SEMIEMPIRICAL NEUTRINO MODEL WITH $m_{\nu_e} \simeq 2m_{\nu_\mu} \simeq m_{\nu_\tau}$ *

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A semiempirical model of neutrinos and charged leptons of three families is presented, predicting nonconventionally $m_{\nu_e} \simeq 2m_{\nu_\mu} \simeq m_{\nu_\tau}$ and successfully $m_\tau = 1776.80$ MeV, the latter when experimental values of m_e and m_μ are used. The model implies for weak-interaction eigenstates the oscillation $\nu_\mu \to \nu_\tau$ 56 times more probable than the oscillation $\nu_e \to \nu_\mu$ (if the latter appears).

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The possible nonzero mass of neutrinos is at present one of crucial problems of particle physics, because neutrinos seem to be natural candidates for the so-called hot component [1] of the mysterious dark matter dominating apparently the massive matter distribution in the universe. In this note we present a semiempirical model of three families of neutrinos ν_e, ν_μ, ν_τ and charged leptons e^-, μ^-, τ^-, predicting for their Dirac masses that

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = 1 : \frac{4}{9} : \frac{24}{25}\,, \tag{1}$$

if $m_{\nu_e} \neq 0$, and

$$m_e : m_\mu : m_\tau = 1 : \frac{4}{9}\left(\frac{80}{\varepsilon^2}+1\right) : \frac{24}{25}\left(\frac{624}{\varepsilon^2}+1\right)\,, \tag{2}$$

where $\varepsilon^2 > 0$ denotes a free parameter. Hence,

$$m_{\nu_e} \simeq m_{\nu_\tau} > m_{\nu_\mu} \simeq \tfrac{1}{2} m_{\nu_\tau} \tag{3}$$

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and, when we use experimental values of m_e and m_μ,

$$m_\tau = 1776.80\,\text{MeV} \;\;,\;\; \varepsilon^2 = 0.172329\,. \tag{4}$$

Note that this prediction for m_τ is remarkably close to its actual experimental value: $m_\tau^{\text{exp}} = 1777.1^{+0.4}_{-0.5}$ MeV [2].

The model consists of two parts which differ in the level of their reliability *a priori*: (i) the theoretical assumption that all kinds of fundamental fermions existing in Nature *can be deduced* from Dirac's square-root procedure, $\sqrt{p^2} \to \Gamma \cdot p$, and ($ii$) an empirical ansatz for the Higgs coupling strength of leptons belonging to three families.

It was shown recently [3] that Dirac's square-root procedure leads in general to the sequence $N = 1, 2, 3, \ldots$ of different (generally reducible) representations

$$\Gamma^{(N)\mu} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \gamma_i^{(N)\mu} \tag{5}$$

of the Dirac algebra

$$\left\{\Gamma^{(N)\mu},\, \Gamma^{(N)\nu}\right\} = 2g^{\mu\nu}\,, \tag{6}$$

defined by the sequence of Clifford algebras

$$\left\{\gamma_i^{(N)\mu}, \gamma_j^{(N)\nu}\right\} = 2\delta_{ij} g^{\mu\nu} \;\; (i,j = 1,2,\ldots,N)\,. \tag{7}$$

Then, the corresponding sequence $N = 1, 2, 3, \ldots$ of Dirac-type equations

$$\left[\Gamma^{(N)} \cdot (p - gA) - M^{(N)}\right] \psi^{(N)} = 0\,, \tag{8}$$

if written down in terms of reduced forms

$$\Gamma^{(N)\mu} = \gamma^\mu \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{(N-1)\text{times}} \tag{9}$$

of the representations (5) (γ^μ and $\mathbf{1}$ are the usual 4×4 Dirac matrices), provides us with the equations

$$\left[\gamma \cdot (p - g\,A) - M^{(N)}\right]_{\alpha_1\beta_1} \psi^{(N)}_{\beta_1\alpha_2\ldots\alpha_N} = 0 \tag{10}$$

for the sequence $N = 1, 2, 3, \ldots$ of wave functions (or fields) $\psi^{(N)}(x) = \left(\psi^{(N)}_{\alpha_1\alpha_2\ldots\alpha_N}(x)\right)$. Here, $\alpha_1, \alpha_2, \ldots, \alpha_N$ are N bispinor indices of which only α_1 is affected by the gauge fields $A_\mu(x)$, while the rest of them, $\alpha_2, \ldots, \alpha_N$, *are decoupled.* For $N = 1$ Eq. (10) is evidently the usual Dirac equation, whereas for $N = 2$ it is known as the Dirac form of [4] the Kähler equation [5]. For $N = 3, 4, 5, \ldots$ we get *new* Dirac-type equations.

