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HIGH ENERGY ASYMPTOTICS OF PERTURBATIVE QCD*,**

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In these lectures some perturbative approaches to the description of high energy scattering processes in QCD are reviewed. It is shown, that the gluon is reggeized and the pomeron is a compound state of two reggeized gluons. We demonstrate, that the equations for compound states of an arbitrary number of reggeized gluons in the multi-colour QCD have remarkable mathematical properties. In the conclusion the effective action describing the gluon-Reggeon interactions is discussed.

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1. Introduction

The Feynman–Bjorken parton model was invented to explain the approximate scaling behaviour of the structure functions $W_{1,2}(x,Q^2)$ for deep-inelastic scattering at the fixed Bjorken variable $x = Q^2/2pq$ and the large photon virtuality $Q^2 = -q^2$. Now this process is investigated in the region of very small values of x. Here the usual GLAP equation [1]

$$\frac{\partial}{\partial \ln Q^2} n_i(x) = -w_i\, n_i(x) + \sum_k \int_x^1 \frac{dx'}{x}\, w_{k\to i}(\frac{x}{x'})\, n_k(x') \tag{1}$$

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describing the Q^2-dependence of parton distributions $n_i(x)$ should be combined with the BFKL equation [2]

$$\frac{\partial}{\partial \ln \frac{1}{x}} n_g(x,k_\perp) = 2\omega(k_\perp^2) n_g(x,k_\perp) + \int d^2k'_\perp K(k_\perp,k'_\perp) n_g(x,k'_\perp) \quad (2)$$

describing their x-dependence for fixed values of the gluon transverse momentum $k_\perp$.

In these lectures it will be demonstrated, that in the Regge limit of large energies $\sqrt{s}$ and fixed momentum transfers $\sqrt{-t}$ the gluon having the spin $j = 1$ at $t = 0$ lies on the Regge trajectory $j = j(t)$. Therefore it is natural to reformulate QCD in terms of the Reggeon effective field theory. Below we show also, that in the generalized leading logarithmic approximation the equations for compound states of several reggeized gluons in the multi-colour QCD have remarkable mathematical properties leading to their exact integrability.

In QCD the gluon has two degrees of freedom corresponding to two possible values $\lambda = \pm 1$ for the helicity which is the projection of its spin $\vec{S}$on the momentum $\vec{k}$. They correspond to two polarization vectors e^λ. Their nonzero components are $e_1^\lambda = 1/\sqrt{2}$ and $e_2^\lambda = \lambda\, i/\sqrt{2}$ for the gluon moving along the third axis.

In accordance with the Lorentz invariance the gluon in the Yang–Mills theory is described by the four-dimensional vector-potential $v_\mu^a(x)$ (μ and a are the Lorentz and colour indices correspondingly). The matrix fields $v_\mu = v_\mu^a t^a$ belong to the self-conjugated representation of the SU(N) group ($N = 3$ for QCD). The quantities t^a are the anti-Hermitian Gell–Mann matrices with the commutation relations $[t^a, t^b] = f_{abc} t^c$ and the tensors f_{abc} are the structure constants of SU(N). The Yang–Mills theory is invariant under the gauge transformation

$$\delta v_\mu = [D_\mu, \chi]\,, \quad D = \partial_\mu + g v_\mu\,, \quad (3)$$

where g is the coupling constant and D is the covariant derivative. The intensity of the Yang–Mills field is given below

$$G_{\mu\nu} = \tfrac{1}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]\,. \quad (4)$$

The action for the gluodynamics is

$$S_{QCD} = -\tfrac{1}{2}\int d^4x\, tr(G_{\mu\nu})^2\,. \quad (5)$$

The elastic scattering amplitude $A(s,t,u)$ for the hadron process $a+b \to a' + b'$ is an analytic function of three invariants:

$$s = (p_a + p_b)^2,\ u = (p_a - p_{b'})^2,\ t = (p_a - p_{a'})^2\,, \quad (6)$$

related as follows:

$$s + u + t = 4m^2 \tag{7}$$

for the case of spinless particles with an equal mass m.

The function $A(s,t,u)$ describes simultaneously three channels. In the s, u and t-channels one has

$$s > 4m^2, \quad u < 0, \quad t < 0\,;$$
$$u > 4m^2, \quad s < 0, \quad t < 0\,;$$

and

$$t > 4m^2, \; u < 0, \quad s < 0\,,$$

correspondingly. For example, the scattering amplitude in the s-channel (where $\sqrt{s}$ is the c.m. energy of colliding particles) is obtained as a boundary value of $A(s,t,u)$ on the upper side of the cut at $s > 4m^2$ in the complex s-plane.

The Regge kinematics in the s-channel is given below

$$s \simeq -u >> m^2 \approx -t = \vec{q}^{\,2}\,, \tag{8}$$

where $\vec{q}$ is the momentum transfer in the c.m. system ($q = p_a - p_{a'}$). The total cross-section is determined by the optical theorem:

$$\sigma_{\rm tot}(s) = \tfrac{1}{s}\,\mathrm{Im}_s\; A(s,0)\,, \tag{9}$$

where $\mathrm{Im}_s\; A(s,t)$ is the imaginary part of the scattering amplitude in the s-channel. In the Regge kinematics the essential s-channel angular momenta $l = \rho\, p$ (ρ is the impact parameter and p is the c.m. momentum) are large and one can write the angular momentum expansion of A in the form of its Fourier transformation:

$$A(s,t) = -2is \int d^2\rho\; [S(s,\rho) - 1] e^{i\vec{q}\vec{\rho}}\,. \tag{10}$$

The quantity $S(s,\rho)$ is parametrized with the eikonal phase $\delta(s,\vec{\rho})$:

$$S(s,\rho) = e^{i\delta(s,\rho)}\,, \tag{11}$$

where $\mathrm{Im}\;\delta > 0$. Because essential values of ρ are fixed from above by the value $\rho_{\max} = c\ln(s)$ the Froissart theorem is valid:

$$\sigma_{\rm tot} < 4\pi c^2 \ln^2(s)\,. \tag{12}$$

Using some additional assumptions one can derive also the Pomeranchuk theorem for the particle-particle and particle-anti-particle total cross sections:

$$\sigma_t^{pp} = \sigma_t^{p\bar{p}} . \tag{13}$$

In the Regge model the asymptotics of the elastic scattering amplitude has the following form

$$A(s,t) = \sum_{p=\pm} \xi^p_{j_p(t)} s^{j_p(t)} g_1^p(t) g_2^p(t) . \tag{14}$$

Here $g_{1,2}(t)$ are the Reggeon couplings with external particles, $\xi^{\pm}_{j(t)}$ is the signature factor (for the signature $p = \pm 1$):

$$\xi_j^p = i - \frac{\cos \pi j + p}{\sin \pi j} \tag{15}$$

and $j_p(t)$ is the Regge trajectory assumed to be linear:

$$j_p(t) = j_0^p + \alpha'_p t , \tag{16}$$

where j_0^p and α'_p are the Reggeon intercept and slope correspondingly.

