and *(ii)* the form factor $F_{fi}(\mathbf{y}^2_{12})$ is also replaced by *the same*, explicitly L–form invariant form factor $G_{fi}(x^2_{12}) \doteq F_{fi}(\mathbf{y}^2_{12})$. Assumption *(i)* is realized by the quantum–relativistic perturbation theory of local fields, as this theory provides us with L-form invariant propagators and their convolutions over the whole 4-space $\mathcal{L}_4(x)$ so the locality of eventism L_4 is overcome. An essential *novum* introduced with relationism R_3 is the nonlocality of the L-form invariant form factors $G(x^2)$, as local fields admit point–particles only with their universal (local) L-form invariant form factor $\delta^{(4)}(x)$. Consequently, in opposition to the form factor $F_{fi}(\mathbf{y}^2_{12})$ from (B.1), the L-form invariant form factor $G_{fi}(x^2_{12}) \doteq F_{fi}(\mathbf{y}^2_{12})$ ceases to be factorizable into initial and final states. In agreement with Landau's opinion [38], the presence of $G_{fi}(x^2_{12})$ in the integrand of S_{fi} shows clearly that the S matrix cannot be obtained from states evolving continuously with the time parameter of any reference frame S in L_4 as required by Moeller's matrices.

The two above assumptions *(i)* and *(ii)* result in the following form of S_{fi}

$$\begin{aligned} S_{fi} =& (A^\dagger_f A_i)(A^\dagger_{3f} A_{3i}) \int d^4X_1 \int d^4X_2 \int d^4X_3 \\ & \times \exp\left\{ \frac{i}{\hbar} \left[P_i(a_i X_1 + b_i X_2) + P_{3i} X_3 \right] \right\} \\ & \times \exp\left\{ -\frac{i}{\hbar} \left[P_f(a_f X_1 + b_f X_2) + P_{3f} X_3 \right] \right\} \\ & \times G_{fi} \left[(X_2 - X_1)^2 \right] U \left[(X_3 - X_1)^2 \right] \\ =& (A^\dagger_f A_i)(A^\dagger_{3f} A_{3i})(2\pi\hbar)^4 \delta^{(4)}(P_i + P_{3i} - P_f - P_{3f}) \\ & \times \tilde{G}_{fi} \left[(b_i P_i - b_f P_f)^2 \right] \tilde{U}(\tilde{t}) , \end{aligned} \tag{B.6}$$

where

$$\tilde{t} = (P_i - P_f)^2 = (P_{3f} - P_{3i})^2 .$$

In the privileged p language of S matrix, the jump–effect manifests itself in the argument $(b_i P_i - b_f P_f)^2$ of $\tilde{G}_{fi}$, *i.e.* in the fact that the four-vector $(b_i P_i - b_f P_f)$ is not parallel to the four-momentum transfer $(P_i - P_f)$ which takes place between A_3 and $\mathcal{M}$.

Equation (16.7) which expresses L-form invariantly the orthonormality of states $\psi_{i,f}(y^2_{1,2})$ for $M_i \neq M_f$ (inelastic collision) results in $\tilde{G}_{fi}(0) = 0$ in the p language and hence, if

$$(b_i P_i - b_f P_f)^2 = 0 \tag{B.7}$$

S_{fi} and the corresponding cross-sections vanish. Assuming that

$$\Delta M = M_f - M_i = \Delta W / c^2 \ll \min(m_1, m_2) \tag{B.8}$$

which is fulfilled in the case of loosely bound atoms, equation (B.7) determines $\tilde{t} = \tilde{t}_0$

$$\tilde{t}_0 = \left[\left(\frac{m_1}{m_2}\right)^2 - 1\right]\left(\frac{\Delta W}{c}\right)^2 . \tag{B.9}$$

If $m_1 = AM_N$ is the nucleus mass (M_N denotes the nucleon mass and A is the mass number) and $m_2 = m_e$ $(m_1 \gg m_2)$, $\tilde{t}_0$ is a positive quantity. Taking ΔW equal to the difference between the internal energy levels of the first excited state and the ground state of a hydrogen-like atom, we obtain $\tilde{t}_0$ equal to

$$\tilde{t}_0 = 360 A^2 Z^2 \quad \left[\frac{\text{keV}}{c^2}\right] , \tag{B.10}$$

where Z is the charge number of the nucleus.