In Eqs. (8) and (10), $A_\mu(x)$ symbolize the the standard-model gauge fields including the SU(3)⊗SU(2)⊗U(1) coupling matrices λ's, τ's, Y and $\Gamma^5 \equiv i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$.

For N odd these imply for each solution to Eqs. (10) the familiar 16 standard-model particle states forming two left-handed weak- isospin doublets or four right-handed weak-isospin singlets, half of them colorless leptonlike and half color quarklike.

Under the asumption that the decoupled Kähler-type indices $\alpha_2, \ldots, \alpha_N$ describe the particle's *undistinguishable* degrees of freedom obeying Fermi statistics along with Pauli exclusion principle, the wave functions $\psi^{(N)}_{\alpha_1\alpha_2\ldots\alpha_N}(x)$ must be fully *antisymmetric* with respect to $\alpha_2, \ldots, \alpha_N$. Thus, the sequence = 1, 2, 3, ... of equations (10) *ought to terminate* at $N = 5$.

Then, applying the theory of relativity *consequently to all* bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N$ and making use of the probabilistic interpretation of wave functions $\psi^{(N)}_{\alpha_1\alpha_2\ldots\alpha_N}(x)$, we are led to the conclusion [3] that in the case of N odd there are *three (and only three)* particle families $N = 1, 3, 5$ satisfying the Dirac-type equations (10). In fact, they correspond to

$$\begin{aligned} &\psi^{(1)}_{\alpha_1}\,, \\ &\psi^{(3)}_{\alpha_1} \equiv \tfrac{1}{4}\left(C^{-1}\gamma^5\right)_{\alpha_2\alpha_3}\psi^{(3)}_{\alpha_1\alpha_2\alpha_3} = \psi^{(3)}_{\alpha_1\,12} = \psi^{(3)}_{\alpha_1\,34}\,, \\ &\psi^{(5)}_{\alpha_1} \equiv \tfrac{1}{24}\varepsilon_{\alpha_2\alpha_3\alpha_4\alpha_5}\psi^{(5)}_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} = \psi^{(5)}_{\alpha_1\,1234}\,, \end{aligned} \tag{11}$$

where on the right-hand side the chiral representation $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \mathrm{diag}(1, 1, -1, -1)$ is used. They all carry spin $^1\!/_2$ and have the familiar 16 standard-model signatures. Thus, it is very natural to identify them with three empirical families of leptons and quarks.

Note that in the sector wave functions $\psi^{(1)}_{\alpha_1}$, $\psi^{(3)}_{\alpha_1\alpha_2\alpha_3}$ and $\psi^{(5)}_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}$ the bispinors (11) are repeated (up to the sign) 1, 4 and 24 times, respectively. Thus, the following overall wave function comprises three sectors $N = 1, 3, 5$ (or three fundamental-fermion families):

$$\Psi = \hat{\rho}\begin{pmatrix}\psi^{(1)}_{\alpha_1} \\ \psi^{(3)}_{\alpha_1} \\ \psi^{(5)}_{\alpha_1}\end{pmatrix}, \quad \hat{\rho} \equiv \frac{1}{\sqrt{29}}\begin{pmatrix}1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24}\end{pmatrix}. \tag{12}$$

Hence, the Dirac-mass matrix for any family triplet of fundamental fermions of a given standard-model signature is suggested to have the form

$$\widehat{M} = \widehat{\rho}\widehat{h}\widehat{\rho}, \tag{13}$$

where $\widehat{h}$ is a Higgs-coupling-strength matrix.

