# VARIATION OF rms RADII IN THE $N = 50$ REGION DUE TO FRAGMENTATION OF PROTON HOLE STRENGTHS NEAR THE FERMI SURFACE

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Isotopic variation of rms charge radii of even isotopes of Zr and Mo has been evaluated using an optimized one body potential and using the experimental occupation probabilities of the proton states near the Fermi surface. We have obtained excellent agreement both in the case of single particle(-hole) energies and rms charge radii of the chain of isotopes of Zr and Mo. Calculated values of equivalent radii and model independent Fourier–Bessel coefficients show good agreement with the experiment.

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## 1. Introduction

Detailed investigation of the rearrangement of the nuclear charge distribution in response to the addition of neutrons or protons throughout a whole series of isotopes provides a valuable and stringent test of one's understanding of the structure of nuclear ground state. The availability of vast body of results from optical, electronic X-ray transitions and muonic X-ray experiments of differences of charge radii of isotopes and isotones provides a sensitive tool to check the validity of the calculated nuclear wave functions. Electron scattering data which essentially gives information about the Fourier transform of the nuclear charge distribution, determine directly the charge distribution and charge distribution differences, provided the measurements are accurate enough to extend over a sufficiently large range of momentum transfer. By using Fourier–Bessel expansion techniques (Dreher *et al.* 1974) realistic error bands can be derived for the charge distributions and their differences. On the other hand, the muonic transition energies, within the limits of present experimental errors, determine certain Barrett moments ($\langle r^k e^{-\alpha r} \rangle$) of the nuclear charge distributions.

The study of charge distribution systematics can serve to impose severe constraints on the saturation properties of nuclear forces and it also displays structural effects related to nuclear shapes, providing information that is often complementary to that given by nuclear spectroscopy.

Out of the eleven nuclei (one of Sr, four from Zr family and six from Mo family) the charge distribution of only $^{90}$Zr had been studied extensively through different theoretical approaches (Tarbutton and Davies 1968, Nemeth 1972, Campi and Sprung 1972, Angeli *et al.* 1980). However, a complete different philosophy was taken by Wesolowski (1984) in reproduction of the second moment of the proton distributions of all the nuclei using a nine parameter formula. Recently Buchinger *et al.* (1994) have calculated the nuclear charge radii of all nuclei from a mass formula. So far there is no calculation which produced the charge distributions of Sr-Zr-Mo isotopic chains after the inclusion of experimental occupation probabilities.

It has been established now (Bhattacharya and Krishan 1993, Bhattacharya 1994) that the occupation probabilities of the proton states near the Fermi surface of the nuclei plays the most important role in the determination of the charge distributions of nuclei. In the present work we are presenting for the first time the calculation of rms charge radii, Barett moments and equivalent charge radii of even isotopes of Sr, Zr and Mo by taking experimentally determined fragmentation of the proton hole strengths (Pfeiffer *et al.* 1986) into account.

The organization of the paper is as follows: in Section 2 we present the model of our calculation. In Section 3 the results are described and compared with the experimental values. The conclusions are presented in Section 4.

## 2. The model

From the experimental evidences (Kocher 1972, Medsker and Horen 1972, Medsker 1974) and from theoretical investigations (Talmi and Unna 1960, Kumar and Bansal 1979) it is apparent that the $^{90}$Zr ground state is substantially two protons outside of a closed $^{88}$Sr core. In order to get the zeroth order shell model states of Sr nucleus we have started our investigation in the Ni region because the shell closure is very prominent there. From the analysis of the observed distribution of the single particle strengths in terms of the weak coupling of the particle motion with the collective vibrational states of Ni core, one can estimate the extent of core polarization effect in the single particle spectrum. To reproduce the observed single particle spectra we have developed an average one body potential of Woods–Saxon form. The form of the local potential is given by

$$V(r) = V_c(r) - V_0 f_0(r) + V_s \frac{1}{r} \frac{d}{dr} f_{so}(r) \bar{L} \cdot \bar{S}\,, \qquad (1)$$

where

$$f(r) = \left[1 + \exp\left\{\frac{r - r_0(A-1)^{1/3}}{a}\right\}\right]^{-1}, \tag{2}$$

and $V_c(r)$ is the Coulomb potential given by

$$\begin{aligned} V_c(r) &= \frac{1.44\,Z}{2R_c}\left(3 - \frac{r^2}{R_c^2}\right), \quad r < R_c \\ &= \frac{1.44\,Z}{r}, \qquad\qquad r > R_c \end{aligned} \tag{3}$$

$R_c = r_c A^{1/3}$.