The Regge asymptotics seems to be natural from the t-channel point of view. Indeed, let us present the t-channel scattering amplitudes as the sum of contributions from different angular momenta j

$$A(s,t) = 16\pi \sum_{j=0}^{\infty} (2j+1) f_j^p P_j(\cos \theta_t) . \tag{17}$$

Here $P_j(z)$ are the Legendre polynomials and the t-channel scattering angle θ_t is related with the invariant s as follows:

$$z = \cos \theta_t = 1 + \frac{2s}{t - 4m^2} . \tag{18}$$

The t-channel partial waves f_j^p in a general case of the particles with non-zero spins are different for two signatures $p = \pm$ corresponding to the analytic continuation from even and odd values of j.

In the physical region of the s-channel the sum over integer j should be replaced by the Watson–Sommerfeld integral

$$A(s,t) = \frac{16\pi}{4i} \sum_p \int_{(L)} dj \, \xi_j^p \, p \, (2j+1) \, f_j^p(t) \, P_j(\cos(-\theta_t)) \tag{19}$$

along the contour L displaced along the line $\mathrm{Re}(j) = \sigma$ parallel to the imaginary axis. It is situated to the right of all singularities of f_j^p.

Using the known asymptotic behaviour of the Legendre polynomials for their large arguments one can write $A(s,t)$ in the Regge limit as follows:

$$A(s,t) = A^{+}(s,t) + A^{-}(s,t),\ A^{\pm}(-s,t) = \pm A^{\pm}(s,t)\,, \tag{20}$$

where

$$A^{p}(s,t) = \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{dj}{2\pi i}\, \xi_j^p\, s^j\, \phi_j^p(t)\,. \tag{21}$$

The functions $\phi_j^p(t)$ are proportional to the partial waves $f_j^p(t)$:

$$\phi_j^p(t) = c_j\, p\,(4m^2 - t)^{-j}\, f_j^p(t),\ c_j = 16\pi^2 4^j\, \frac{\Gamma(j+\frac{3}{2})}{\Gamma(\frac{1}{2})\Gamma(j+1)} \tag{22}$$

and are real in the physical region of the s-channel.

Thus, for the imaginary parts of the signatured amplitudes A^p one obtains the simple formulae, corresponding to the Mellin transformation:

$$\mathrm{Im}_s\, A^p(s,t) = \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{dj}{2\pi i}\, s^j\, \phi_j^p(t)\,. \tag{23}$$

The inverse Mellin transformation

$$\phi_j^p = \int_0^\infty d\xi\, e^{-j\xi}\, \mathrm{Im}_s\, A^p(s,t),\ \xi = \ln(s) \tag{24}$$

is a simplified version of the Gribov–Froissart representation for $f_j^p(t)$.

The t-channel elastic unitarity condition for partial waves ϕ_j^p analytically continued to complex j for $t > 4m^2$ takes the form

$$\frac{\phi_j^p(t+i\varepsilon) - \phi_j^p(t-i\varepsilon)}{2i} = c_j^{-1}(t-4m^2)^{j+\frac{1}{2}}\, t^{-\frac{1}{2}}\, \phi_j^p(t+i\varepsilon)\, \phi_j^p(t-i\varepsilon)\,. \tag{25}$$

The physical unitarity condition for f_j^p with integer j is its particular case. Experimentally all light hadrons belong to the Regge families with almost linear trajectories and an universal slope $\alpha' \approx 1\mathrm{GeV}^{-2}$.

Total cross-sections for hadron-hadron interactions at high energies are approximately constant (up to possible logarithmic terms). To reproduce such behaviour in the Regge model a special Reggeon called Pomeron was invented many years ago. Its trajectory is assumed to be close to 1:

$$j(t) = 1 + \omega(t) \ , \ \omega = \Delta + \alpha' t \, . \tag{26}$$

where $\Delta \approx 0.1$ and $\alpha' \approx 0.3\,\mathrm{GeV}^{-2}$. The real part of $A(s,t)$ for the Pomeron contribution is small. The quantities Δ and α' are the so-called bare parameters of the Pomeron.

In the j-plane there should be other moving singularities of $\phi_j^+(t)$ — the Mandelstam cuts arising as a result of simultaneous exchanges of several Pomerons. V. Gribov constructed the Reggeon field theory for calculating their contributions which renormalize in particular the Pomeron parameters. The Mandelstam cuts are taken into account approximately if one writes the eiconal phase in the form

$$\delta(s,\rho) = \frac{1}{2} \int \frac{d^2 q}{(2\pi)^2} \, e^{-i\vec{q}\vec{\rho}} \, i \, g^2(t) \, s^{\Delta - \alpha' \vec{q}^2} \, . \tag{27}$$

In this case the resulting amplitude satisfies the Froissart requirements. The analogous unitarization procedure can be used also in other cases when the scattering amplitude obtained in some approximation grows more rapidly than any power of $\ln(s)$.

The high energy theorems do not forbid the existence of another Regge pole with the flavour vacuum quantum numbers — the Odderon which has the negative signature and the negative charge parity. It could be situated also near $j = 1$, which would lead to a large real part of scattering amplitudes at high energies and to a significant difference between proton-proton and proton-antiproton interactions. Such singularity appears in the perturbative QCD simultaneously with the Pomeranchuk singularity and, therefore, the discovery of the Odderon effects would be very important.