Thus, the detection of the jump effect does not require impinging electrons to be very high–energetic ones. A characteristic Fermi–momentum square q_0^2 of hydrogen-like atom is equal to

$$q_0^2 \simeq 13\, Z^2 \quad \left[\frac{\text{keV}}{c}\right]^2 , \tag{B.11}$$

hence $\tilde{t}_0$ is much greater than q_0^2,

$$\frac{\tilde{t}_0}{q^2} \approx 30\, A^2 , \tag{B.12}$$

but it does not conflict with the NR approximation used in evaluating $F_{fi} \doteq G_{fi}$. Indeed, as it follows from (B.7), the jump effect manifests itself in the vicinity of $\tilde{t}_0$ which corresponds to the argument q^2 of $\tilde{F}_{fi}(q^2)$ being equal to

$$q^2 = (b_i P_i - b_f P_f)^2 = (b^G)^2(\tilde{t} - \tilde{t}_0), \quad b^G = \frac{m_2}{m_1 + m_2}. \qquad \text{(B.13)}$$

Thus even for $\tilde{t} = 2\tilde{t}_0$ the argument q^2 of $\tilde{F}_{fi}(q^2)$ takes the value

$$q^2 = (b^G)^2 \tilde{t}_0 = \left(\frac{\Delta W}{c}\right)^2 \ll 2m_e \Delta W \simeq q_0^2 . \qquad \text{(B.14)}$$

Note that the jump effect affects a very small fraction of the total electron–atom cross-section, namely it affects these cases when electron collides with nucleus inducing an excitation of atom from the atom's ground state to the first excited one.

Of course, the jump effect disappears for elastic collisions when $a_i = a_f$ and also for inelastic collisions with an $\mathcal{M}$ composed, like positronium, of two particles of equal masses as in such case $a_i = a_f = {}^1\!/_2$, no matter whether the collision is inelastic or elastic. As it has been said before and can be seen from (B.9), the jump effect disappears in the NR limit ($c \to \infty$). There is no room for it within the local field theory either, as it is strictly connected with the structure of a particle $\mathcal{M}$ synthetized mechanically from other *more elementary* particles.

Let us analyze two different Lorentz limits of S_{fi}, namely the case when A_3 becomes infinitely heavy ($m_3 \to \infty$) and another case, when A_2 and, consequently, $\mathcal{M} = A_1 + A_2$ become infinitely heavy. As it has been shown in Section 17, in the first case the configuration subspace $R_3(\boldsymbol{y}_{13})$ becomes isomorphic with the E_3^* space of the rest frame S^* of infinitely heavy A_3 embedded in L_4. By identifying the world–line of infinitely heavy A_3 with the time axis of S^*, one can replace the relational coordinate $\boldsymbol{y}_{13}$ with $\boldsymbol{X}_1^*$ and the internal time τ coincides (up to an additive constant) with the time t^* of S^*.

The Lorentz limit of the configuration space $R_3(\boldsymbol{y}_{13})$, however, does not affect the relational nature of the second configuration subspace $R_3(\boldsymbol{y}_{12})$ in which relational shape $F_{fi}(\boldsymbol{y}_{12}^2)$ ($\doteq G_{fi}(\mathrm{x}_{12}^2)$) is embedded. This is the fact which points to an essential difference between eventism L_4 and relationism R_3. It should be noted that this aspect of relationism could not be explicitly perceived in the Lorentz limit of the two-body problem discussed in Section 17. Thus, an infinitely heavy A_3 becomes the source of an external dynamical field given by event shape $U(X_1)$ which, being such a shape, ceases to represent the L-form invariant shape. Of course, the external field

$U(X_1)$ breaks the L-form invariance of S_{fi} which reveals itself in the violation of the momentum conservation for the system $\mathcal{M}$ in S^*. Nevertheless, the L covariant expression of S_{fi} guarantees the described process to remain consistent with STR which requires only a passive interpretation of symmetry L.

