ELECTROWEAK PROCESSES INVOLVING (0^+0) EXCITATIONS IN NUCLEI*

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Within the Standard Model, strong isospin invariance, and the nuclear domain of u, d quarks, the parity violating asymmetry in polarized electron scattering is predicted and the neutrino scattering cross section is directly related to the electron scattering cross section, for inelastic $(0^+0)_{\rm gnd} \rightarrow (0^+0)^*$ nuclear excitations (assuming pure quantum numbers for both states). With the inclusion of strange quarks, the asymmetry measures a new nuclear matrix element of the strangeness current, if the inelastic charge form factor for that transition is large enough for the experiment. The ground and first excited states of ^{4}He have $(J^\pi = 0^+, T = 0)$; thus the analysis is applicable to future CEBAF experiments on parity violation and possible neutrino scattering experiments on this nucleus. Existing low momentum transfer q^2 data on the inelastic charge form factor for the $(0^+0)_{\rm gnd} \rightarrow (0^+0)^*$ transition in ^{4}He (which show it growing relative to the elastic one) are fit within simple nuclear models, and predictions are made for higher q^2. The more quantitative analysis for ^{4}He is significantly complicated by the fact that the excited state lies just above the break-up threshold. It is desirable to first have an experimental measurement of this form factor to higher q^2, using the predicted magnitude as a guide.

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1. Introduction

Simple ground state quantum numbers of $(0^+0)_{\rm gnd}$ nuclei are known to generate relations, connecting various elastic electroweak processes for the same nucleus. Some of these relations provide new unique tools to study nuclear and nucleon structure. For example, strange quark pairs $(s, \bar{s})$ appear to contribute significantly to the properties of a nucleon [1]. So far

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little is known about various strange quark matrix elements of the nucleon, and ways to obtain experimental information on these matrix elements are intensively discussed in the literature [2–6]. Parity violating (PV) elastic polarized electron scattering and elastic neutrino scattering experiments on light ($J^\pi = 0^+$, $T = 0$) nuclei have been proposed as probes of the electric strange form factor of the nucleon [7]. As noted in [8], because of potential isospin mixing and decrease in the figure of merit, only ^{4}He, ^{12}C (and possibly ^{16}O) nuclei appear to be suitable targets for such experiments. Experiments to determine the ground-state nuclear matrix element of the weak strange current from the PV asymmetry in elastic polarized electron scattering on ^{4}He are planned for CEBAF [9, 10]. All the foregoing nuclei have low lying $(0^+0)^*$ excited states. Analysis shows that one can obtain results similar to the elastic case when the same electroweak processes excite a $(0^+0)_{\rm gnd}$ nucleus to its $(0^+0)^*$ state. In the following discussion, inelastic processes will mean just such excitations. The present work has been motivated by the questions of what new issues of nuclear structure can be addressed, and whether any of the proposed measurements can be enhanced if, in addition to an elastic electroweak experiment, one measures excitation of a $(0^+0)^*$ state in the same nucleus.

Section 2 of this paper describes in detail general relations for the cross sections of inelastic electron and neutrino scattering. It is shown, for example, that within the *single nucleon picture* of the nucleus the inelastic and elastic PV asymmetries are identical. Measuring the PV asymmetry in the inelastic polarized electron scattering cross section could enable one to extract information about a new transition nuclear matrix element of the vector strange quark current in the nucleus under consideration. Knowledge of the inelastic charge form factor of the nucleus for the region of intermediate transferred momentum q^2 (here $q \equiv |\vec{q}\,|$ is the absolute value of transfered three-momentum) will be necessary to design such experiments. A central goal of the present paper is to urge experimental investigation of how well the inelastic $(0^+0)^*$ resonance in electron scattering can be seen at intermediate q^2, and to provide some theoretical guidance for such experiments.

In Sec. 3 of this paper existing low momentum transfer ($q^2 < 2.4\,\text{fm}^{-2}$) data on the inelastic charge form factor of ^{4}He is explained, and its higher q^2 (up to 10 fm^{-2}) behavior is predicted, with the help of three simple models used for the ^{4}He nucleus. While the $(0^+0)^*$ state is indeed the first excited state of ^{4}He, the situation here is complicated by the fact that this state lies just above the break-up threshold. Although the $(0^+0)^*$ state in ^{12}C and ^{16}O are distinct bound states, we concentrate here on ^{4}He because the PV electron experiment *will* actually be done on this nucleus, and the corresponding neutrino experiments are under consideration. The relatively crude estimates of the inelastic electron scattering form factor in the present paper are aimed at the determination of the scale of the form factor before attempting to perform state-of-the-art calculations of the structure of $(0^+0)^*$ excitation of the ^{4}He nucleus. Thus we set aside questions of the break-up background, radiative corrections, parity and isospin mixing and meson exchange currents (MEC) contributions. The form factor is shown to be large enough to be seen in future CEBAF experiments. CEBAF will have luminosity and resolution sufficient to measure the inelastic charge form factor and PV asymmetry in the inelastic polarized electron scattering on ^{4}He up to high momentum transfers q^2.

The role of the inelastic transition in PV experiments aimed at extracting information about the small strangeness current contribution is estimated to be only marginal, however, since experiments of sufficient accuracy would be very difficult, and their interpretation complicated. Conclusions are summarized in Sec. 4.