The asymptotic behaviour of scattering amplitudes in the Born approximation is determined by the spin σ of the particle, exchanged in the crossing channel:

$$A_{\mathrm{Born}} \sim s^{\sigma} \tag{28}$$

and the Regge asymptotics is a generalization of this rule to continuous values of the spin: $\sigma \to j = j(t)$. In higher orders of the perturbation theory the scattering amplitude behaves (apart from possible logarithmic terms) as s^{σ}, where the value $\sigma = 1 + \sum_i (\sigma_i - 1)$ is determined by the spins σ_i of the particles in the t-channel intermediate states.

In QCD the gluon spin is 1 and therefore here the most important processes are governed by the gluon exchanges. For example, the Born amplitude for the parton-parton scattering is [2]

$$A(s,t) = \, 2s \, g \delta_{\lambda_a, \lambda_{a'}} T^c_{A'A} \, \tfrac{1}{t} \, g \delta_{\lambda_b, \lambda_{b'}} T^c_{B'B} \, , \tag{29}$$

where λ_i are helicities of the initial and final particles; A, A', B, B' are their colour indices and T^c_{ij} are the colour group generators. The s-channel helicity for each colliding particle is conserved because the virtual gluon in the t-channel for small q interacts with the total colour charge Q^c commuting with the QCD Hamiltonian.

Let us consider now the hadron-hadron scattering amplitude described by the Feynman diagrams containing two gluons in the t-channel without any other pure gluonic intermediate states. The polarization matrix for each of the gluon propagators with momenta k, $q-k$ can be written at large energies $s=(p_a+p_b)^2 \gg m^2$ as follows

$$\delta^{\mu\nu} = \delta^{\mu\nu}_{\parallel} + \delta^{\mu\nu}_{\perp} \simeq \delta^{\mu\nu}_{\parallel} = \frac{p_a^\mu p_b^\nu + p_a^\nu p_b^\mu}{p_a p_b} . \tag{30}$$

Moreover, if the indices μ and ν belong to the blobs with incoming particles a and b correspondingly, then with a good accuracy we have

$$\delta^{\mu\nu} \rightarrow \frac{p_b^\mu p_a^\nu}{p_a p_b} . \tag{31}$$

By introducing the Sudakov parameters

$$\alpha = -\frac{kp_a}{p_a p_b} = -s_a/s\,,\ \beta = \frac{kp_b}{p_a p_b} = s_b/s\,,\ d^4k = d^2k_\perp \frac{d\,s_a d\,s_b}{2\,s} \tag{32}$$

for the virtual gluon momenta $k\,, q-k$ one obtains for the contribution of these diagrams the following impact-factor representation:

$$A(s,t) = 2i|s|\frac{1}{2!}\int \frac{d^2k}{(2\pi)^2} \left(\vec{k^\perp}\right)^{-2} \left(\vec{q}-\vec{k^\perp}\right)^{-2} \Phi^a\left(\vec{k^\perp},\vec{q}-\vec{k^\perp}\right)\Phi^b\left(\vec{k^\perp},\vec{q}-\vec{k^\perp}\right), \tag{33}$$

The sum over the gluon colour indices is implied. Note, that we neglected the longitudinal momenta in gluon propagators in comparison with the corresponding transverse components because in the essential integration region for the impact factors

$$\Phi^{a,b}\left(\vec{k^\perp},\vec{q}-\vec{k^\perp}\right) = \int_{-\infty}^{\infty} \frac{ds_{a,b}}{2\pi i}\frac{p^\mu_{b,a}}{s}\frac{p^\nu_{b,a}}{s} f^{a,b}_{\mu\nu}\left(s_{a,b},\vec{k^\perp},\vec{q}-\vec{k^\perp}\right), \tag{34}$$

we have

$$s_{a,b} \sim m^2,\ \left(\vec{k^\perp}\right)^2 \sim \left(\vec{q}-\vec{k^\perp}\right)^2 \sim m^2\,, \tag{35}$$

and therefore $(k^{\|})^2 \ll (k^{\perp})^2$. The impact factors are real functions of $\vec{k^{\perp}}$, $\vec{q} - \vec{k^{\perp}}$ vanishing for small $|\vec{k^{\perp}}|$ and $|\vec{q} - \vec{k^{\perp}}|$, which is a consequence of the gauge invariance. In particular the total cross-section for the photon-photon scattering does not contain any infrared divergency in the integral over $\vec{k^{\perp}}$. In the leading logarithmic approximation ones also obtains the infrared-stable result [2]:

$$A_{\mathrm{BFKL}}(s,t) = i|s| \int \frac{d^2k^{\perp}}{(2\pi)^2} \frac{d^2k'^{\perp}}{(2\pi)^2} \Phi^a\left(\vec{k^{\perp}}, \vec{q} - \vec{k^{\perp}}\right) \Phi^b\left(\vec{k'^{\perp}}, \vec{q} - \vec{k'^{\perp}}\right) f_{\mathrm{BFKL}}(k, k', q), \tag{36}$$

where the function f_{BFKL} is expressed in terms of the t-channel partial wave f_ω for the virtual gluon-gluon scattering with the use of the Mellin transformation

$$f_{\mathrm{BFKL}}(k, k', q) = \int \frac{d\omega}{2\pi} s^{\omega} f_\omega\left(\vec{k^{\perp}}, \vec{q} - \vec{k^{\perp}};\, \vec{k'^{\perp}}, \vec{q} - \vec{k'^{\perp}}\right). \tag{37}$$

The BFKL equation for f_ω will be discussed later.

2. Multi-Regge processes in QCD

In the case of the deep-inelastic scattering at small Bjorken variable x the gluon distribution $g(x, k_\perp)$ depending on the longitudinal Sudakov component x of the gluon momentum k and on its transverse projection $k_\perp$ in the infinite momentum frame of the proton $|\vec{p}_A| \to \infty$ can be expressed in terms of the imaginary part of the gluon-gluon scattering amplitude at $t = 0$ in the Regge regime of high energies $\sqrt{s} = \sqrt{2p_Ap_B}$ and fixed momentum transfers $q = \sqrt{-t}$. The most probable process at large s is the gluon production in the multi-Regge kinematics for final state particle momenta $k_0 = p_{A'}, k_1 = q_1 - q_2, \ldots, k_n = q_n - q_{n+1}, k_{n+1} = p_{B'}$:

$$s \gg s_i = 2k_{i-1}k_i \gg t_i = q_i^2 = (p_A - \sum_{r=0}^{i-1} k_r)^2, \prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} \vec{k}_i^{\,2}, k_\perp^2 \equiv -\vec{k}^2 . \tag{38}$$