The infinitely heavy centre A_3 ceases to participate in the motion of $\mathcal{M} + A_3$ and *freezes* four degrees of freedom of the initially isolated system $\mathcal{M}+A_3$. Hence, S_{fi} is now given by an 8-fold integral over $X_{1,2}$ coordinates of L_4. The L–covariance of S_{fi} justifies to evaluate S_{fi} in S^* where $U(X_1)$ takes the form of a static event shape which leads to energy conservation of $\mathcal{M}$ in S^*. Thus we come to deal with

$$\begin{aligned} S_{fi} =&(A_f^\dagger A_i) \int d^4X_1^* \int d^4X_2^* \\ &\times \exp\left\{\frac{i}{\hbar}\left[P_i^*(a_iX_1^* + b_iX_2^*) - P_f(a_fX_1^* + b_fX_2^*)\right]\right\} \\ &\times G_{fi}\left[(X_2^* - X_1^*)^2\right] U(\boldsymbol{X}_1^{*2}) \\ =&(A_f^\dagger A_i)(2\pi\hbar)\delta^{(1)}(P_{i0}^* - P_{f0}^*)\tilde{G}_{fi}\left[(b_iP_i - b_fP_f)^2\right]\tilde{U}(\tilde{t})\,. \end{aligned} \tag{B.15}$$

As it was to be expected, the jump effect survives the Lorentz limit $m_3 \to \infty$ while the energy conservation of $\mathcal{M}$ in S^* takes the form

$$E_i^* = c\sqrt{M_i^2c^2 + \boldsymbol{P}_i^{*2}} = c\sqrt{M_f^2c^2 + \boldsymbol{P}_f^{*2}} = E_f^*, \tag{B.16}$$

$$\tilde{t} = (P_i - P_f)^2 = (\boldsymbol{P}_i^* - \boldsymbol{P}_f^*)^2\,.$$

We are going to analyze the second Lorentz limit of S_{fi} from (B.6), when the constituent A_2 of $\mathcal{M} = A_1 + A_2$ becomes infinitely heavy ($m_2 \to \infty$) but A_3 has still a finite mass m_3. The configuration subspace $R_3(y_{12})$ becomes now isomorphic with the E_3^* space of the rest frame S^* of A_2 (and, at the same time, of $\mathcal{M} = A_1 + A_2$). Four degrees of freedom of A_2 become frozen, as the infinitely heavy A_2 acquires a given classical world–line which can be identified with the 0–t^* axis of S^*. Consequently, $y_{12} \to \boldsymbol{X}_1^*$ and the L-absolute relational shape of form factor $F_{fi}(y_{12}^2)$ converts into an L-scalar event shape represented in S^*, where it takes the form of a static and spherically symmetric shape similarly as the interaction $V(y_{13}^2 = \boldsymbol{X}_1^{*2})$ in the Lorentz limit of the previous example,

$$F_{fi}(y_{12}^2) \longrightarrow F_{fi}(y_{12}^2 = \boldsymbol{X}_1^{*2})\,. \tag{B.17}$$

Again, the L covariant form of S_{fi} justifies one to evaluate S_{fi} in S^* with the 8-fold integral taking the form

$$\begin{aligned} S_{fi} =&(A^\dagger_{3f}A_{3i}) \int d^4X_1^* \int d^4X_3^* \\ &\times \exp\left\{\frac{i}{\hbar}\left[(a_iP_i^* - a_fP_f^*)X_1^* + (P_{3i}^* - P_{3f}^*)X_3^*\right]\right\} \\ &\times F_{fi}\left[(X_1^*)^2\right] U\left[(X_1^* - X_3^*)^2\right] . \end{aligned} \tag{B.18}$$

Although in the limit, $m_2 \to M_{i,f} \to \infty$, and hence

$$a_{i,f} = \tfrac{1}{2}\left[1 - \frac{m_2^2 - m_1^2}{M_{i,f}^2}\right] \xrightarrow[m_2\to\infty]{} 0\,, \tag{B.19}$$

the four-vectors $(a_{i,f}P_{i,f}^*)$ in the exponent in (B.18) do not vanish in the limit $m_2 \to \infty$, because the quantities $(P_{i,f}^*)_0$ become infinitely large.