2. General electroweak relations

In this section we consider two inelastic electroweak processes causing a $(0^+0)_{\text{gnd}} \rightarrow (0^+0)^*$ transition of a target nucleus (assuming pure quantum numbers for both states): polarized electron scattering and neutrino scattering. We follow the metric conventions of Bjorken and Drell [11]. The only assumptions that we make in the analysis are the validity of the Standard Model and strong isospin symmetry. Spin and isospin selection rules then allow one to derive simple relations between weak and electromagnetic properties of a nucleus for inelastic processes in the same way as for elastic processes. The fact that isospin $T = 0$ for initial and final states of the target implies that only the isoscalar part of the currents will contribute to hadronic matrix elements. In the approximation of the nuclear domain, where a nucleus is assumed to contain only u, d quarks and their antiquarks, this implies that only the following term of the weak neutral current of quarks will contribute [12]

$$J_\mu^{(0)} \doteq -2 \, \sin^2 \Theta_{\text{w}} \, J_\mu^\gamma \, . \tag{1}$$

Thus, in the nuclear domain, the PV asymmetry in polarized electron scattering is independent of the nuclear structure and the neutrino cross section is proportional to the electron cross section. Corresponding formulae are discussed below.

In the real world, heavy quarks of other flavors contribute to the isoscalar part of the weak current. These quarks can exist as virtual $q, \overline{q}$ pairs in a nucleus. We take into account only the contribution of the $s, \overline{s}$ quarks because quarks of other flavors are much heavier and their influence can be neglected. The additional isoscalar piece of the weak neutral current is then

$$\delta J_\mu^{(0)} = -\frac{1}{2}\,\bar{s}\gamma_\mu(1-\gamma_5)s\,. \tag{2}$$

The axial-vector part of this current cannot contribute to the processes considered because initial and final states of the target have $J^\pi = 0^+$. Thus the vector part of the strange current can be studied by observing the contributions it makes to the processes discussed here.

Let us consider the PV part of the inelastic polarized electron scattering. The PV asymmetry A is defined in the usual way:

$$A \equiv \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow}\,. \tag{3}$$

Then the same way of reasoning that was used in the case of the PV in elastic scattering [12] generates the result (within the one-photon-exchange approximation)

$$A_{\rm inel} = \left(\frac{GQ^2}{\sqrt{2}\pi\alpha}\right)\sin^2\Theta_{\rm w}\;\xi\,, \tag{4}$$

where ξ is defined by

$$\xi \equiv \left[1 - \frac{\delta F_{\rm inel}^{(0)}(q^2)}{2\,\sin^2\Theta_{\rm w}\,F_{\rm inel}^{\rm ch}(q^2)}\right]. \tag{5}$$

Precise definition of the form factors is given in the next section. By Q^2 we designate the absolute value of the space-like four-vector of transferred momentum squared $Q^2 = -q_\mu q^\mu$. For energies and transferred momentum in the range considered in this work $Q^2 \simeq q^2$.

The same formula without the ξ factor represents the asymmetry in the nuclear domain. Deviation of $A_{\rm inel}$ from the simple nuclear domain result measures either a strange quark contribution to the nuclear transition considered, or the degree of strong isospin symmetry breakdown. For light nuclei, isospin symmetry holds well [8] and the deviation that we see comes from the strange quark effects. $\delta F_{\rm inel}^{(0)}(q^2)$ measures directly the new nuclear matrix element of the vector strange current for all q^2 considered. To perform an informative PV electron scattering experiment, one would have to measure the inelastic charge form factor at least up to q^2 around which the figure-of-merit for the transition reaches its maximum (in reality — to still higher q^2, so that the elastic scattering radiation tail can be separated).

For neutrino-nucleus inelastic scattering, the same argument as the one used for the elastic process generates the result

$$\left(\frac{d^2\sigma}{d\Omega d\epsilon_2}\right)_{\rm inel}^{\nu\nu'} = \left(\frac{GQ^2}{\sqrt{2}\pi\alpha}\right)^2 \sin^4\Theta_{\rm w}\left(\frac{d^2\sigma}{d\Omega d\epsilon_2}\right)_{\rm inel}^{ee'}\xi^2\,, \tag{6}$$

where the factor ξ has been defined above. Here we encounter the second power of the small quantity G on the right hand side, so the effect is very small. However, in principle, this relation can be used to make a model-independent prediction of the inelastic neutrino scattering cross section in the nuclear domain approximation[1] (the same formula with no ξ^2 factor). One can use the corresponding elastic relation to determine the neutrino flux (which is the largest source of uncertainty in neutrino experiments [13, 14]) and thus to predict the counting rate for the inelastic case.

It is important to note that the above relations are true to all orders in QCD.

If one neglects meson exchange currents (MEC) the Coulomb multipole operator becomes a one-body operator. If one further assumes both ground and excited (0^+0) states to consist of nucleons in S-states only, then the spin-orbit part of the Coulomb operator does not contribute to the inelastic form factor (spin-orbit corrections to $A_{\rm el}$ due to 15% D-state admixture in $(0^+0)_{\rm gnd}$ have been estimated as negligible [15]). Then the nuclear structure cancels from the ratio of the form factors leaving only single nucleon form factors behind:

$$\left.\frac{\delta F^{(0)}_{\rm inel}(q^2)}{F^{\rm ch}_{\rm inel}(q^2)}\right|_{S-\rm state} \rightarrow \frac{G^{(s)}_{\rm E}(q^2)}{G^{(T=0)}_{\rm E}(q^2)} . \qquad (7)$$

Here $G_{\rm E}$ are Sachs electric form factors. In this limit, nuclear structure does not enter into results for the PV asymmetry and for the same q^2

$$\frac{A_{\rm inel}}{A_{\rm el}} = 1 . \qquad (8)$$

Deviations of the magnitude of the ratio from unity could allow one to test the validity of the picture neglecting MEC (independently of the nucleon strangeness). In this test one compares (for a pure excited state) two experimental quantities rather than experiment to (model dependent) impulse approximation calculation as usual. If one measures elastic and inelastic asymmetries in the same experiment, their ratio is independent of the polarization of the electron beam. Some of the helicity-beam-parameters correlation, which constitute the most important class of systematic errors in asymmetry measurements [16], will also be reduced in this ratio.