The potential parameters were optimized using a six parameter search program. In the optimization procedure we have enforced a self consistency criterion. Initially we had started our calculation using the potential parameters of the Ca-isotopes as reported in our work (Bhattacharya and Krishan 1993). Using Numerov's technique we have evaluated single particle energies and the wave functions corresponding to each $nlj$. To bring out the details of the wave function we have chosen small (0.05 fm) integration steps. The local wave function was subjected to non-locality correction using the prescription of Perey and Buck (1962) to take the momentum dependence of the nuclear interaction into account. Point proton distribution was calculated using the formula

$$\rho(r) = \sum_j u_j^2 (2j+1)\psi_{nlj}^2(r)\,. \tag{4}$$

The sum is over all the occupied states and $u_j^2$'s are the *experimental occupancies* as provided by Pfeiffer *et al.* (1986). The point proton distribution was then folded with the proton charge distribution and corrected for centre of mass motion. Finally the corrected charge distribution was subjected to Fourier–Bessel analysis. For the normalization we have adopted the convention that the integral over the charge distribution equals the nuclear charge Ze. This normalization has the advantage that the difference between different nuclei can be deduced directly from the Fourier–Bessel (FB) coefficients provided that both sets of coefficients have been determined for the same value of the cutoff radius. Next we have compared the FB coefficients extracted from the calculated charge distribution with the experimental ones (de Vries *et al.* 1987). The whole exercise of the above procedure was repeated several times in order to get a good reproduction of both energy levels and FB coefficients.

After optimizing the potential parameters in the Ni region, the central potential for the nuclei under consideration was found out by using an isobaric dependence according to Bohr and Mottelson (1975)

$$V_p(r) = \left(V_{0p} - K_p \frac{N-Z}{A}\right) f_p(r). \tag{5}$$

After a thorough evaluation of the single particle(-hole) energies in the Ca region and $f-p$ shell region the value of $K_p$ was fixed at 28.0 MeV. For the strength of the spin-orbit potential, we have assumed proportional dependence on the depth of the central potential. The geometry of the potential for all the nuclei was kept constant, thus minimizing the number of parameters.

## 3. Results and analysis

The values of the potential parameters after optimization is presented in Table I.

TABLE I

List of potential parameters used in the calculation

| $V_0$ (MeV) | $r_0$ (fm) | $a_0$ | $V_s$ (MeV) | $r_s$ (fm) | $a_s$ | $W_c$ |
|---|---|---|---|---|---|---|
| 59.1087 | 1.26 | 0.5592 | 30.038 | 1.06799 | 0. 86238 | 28.0 |

The value of $r_0$ is a little bit on the higher side compared to the reported value of $r_0$ used in the DWBA calculation of Mairle *et al.* (1984), but as these set of parameters produced excellent agreement in the $f-p$ shell region we did not make any change. The comparison of calculated single particle energies with the experimental values taken from Pfeiffer *et al.* (1986) is presented in Table II. The agreement between the calculated values and the experimental ones is very good if we keep in mind that the uncertainties associated with the experimental values are of the order of 15–20% and furthermore in the evaluation of the energies of the single particle states

$$E_{lj}(A) = \frac{\sum_f [E_f^+ G_f^+(lj) + E_f^- G_f^-(lj)]}{\sum_f [G_f^+(lj) + G_f^-(lj)]} \tag{6}$$

it was assumed that $E_f^+$, $E_f^-$ and spectroscopic strength $G_f^+(lj)$, $G_f^-(lj)$ measured in proton stripping $(+)$ and pickup $(-)$ reactions form a complete set. The behaviours of calculated and experimental $E_{lj}(A)$ are qualitatively very similar for different $j$-values. With the number of nucleons $E_{lj}(A)$ increases slightly for a given isospin $T_0$ and $E_{lj}(A)$ increases sharply with $T_0$ for nuclei with given mass number.