First, consider the family triplet of charged leptons e^-, μ^-, τ^- and try for them the diagonal ansatz $\widehat{h} = \mathrm{diag}(h^{(1)}, h^{(3)}, h^{(5)})$ with [6]

$$h^{(N)} \equiv \mu^{(e)}\left(N^2 - \frac{1-\varepsilon^2}{N^2}\right) \quad (N = 1,3,5), \tag{14}$$

where $\mu^{(e)} > 0$ and ε^2 are two free parameters. Then, the eigenvalues of mass matrix (13) are

$$\begin{aligned} m_e \equiv M^{(1)} &= \frac{\mu^{(e)}}{29}\varepsilon^2, \\ m_\mu \equiv M^{(3)} &= \frac{4}{9}\frac{\mu^{(e)}}{29}\left(80+\varepsilon^2\right), \\ m_\tau \equiv M^{(5)} &= \frac{24}{25}\frac{\mu^{(e)}}{29}\left(624+\varepsilon^2\right). \end{aligned} \tag{15}$$

Hence the prediction (4) for m_τ and ε^2, and

$$\mu^{(e)} = 85.9924\,\mathrm{MeV}, \tag{16}$$

when experimental values of m_e and m_μ are used. The excellent agreement of the predicted m_τ with the experiment seems to justify the ansatz (14).

The ansatz (14) can be expressed also as

$$h^{(N)} \equiv \mu^{(e)}\left(g^2N^2 - \frac{g^2-\varepsilon^2}{N^2}\right) \quad (N = 1,\,3,\,5), \tag{17}$$

where $g \equiv Q^2$ with $Q = -1$ being the electric charge. If extended in this form to the family triplet of neutrinos ν_e, ν_μ, ν_τ, the ansatz (17) gives for them

$$h^{(N)} \equiv \mu^{(\nu)}\frac{\varepsilon^2}{N^2} \quad (N = 1,3,5), \tag{18}$$

where $\mu^{(\nu)} \geq 0$ is a free parameter (multiplied by the universal ε^2). The idea of this extension relates the difference between masses of charged leptons and neutrinos to the electric charge $Q = -1$ and 0, respectively, as well as to an overall mass scale $\mu^{(e)}$ and $\mu^{(\nu)}$. Then, within both tribes of leptons, mass differences are caused by the family number $N = 1, 3, 5$. (Here, the popular see-saw mechanism might work for the overall mass scale $\mu^{(\nu)}$ if neutrinos got appropriate Majorana masses beside Dirac ones.) Thus, from Eqs. (13) and (18) the neutrino Dirac masses are

$$\begin{aligned} m_{\nu_e} &\equiv M^{(1)} = \frac{\mu^{(\nu)}}{29}\,\varepsilon^2\,, \\ m_{\nu_\mu} &\equiv M^{(3)} = \frac{4}{9}\frac{\mu^{(\nu)}}{29}\varepsilon^2\,, \\ m_{\nu_\tau} &\equiv M^{(5)} = \frac{24}{25}\frac{\mu^{(\nu)}}{29}\varepsilon^2\,. \end{aligned} \tag{19}$$

Hence, the prediction (1) for $m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau}$ if $\mu^{(\nu)} > 0$.

The diagonal ansatz (18) for the family triplet of neutrinos ν_e , ν_μ, ν_τ does not imply their mixing. Such an absence of neutrino mixing may be the case when fermion families mix due to the baryon number which appears only for quarks [6]. Nevertheless, in order to include a possibility of neutrino mixing we may introduce the family-number matrix

$$\widehat{N} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \widehat{1} + 2\widehat{n}, \quad \widehat{n} \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \tag{20}$$

and the (*truncated*) annihilation and creation matrices (for pairs of decoupled Kähler-type indices)

$$\widehat{a} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \widehat{a}^\dagger \equiv \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix} \tag{21}$$

giving

$$\widehat{n} = \widehat{a}^\dagger \widehat{a}\,. \tag{22}$$

Here, $\widehat{a}^3 = 0 = \widehat{a}^{\dagger\,3}$ and so $\widehat{a}^\dagger|n\rangle = 0$ for $n = 2$, when $\widehat{n}|n\rangle = n|n\rangle$ ($N = 1 + 2n$, $n = 0$, 1, 2). Then, we may try for neutrinos the nondiagonal ansatz

$$\widehat{h} \equiv \mu^{(\nu)} \left[\frac{\varepsilon^2}{\widehat{N}^2} + f\varepsilon\left(\widehat{a}e^{i\varphi} + \widehat{a}^\dagger e^{-i\varphi}\right)\right], \tag{23}$$

where f and φ are free parameters (if $f \to 0$, we return to the previous diagonal ansatz). Thus, from Eq. (13)