The foregoing analysis is valid for any light (0^+0) nucleus. In the rest of this paper we shall consider the ^{4}He nucleus, however, as a practically

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[1] This excitation is, in principle, easier to detect than the elastic scattering. The excited state is unstable and one will observe two new slow charged particles in the final state p+^{3}H (one of which, ^{3}H, experiences β-decay) rather than just recoiling neutral ^{4}He in the elastic case.

important example. Experiments to determine the ground-state nuclear matrix element of the strange current from the asymmetry in elastic polarized electron scattering on that nucleus are planned for CEBAF [9, 10]. It could be possible to use the same equipment to measure the analogous inelastic process. The possibility of measuring inelastic neutrino scattering on ^{4}He is also discussed [17]. We note that isospin symmetry holds well for the ^{4}He nucleus [8]. To be able to make a real use of the relations of this section between elastic and inelastic processes, one should measure $F^{\rm ch}_{\rm inel}(q^2)$ for higher momentum transfers. To make an estimate of where in q^2 the figure-of-merit for PV asymmetry has a maximum, and whether it is reasonable to expect that the inelastic charge form factor is large enough at this q^2 to allow measuring the cross section, we will explain existing low q^2 data for ^{4}He within three simple models of the $(0^+0)^*$ state and project the curves for the form factor to higher q^2. Very few measurements of $F^{\rm ch}_{\rm inel}(q^2)$ have been performed so far [18].

3. Inelastic charge form factor of ^{4}He

In the energy level diagram of ^{4}He in Fig. 1 taken from [19], one can see that this nucleus actually has the ground and the first excited states with the required quantum numbers. The $(0^+0)^*$ excited state lies at 20.21 MeV, just above the threshold of the break-up into p+^{3}H at 19.81 MeV and just below the threshold for n+^{3}He at 20.57 MeV. Discussion of the accuracy of the quantum numbers determination for this state is presented in [20, 18]. It is the lowest excited state of ^{4}He with the next closest resonant level (0^-0) at 21.01 MeV. Thus to detect the excitation of the $(0^+0)^*$ level we need the energy resolution better than 0.9 MeV. CEBAF's Hall A detector will have the high momentum resolution of $\delta p/p \simeq 10^{-4}$, thus making the detection of the first excited state possible. This state was observed as a narrow Breit–Wigner resonance with $\Gamma \simeq 240$ keV in the inelastic electron scattering experiment in Mainz [18] (see Fig. 2). Radiative corrections were subtracted while analyzing the data and the break-up background was separated by fitting it with a smooth curve. The inelastic charge form factor of ^{4}He was measured in this experiment only up to $q^2 < 2.4$ fm^{-2} and experimental errors are large.

In the analysis of the $(0^+0)^*$ resonance there arise questions of how well one can take into account the presence of the break-up background and intrinsic parity and isospin mixing with the neighboring states and continuum. These questions need to be answered for obtaining quantitative predictions for experiments with the $(0^+0)^*$ state. At the same time, their consideration makes the analysis significantly more difficult. It would seem reasonable to have an experimental result if the resonance can be discerned

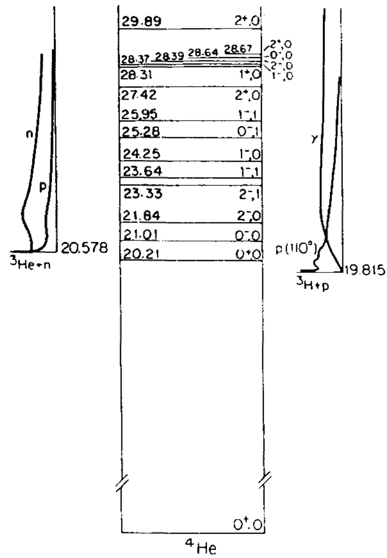

Fig. 1. Energy level diagram of ^{4}He. The (J^π, T) quantum numbers are shown on the right. All the energies on the left are in MeV (from [19]).

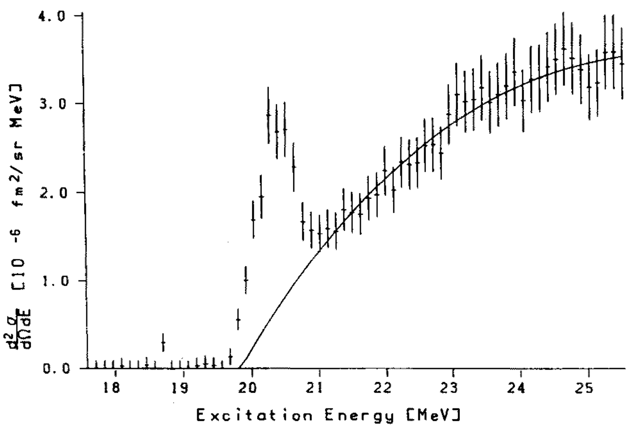

Fig. 2. Double differential cross section for the inelastic ^{4}He(e,e′) ^{4}He scattering at the incident energy $E_0 = 320$ MeV and the scattering angle $\Theta = 44.96°$. Solid line represents smooth break-up background. The (0^+0) excitation at 20.21 MeV is seen as a sharp resonance. Taken from [18].

from the background at the q^2 of interest first. Since the goal of the present work is to provide a qualitative estimate what these q^2 are and whether the $(0^+0)_{\rm gnd} \to (0^+0)^*$ transition can be used in future electroweak experiments, we shall not consider these complications here. Pure quantum numbers and a resonant character are assumed for the (0^+0) state. Just one note can be made, that admixture of a (0^-0) state cannot contribute to the inelastic electron scattering cross section or inelastic PV asymmetry since each of them contains the matrix element of EM current at least once. This current has no multipole capable of connecting (0^+0) and (0^-0) states. For neutrino scattering, there exists a pure axial current term and one still faces the question of the parity-mixing.