In LLA the production amplitude in this kinematics has the multi-Regge form [2]:

$$A^{\mathrm{LLA}}_{2\to 2+n} = A^{\mathrm{tree}}_{2\to 2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)} . \tag{39}$$

Here $s_i^{\omega(t_i)}$ are the Regge-factors appearing from the radiative corrections to the Born production amplitude $A^{\mathrm{tree}}_{2\to 2+n}$. The gluon Regge trajectory

$j = 1 + \omega(t)$ is expressed in terms of the quantity:

$$\omega(t) = -\frac{g^2 N_c}{16\pi^3} \int d^2k \frac{\vec{q}^{\,2}}{\vec{k}^2 (\vec{q}-\vec{k})^2}\,, \quad t = -\vec{q}^{\,2}\,. \tag{40}$$

Infrared divergencies in the Regge factors cancel with analogous divergencies in $\sigma_{\rm tot}$ from the contributions of real gluons. The production amplitude in the tree approximation has the following factorized form [2]

$$A^{\rm tree}_{2\to 2+n} = 2gT^{c_1}_{A'A}\Gamma_1 \frac{1}{t_1} gT^{d_1}_{c_2c_1}\Gamma^1_{2,1}\frac{1}{t_2}\dots gT^{d_n}_{c_{n+1}c_n}\Gamma^n_{n+1,n}\frac{1}{t_{n+1}} gT^{c_{n+1}}_{B'B}\Gamma_2\,. \tag{41}$$

Here A, B and A', B', d_r $(r = 1, 2...n)$ are colour indices for initial and final gluons correspondingly. $T^c_{ab} = -if_{abc}$ are generators of the gauge group SU(N_c) and g is the Yang–Mills coupling constant. Further,

$$\Gamma_1 = \tfrac{1}{2} e^{\lambda}_{\nu} e^{\lambda' *}_{\nu'} \Gamma^{\nu\nu'}\,,\ \Gamma^r_{r+1,r} = -\tfrac{1}{2}\Gamma_\mu(q_{r+1}, q_r) e^{\lambda_r *}_{\mu}(k_r) \tag{42}$$

are the Reggeon-Particle-Particle (RPP) and Reggeon-Reggeon-Particle (RRP) vertices correspondingly. The quantities $\lambda_r = \pm 1$ are the s-channel helicities of gluons in the c.m. system. They are conserved for each of two colliding particles: $\Gamma_1 = \delta_{\lambda'\lambda}$, which is not valid in the one loop approximation [3]. The tensor $\Gamma^{\nu\nu'}$ can be written as a sum two terms:

$$\Gamma^{\nu\nu'} = \gamma^{\nu\nu'+} - q^2 (n^+)^\nu \frac{1}{p^+_A} (n^+)^{\nu'}\,, \tag{43}$$

where we introduced the light cone vectors

$$n^- = \frac{p_A}{E},\, n^+ = \frac{p_B}{E},\, E = \sqrt{s}/2,\, n^+ n^- = 2\,, \tag{44}$$

and the light cone projections $k^\pm = k^\sigma n^\pm_\sigma$ of the Lorentz vectors k^σ. The first term is the light cone component of the Yang–Mills vertex:

$$\gamma^{\nu\nu'+} = (p^+_A + p^+_{A'})\delta^{\nu\nu'} - 2p^{\nu'}_A (n^+)^\nu - 2p^\nu_{A'}(n^+)\,. \tag{45}$$

The second (induced) term is a coherent contribution of the Feynman diagrams in which the pole in the t-channel is absent. Indeed, it is proportional to the factor q^2 cancelling the neighboring propagator.

Similarly, the effective RRP vertex $\Gamma(q_2, q_1)$ can be presented as follows [2]

$$\Gamma^\sigma(q_2, q_1) = \gamma^{\sigma -+} - 2q_1^2 \frac{(n^-)^\sigma}{k_1^-} + 2q_2^2 \frac{(n^+)^\sigma}{k_1^+}\,, \tag{46}$$

where

$$\gamma^{\sigma+-} = 2q_2^\sigma + 2q_1^\sigma - 2(n^-)^\sigma k_1^+ + 2(n^+)^\sigma k_1^- \tag{47}$$

is the light-cone component of the Yang–Mills vertex.

Due to the gluon reggeization the above expression for the production amplitude in LLA has the important property of the two-particle unitarity in each of the t_i channels. Furthermore, it satisfies approximately the unitarity conditions in the direct channels s_i with the intermediate particles being in the multi-Regge kinematics. These conditions lead to the so called "bootstrap" equations.

Note, that Γ^σhas the important property:

$$(k_1)^\mu \Gamma_\mu(q_2, q_1) = 0\,, \tag{48}$$

which gives us a possibility to chose an arbitrary gauge for each of the produced gluons. In the left (l) light cone gauge where $p_A e^l(k) = 0$ the polarization vector $e^l(k)$ is parametrized in terms of the two-dimensional vector $e^l_\perp$

$$e^l = e^l_\perp - \frac{k_\perp e^l_\perp}{k p_A} p_A\,, \tag{49}$$

and the Reggeon-Reggeon-particle vertex Γ takes an especially simple form

$$\Gamma^1_{2,1} = Ce^* + C^* e,\ \ C = \frac{q_1^* q_2}{k_1^*}\,, \tag{50}$$

if we introduce the complex components

$$e = e_x + ie_y\,,\ e^* = e_x - ie_y\,;\ k = k_x + ik_y\,,\ k^* = k_x - ik_y \tag{51}$$

for transverse vectors $\vec{e_\perp}$, $\vec{k_\perp}$. The factors q_1^* and q_2 in the expression for C suppress the inelastic amplitude at small momentum transfers. The singularity $1/k_1^*$ in C reproduces correctly the bremstrahlung factor in the soft gluon emission theorem.

The above complex representation was used in [4] to construct an effective scalar field theory for the multi-Regge processes. This theory was derived recently from the Yang–Mills Lagrangian by integrating over the fields corresponding to the highly virtual particles [5].