$$\widehat{M} = \widehat{M}_0 + \widehat{M}_1\,, \widehat{M}_0 \equiv \frac{\mu^{(\nu)}\varepsilon^2}{29} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{4}{9} & 0 \\ 0 & 0 & \frac{24}{25} \end{pmatrix},$$

$$\widehat{M}_1 \equiv \frac{\mu^{(\nu)} f\varepsilon}{29} \begin{pmatrix} 0 & 2e^{i\varphi} & 0 \\ 2e^{-i\varphi} & 0 & 8\sqrt{3}e^{i\varphi} \\ 0 & 8\sqrt{3}e^{-i\varphi} & 0 \end{pmatrix}. \tag{24}$$

Assuming that $|f|$ is small enough (say, $f^2 = O(10^{-5})$ or smaller) we can apply the perturbation calculation with respect to f to the Dirac-mass matrix (24). Then, in the lowest perturbative order we obtain the corrected neutrino Dirac masses

$$m_{\nu_e} \equiv M^{(1)} = \frac{\mu^{(\nu)}}{29}\left(\varepsilon^2 + \frac{36}{5}f^2\right),$$
$$m_{\nu_\mu} \equiv M^{(3)} = \frac{\mu^{(\nu)}}{29}\left[\frac{4}{9}\varepsilon^2 - \left(\frac{36}{5} + \frac{10800}{29}\right)f^2\right],$$
$$m_{\nu_\tau} \equiv M^{(5)} = \frac{\mu^{(\nu)}}{29}\left(\frac{24}{25}\varepsilon^2 + \frac{10800}{29}f^2\right), \tag{25}$$

whereas the diagonalizing matrix $\widehat{U}^{-1}$, such that

$$\widehat{U}^{-1}\widehat{M}\widehat{U} = \mathrm{diag}\left(M^{(1)}, M^{(3)}, M^{(5)}\right), \tag{26}$$

gets in the lowest order the form

$$\widehat{U}^{-1} = \begin{pmatrix} 1 - \frac{1}{2}\left(\frac{18f}{5\varepsilon}\right)^2 & \frac{18f}{5\varepsilon}e^{-i\varphi} & \frac{720\sqrt{3}f^2}{\varepsilon^2}e^{-2i\varphi} \\ -\frac{18f}{5\varepsilon}e^{i\varphi} & 1 - \frac{1}{2}\left[\left(\frac{18f}{5\varepsilon}\right)^2 + \left(\frac{450\sqrt{3}f}{29\varepsilon}\right)^2\right] & -\frac{450\sqrt{3}f}{29\varepsilon}e^{-i\varphi} \\ -\frac{22500\sqrt{3}f^2}{29\varepsilon^2}e^{2i\varphi} & \frac{450\sqrt{3}f}{29\varepsilon}e^{i\varphi} & 1 - \frac{1}{2}\left(\frac{450\sqrt{3}f}{29\varepsilon}\right)^2 \end{pmatrix} \tag{27}$$

($\widehat{U}^{-1}$ of this form is unitary, $\widehat{U}^{-1} = \widehat{U}^\dagger$, up to $O(f^2)$). If there is no mixing of charged leptons or if it can be neglected in comparison with neutrino mixing, then $\widehat{U}$ plays for neutrinos the role of Cabibbo–Kobayashi–Maskawa matrix $\widehat{V}$, giving the transformation

$$\begin{pmatrix} \nu_{e\,0} \\ \nu_{\mu\,0} \\ \nu_{\tau\,0} \end{pmatrix} = \widehat{V}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{28}$$

from the mass eigenstates ν_e, ν_μ, ν_τ to the weak-interaction eigenstates $\nu_{e\,0}$, $\nu_{\mu\,0}$, $\nu_{\tau\,0}$ representing here unperturbed states (corresponding to $f \to 0$). Of course, only $\nu_{e\,0\,L}$, $\nu_{\mu\,0\,L}$, $\nu_{\tau\,0\,L}$ interact with electroweak vector bosons, though also $\nu_{e\,0\,R}$, $\nu_{\mu\,0\,R}$, $\nu_{\tau\,0\,R}$ are included in the neutrino Dirac-mass terms, both unperturbed (when $f \to 0$) and perturbed.