TABLE II

Calculated and experimental single particle energies for the proton hole states

| Nucleus/State | $1f_{7/2}$ | $2p_{3/2}$ | $1f_{5/2}$ | $2p_{1/2}$ | $1g_{9/2}$ |
|---|---|---|---|---|---|
| $^{88}$Sr (Ex) | - | 9.80 | 10.43 | 7.98 | 6.17 |
| (Th) | 16.08 | 10.25 | 10.78 | 8.63 | 7.11 |
| $^{90}$Zr (Ex) | 16.16 | 9.02 | 9.77 | 7.17 | 5.51 |
| (Th) | 15.30 | 9.52 | 10.12 | 7.93 | 6.49 |
| $^{92}$Zr (Ex) | 16.66 | 9.67 | 11.00 | 7.55 | 6.31 |
| (Th) | 16.31 | 10.41 | 11.03 | 8.61 | 7.07 |
| $^{94}$Zr (Ex) | - | 10.82 | 11.25 | 8.92 | 7.08 |
| (Th) | 17.10 | 11.27 | 11.39 | 9.33 | 7.19 |
| $^{96}$Zr (Ex) | 18.61 | 12.47 | 12.20 | 10.75 | 7.95 |
| (Th) | 17.96 | 12.11 | 12.57 | 10.52 | 8.35 |
| $^{92}$Mo (Ex) | 15.01 | 7.94 | 8.92 | 6.12 | 4.94 |
| (Th) | 13.90 | 8.02 | 8.65 | 6.48 | 5.08 |
| $^{94}$Mo (Ex) | - | 8.64 | - | 7.55 | 5.84 |
| (Th) | 14.60 | 8.88 | 9.54 | 7.35 | 6.05 |
| $^{96}$Mo (Ex) | - | 9.70 | 11.02 | 7.98 | 6.48 |
| (Th) | 15.48 | 9.73 | 10.41 | 8.19 | 6.98 |
| $^{98}$Mo (Ex) | - | 10.71 | 11.03 | 9.46 | 7.07 |
| (Th) | 16.32 | 10.56 | 11.25 | 9.02 | 7.89 |
| $^{100}$Mo (Ex) | - | 10.98 | - | 8.95 | 8.10 |
| (Th) | 17.15 | 11.36 | 12.08 | 9.82 | 8.78 |

Experimental values are taken from Pfeiffer *et al.* (1986).

The orbit radii of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ and $1g_{9/2}$ valence proton states were calculated and their values are presented in Table III. For lack of experimental results comparison could only be done for $1g_{9/2}$ states of $^{88}$Sr, $^{90}$Zr and $^{92}$Mo and for $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ states of $^{90-92}$Zr. The calculated values for $2p$ and $1g$ states are slightly on the lower side, but for $1f$ states we have obtained a better agreement.

TABLE III

Valence orbit radii for the even isotopes of Sr, Zr and Mo

| Nucleus/State | $1f_{7/2}$ | $2p_{3/2}$ | $1f_{5/2}$ | $2p_{1/2}$ | $1g_{9/2}$ |
|---|---|---|---|---|---|
| $^{88}$Sr (Th) | 4.47 | 4.21 | 4.42 | 4.25 | 4.70 |
| (Ex) | - | - | - | - | 4.95 [b] |
| $^{90}$Zr (Th) | 4.51 | 4.29 | 4.53 | 4.34 | 4.80 |
| (Ex) | - | $4.51^{+.12}_{-.10}$ [a] | - | $4.64^{+.12}_{-.11}$ [a] | 5.03 [b] |
| $^{92}$Zr (Th) | 4.55 | 4.31 | 4.57 | 4.36 | 4.84 |
| (Ex) | - | $4.58^{+.09}_{-.11}$ [a] | $4.55^{+.10}_{-.12}$ [a] | $4.60^{+.09}_{-.11}$ [a] | - |
| $^{94}$Zr (Th) | 4.58 | 4.32 | 4.59 | 4.37 | 4.87 |
| (Ex) | - | - | - | - | - |
| $^{96}$Zr (Th) | 4.60 | 4.33 | 4.60 | 4.37 | 4.88 |
| (Ex) | - | - | - | - | - |
| $^{92}$Mo (Th) | 4.55 | 4.34 | 4.57 | 4.39 | 4.85 |
| (Ex) | - | - | - | - | 5.06 [b] |
| $^{94}$Mo (Th) | 4.56 | 4.34 | 4.58 | 4.39 | 4.86 |
| (Ex) | - | - | - | - | - |
| $^{96}$Mo (Th) | 4.65 | 4.39 | 4.66 | 4.43 | 4.94 |
| (Ex) | - | - | - | - | - |
| $^{98}$Mo (Th) | 4.68 | 4.40 | 4.69 | 4.44 | 4.96 |
| (Ex) | - | - | - | - | - |
| $^{100}$Mo (Th) | 4.70 | 4.40 | 4.70 | 4.44 | 4.97 |
| (Ex) | - | - | - | - | - |

[a] Simcock *et al.* (1981); [b] Milliman *et al.* (1985).