If we put

$$M_{i,f} = m_1 + m_2 - \Delta M_{i,f}, \quad \Delta M = M_f - M_i = \Delta M_i - \Delta M_f\,, \tag{B.20}$$

we find easily that

$$\lim_{m_2\to\infty} a_{i,f}P_{i,f}^* = (0,0,0;-c\,M_{i,f})\,. \tag{B.21}$$

Finally, after inserting (B.21) into (B.18), S_{fi} takes the form

$$\begin{aligned} S_{fi} = &(2\pi\hbar)(A^\dagger_{3f}A_{3i})\delta^{(1)}\left[(P_{3i}^*)_0 - (P_{3f}^*)_0 - c\Delta M\right] \\ &\times \tilde{F}_{fi}\left[(\boldsymbol{P}_{3i}^* - \boldsymbol{P}_{3f}^*)^2\right]\tilde{U}(\tilde{t})\,. \end{aligned} \tag{B.22}$$

According to the energy conservation, the argument $(\boldsymbol{P}_{3i}^* - \boldsymbol{P}_{3f}^*)^2$ of the L–absolute form factor $\tilde{F}_{fi}$ is equal to

$$(\boldsymbol{P}_{3i}^* - \boldsymbol{P}_{3i}^*)^2 = \tilde{t} + (\Delta Mc)^2 = \tilde{t} + \left(\frac{\Delta W}{c}\right)^2, \quad \tilde{t} = (P_{3i} - P_{3f})^2. \tag{B.23}$$

Thus the energy gap ΔW between M_f and M_i shifts the argument $\tilde{t}$ of $\tilde{F}_{fi}$ towards a larger value $\tilde{t} + (\Delta W/c)^2$ which is but very little different from $\tilde{t}$. Nevertheless, this is also an R_3–effect ($\hbar/c \neq 0$) which vanishes in the NR framework ($1/c = 0$), as $\lim_{c\to\infty}(\Delta W/c) = 0$.

The two Lorentz limits (B.15) and (B.22), admitted by the 3-body problem, of the S matrix element S_{fi} established in (B.6) show clearly the reason for the priority of relationism R_3 and its configuration spaces $R_3 \otimes R_3 \ldots$ over eventism L_4, namely the fact that the universal spacetime L_4 of measurement induces *a priori* all possible configuration spaces $\mathcal{L}_4 \otimes \mathcal{L}_4 \otimes \ldots$. Let us emphasize once more that the hypothesis of *hidden* relational space is possible because of the finiteness of universal constant $\hbar/c$ and vice versa: all R_3 effects follow from the finite value of this constant.

Appendix C

Decay mode of meso-atom

The decay mode of a meso-atom in its ground state ψ_0 and the value of its proper mean life-time analyzed in paper [59] provide us with some repeatable observables which can be used for deciding whether the internal motion of the constituents of a composite bound structure $\mathcal{M}$ takes place on the background of external spacetime L_4 (eventism) or in the internal relational space–and–time I_4 induced by the elementary two-body system (relationism) and vanishing together with $\mathcal{M}$. We shall consider the ground state ψ_0 of a meso-atom whose instability is due uniquely to the instability of the meson component.