The weak-interaction eigenstates (28)

$$\nu_{i\,0} = \sum_j V_{ij}\nu_j\,, \quad \widehat{V} \equiv (V_{ij})\,, \tag{29}$$

oscillate in time according to the formula

$$\nu_{i\,0}(t) = \sum_j V_{ij}\nu_j e^{-iE_j t}\,, \tag{30}$$

if $\nu_{i\,0}(0) = \nu_{i\,0}$ [7]. Here, $E_i - E_j \simeq (m^2_{\nu_i} - m^2_{\nu_j})/2|\vec{p}|$ when we consider a neutrino beam with momentum $|\vec{p}|$ much larger than masses m_{ν_i}. Then, the probability in time of the oscillation $\nu_{i\,0} \to \nu_{j\,0}$ (in the vacuum) is given by

$$P(\nu_{i\,0} \to \nu_{j\,0}, t) = |\langle \nu_{j\,0}|\nu_{i\,0}(t)\rangle|^2 = \sum_{lk} V_{jl}V^*_{il}V^*_{jk}V_{ik} \exp\left(i\frac{m^2_{\nu_l} - m^2_{\nu_k}}{2|\vec{p}|}\,t\right). \tag{31}$$

Hence, making use of $V_{ij} = (\widehat{V}^{-1\dagger})_{ij} = (\widehat{V}^{-1})^*_{ji}$ as they follow from Eq. (27) for $\widehat{V}^{-1} = \widehat{U}^{-1}$, we obtain

$$\begin{aligned}
P(\nu_{e\,0} \to \nu_{\mu\,0}, t) =& 75.2 f^2 \sin^2\left(\frac{m^2_{\nu_e} - m^2_{\nu_\mu}}{4|\vec{p}|}\,t\right) + O(f^4)\,,\\
P(\nu_{\mu\,0} \to \nu_{\tau\,0}, t) =& 4190 f^2 \sin^2\left(\frac{m^2_{\nu_\mu} - m^2_{\nu_\tau}}{4|\vec{p}|}\,t\right) + O(f^4)\,,\\
P(\nu_{e\,0} \to \nu_{\tau\,0}, t) =& 2260 \times 10^5 f^4 \sin^2\left(\frac{m^2_{\nu_e} - m^2_{\nu_\tau}}{4|\vec{p}|}\,t\right)\\
&- 163 \times 10^5 f^4 \sin^2\left(\frac{m^2_{\nu_e} - m^2_{\nu_\mu}}{4|\vec{p}|}\,t\right)\\
&+ 175 \times 10^5 f^4 \sin^2\left(\frac{m^2_{\nu_\mu} - m^2_{\nu_\tau}}{4|\vec{p}|}\,t\right).
\end{aligned} \tag{32}$$

Here, $m_{\nu_e} \simeq 2m_{\nu_\mu} \simeq m_{\nu_\tau}$ due to Eq. (25) and, say, $f^2 = O(10^{-5})$ or smaller (in order to make the applied perturbation calculation acceptable; otherwise one must diagonalize the mass matrix exactly). Thus, in our model, the oscillation $\nu_{\mu\,0} \to \nu_{\tau\,0}$ is 56 times more probable than $\nu_{e\,0} \to \nu_{\mu\,0}$ or $\nu_{\mu\,0} \to \nu_{e\,0}$ (independently of the particular value of f^2).

If neutrinos are responsible for the hot part of dark matter, their masses are estimated to satisfy the cosmological bound [1, 8]:

$$\sum_i m_{\nu_i} \simeq 5 \text{ to } 7\ \text{eV}\,. \tag{33}$$

Thus, in our case $m_{\nu_e} \simeq 2m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2$ to 2.8 eV when $\nu_i = \nu_e\,,\,\nu_\mu\,,\,\nu_\tau$. Then, $\mu^{(\nu)} \simeq 350$ to 490 eV from Eq. (25). However, if neutrino oscillations $\nu_{e\,0} \to \nu_0$ and $\nu_{\mu\,0} \to \nu'_0$ have to solve the solar ν_e deficit (when including also their oscillations in Sun matter) and the possible atmospheric ν_μ deficit, the following requirements are suggested for neutrino masses [1, 8]:

$$|m^2_{\nu_e} - m^2_\nu| = O(10^{-5})\,\mathrm{eV}^2\,, \quad |m^2_{\nu_\mu} - m^2_{\nu'}| = O(10^{-2})\,\mathrm{eV}^2 \tag{34}$$