Only the Coulomb multipole $\hat{\mathcal{M}}^{\rm coul}_{0,0}$ can contribute to the excitation of the $(0^+0)^*$ level in electron scattering, so by measuring a differential cross section (with the break-up background subtracted) we measure the inelastic charge form factor of the transition. To define what exactly is meant here by the charge form factor, we write the formula for the electron scattering cross section integrated over the resonance (with the break-up background and radiative corrections excluded)

$$\left(\frac{d\sigma}{d\Omega}\right)^{ee'}_{(0^+0)_{\rm gnd}\to(0^+0)^*} = Z^2\sigma_{\rm M}\frac{Q^4}{\vec{q}^4}\,|\,F^{\rm ch}_{\rm inel}(q^2)\,|^2 r\,. \tag{9}$$

Here $r^{-1} = 1 + 2\epsilon_1/M_{\rm T}\,\sin^2(\theta/2)$ is a recoil factor and

$$\sigma_{\rm M} \equiv \frac{\alpha^2\cos^2(\theta/2)}{4{\epsilon_1}^2\sin^4(\theta/2)} \tag{10}$$

is the Mott cross section. The first two factors on the right hand side are chosen to normalize the form factor in the same formula for the elastic case to $F^{\rm ch}_{\rm el}(0) = 1$.

The elastic charge form factor of ^{4}He has been measured up to $q^2 \simeq 45\,{\rm fm}^{-2}$ [21]. It was parametrized well analytically in the region below $q^2 \simeq 12\ {\rm fm}^{-2}$ by the formula [22]

$$F^{\rm ch}_{\rm el}(q^2) = \left(1 - (a^2q^2)^6\right)\,\exp\left(-b^2q^2\right), \tag{11}$$

where $a = 0.316$ fm and $b = 0.675$ fm. Figure 3 shows the existing data on the elastic and inelastic form factors of ^{4}He in this region.

We shall try to explain the measured low-q^2 data on the inelastic charge form factor, and to predict its behavior at higher q^2, using the following three simple models of the ^{4}He nucleus:

- Collective model of Werntz and Überall [25],

and two shell models with potentials of

- Simple harmonic oscillator
- Finite square-well.

More sophisticated calculations of the structure of the $(0^+0)^*$ state in ^{4}He have been performed [23, 24], however the objectives of the present investigation can be met with the above simple models.

3.1. Collective model ("breathing mode")

This model of the (0^+0) excitation of ^{4}He was developed first by Werntz and Überall in 1964 [25]. They approximated the ^{4}He nucleus by a system with a continuous matter distribution $\rho_0(r)$. The first excited $(0^+0)^*$ state is modelled by the system experiencing radial scaling oscillations ("breathing mode"). The scaling factor is assumed to be small and to change harmonically. This breathing mode motion is quantized, and the position of the first excitation of the harmonic oscillator is fit to the measured energy of the lowest $(0^+0)^*$ state of ^{4}He. All the parameters of the model are then determined and we obtain the following formula for the inelastic charge form factor:

$$F^{\rm ch}_{\rm inel}(q^2) = {\rm const} \times \; q \, \frac{dF^{\rm ch}_{\rm el}(q^2)}{dq} \,. \tag{12}$$

It seems however to be an oversimplification to treat ^{4}He, consisting of just four nucleons, as a continuous matter distribution experiencing, as a whole, scaled oscillations. Thus we leave an overall constant factor to be fit to the existing data for $F^{\rm ch}_{\rm inel}(q^2)$ at low q^2 [18]. For $F^{\rm ch}_{\rm el}(q^2)$ the analytic expression mentioned in the previous section is used. The data is explained well by const=0.04 with χ^2/N=0.98, if we exclude the first three points with the lowest q^2 (see results in Fig. 6). We cannot fit all the experimental points with this model. The first three points actually come from a different experiment and, keeping in mind the difficulty of the experiment, this fact can be a possible justification to consider the curve that fits well the rest of the experimental data. We shall return to this issue while discussing the finite square-well model.

3.2. Single particle models

3.2.1. General discussion

We start by considering the $(0^+0)^*$ state of ^{4}He to be discrete, and we apply the general formula for the multipole decomposition of the electron scattering cross section. Only the $J = 0$ Coulomb multipole will contribute to the process considered. The Coulomb operator is taken to be a single-particle operator, since meson exchange currents are known to make a minor contribution to electron scattering through the isoscalar charge multipoles, for the intermediate q^2 that we consider. We then decompose the Coulomb monopole matrix element in a single-particle basis [26]

$$\frac{Z}{\sqrt{4\pi}}F^{\rm ch}(q^2) \equiv \langle 0^+0; f \parallel \hat{\mathcal{M}}_0^{\rm coul}(q) \parallel 0^+0; i\rangle$$
$$= \frac{1}{\sqrt{2}}\sum_{a,b}\langle a \parallel \mathcal{M}_0^{(0)}(q) \parallel b\rangle\, \Psi_{00}^{fi}(ab)\,, \tag{13}$$

where

$$\mathcal{M}_0^{(0)}(q) = \frac{1}{\sqrt{4\pi}}j_0(qr) \tag{14}$$

is the single-particle matrix element and

$$\Psi_{00}^{fi}(ab) = \langle 0^+0; f \mid c_a^+ c_b \mid 0^+0; i\rangle\,. \tag{15}$$