In Table IV the comparison of the experimental rms charge radii with the corresponding calculated values of De Vries *et al.* (De Vries 1987) is shown. The agreement with the experimental values is excellent. This actually proves our hypothesis that the determination of the zeroth order shell model states near a shell closed nucleus is very important in finding out the variation of charge distributions for a family of isotopes. However, one should not forget about the important contributions of the experimental occupation probabilities.

TABLE IV

Comparison of calculated rms charge radii with the experimental values

| Nucleus | $\langle r^2 \rangle_{ex}^{1/2}$ (fm) | $\langle r^2 \rangle_{th}^{1/2}$ (fm) |
|---|---|---|
| $^{88}$Sr | 4.197(6) | 4.198 |
| $^{90}$Zr | 4.258(8) | 4.259 |
| $^{92}$Zr | 4.295(11) | 4.294 |
| $^{94}$Zr | 4.315(10) | 4.316 |
| $^{96}$Zr | - | 4.324 |
| $^{92}$Mo | 4.294(16) | 4.291 |
| $^{94}$Mo | 4.331(16) | 4.337 |
| $^{96}$Mo | 4.364 | 4.367 |
| $^{98}$Mo | 4.388(16) | 4.385 |
| $^{100}$Mo | 4.430(16) | 4.431 |

Experimental values are taken from De Vries *et al.* (1987).

We would like to make a comment here that the comparison of the calculated values with the experimental ones deteriorates if one takes into account the recent compilation of the charge radii data by Nadjakov *et al.* (Nadjakov 1994). But we must emphasize the fact that we have made the comparison with the electron scattering data in order to find out the details of the charge distribution. The Nadjakov compilation is based on muonic transition data.

The energy shift of a muonic transition between the levels $i$ and $f$ in the first order approximation is given by

$$\Delta E = 4\pi \int_0^\infty \Delta\rho(r)[V_\mu^i(r) - V_\mu^f(r)]r^2 dr\,. \tag{7}$$

It has been shown by Barrett (1970) that the region of overlap with the nuclear charge distribution the potential difference can be approximated with good accuracy by

$$V_\mu^i(r) - V_\mu^f(r) = A + Br^k e^{-\alpha r}\,, \tag{8}$$

so that the Barret moment

$$\langle r^k e^{-\alpha r} \rangle = \left(\frac{4\pi}{Ze}\right) \int_0^\infty \rho(r) r^k e^{-\alpha r} r^2 dr \tag{9}$$

can be determined model independently from muonic atom transition energies. The Barret moment is used to define the equivalent nuclear charge radius $R_k$ of a homogeneously charged sphere by means of the relation

$$\left(\frac{3}{R_k^3}\right)\int_0^{R_k} r^k e^{-\alpha r} r^2 dr = \langle r^k e^{-\alpha r}\rangle . \tag{10}$$

Using the calculated charge distribution defined in Eq. (4) we have evaluated the Barrett moments and the equivalent radii $R_k$. The results are shown in Table V. We have not included the correction due to nuclear polarization. For Mo isotopes we have used $k = 2.161$ and $\alpha = 0.096$ fm$^{-1}$. Though the calculated values are a little bit on the higher side, here we have got the right trend. The Barrett moments could not be compared with experiment due to nonavailability of data.