(when mixing of only two neutrino mass eigenstates is relevant). Here, $m_{\nu_e} < m_\nu$ in order to allow for the resonant enhancement of the oscillation $\nu_{e\,0} \to \nu_0$ in the Sun. (In the inverted case of $m_{\nu_e} > m_\nu$ there would be possible a resonant enhancement of the antineutrino oscillation $\bar{\nu}_{e\,0} \to \bar{\nu}_0$ irrelevant in the Sun; in contrast, this option might play a relevant role in supernovae by excluding there the resonant oscillation $\nu_0 \to \nu_{e\,0}$ unfavourable for the so-called r-process needed for nucleosynthesis of heavy elements [8].)

When $\nu = \nu_\mu$ and $\nu' = \nu_\tau$, the estimations (33) and (34) suggest that $m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 1.7$ to 2.3 eV (with $m_{\nu_e} < m_{\nu_\mu}$), where the mass differences are much smaller than the masses themselves. We can see, therefore, that in our case of inverted neutrino-mass hierarchy, $m_{\nu_e} \simeq 2m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2$ to 2.8 eV, the requirements (34) cannot be satisfied for $\nu = \nu_\mu$ and $\nu' = \nu_\tau$.

When $\nu = \nu_\tau$ (and, somehow, in the Sun only the mixing $\nu_e - \nu_\tau$ is relevant), the first of requirements (34) may be fulfilled in our case of $m_{\nu_e} \simeq m_{\nu_\tau}$, since from Eq. (25) $0 < m^2_{\nu_\tau} - m^2_{\nu_e} \simeq 10^{-5}\,\mathrm{eV}^2$ for $\mu^{(\nu)} \simeq 350$ to 490 eV if we choose $f^2 \simeq (1.9286$ to $1.9285)\times 10^{-5}$. Then, the coefficient at $\sin^2\left[(m^2_{\nu_e} - m^2_{\nu_\tau})\,t/4|\vec{p}|\right]$ in Eq. (32) (valid in the vacuum) is estimated as 0.84. The second requirement (34) pertaining to the atmospheric ν_μ deficit — that is not very well established yet — may be perhaps ignored.

Still another option for ν and/or ν' might be some steril neutrinos [1, 8], not interacting with electroweak vector bosons, if such neutrinos participated in oscillations $\nu_{e\,0} \to \nu_0$ and/or $\nu_{\mu\,0} \to \nu'_0$ caused by an appropriate neutrino mass matrix. Then, steril neutrinos might also contribute nontrivially to the lhs. of the bound (33). Note, however, that the existence of neutrino oscillations is not necessarily implied by their masses, even nondegenerated (*cf.* the case (19)).

Notice finally that, in principle, the charged leptons may also mix. To allow for such a possibility we may try for the family triplet of charged leptons $e^-\,,\,\mu^-\,,\,\tau^-$ the nondiagonal ansatz

$$\widehat{h} \equiv \mu^{(e)}\left[\widehat{N}^2 - \frac{1-\varepsilon^2}{\widehat{N}^2} + f\varepsilon\left(\widehat{a}e^{i\varphi} + \widehat{a}^\dagger e^{-i\varphi}\right)\right]\,, \tag{35}$$

where f and φ are free parameters different, in general, than those in the ansatz (23) for neutrinos. Analogical perturbative calculation as in the neutrino case leads now in the lowest perturbative order to the corrected masses

$$\begin{aligned} m_e \equiv M^{(1)} &= \frac{\mu^{(e)}}{29}\left(\varepsilon^2 - \frac{36f^2\varepsilon^2}{320-5\varepsilon^2}\right), \\ m_\mu \equiv M^{(3)} &= \frac{\mu^{(e)}}{29}\left[\frac{4}{9}\left(8+\varepsilon^2\right) + \frac{36f^2\varepsilon^2}{320-5\varepsilon^2} - \frac{10800f^2\varepsilon^2}{31696+29\varepsilon^2}\right], \\ m_\tau \equiv M^{(5)} &= \frac{\mu^{(e)}}{29}\left[\frac{24}{25}\left(624+\varepsilon^2\right) + \frac{10800f^2\varepsilon^2}{31696+29\varepsilon^2}\right], \end{aligned} \tag{36}$$