For a ground state of ^{4}He we assume a closed-shell configuration with all four nucleons in the (1s) state. Under this assumption the elastic charge form factor will be approximated well in the region of interest by both models which we shall consider (see Fig. 3). Thus for calculating $F_{\rm el}^{\rm ch}(q^2)$ we consider the filled s-shell, so $\Psi_{00}^{fi} = 2\delta_{ab}$ and

$$F_{\rm el}^{\rm SM}(q^2) = \langle 1{\rm s} \mid j_0(qr) \mid 1{\rm s}\rangle\,. \tag{16}$$

Let us apply selection rules to the states that Coulomb monopole operator, as a single-particle operator, will see among those comprising the $(0^+0)^*$ excited state:

$| (1{\rm s})^{-1}(1{\rm p}) >$ is ruled out by parity conservation,

$| (1{\rm s})^{-1}(2{\rm s}) >$ is allowed (as well as $| (1{\rm s})^{-1}(m{\rm s}) >$ in general),

$| (1{\rm s})^{-1}(1{\rm d}) >$ and higher l excitations are ruled out since their angular momentums cannot add to produce J=0.

In models with a continuum spectrum, $l = 0$ states from the continuum can also contribute.

For each allowed single-particle contribution to the excited state of ^{4}He we have a particle-hole transition, and for any pure particle-hole transition $\Psi_{00}^{fi}(ab) = \delta_{a(ms)}\delta_{b(1s)}$ in the calculation of $F_{\rm inel}^{\rm ch}(q^2)$. One can use any complete system of states for the decomposition of the $(0^+0)^*$ state of ^{4}He, but we would like to find a model in which the contributions of the few lowest excited states approximate well the inelastic charge form factor.

While comparing form factors calculated in the shell model to experimental results, we have to multiply the former by a single-nucleon form factor described by

$$F_{\rm sn}(Q^2) = \left[1 + \frac{Q^2}{18.84\ {\rm fm}^{-2}}\right]^{-2} \tag{17}$$

and corrections due to the center-of-mass motion have to be taken into account [26] (see below).

3.2.2. Simple harmonic oscillator model

There are two main reasons to start the analysis by considering a simple harmonic oscillator potential model:

1. The necessary matrix elements are easy to calculate in a closed form.
2. The center-of-mass corrections to the form factor can be treated exactly.

In this model

$$F_{\rm el}^{\rm SM}(q^2) = {\rm e}^{-y}\,, \tag{18}$$

where

$$y = \frac{b_{\rm osc}^2 q^2}{4} \tag{19}$$

and $b_{\rm osc}$ is the oscillator parameter.

Then

$$F_{\rm el}^{\rm ch}(q^2) = F_{\rm sn}\; F_{\rm CM}\; F_{\rm el}^{\rm SM}\;(q^2)\,. \tag{20}$$

Here $F_{\rm CM} = {\rm e}^{y/4}$. It is a correction subtracting the spurious effect of the center-of-mass motion [26].

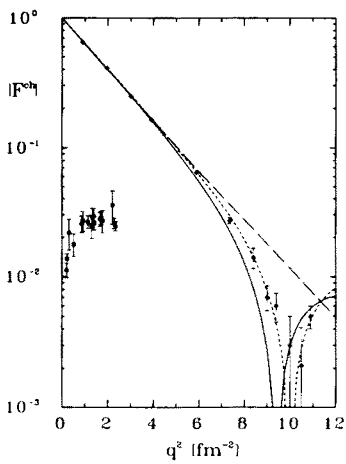

Fig. 3. Experimental data on the ^{4}He form factors. Points with circles represent $F_{\rm inel}^{\rm ch}(q^2)$ [18], while points with diamonds represent $F_{\rm el}^{\rm ch}(q^2)$. Short-dashed line shows the analytical approximation of $F_{\rm el}^{\rm ch}(q^2)$, Ref. [22]. Long-dashed and solid lines represent the s.h.o. model and the finite square well model fits to $F_{\rm el}^{\rm ch}(q^2)$, respectively.

The oscillator parameter is determined by fitting the experimental $F_{\rm el}^{\rm ch}(q^2)$ for $q^2 < 10\ {\rm fm}^{-2}$ (see Fig. 3). This is about how far in q^2 the simple harmonic oscillator model can approximate the experimental $F_{\rm el}^{\rm ch}(q^2)$, and thus how far $F_{\rm inel}^{\rm ch}(q^2)$ can be predicted in this model. The result of fitting is $b_{\rm osc} = 1.39$ fm. Then for the relative energy of the (2s) state one obtains $E_{\rm exc} = 2\hbar\omega \simeq 43$ MeV. This is about twice the energy of the $(0^+0)^*$ in ^{4}He. We shall try to model the excited nucleus by considering it to consist of any number of nucleons promoted to the three lowest shell model states (*i.e.* 1p, 2s, 1d).