TABLE V

Barrett moments and equivalent radii of even isotopes of Sr, Zr and Mo

| Nucleus | $\langle r^k e^{-\alpha r}\rangle$ (fm)$^k$ | $R_k$ (fm) | $R_k$ (fm) |
|---|---|---|---|
| $^{88}$Sr | 14.868 | 5.52 | - |
| $^{90}$Zr | 15.259 | 5.59 | 5.46[a)] |
| $^{92}$Zr | 15.515 | 5.65 | - |
| $^{94}$Zr | 15.675 | 5.69 | - |
| $^{96}$Zr | 15.718 | 5.70 | - |
| $^{92}$Mo | 15.431 | 5.65 | 5.53[b)] |
| $^{94}$Mo | 15.712 | 5.69 | 5.57[b)] |
| $^{96}$Mo | 15.943 | 5.75 | 5.61[b)] |
| $^{98}$Mo | 16.071 | 5.77 | 5.64[b)] |
| $^{100}$Mo | 16.419 | 5.85 | 5.69[b)] |

[a)] Phan *et al.* (1985); [b)] Schellenberg *et al.* (1980).

The calculated charge distribution was subjected to Fourier–Bessel analysis after being appropriately modified for the finite extension of the proton charge distribution and corrected for centre of mass motion. The Fourier–Bessel expansion for the charge distribution is given by

$$\rho(r) = \sum_\nu a_\nu j_0\left(\frac{\nu\pi r}{R}\right), \quad r \le R$$
$$= 0, \quad r > R, \tag{11}$$

where $R$ is the cutoff radius and $j_0$ is the spherical Bessel function of zeroth order. The FB coefficients for all the nuclei studied are presented in Table VI along with their experimental values.

TABLE VI

Comparison of theoretical and experimental Fourier-Bessel coefficients

| Nucleus | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $a_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $^{88}$Sr (Th) | .52157e-1 | .59874e-1 | -.26749e-1 | -.36942e-1 | .14572e-1 | .18367e-1 | -.43182e-2 | -.71490e-2 | .60683e-2 | -.45527e-3 |
| (Ex) | .56435e-1 | .55072e-1 | -.33363e-1 | -.26061e-1 | .15749e-1 | .75233e-2 | -.55044e-2 | -.23643e-2 | .39362e-3 | -.22733e-3 |
| $^{90}$Zr (Th) | .46414e-1 | .65584e-1 | -.92682e-2 | -.41327e-1 | .15258e-2 | .21861e-1 | -.19894e-2 | -.99271e-2 | -.45227e-2 | .13263e-3 |
| (Ex) | .46188e-1 | .61795e-1 | -.12315e-1 | -.36915e-1 | .25175e-2 | .15234e-1 | -.55146e-3 | -.60631e-2 | -.12198e-2 | .36200e-3 |
| $^{92}$Zr (Th) | .48148e-1 | .65775e-1 | -.12991e-1 | -.42022e-1 | .43160e-2 | .22293e-1 | -.47430e-3 | -.10035e-1 | -.36351e-2 | .14083e-2 |
| (Ex) | .45939e-1 | .60104e-1 | -.13341e-1 | -.35106e-1 | .37160e-2 | .113753e-1 | -.82682e-3 | -.53001e-2 | -.97579e-3 | .26489e-3 |
| $^{94}$Zr (Th) | .48465e-1 | .66047e-1 | -.12750e-1 | -.40618e-1 | .56369e-2 | .21527e-1 | -.11449e-3 | -.10637e-1 | -.34336e-2 | .14593e-2 |
| (Ex) | .45798e-1 | .59245e-1 | -.13389e-1 | -.33252e-1 | .39888e-2 | .12750e-1 | -.15793e-2 | -.56692e-2 | -.15698e-2 | .54394e-4 |
| $^{92}$Mo (Th) | .30928e-1 | .60432e-1 | .22114e-1 | -.31239e-1 | -.29352e-1 | .62086e-2 | .19942e-1 | .58741e-2 | -.72810e-2 | -.74077e-2 |
| (Ex) | .30782e-1 | .59896e-1 | .22016e-1 | -.28945e-1 | -.26707e-1 | .40426e-2 | .14429e-1 | .31696e-2 | -.63061e-2 | -.45119e-2 |
| $^{94}$Mo (Th) | .30342e-1 | .59622e-1 | .22245e-1 | -.30844e-1 | -.29597e-1 | .59555e-2 | .20282e-1 | .61870e-2 | -.74808e-2 | -.81278e-2 |
| (Ex) | .30661e-1 | .58828e-1 | .20396e-1 | -.28830e-1 | -.25077e-1 | .44768e-2 | .13127e-1 | .19548e-2 | -.61403e-2 | -.35825e-2 |
| $^{96}$Mo (Th) | .31796e-1 | .61706e-1 | .21446e-1 | -.33271e-1 | -.29760e-1 | .77472e-2 | .20958e-1 | .54125e-2 | -.80956e-2 | -.69936e-2 |
| (Ex) | .30564e-1 | .58013e-1 | .19255e-1 | -.28372e-1 | -.23304e-1 | .49894e-2 | .12126e-1 | .10496e-2 | -.62592e-2 | -.32814e-2 |
| $^{98}$Mo (Th) | .29669e-1 | .60529e-1 | .27101e-1 | -.26990e-1 | -.31075e-1 | .26966e-2 | .20124e-1 | .77138e-2 | -.69364e-2 | -.10065e-1 |
| (Ex) | .30483e-1 | .57207e-1 | .17888e-1 | -.28388e-1 | -.21778e-1 | .56780e-2 | .11236e-1 | .82176e-3 | -.50390e-2 | -.23877e-2 |
| $^{100}$Mo (Th) | .32570e-1 | .62843e-1 | .20660e-1 | -.35727e-1 | -.30960e-1 | .88099e-2 | .22332e-1 | .57946e-2 | -.82242e-2 | -.64491e-2 |
| (Ex) | .30353e-1 | .56087e-1 | .16057e-1 | -.28767e-1 | -.20683e-1 | .62429e-2 | .11058e-1 | .11502e-2 | -.39395e-2 | -.14978e-2 |