The effective action describing multi-Regge processes can be written in the form invariant under the Abelian gauge transformations $\delta V^a_\mu = i\partial_\mu \chi^a$ for the physical fields V_μ provided that the Reggeon fields $A_\pm$ are gauge invariant ($\delta A_\pm = 0$):

$$S_{m\,R} = \int d^4x \Big\{ \tfrac{1}{4}(F^a_{\mu\nu})^2 + \tfrac{1}{2}(\partial_{\perp\sigma}A^a_+)(\partial_{\perp\sigma}A^a_-)$$
$$+\tfrac{1}{2}g\Big[-A^a_+(F_{-\sigma}T^a i\partial_-^{-1}F_{-\sigma}) - A^a_-(F_{+\sigma}T^a i\partial_+^{-1}F_{+\sigma})$$
$$+(\partial_-^{-1}F^a_{-\sigma})(A_-T^a i\partial_\sigma A_+) + (\partial_+^{-1}F^a_{+\sigma})(A_+T^a i\partial_\sigma A_-)$$
$$+i\Big(\frac{1}{\partial_+}\frac{1}{\partial_-}F^a_{+-}\Big)(\partial_\sigma A_+)T^a(\partial_\sigma A_-) + iF^a_{+-}(A_-T^aA_+)\Big]\Big\}, \qquad (52)$$

where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $N_\pm = N_0 \pm N_3$ are the light-cone components of the vectors N_μ. The fields $A_\pm$ satisfy the kinematical constraints $\partial_\pm A_\mp = 0$ equivalent to the condition that in the multi-Regge kinematics the reggeized gluon transfers the negligible part of energy from the colliding particles. The Feynman vertices which are generated by this action coincide on the gluon mass shell with the above effective vertices for the Reggeon-gluon interactions.

3. BFKL pomeron

Using the explicit expressions for production amplitudes in the multi-Regge kinematics one can calculate the imaginary part of the elastic scattering amplitude with the vacuum quantum numbers in the crossing channel. The contribution from the real particle production is expressed in terms of the product of the effective vertices calculated in the light cone gauge [4]:

$$C(p_1,p_{1'})\,C^*(p_2,p_{2'}) + \text{h.c.} = \frac{p_1^* p_2\, p_{1'} p_{2'}^*}{|\,k\,|^2} + \text{h.c.}, \qquad (53)$$

where p_1, p_2 and $p_{1'}, p_{2'}$ are the corresponding complex transverse components of initial and final momenta in the t-channel ($q = p_1 + p_2 = p_{1'} + p_{2'}$). In turn, the contribution related to virtual corrections to the production amplitudes is proportional to the sum of the Regge trajectories of two gluons:

$$\omega(-\vec{p_1}^{\,2}) + \omega(-\vec{p_2}^{\,2}) \sim \ln|\,p_1\,|^2 + \ln|\,p_2\,|^2 + c\,, \qquad (54)$$

where the constant c contains the infrared divergent terms which are cancelled with the analogous terms from the real contribution after its integration in k. The final homogeneous equation for the t-channel partial wave $f_\omega(k, q-k)$ takes the form

$$E\Psi = H_{12}\Psi,\; E = -\frac{8\omega\pi^2}{g^2}\,. \qquad (55)$$

Here the "Hamiltonian" H_{12} is

$$H_{12} = \ln|p_1|^2 + \ln|p_2|^2 + \frac{1}{|p_1|^2|p_2|^2}(p_1^* p_2 \ln|\rho_{12}|^2 p_1 p_2^* + \text{h.c.}) - 4\psi(1)\,, \tag{56}$$

where $\psi(x) = \frac{d}{dx}\ln\Gamma(x)$ and $\Gamma(x)$ is the Euler Γ-function. In the above expression $1/\vec{p_i}^{\,2}$ are the gluon propagators. We introduced the complex components $\rho_k = x_k + iy_k$ for the impact parameters canonically conjugated to the momenta $p_k = i\frac{\partial}{\partial\rho_k}$ and performed the Fourier transformation:

$$\frac{1}{|k|^2} \to \ln|\rho_{12}|^2\,, \tag{57}$$

where $\rho_{ik} = \rho_i - \rho_k$. The expressions

$$\ln|p_i|^2, \qquad |p_i|^{-2} \tag{58}$$

are the integral operators in the impact parameter representation. The Hamiltonian has the property of the holomorphic separability:

$$H_{12} = h_{12} + h_{12}^*\,, \qquad E = \varepsilon + \widetilde{\varepsilon}\,, \tag{59}$$

where ε and $\widetilde{\varepsilon}$ are the energies correspondingly in the holomorphic and anti-holomorphic subspaces:

$$\begin{aligned} \varepsilon\psi(\rho_1,\rho_2) &= h_{12}\psi(\rho_1,\rho_2)\,, \\ \widetilde{\varepsilon}\widetilde{\psi}(\rho_1^*,\rho_2^*) &= h_{12}^*\widetilde{\psi}(\rho_1^*,\rho_2^*)\,, \\ \Psi(\vec{\rho_1},\vec{\rho_2}) &= \psi\,\widetilde{\psi}\,. \end{aligned} \tag{60}$$

The holomorphic Hamiltonian is

$$h_{12} = \frac{1}{p_1}\ln(\rho_{12})\,p_1 + \frac{1}{p_2}\ln(\rho_{12})\,p_2 + \ln(p_1 p_2) - 2\psi(1)\,. \tag{61}$$

One can verify the validity of another representation for H_{12}:

$$h_{12} = \rho_{12}\ln(p_1 p_2)\,\rho_{12}^{-1} + 2\ln(\rho_{12}) - 2\psi(1)\,. \tag{62}$$

Further, using the following identities:

$$2\ln\partial + 2\ln\rho = \psi(-\rho\partial) + \psi(\partial\rho) = \psi(-\rho\partial) + \psi(1+\rho\partial)\,, \tag{63}$$

and

$$2\ln(\rho^2\partial) - 2\ln(\rho) = \psi(\rho\partial) + \psi(-\rho^2\partial\rho^{-1}) = \psi(\rho\partial) + \psi(1-\rho\partial)\,, \tag{64}$$

we can derive the following formulas:

$$h = \ln(\rho_{12}^2 p_1) + \ln(\rho_{12}^2 p_2) - 2\ln(\rho_{12}) - 2\psi(1), \tag{65}$$

$$h = \tfrac{1}{2}\psi(\rho_{12}\partial_1) + \tfrac{1}{2}\psi(\rho_{21}\partial_2) + \tfrac{1}{2}\psi(1+\rho_{21}\partial_1) + \tfrac{1}{2}\psi(1+\rho_{12}\partial_2) - 2\psi(1). \tag{66}$$