Loosely bound atoms justify the use of NR approximation of the internal structure of meso-atom which results (by virtue of the p–x duality) in internal motion of the constituents. According to STR, this motion should result in dilatation of meson's mean life-time and, consequently, of the mean life-time of the meso-atom as a whole. In the assumed NR approximation, relational Fermi momenta $\boldsymbol{q}$ parametrizing internal state $\tilde{\psi}_0$ of meso-atom in the p representation determine velocities $\boldsymbol{v}^*$ of meson in the rest frame S^* of meso-atom, as we have

$$\boldsymbol{v}^* = \frac{\boldsymbol{q}}{\overset{o}{m}}, \quad \gamma^* \simeq 1 + \tfrac{1}{2}\left(\frac{\boldsymbol{v}^*}{c}\right)^2 . \tag{C.1}$$

Here, $\overset{o}{m}$ is the mass of muon and γ^* is the corresponding dilatation factor (in the NR approximation). The spectrum of Fermi relational momentum squares $\boldsymbol{q}^2$ contained in $\tilde{\psi}_0(\boldsymbol{q})$ results then in a spectrum of Lorentz factors γ^* which should affect the life time and the decay mode of the meso-atom.

From experiments we know that a free muon decays according to the one-exponential (OE) law. In Gamov's phenomenological description of meta-stable states or particles, the OE decay mode of such systems results from the complex *internal energy* w which replaces the real internal energy $\overset{o}{w}$ corresponding to stable particles (stationary states). Thus

$$w = \overset{o}{w} - \frac{i\,\delta w}{2} = \left(\overset{o}{m} - \frac{i\,\delta m}{2}\right) c^2 , \tag{C.2}$$

where the well–defined uncertainty δw of internal energy results in the OE decay mode of the state. The experimentally determined value of the proper mean life-time T_0 of muon and hence, that of the internal energy uncertainty δw, amounts to [60]

$$T_0 = \frac{\hbar}{\delta w} = 2.19703 \times 10^{-6} \text{ sec.} \tag{C.3}$$

From (C.1) we obtain the velocity distribution of meson in S^* given by

$$\pi_0(\boldsymbol{v}^*) = \overset{o}{m}{}^3 |\tilde{\psi}_0(\boldsymbol{q} = \overset{o}{m}\,\boldsymbol{v}^*)|^2 , \tag{C.4}$$

which — according to eventism — determines the spectrum of Lorentz factors γ^*. Consequently, as the instability of meso-atom is due to the instability of muon only, the fractional–potential existence of meso-atom at the instant $t^* \geq 0$ is given by

$$\Pi(t^*; 0) = \int d^3v^* \pi_0(\boldsymbol{v}^*) \mathrm{e}^{-(t^*/T^*)}, \quad t^* \geq 0 , \tag{C.5}$$

$$T^* = \gamma^* T_0 ,$$

under assumption that the meso-atom existed at the instant $t^* = 0$. Since $\pi_0(\boldsymbol{v}^*)$ deals with different values of $\boldsymbol{v}^{*2}$, the dependence of $\Pi(t^*; 0)$ deviates from an exponential one which means that the eventistic picture results in a violation of the OE decay mode of meso-atom.

The quantum-potential picture of the relational motion of meson and nucleus in I_4 leads to quite a different description of the decay mode of meso-atom. This difference is strictly connected with the fundamental difference between bound and scattering states of a composite system $\mathcal{M} = A_1 + A_2$ analyzed in Section 18. Let us recall that, according to relationism, four-momenta $P_{1,2}$ are undefined — *cf.* (18.9) — which is due to the 4–degree freedom of the relative four-momentum p. In consequence, $\boldsymbol{v}^* = \boldsymbol{q}/\overset{o}{m}$ cannot be interpreted as the velocity of meson in spacetime L_4 and represented in S^*. Below we show that the quantum-potential motion of meson inside atom maintains the OE decay mode of meso-atom, although the proper mean life-time of meso-atom $T^{(0)}$ is longer than T_0, attesting to the fact that the meson is not at rest in the meso-atom.