and to the corresponding diagonalizing matrix $\widehat{U}^{-1}$ having the elements

$$\begin{aligned} &(\widehat{U}^{-1})_{11} = 1 - \frac{1}{2}\left(\frac{18f\varepsilon}{320-5\varepsilon^2}\right)^2, \quad (\widehat{U}^{-1})_{12} = -(\widehat{U}^{-1})^*_{21} = -\frac{18f\varepsilon}{320-5\varepsilon^2}e^{-i\varphi}, \\ &(\widehat{U}^{-1})_{13} = \frac{3600\sqrt{3}f^2\varepsilon^2}{(320-5\varepsilon^2)(14976-\varepsilon^2)}e^{-2i\varphi}, \quad (\widehat{U}^{-1})_{31} = \frac{22500\sqrt{3}f\varepsilon}{(31696+29\varepsilon^2)(14976-\varepsilon^2)}e^{2i\varphi}, \\ &(\widehat{U}^{-1})_{22} = 1 - \frac{1}{2}\left[\left(\frac{18f\varepsilon}{320-5\varepsilon^2}\right)^2 + \left(\frac{450\sqrt{3}f\varepsilon}{31696+29\varepsilon^2}\right)\right], \\ &(\widehat{U}^{-1})_{23} = -(\widehat{U}^{-1})^*_{32} = -\frac{450\sqrt{3}f\varepsilon}{31696+29\varepsilon^2}e^{-i\varphi}, \quad (\widehat{U}^{-1})_{33} = 1 - \frac{1}{2}\left(\frac{450\sqrt{3}f\varepsilon}{31696+29\varepsilon^2}\right)^2 \end{aligned} \tag{37}$$

($\widehat{U}^{-1}$ of this form is unitary, $\widehat{U}^{-1} = \widehat{U}^\dagger$, up to $O(f^2)$).

For the corrected masses (36) we can derive the formula

$$m_\tau = \frac{6}{125}\left(351m_\mu - 136m_e\right) + 1.76\, f^2 \text{ MeV}, \tag{38}$$

where in the term $O(f^2)$ the figures (4) for ε^2 and (16) for $\mu^{(e)}$ are used. Inserting into this formula experimental values of m_e and m_μ we predict that

$$m_\tau = \left(1776.80 + 1.76f^2\right) \text{ MeV}. \tag{39}$$

Thus, the difference

$$m_\tau - m_\tau^{\text{exp}} = \left(1.76f^2 - 0.3^{+0.4}_{-0.5}\right) \text{ MeV} \tag{40}$$

becomes zero for

$$1.76f^2 = 0.3^{+0.4}_{-0.5}\,, \tag{41}$$

what cannot exclude the option $f = 0$ for charged leptons.

The analogue for leptons of the Cabibbo–Kobayashi–Maskawa matrix gets now the form $\widehat{V} = \widehat{U}^{(e)\,\dagger}\widehat{U}^{(\nu)}$, where $\widehat{U}^{(\nu)\,-1}$ and $\widehat{U}^{(e)\,-1}$ are the diagonalizing matrices $\widehat{U}^{-1}$ for neutrinos [Eq. (27)] and charged leptons [Eq. (37)], respectively (here, the coupling constants f ought to be denoted by $f^{(\nu)}$ and $f^{(e)}$, respectively, while ε^2 is universal). It can be easily seen, however, that due to large denominators in the elements (37) of $\widehat{U}^{(e)\,-1} = \widehat{U}^{(e)\dagger}$ one can put effectively $\widehat{U}^{(e)\dagger} \simeq \widehat{1}$ and so $\widehat{V} \simeq \widehat{U}^{(\nu)}$ if $|f^{(e)}|$ is of the same order as $|f^{(\nu)}|$ or smaller. Then, $\widehat{V}$ is given effectively by the Hermitian conjugate of the matrix (27).

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[8] For a recent review *cf.* G. Gelmini, E. Roulet, *Neutrino Masses*, CERN-TH.7541/94, December 1994; D.O. Caldwell, *Evidence for Neutrino Mass*, talk to the 2nd Educational Tempus Workshop: *Cosmology and Particle Physics*, Warsaw, April 19-22, 1995.

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* In the latter paper, please correct in Eq. (1) the misprint $-\varepsilon^2$ to $+\varepsilon^2$.