Then, in accord with the general discussion,

$$F_{\rm inel}^{\rm SM}(q^2) = \frac{\alpha}{2} < 2{\rm s} \mid j_0(qr) \mid 1{\rm s} > \qquad (21)$$

or, upon integration of the matrix element,

$$F_{\rm inel}^{\rm SM}(q^2) = \frac{\alpha}{\sqrt{6}} y\ {\rm e}^{-y}\,. \qquad (22)$$

Here α is the probability amplitude of the $(0^+0)^*$ state to contain a $\mid (1{\rm s})^{-1}(2{\rm s})\rangle$ excitation of the shell model. If the $(0^+0)^*$ state were a pure $\mid (1{\rm s})^{-1}(2{\rm s})\rangle$ state of the shell model, α would be equal to 1. Fitting all experimental points, excluding the three with the lowest q^2, we determine $\alpha = 0.18$ with $\chi^2/N = 0.92$. Again we cannot fit all the experimental points. One possible reason for that was mentioned in connection with the collective model, another one will be discussed below. The resulting curve for $F_{\rm inel}^{\rm ch}(q^2)$ is shown in Fig. 6. It is seen to follow closely the curve obtained in the "breathing mode" collective model.

There are reasons to take the results of the simple harmonic oscillator model with a grain of salt. In calculating the charge form factor, the charge density matrix element in the integral is weighted by the square of the radial distance r^2, so the tail of the wave function makes a significant contribution. All states in the simple harmonic oscillator potential are bound, while the $(0^+0)^*$ state in ^{4}He lies, in fact, above the threshold of the break-up continuum. Thus the simple harmonic oscillator model makes a poor approximation of the region of large r that is important in the problem. In momentum space, this corresponds to the low q^2 region, so the inability to fit the first three points by a theoretical curve can be attributed to the shortcomings of the model chosen. To improve our results, we have to consider the $(0^+0)^*$ state as a resonance and choose a shell model potential that has a continuum spectrum. The shape of the potential that we use for small r is relatively unimportant. To make the formulae tractable, we choose the model potential to have the shape of a finite square-well.

3.2.3. Finite square-well model

Now we consider the problem in a more general fashion, taking into account that the final $(0^+0)^*$ state is actually a resonance in the break-up continuum. The final nuclear state is taken to be $| f\rangle = | p_2, \kappa^{(-)}\rangle$ which is the exact two-particle scattering state of p+^{3}H. All kinematical variables are defined in Fig. 4.

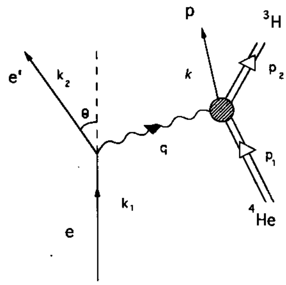

Fig. 4. Diagram for the break-up scattering ^{4}He(e,e'p)^{3}He

Then we can follow the analysis of coincidence experiments given in [26]. The general formula for the coincidence cross section can be found in [26] expressed in terms of

$$\mathcal{J}_\mu \equiv \frac{1}{4\pi W}\left(2\omega_\kappa E_1 E_2 \Omega^3\right)^{1/2} \langle p_2, \kappa^{(-)} \mid \int e^{i\vec{q}\cdot\vec{x}} J_\mu(\vec{x}) d\vec{x} \mid p_1\rangle\,, \qquad (23)$$

where W is the final energy of hadrons in the c.m. frame.

On the other hand, from experiment we know that the scattering is resonant in the $(0^+0)_{\rm gnd} \to (0^+0)^*$ channel and can be parametrized in the Breit–Wigner form:

$$\left(\frac{d^2\sigma}{d\Omega d\epsilon_2}\right)^{ee'}_{\rm inel} = 4\pi\sigma_{\rm M}\left[\frac{M_{\rm T}}{W}\,\frac{\Gamma}{2\pi}\,\frac{1}{(W-W_{\rm R})^2+\Gamma^2/4}\right] \times \frac{Q^4}{\vec{q}^4} \left| \langle 0 \,\|\, \hat{\mathcal{M}}_0^{\rm coul}(q) \,\|\, 0\rangle_{\rm res} \right|^2 . \qquad (24)$$

After integration over the resonance, the term in the square brackets gives the recoil factor and the formula (9) is reproduced.

The experimental formula (24) for the double-differential cross section can be deduced from the general result for the coincidence cross section if one assumes that the hadronic current matrix element has the following form

$$\left(\frac{\kappa}{\pi}\right)^{1/2} \mathcal{J}_c = \left(\frac{\Gamma}{2\pi}\right)^{1/2} \frac{1}{W - W_{\rm R} + i\Gamma/2} \langle 0 \,\|\, \hat{\mathcal{M}}_0^{\rm coul}(q) \,\|\, 0\rangle_{\rm res} \,. \qquad (25)$$

In this formula the q-dependence is separated from the W-dependence, since the matrix element of the Coulomb operator is evaluated at the resonant energy; however, we still do not know the wave functions necessary to evaluate this matrix element.

The question is whether we can convert the general matrix element of the current into this form with the help of the single particle decomposition of the $J = 0$ Coulomb operator, which is the only operator to contribute to $\mathcal{J}_c$ in the process that we consider

$$\mathcal{J}_c = \frac{M_{\mathrm{T}}}{4\pi W} (2\omega_\kappa \Omega)^{1/2} \langle 0; f, W \parallel \hat{\mathcal{M}}_0^{\mathrm{coul}}(q) \parallel 0; i \rangle , \tag{26}$$

where $M_{\mathrm{T}} \simeq \sqrt{E_1 E_2}$. We use the decomposition of Eq. (13) which is exact for any single-particle operator with any complete single-particle basis.