Using above expressions for h it is possible to verify that h is invariant under the Möbius transformations:

$$\rho_k \to \frac{a\rho_k + b}{c\rho_k + d} \tag{67}$$

for arbitrary complex values of a, b, c, d. It means, that solutions of the BFKL equation belông to the irreducible unitary representations of the Möbius group. The generators of this group for an arbitrary number n of particles are

$$M^z = \sum_{k=1}^{n} \rho_k\partial_k\,,\ M^- = \sum_{k=1}^{n} \partial_k\,,\ M^+ = -\sum_{k=1}^{n} \rho_k^2\partial_k\,. \tag{68}$$

Its Casimir operator is

$$M^2 = (M^z)^2 - \tfrac{1}{2}(M^+M^- + M^-M^+) = -\sum_{r<s} \rho_{rs}^2\partial_r\partial_s\,. \tag{69}$$

In the case of two particles we can use the Polyakov ansatz for the wave function:

$$\psi_m(\rho_{10}, \rho_{20}) = \langle 0 \mid \varphi(\rho_1)\varphi(\rho_2)O_m(\rho_0) \mid 0\rangle = (\frac{\rho_{12}}{\rho_{10}\rho_{20}})^m\,. \tag{70}$$

Here $m = \frac{1}{2} + i\nu + \frac{n}{2}$ is the conformal weight of the composite operator O_m which has the anomalous dimension $d = 1 + 2i\nu$ and the conformal spin n. This operator belongs to the basic series of the unitary representations if ν is real and n is integer. The fields $\varphi(\rho_i)$ describe the reggeized gluons and have the trivial quantum numbers $d = n = 0$. The holomorphic factor ψ_m is an eigen-function of the corresponding Casimir operator:

$$M^2\psi_m = m(m-1)\psi_m\,. \tag{71}$$

Due to the Möbius invariance of the Hamiltonian ψ_m is also an eigen-function of the Schrödinger equation in the holomorphic subspace:

$$h\psi_m = \varepsilon\psi_m\,, \qquad \varepsilon = \psi(m) + \psi(1-m) - 2\psi(1)\,. \tag{72}$$

The eigen-value ε can be obtained from the above representations for h if one will integrate the both sides of the Schrödinger equation over the coordinate ρ_0 with the use of the relation:

$$\int d\rho_0\, \psi(\rho_{10}, \rho_{20}) \sim \rho_{12}^{1-m} \,. \tag{73}$$

The second Casimir operator M^{2*} is expressed through the conformal weight $\widetilde{m} = \frac{1}{2} + i\nu - \frac{n}{2}$. The total energy is

$$E = \psi(m) + \psi(1-m) + \psi(\widetilde{m}) + \psi(1-\widetilde{m}) - 4\psi(1)\,. \tag{74}$$

One can rewrite E as follows

$$E = 4\, Re\, \psi\left(\frac{1}{2} + i\nu + \frac{|\, n\,|}{2}\right) - 4\psi(1)\,. \tag{75}$$

The minimum of the energy is obtained for $\nu = n = 0$ and equals $E_0 = -8\ln 2$. Therefore the total cross-section calculated in LLA using the above expressions for production amplitudes grows very rapidly as s^{Δ} (where $\Delta = (g^2 N_c/\pi^2)\ln\ 2$), which violates the Froissart bound $\sigma_{tot} < c\ln^2 s$ [2]. One of the possible ways of improving the LLA results is to use the above effective field theory [4, 5]. We shall discuss this approach later.

4. Multi-Reggeon compound states

The simple method of unitarizing the scattering amplitude obtained in LLA is related with the solution of the BKP equations [6] for compound states of n reggeized gluons:

$$E\Psi = \sum_{i<k} H_{ik}\Psi\,, \tag{76}$$

where the eigen-value E is proportional to the position $\omega = j - 1$ of the singularity of the t-channel partial wave:

$$E = -\frac{8\pi^2}{g^2 N_c}\omega\,, \tag{77}$$

and the pair Hamiltonian H_{ik} has the property of the homomorphic separability:

$$H_{ik} = -\frac{T_i^a T_k^a}{N_c}(h_{ik} + h_{ik}^*)\,. \tag{78}$$

where T_i^a are the group generators acting on colour indices of the gluon i. The holomorphic two-body Hamiltonian is

$$h_{ik} = \frac{1}{p_i}\ln(\rho_{ik})p_i + \frac{1}{p_k}\ln(\rho_{ik})p_k + \ln(p_i p_k) + 2\gamma\,. \tag{79}$$

where $\gamma = -\psi(1)$ is the Euler constant. Similar to the Pomeron case we introduced the complex coordinates $\rho_k = x_k + iy_k$ $(k = 1, 2, \ldots n)$ and the canonically conjugated momenta $p_k = i\frac{\partial}{\partial(\rho_k)}$ in the impact parameter space (note, that $\rho_{ik} = \rho_i - \rho_k$). The above Schrödinger equation for Ψ is invariant [7] under the Möbius transformations:

$$\rho_k' = \frac{a\,\rho_k + b}{c\,\rho_k + d}$$

for any complex values of a, b, c, d.

In the multi-colour QCD ($N_c \to \infty$) only planar diagrams in the colour space are important according to 't Hooft and therefore the total Hamiltonian H can be written as a sum of the mutually commuting holomorphic and anti-holomorphic operators [8]:

$$H = \tfrac{1}{2}(h + h^*)\,, \qquad [h, h^*] = 0\,, \tag{80}$$

where $\frac{1}{2}$ is a colour factor and h contains only the neighboring gluon interaction:

$$h = \sum_{i=1}^{n} h_{i,i+1}\,. \tag{81}$$

Thus, in this case the solution of the Schrödinger equation has the property of the holomorphic factorization:

$$\Psi = \sum c_k \psi_k(\rho_1, \ldots \rho_n)\, \widetilde{\psi}_k(\rho_1^*, \ldots \rho_n^*)\,. \tag{82}$$

where ψ and $\widetilde{\psi}$ are correspondingly the analytic and anti-analytic functions of their arguments and the sum is performed over the degenerate solutions of the Schrödinger equations in the homomorphic and anti-holomorphic subspaces:

$$\varepsilon\psi = h\psi,\ \varepsilon^*\psi^* = h^*\psi^*,\ E = \tfrac{1}{2}(\varepsilon + \varepsilon^*)\,. \tag{83}$$