Let us remark that the bound eigenstates of internal Hamiltonian $\widehat{h}$ in R_3 determine internal energy levels $W_n = M_n c^2$ $(M_n < m = m_1 + m_2)$ of $\mathcal{M} = A_1 + A_2$ which are well defined functions of: *(i)* masses m_J of $\mathcal{M}$'s constituents; *(ii)* appropriate dynamical constants and *(iii)* universal constants $\hbar$ and c. Thus we have

$$W_n = M_n c^2 = W_n(m_J; \beta) , \tag{C.6}$$

where β symbolizes the dependence of W_n on *(ii)* and *(iii)*. Let us assume that— similarly as in the case of a meso-atom in its ground state ψ_0 — the instability of a bound state ψ_n of $\mathcal{M}_n$ is due to the instability of one of the $\mathcal{M}_n$'s constituents, say the A constituent. The A constituent is characterized, when free, by a mass $\overset{o}{m}$ and a mass uncertainty δm which means, in accordance with Gamov's phenomenology, that a free A has the OE decay mode with a proper mean life-time $T_0 = \hbar/(\delta m\, c^2)$.

The point is that according to (C.6) the well-defined uncertainty δm, which underlies the OE decay mode of free particle A, results in an also well-defined uncertainty of mass M_n of $\mathcal{M}_n$ as

$$\delta W_n = \left.\frac{\partial W_n}{\partial m}\right|_{m=\overset{o}{m}} (\delta m) = \delta M_n\, c^2\,. \tag{C.7}$$

Thus $\mathcal{M}_n$ decays in an OE–mode with the proper mean life-time $T^{(n)}$ equal to

$$T^{(n)} = \frac{\hbar}{\delta W_n} = \left.\left(\frac{\partial M_n}{\partial m}\right)^{-1}\right|_{m=\overset{o}{m}} \left(\frac{\hbar}{\delta w_n}\right) = \gamma^{(n)} T_0, \tag{C.8}$$

$$\gamma^{(n)} = \left.\left(\frac{\partial M_n}{\partial m}\right)^{-1}\right|_{m=\overset{o}{m}} > 1\,.$$

The inequality $\gamma^{(n)} > 1$, which can be proved quite generally, is illustrated below on the example of a hydrogen-like atom.

Apart from the instability of meson, the ground state ψ_0 of meso-atom deals with internal energy $\overset{o}{W}$ of $\mathcal{M}_0$ which, in our NR approximation, amounts to

$$\overset{o}{W} = \overset{o}{M} c^2 = [(M_J + \overset{o}{m}) - \tfrac{1}{2}\alpha^2 Z^2 \mu + 0(\alpha^3)]c^2\,, \tag{C.9}$$

where M_J, $\overset{o}{m}$ denote the masses of nucleus and muon, respectively, μ is their reduced mass and $\overset{o}{M}_k$ is the also L-absolute mass of meso-atom in the ground state ψ_0. The term $0(\alpha^3)$ stands for all higher than second order corrections of $\alpha = 1/137$. These corrections transgress the boundaries of the NR approximation used here. If instead of real mass $\overset{o}{m}$ of muon we insert $m = \overset{o}{m} - i\delta m/2$, which would account for the muon's OE decay mode, the stationary ground state converts into an unstable one, leading — as it can be seen from (C.8) — to the OE decay mode of $\mathcal{M}_0$. Equation (C.9) is a particular case of equality (C.6), hence we end up with a proper mean life-time $T^{(0)}$ of $\mathcal{M}_0$ which, by virtue of (C.8), is equal to

$$T^{(0)} = \gamma^{(0)} T_0, \quad \gamma^{(0)} = 1 + \frac{\frac{\alpha^2 Z^2}{2}}{(1 + \frac{\overset{o}{m}}{M_J})^2} > 1\,. \tag{C.10}$$

Thus, the quantum–relational picture, in opposition to the eventistic one, promotes the OE decay mode of meso-atom as *inherited* from the constituent muon, although $T^{(0)} > T_0$. In paper [59] some measurable characteristics have been presented which make possible to distinguish between the OE decay mode of meso-atom predicted by quantum relationism and the non-OE decay mode following from eventism L_4. Of course, the established geometrical background (I_4 or L_4) of the meson motion inside atom would also be valid for the electron motion inside an *ordinary* atom.

REFERENCES

[1] H. Minkowski, *Physikalische Zeitschrift* **10**, 104 (1909).