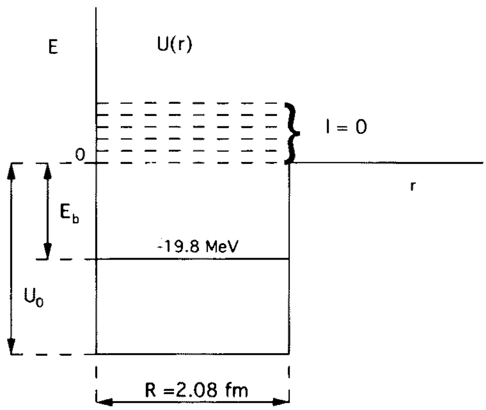

Fig. 5. Potential of the finite square-well model. Dashed lines represent $l = 0$ continuum states from which the $(0^+0)^*$ excitation is made.

We shall consider the finite square-well model potential shown in Fig. 5. The well parameters are adjusted to give the correct single nucleon binding energy in ^{4}He. We assume that the $(0^+0)_{\mathrm{gnd}}$ is a closed (1s) shell and build the $(0^+0)^*$ out of the $l = 0$ continuum states. The depth of the well $U_0(R)$ is determined first as a function of R by matching the wave functions at the edge of the well. Then we determine $F_{\mathrm{el}}^{\mathrm{SM}}(q^2, R)$ and fit the experimental data on the elastic charge form factor up to $q^2 = 10\ \mathrm{fm}^{-2}$ to determine R (see Fig. 3). $R = 2.08$ fm approximates the data on $F_{\mathrm{el}}^{\mathrm{ch}}(q^2)$ well. Then U_0 is determined to be $U_0 = 45$ MeV. The Coulomb interaction between the p and ^{3}H in the final state, which has the potential $U_{\mathrm{coul}}(R) \simeq 0.7$ MeV at the edge of the square-well, is neglected [2].

[2] It is assumed here, as in the previous two models, that strong isospin is a good

The Coulomb monopole matrix element of the transition can then be expanded as

$$\langle 0; f, W \mid \hat{\mathcal{M}}_0^{\mathrm{coul}}(q) \mid 0; i\rangle = \frac{1}{\sqrt{2}} \int_0^{\infty} dE\, \mathcal{C}(q,E)\, \Psi^{fi}(E,W)\,, \qquad (27)$$

where

$$\mathcal{C}(q,E) \equiv \langle l = 0,\ E \mid \mathcal{M}_0(q) \mid 1\mathrm{s}\rangle\,, \qquad (28)$$

is the single-particle matrix element and

$$\Psi^{fi}(E,W) \equiv \langle 0^+0; f, W \mid c_{\mathrm{E}}^{+} c_{1\mathrm{s}} \mid 0^+0; i\rangle\,. \qquad (29)$$

These relations are exact if the Coulomb monopole is a single-particle operator and if the ground state of ^{4}He is a closed (1s) shell. We remember that the q-dependence should be separated from the W-dependence in order to cast the matrix element into the form reproducing the cross section of the Breit-Wigner type. This can be achieved most simply if the single-particle energy E dependence factors from the $\mathcal{C}(q,E)$. If this separation occurs, the integral over the energy E can be calculated, producing a constant. The shape of the charge form factor of the transition will then coincide with the shape of the single-particle Coulomb monopole matrix element.

Analysis shows that for the single-particle energies up to $E \simeq 10$ MeV, and for momentum transfers q^2 ranging from 1 to about 10 fm^{-2}, the E-dependence of the single-particle matrix element can be separated, with an accuracy better than 20 percent, in the following form:

$$\mathcal{C}(q,E) \simeq \sqrt{\frac{E}{W_{\mathrm{R}}}}\, \mathcal{C}(q, W_{\mathrm{R}})\,. \qquad (30)$$

Then if we cut off the integral over the energy in Eq. (27) at 10 MeV, we obtain the following formula for the charge form factor for transferred momentum below 10 fm^{-2}:

$$F_{\mathrm{inel}}^{\mathrm{SM}}(q^2) = \mathrm{const} \times \mathcal{C}(q, W_{\mathrm{R}})\,. \qquad (31)$$

While comparing this result with the experimental $F_{\mathrm{inel}}^{\mathrm{ch}}(q^2)$, we take the c.m. corrections from the simple harmonic oscillator model. Fitting *all* the experimental data on $F_{\mathrm{inel}}^{\mathrm{ch}}(q^2)$, we obtain const $= 2.5$ with $\chi^2/N = 1.14$.

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symmetry for the nuclear transition matrix element (even though it is broken by Coulomb and mass effects in the decay channels).

Let us consider the first three points of lowest q^2 that troubled the first two models. Even if we forget about them for a moment and fit only the remaining data, the curve that we obtain predicts these points to be in the place where they actually have been measured. This fact adds to the confidence in the model considered. The results are presented in Fig. 6.

3.2.4. Results

Figure 6 shows how well the inelastic charge form factor curves calculated in the three different models of the ^{4}He nucleus explain the experimental results at low q^2. Predictions for higher q^2 are projected also by these curves. The correct behavior of the form factor at $q^2 \to 0$ is preserved in the single-particle models because the same potential has been used to calculate the ground and excited states of the nucleus. One can see that the best fit to the data is given by the model with the finite square-well potential. The inelastic charge form factor predicted by all the models is seen to be 4 to 10 times smaller than the elastic one in the region of interest around $4 - 7$ fm^{-2}.