These equations have nontrivial integrals of motion [9]:

$$t(\theta) = tr\,T(\theta)\,, \qquad [t(u), t(v)] = [t(\theta), h] = 0\,, \tag{84}$$

where θ is the spectral parameter of the transfer matrix $t(\theta)$. The monodromy matrix $T(\theta)$ is constructed from the product

$$T(\theta) = L_1(\theta)L_2(\theta)\dots L_n(\theta) \tag{85}$$

of the L-operators expressed in terms of the Möbius group generators:

$$L_k(\theta) = \begin{pmatrix} \theta + i\rho_k\partial_k & i\partial_k \\ -i\rho_k^2\partial_k & \theta - i\rho_k\partial_k \end{pmatrix} . \tag{86}$$

Thus, the solution of the Schrödinger equation is reduced to a pure algebraic problem of finding the representation of the Yang–Baxter commutation relations [10]:

$$T_{i_1 i_1'}(u)T_{i_2 i_2'}(v)(v-u+iP_{12}) = (v-u+iP_{12})T_{i_2 i_2'}(v)T_{i_1 i_1'}(u) , \tag{87}$$

where the operator P_{12} in the left and right hand sides of the equation transmutes correspondingly the right and the left indices of the matrices $T(u)$ and $T(v)$. Moreover [10], the Hamiltonian for the Schrödinger equation coincides with the Hamiltonian for a completely integrable Heisenberg model with the spins belonging to an infinite dimensional representation of the noncompact Möbius group and all physical quantities can be expressed in terms of the Baxter function $Q(\lambda)$ satisfying the equation:

$$t(\lambda)Q(\lambda) = (\lambda+i)^n Q(\lambda+i) + (\lambda-i)^n Q(\lambda-i) , \tag{88}$$

where $t(\lambda)$ is an eigen-value of the transfer matrix. The solution of the Baxter equation is known for $n=2$ [10]. In a general case $n > 2$ one can present it as a linear combination of the solutions for $n=2$ by obtaining a recurrence relation for their coefficients. For $n=3$ this relation takes the form:

$$A d_k(A) = \frac{k(k+1)}{2(2k+1)}(k-m+1)(k+m)(d_{k+1}(A) + d_{k-1}(A)) \tag{89}$$

with the initial conditions $d_0 = 0, d_1 = 1$. For integer values of the conformal weight m the quantization condition for eigen-values A is $d_{m-1}(A) = 0$. Although the orthogonality and completeness conditions for the polynomials $d_k(A)$ are known:

$$\sum_{k=1}^{m-2} \frac{2(2k+1)}{k(k+1)(k-m+1)(k+m)} d_k(A)\, d_k(\widetilde{A}) = \delta_{A\widetilde{A}} d'_{m-1}(A)\, d_{m-2}(A) , \quad A \neq 0 , \tag{90}$$

$$\sum_{A\neq 0} \frac{d_k(A)\, d_{\tilde{k}}(A)}{d'_{m-1}(A) d_{m-2}(A)} = \delta_{k\tilde{k}} \frac{k(k+1)(k-m+1)(k+m)}{2(2k+1)}, \tag{91}$$

their complete theory is not constructed yet. It does not allow us to calculate analytically the intercept and wave function of the Odderon in QCD.

All these results are based on calculations of effective Reggeon vertices and the gluon Regge trajectory in the first nontrivial order of perturbation theory. Up to now we do not know the region of applicability of LLA including the intervals of energies and momentum transfers fixing the scale for the QCD coupling constant.

Therefore, it is needed to generalize the effective field theory of Ref. [4] to processes for which the final state particles are separated in several groups consisting of an arbitrary number of gluons with a fixed invariant mass; each group is produced with respect to others in the multi-Regge kinematics. These conditions are more general than the requirements for the quasi-multi-Regge kinematics of Ref. [11] where only one additional group consisting of two gluons was considered.

5. Effective action for high energy processes in QCD

To begin with, let us consider the quasi-elastic process in which the final state contains apart from the particle B' with momentum $p_{B'} \simeq p_B$ also several gluons with a fixed invariant mass in the fragmentation region of the initial gluon A. It is convenient to denote the colour indices of the produced gluons by $a_1, a_2, \ldots a_n$ leaving the index a_0 for the particle A. Further, the momenta of the produced gluons and of the particle A are denoted by $k_1, k_2, \ldots k_n$ and $-k_0$ correspondingly. The quantity $q = -\sum_{i=0}^{n} k_i$ is the momentum transfer. Omitting the polarization vectors $e_{\nu_i}(k_i)$ for the gluons $i = 0, 1, \ldots n$ we can write the production amplitude related with the single gluon exchange in the tensor representation as follows

$$A^{\nu_0\nu_1\ldots\nu_n}_{a_0a_1\ldots a_n B'B} = -\phi^{\nu_0\nu_1\ldots\nu_n +}_{a_0a_1\ldots a_n c} \, \frac{1}{t}\, g\, p_B^- T^c_{B'B} \delta_{\lambda_{B'},\lambda_B} \, . \tag{92}$$

Here the form-factor ϕ depends on the invariants constructed from the momenta $k_0, ..k_n$.