[2] N. Maxwell, *Philosophy of Science* **52**, 23 (1985).

[3] A. Einstein, *The Meaning of Relativity*, Princeton Univ. Press, Princeton, New Jersey 1945, 2nd ed.; A. Einstein, *Geometria i doświadczenie*, "Renaissance", David Erdtracht, Wiedeń–Lwów–Berlin–New York, in Polish.

[4] W. Heisenberg, *Physics and Philosophy*, Książka i Wiedza, Warszawa 1965, in Polish; W. Heisenberg *Quantum Theory and its Interpretation* in *Niels Bohr*, North–Holland, Wiley, New York 1967.

[5] J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik*, Springer Verlag, Berlin 1932.

[6] L. Landau, R.E. Peierls, *Z. Phys.* **69**, 56 (1931).

[7] N. Bohr, *Atomic Theory and the Description of Nature*, Cambridge University Press 1934; N. Bohr, *Atomic Physics and Human Knowledge*, Wiley, New York 1958.

[8] M.L. Goldberger, K.M. Watson, *Collision Theory*, Wiley, New York 1964.

[9] A. Einstein, B. Podolsky, N. Rosen, *Phys. Rev.* **47**, 777 (1935).

[10] J.S. Bell, *Rev. Mod. Phys.* **38**, 447 (1966); *Physics* (NY) **1**, 195 (1964).

[11] A. Aspect, P. Grangier, G. Roger, *Phys. Rev. Lett.* **47**, 460 (1981); A. Aspect, J. Dalibard, G. Roger, *Phys. Rev. Lett.* **49**, 1804 (1982).

[12] J.F. Clauser, A. Shimonny, *Rep. Prog. Phys.* **41**, 1881 (1978).

[13] P.W. Bridgman, *The Nature of Physical Theory*, Dover Publ., Princeton University Press 1936.

[14] E. Wigner, *Gruppentheorie und ihre Anwendungen*, Fried. Vieweg, Braunschweig 1931; *Phys. Rev.* **40**, 749 (1932).