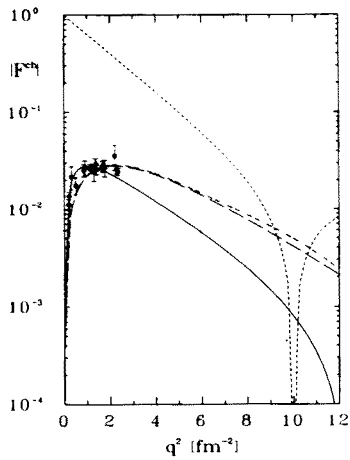

Fig. 6. The ^{4}He inelastic charge form factor. Experimental data are given by the points with circles. Long- and double-dashed lines represent the predictions of the collective ("breathing mode") model and the s.h.o. model correspondingly. The solid line is predicted by the finite square-well model. The $F_{\rm el}^{\rm ch}(q^2)$ of ^{4}He shown by the short-dashed line is given for comparison.

To see how useful the $(0^+0)_{\rm gnd} \to (0^+0)^*$ transition in ^{4}He can be in PV experiments, we discuss the corresponding figure-of-merit. The figure-of-merit $\mathcal{F}$ is defined as [2]

$$\mathcal{F} = \left(\frac{d\sigma}{d\Omega}\right) A^2 . \tag{32}$$

It represents a contribution of the internal properties of the target and of the kinematics of the experiment to the statistical uncertainty in the PV asymmetry. The latter can be calculated as

$$\frac{\delta A}{A} = [\mathcal{F} X_0]^{-1/2} ,$$
$$X_0 = \mathcal{L}\ \Delta\Omega T_0 , \tag{33}$$

where $\mathcal{L}$ is the luminosity, $\Delta\Omega$ is the detector solid angle, and T_0 is the running time. The inelastic PV asymmetry figure-of-merit estimates are shown in Fig. 7 for $\theta = 10°$. Let us assume the highest CEBAF luminosity for the ^{4}He target of $\mathcal{L} = 5 \times 10^{38}$ cm^{-2}s^{-1}, $\Delta\Omega = 10$ msr (angular acceptance of the CEBAF Hall A high resolution spectrometer), 1000 hours of running time and a 100% polarization of the incident electron beam. Then a statistical error of $\delta A/A \simeq 9 - 13\%$ in measuring the PV asymmetry at θ=10° can be achieved in an experiment performed at the incident beam energy $E \simeq 1.7 - 2.2$ GeV (which corresponds to $q^2 = 2.2 - 3.7$ fm^{-2}) where the figure-of-merit curve has its maximum, depending on the nuclear state model chosen. However, such an experiment on ^{4}He would not be easy to interpret due to the complicated nuclear structure of the $(0^+0)^*$ state.

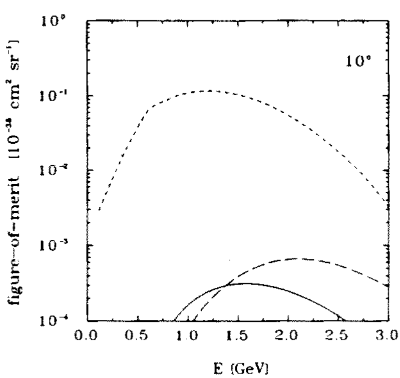

Fig. 7. The figure-of-merit for the PV asymmetry measurements on ^{4}He. The short-dashed line represents the elastic scattering figure-of-merit. Long-dashed and solid curves are predicted by the s.h.o. and finite square-well models for the $(0^+0)_{\rm gnd} \to (0^+0)^*$ transition in ^{4}He, correspondingly.

4. Conclusions

1. General relations between PV electron asymmetries and neutrino cross sections for inelastic $(0^+0)_{gnd} \to (0^+0)^*$ transitions in nuclei have been obtained. It is shown, for example, that within the single-nucleon picture of the nucleus the inelastic PV asymmetry is identical to that in the elastic scattering. For an isolated nuclear state with pure quantum numbers, the deviation of this ratio from unity signals the presence of exchange currents.

2. The inelastic neutrino scattering cross section is predicted to be proportional to the inelastic electron scattering cross section. This prediction can in principle be tested experimentally free of an uncertainty in the neutrino flux, if the flux is determined from the corresponding elastic neutrino scattering experiment.

3. The magnitude of the inelastic electron scattering charge form factor determines whether such experiments are feasible. The low-q^2 inelastic charge form factor data for the $(0^+0)_{gnd} \to (0^+0)^*$ transition in ^{4}He has been explained within three simple models of the excited state. An estimate of this form factor for intermediate transfered momentum (for q^2 from 3 to 10 fm^{-2}) has been made. This estimate can serve as a zeroth order approximation to the real situation. It is predicted that the inelastic PV asymmetry can be measured with a statistical accuracy of about 9-13% in the favorable experimental setup considered.

4. Knowledge of the inelastic charge form factor $F^{ch}_{inel}(q^2)$ for ^{4}He is the key ingredient to all the above predictions. It is important to measure $F^{ch}_{inel}(q^2)$ accurately to higher momentum transfers. The non-resonant background probably increases with increasing q^2. Can one still see the resonant $(0^+0)^*$ peak at the q^2 that would be considered for measuring the inelastic PV asymmetry? If electron scattering experiments at intermediate q^2 give a positive answer, performing a state-of-the-art calculation of the inelastic charge form factor with inclusion of the break-up continuum, wrong parity and isospin admixtures, and effects of MEC would be in order.

5. To the extent to which radiative corrections and non-resonant background can be subtracted, and the wrong parity and isospin admixtures taken into account, measurement of the inelastic PV asymmetry provides a determination of a new *nuclear* transition matrix element of the vector strange current. However due to the excited state structure complexity, and the fact that the transition form factor is smaller than the elastic one, the inelastic PV experiment is significantly more difficult to perform and interpret. Such an experiment will not allow one to gain a better understanding of the *nucleon* strangeness than can be obtained from the corresponding elastic experiments.

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