Experimental values are taken from De Vries et al (1987)

The finding here is that our analysis could reproduce the important first ten coefficients fairly well with their signatures.

In the spirit of Myers and Rozmej (1987) we have done a droplet model type calculation for the series of isotopes of Molybdenum. It was found that to a good approximation we can write

$$R_z = 4.294 + 0.7\frac{(N-50)}{4}. \tag{12}$$

The droplet model contribution to $\delta\langle r^2\rangle$ is given by

$$\delta\langle r^2\rangle_{\text{droplet}} = \tfrac{3}{5}[R_z^2 - (R_z^2)_{N=50}], \tag{13}$$

which approximates to

$$\delta\langle r^2\rangle_{\text{droplet}} \sim 0.09(N-50).$$

TABLE VII

Variation of $\Delta\langle r^2\rangle$ of Mo isotopes with mass number

| $\Delta\langle r^2\rangle^{A,A+2}$ | Droplet model ($\text{fm}^2$) | Present calculation ($\text{fm}^2$) | Experiment ($\text{fm}^2$) |
|---|---|---|---|
| 94–92 | .18 | .39 | .34 |
| 96–94 | .18 | .26 | .27 |
| 98–96 | .18 | .16 | .21 |
| 100–98 | .18 | .41 | .37 |

Experimental values are taken from De Vries *et al.* (1987).

In Table VII, we present the comparison of $\delta\langle r^2\rangle$ for Mo isotopes as calculated

*(i)* from the theoretical charge distribution;

*(ii)* from droplet model and from the optical and muonic studies.

As a test of theoretical nuclear wave functions, the isotone shifts are particularly interesting, since in the single particle model the change in density is equal to the sum of the squares of the wave functions of the added protons. In figures 1 and 2, we show $\Delta\rho(r)r^2 \times 10^2$ for $^{90}$Zt–$^{88}$Sr and $^{92}$Mo–$^{90}$Zr, where we have tried to show from isotone charge distribution differences the rearrangement effect of the proton core due to added protons.

Finally in Fig. 3 the Brix–Kopfermann plot of rms radii of even isotopes of Molybdenum is presented. It is apparent that both the calculated graph

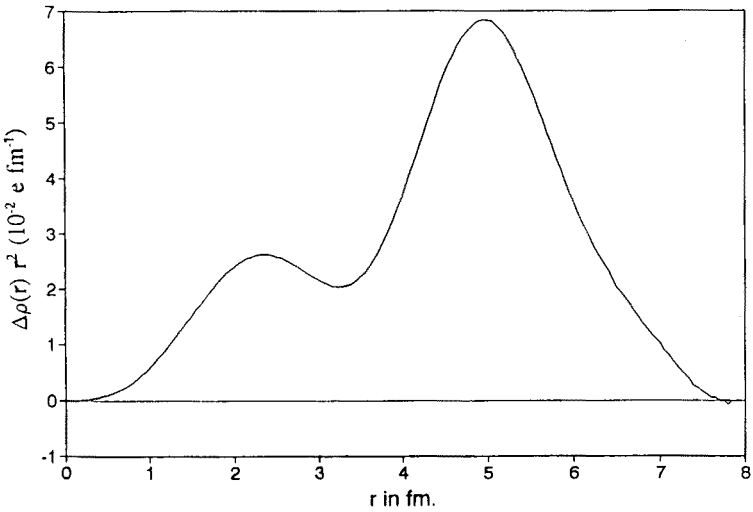


Fig. 1. Rearrangement effect due to $2p_{3/2}$ and $2p_{1/2}$ protons in $^{90}$Zr–$^{88}$Sr.