[15] C.F. v. Weizsaecker, Symposium on the Foundation of Modern Physics, ed. Peter Mittelstaedt, Univ. of Cologne, 1985, pp. 223–237.
[16] S.G. Currie, T.F. Jordan, E.C.G. Sudarshan, *Rev. Mod. Phys.* **35**, 350 (1963).
[17] E.H. Kerner, *The Theory of Action-at-a-Distance in Particle Dynamics*, ed. E. H. Kerner, Gordon and Breach, New York 1973.
[18] R. Feynman, *Theory of Fundamental Processes*, W. A. Benjamin Inc., New York 1961, p.145.
[19] R. Haag, *Mat. Phys. Danske Vid. Selsk.* **29** (1955); R. Haag, B. Schroer, *J. Math. Phys.* **3**, 248 (1962).
[20] J. Froehlich, *Nucl. Phys.* **B200**, 281 (1982).
[21] D. Bohm, *Ukryty Porządek*, Pusty Obłok, Warszawa 1988, in Polish.
[22] H.A. Bethe, E.E. Salpeter, *Phys. Rev.* **84**, 1231 (1951).
[23] F. Gross, *Phys. Rev.* **140**, B410 (1965); **142**, 1025 (1966); B.M. Casper, F. Gross, *Phys. Rev.* **155**, B410 (1967); **142** 1607 (1966).
[24] A.L. Licht, A. Pagnamenta, *Phys. Rev.* **D2**, 1150 (1970).
[25] W. Glaser, W. Zimmerman, *Z. Phys.* **134**, 346 (1952).
[26] H. Rauch *et al.*, *Phys. Lett.* **54A**, 425 (1975).
[27] K.A. Steinhauser *et al.*, *Phys. Rev. Lett.* **44**, 1306 (1980).
[28] The reference could not be indentified.
[29] The reference could not be indentified.
[30] P.A.M. Dirac, *Rev. Mod. Phys.* **21**, 392 (1949).
[31] P.A.M. Dirac, R.E. Peierls, M.H.L. Pryce, *Proc. Camb. Phil. Soc.* **38**, 193 (1942).
[32] L.L. Foldy, *Phys. Rev.* **122**, 275 (1961); L.L. Foldy, R.A. Krajcik, *Phys. Rev. Lett.* **32**, 1025 (1974).
[33] A.S. Eddington, *Proc. Camb. Phil. Soc.* **35**, 186 (1939).
[34] R. Blankenbecler, L. Cook, *Phys. Rev.* **119**, 745 (1960).
[35] R.E. Cutkosky, *Phys. Rev.* **125**, 745 (1962).
[36] V. A. Fock, *Usp. Fiz. Nauk* **46**, 162 (1951); **66**, 599 (1958).
[37] C. Moeller, *Kgl. Danske, Vid. Sels, Math. Phys. Medd.* **23**, no 1 (1945); **26**, no 19 (1946); M. Gell–Mann, M.L. Goldberger, *Phys. Rev.* **91**, 398 (1953).
[38] W.B. Bierestecki, E.M. Lifszyc, L.P. Pitajewski, *Relatywistyczna Teoria Kwantów,* PWN, Warszawa 1972, pp. 15–18, in Polish.
[39] The reference could not be indentified.
[40] A. Grünbaum, *Basic Issue in the Philosophy of Time*, ed. E. Freeman and W. Salers, Open Court, Lassale III 1971; A. Grünbaum, *Philosophical Problems of Space and Time*, D. Reidel, Dordrecht 1973; H. Putnam, *The Journal of Philosophy* **64**, 240 (1967).
[41] H. Weyl, *Raum, Zeit und Materie*, Springer Verlag, Berlin 1921.
[42] K. Goedel, The paper in *Albert Einstein: Philosopher–Scientist*, ed. P.A. Schilpp, Cambridge University Press, London 1970.
[43] M.N. Rosenbluth, *Phys. Rev.* **79**, 615 (1950).
[44] S.D. Drell, F. Zachariasen, *Electromagnetic Structure of Nucleons*, Oxford University Press 1961, pp. 7–18.

[45] W.K.P. Panofsky, Proc. Heidelberg Int. Conf. on Elementary Particles, North–Holland, Amsterdam 1968, pp. 371–382.
[46] A. Messiah, *Mécanique Quantique Vol.1*, Dunod, Paris 1959, pp. 121–131.
[47] N. Bohr, *Phys. Rev.* **48**, 696 (1935).
[48] L. Landau, E. Lifszyc, in *Mechanika kwantowa — teoria nierelatywistyczna*, PWN, Warszawa 1979, pp. 13–17, in Polish.
[49] G.C. Wick, *Phys. Rev.* **96**, 1124 (1954).
[50] L. Landau, E. Lifszyc, in *Fizyka statystyczna*, PWN, Warszawa 1959, pp. 2–29, in Polish.
[51] see *e.g.* V. Rojansky, *Introductory Quantum Mechanics*, Prentice Hall, Inc., New York 1947, p. 457.
[52] H.W. Lewis, J.R. Oppenheimer, S.A. Wouthuysen, *Phys. Rev.* **73**, 127 (1948); R. Glauber, *Phys. Rev.* **84**, 395 (1951).
[53] Z. Chyliński, *Nucl. Phys.* **44**, 58 (1963).
[54] W. Heitler, *The Quantum Theory of Relation*, 2nd edition, Oxford University Press, 1944.
[55] Z. Chyliński, *Phys. Rev.* **A32**, 764 (1985).
[56] G.W. Leibniz, *Trzecie pismo Leibniza (25 II 1716)*, *Wyznanie wiary Filozofa*, PWN, Warszawa 1969, in Polish.
[57] Redhead, the reference could not be identified.
[58] L.W. Jones, *Rev. Mod. Phys.* **49**, 717 (1977).
[59] Z. Chyliński, *Phys. Lett.* **A134**, 152 (1988).
[60] L. Montanet *et al.*, *The Particle Data Group*, *Phys. Rev.* **D50**, 1351 (1994).