For the simplest case of the single gluon production ϕ was calculated in the Born approximation in [11]. We present this result in the form:

$$\phi^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} = g^2 \Bigg\{ \Gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} - T^a_{a_1a_0} T^c_{a_2a} \frac{\gamma^{\nu_1\nu_0\sigma}(k_1,-k_0)\Gamma^{\nu_2\sigma+}(k_2,k_2+q)}{(k_0+k_1)^2}$$
$$- T^a_{a_2a_0} T^c_{a_1a} \frac{\gamma^{\nu_2\nu_0\sigma}(k_2,-k_0)\Gamma^{\nu_1\sigma+}(k_1,k_1+q)}{(k_0+k_2)^2}$$
$$- T^a_{a_2a_1} T^c_{a_0a} \frac{\gamma^{\nu_2\nu_1\sigma}(k_2,-k_1)\Gamma^{\nu_0\sigma+}(k_0,k_0+q)}{(k_1+k_2)^2} \Bigg\} . \tag{93}$$

The three last terms in the brackets correspond to the contributions constructed from the gluon propagator combining the usual Yang–Mills vertex γ and the effective RPP vertex Γ. The first term can be written as

$$\Gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} = \gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} + \Delta^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c}, \tag{94}$$

where γ is the light-cone projection of the usual quadri-linear Yang–Mills vertex

$$\begin{aligned}\gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} =& T^a_{a_1a_0}T^c_{a_2a}(\delta^{\nu_1\nu_2}\delta^{\nu_0+} - \delta^{\nu_1+}\delta^{\nu_0\nu_2})\\ &+ T^a_{a_2a_0}T^c_{a_1a}(\delta^{\nu_2\nu_1}\delta^{\nu_0+} - \delta^{\nu_2+}\delta^{\nu_0\nu_1})\\ &+ T^a_{a_2a_1}T^c_{a_0a}(\delta^{\nu_2\nu_0}\delta^{\nu_1+} - \delta^{\nu_2+}\delta^{\nu_1\nu_0}),\end{aligned} \tag{95}$$

and Δ is a new induced vertex

$$\Delta^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c}(k_0^+, k_1^+, k_2^+) = -t(n^+)^{\nu_0}(n^+)^{\nu_1}(n^+)^{\nu_2}\left\{\frac{T^a_{a_2a_0}T^c_{a_1a}}{k_1^+k_2^+} + \frac{T^a_{a_2a_1}T^c_{a_0a}}{k_0^+k_2^+}\right\}. \tag{96}$$

In the general case of the multi-gluon production in the fragmentation region of the initial particle one should introduce an infinite number of the effective vertices Δ for the gluon-Reggeon interactions to satisfy the condition of the gauge invariance of the inelastic amplitude A.

It turns out, that we can construct the effective action reproducing these vertices for the fragmentation processes:

$$S_{\mathrm{fragm}} = -\int d^4x \ \mathrm{Tr}\left[\tfrac{1}{2}G^2_{\mu\nu} + j_-(V)A_+ + j_+(V)A_-\right], \tag{97}$$

where $G_{\mu\nu} = [D_\mu, D_\nu]$ and $D_\mu = \partial_\mu + gV_\mu$ is the covariant derivative for the Yang–Mills field $V = t_aV^a$, $[t_a, t_b] = f_{abc}t_c$ describing the real gluons. The fields $A_\pm = t_aA^a_\pm$ correspond to the reggeized gluons (*cf.* [12]). The currents $j_\pm$ are given below:

$$j_\pm(V) = j^{\mathrm{mYM}}_\pm(V) + j^{\mathrm{ind}}_\pm(V), \tag{98}$$

where the modified Yang–Mills current j^{mYM} and the induced current j^{ind} equal:

$$j^{\mathrm{mYM}}_\pm = U^{-1}(V_\pm)\, j^{\mathrm{YM}}_\pm\, U(V_\pm), \quad j^{\mathrm{ind}}_\pm = -\partial^2_\perp\partial_\pm U(V_\pm). \tag{99}$$

Here $j^{YM}_\pm = -[D_\mu, G_{\mu\pm}]$ is the usual Yang–Mills current and

$$U(V_\pm) = P\ \exp\left(-\frac{g}{2}\int_{-\infty}^{x^\pm} dx'^\pm V_\pm\right) = \frac{1}{1+g\partial_\pm^{-1}V_\pm} \tag{100}$$

is the path-ordered Wilson exponent. According to equations of motion $j^{YM} = 0$ we obtain $j_\pm = j_\pm^{ind}$. Therefore, one can express the effective action for double quasi-elastic processes after integrating over $A_\pm$ in terms of the action for a two-dimensional σ-model:

$$S_{\text{double}} = S^{YM}(V) - \frac{2}{g^2}\int d^2x_\perp \, tr \, (\partial_{\perp\sigma}T_-)(\partial_{\perp\sigma}T_+), \tag{101}$$

where

$$T\pm = P \, \exp(-\frac{g}{2}\int_{-\infty}^{\infty} dx^\pm V_\pm). \tag{102}$$

This σ-model was derived earlier by E. Verlinde and H. Verlinde using other arguments [13].

For a general case of the Reggeon-gluon interaction local in the rapidity interval $(y-\eta, y+\eta)$ (where $y = \frac{1}{2}\ln\frac{k^+}{k^-}$ and η is an intermediate parameter) the effective action has the form [14]:

$$S_{\text{eff}} = S^{YM}(v) - \int d^4x \, \text{Tr}\left[(A_+^{\text{reg}}(v) - A_+)\partial_{\perp\sigma}^2 A_- + (A_-^{\text{reg}}(v) - A_-)\partial_{\perp\sigma}^2 A_+\right], \tag{103}$$

where

$$A_\pm^{\text{reg}}(v) = -\frac{\partial_\pm}{g}U(v_\pm) = v_\pm - g\, v_\pm \frac{1}{\partial_\pm} v_\pm + \dots \tag{104}$$

is a composite Reggeon field and $j_\pm = \partial_{\perp\sigma}^2 A_\pm$ is the Reggeon current satisfying the kinematical restriction $\partial_\pm j_\mp = 0$ which is important for the gauge invariance of S_{eff}. Taking the functional integral over v one can obtain the pure Reggeon effective action describing all possible processes of production and annihilation of Reggeons in the t-channel. Because $A_\pm^{\text{reg}}(v)$ contains the terms linear in $v_\pm$ there is a non-trivial solution of the Euler–Lagrange equations for S_{eff} which can be constructed as a series in $A_\pm$. By calculating the quantum fluctuations around this solution one can find the gluon Regge trajectory and multi-Reggeon vertices in the one loop approximation. The subsequent functional integration over $A_\pm$ corresponds to the solution of the Reggeon field theory acting in the two-dimensional impact parameter subspace with the time coinciding with the rapidity. It is important, that in the above approach the t-channel dynamics of the Reggeon interactions turns out to be in the agreement with the s-channel unitarity of the S-matrix in the initial Yang–Mills model. In the Hamiltonian formulation of this Reggeon calculus the wave function will contain the components with an arbitrary number of reggeized gluons. Nevertheless, one can hope that at least some of the remarkable properties of the BFKL equation will remain in the general case of the non-conserving number of reggeized gluons.

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