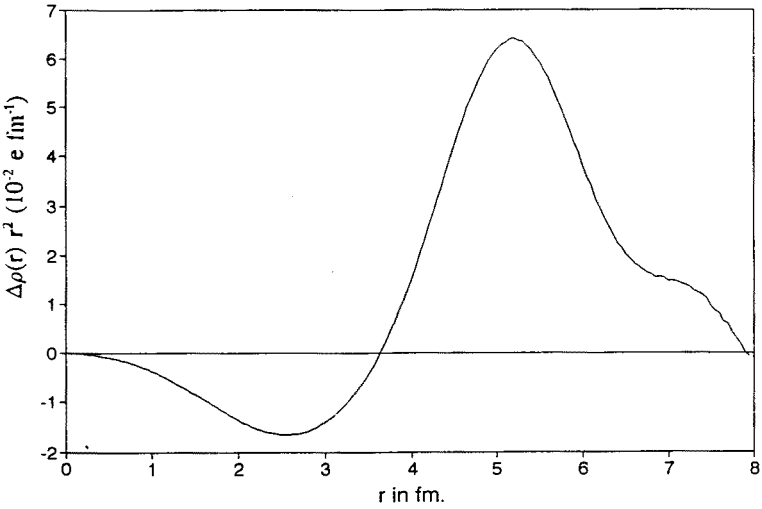


Fig. 2. Rearrangement effect due to $2p_{1/2}$ and $1g_{9/2}$ protons in $^{92}$Mo–$^{90}$Zr.

and experimental graph show very similar behaviour and both of them show a dip at $A = 97$, where as the droplet model predicts no variation at all. May be, this is due to a subshell effect. It needs further investigation.

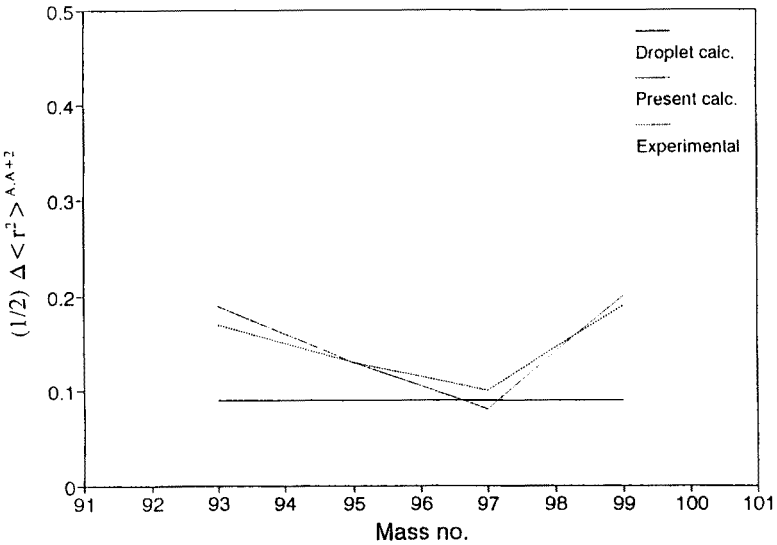


Fig. 3. Brix–Kopfermann plot. Plotted are the differences $(1/2)\Delta\langle r^2\rangle^{A,A+2}$ at $A+1$.

## 4. Conclusion

Through the calculation of rms charge radii, valence orbit radii, Barett moments and corresponding equivalent charge radii with the help of an average one body potential which reproduced the zeroth order shell model states of closed shell nuclei, further support of the conjecture that for reproduction of isotopic variation of charge distributions correct identification of single particle(-hole) states is essential, was obtained. The effect of the correlation in the ground state charge distribution can be taken into account if one uses the experimental occupation probabilities.

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