QUARK-HADRON DUALITY AND INTRINSIC TRANSVERSE MOMENTUM*

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It is demonstrated that local quark-hadron duality prescription applied to several exclusive processes involving the pion, is equivalent to using an effective $\bar{q}q$ (two-body) light-cone wave function $\Psi^{(LD)}(x, k_\perp)$ for the pion. This wave function models soft dynamics of all higher $\bar{q}G \ldots Gq$ Fock components of the standard light-cone approach. Contributions corresponding to higher Fock components in a hard regime appear in this approach as radiative corrections and are suppressed by powers of α_s/π.

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1. Introduction

Study of effects due to finite sizes of the hadrons and incorporation of the transverse momentum degrees of freedom is a notoriously difficult problem for the QCD analysis of inclusive (see, *e.g.*, [1]) and exclusive processes. Since the advent of the parton model [2], it is taken for granted that the hadron can be viewed as a collection of quarks and gluons, each of which carries a finite fraction $x_i P$ of the large "longitudinal" momentum P of the hadron, and also some "transverse" momentum $k_{i\perp}$. However, a justification of such a picture from the basic principles of QCD is not a straightforward exercise. To begin with, the coordinate-representation version of $k_\perp$ is a derivative $\partial_\perp$ in the transverse direction. In a gauge theory,

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$\partial_\perp$ always comes together with the gauge field $A_\perp$ in the form of a covariant derivative $\partial_\perp \to D_\perp = \partial_\perp - igA_\perp$, *i.e.*, the finite-size effects are mixed with those due to extra gluons.

In the operator product expansion (OPE) approach [3], the nonperturbative aspects of the hadron dynamics are described/parameterized by matrix elements of local operators. In particular, the longitudinal momentum distribution is related to the lowest-twist composite operators in which all the covariant derivatives appear in traceless-symmetric combinations $\{D_{\mu_1} D_{\mu_2} \ldots D_{\mu_n}\}$ [4–6]. To take into account transverse-momentum effects, one may wish to consider matrix elements of higher-twist composite operators in which some of the covariant derivatives appear in a contracted form like $D^2 = D_\mu D^\mu$. Attempting to relate them to transverse-momentum distributions (*cf.* [7]), one would immediately notice, however, that D^2 looks more like analogue of the quark virtuality k^2. Furthermore, the presence of the gluonic field A_μ in the covariant derivative obscures such an interpretation as well. In particular, using the equation of motion $\gamma^\mu D_\mu q = 0$, one can convert a two-body quark-antiquark operator $\bar{q}\{\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_n}\} D^2 q$ with extra D^2 into the "three-body" operator $\bar{q}\{\gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_n}\}(\sigma^{\mu\nu} G_{\mu\nu})q$ with an extra gluonic field $G_{\mu\nu}$. Moreover, the two contracted covariant derivatives $D_\mu \ldots D^\mu$ can be separated by D_{μ_k}'s forming the traceless combination and, to put D_μ and D^μ next to each other, one should perform commutations, each producing a G-field again [8]. By choosing an optimized basis, one can avoid some of the complications [1, 9], but the observations listed above clearly show that the OPE-inspired approach is unlikely to produce a simple and intuitively appealing basis for constructing phenomenological functions describing the transverse-momentum degrees of freedom.

Still, the OPE approach has many evident bonuses. In particular, it is based on a covariant perturbation theory in 4 dimensions and provides an explicitly gauge-invariant and Lorentz-covariant description. In this respect, it is analogous to the Bethe–Salpeter formalism for bound states. However, the well-known problem of the Bethe–Salpeter formalism is the presence of the unphysical variable of the relative time. This variable is unnecessary and, without a loss of information, one can describe bound states in a 3-dimensional formalism. This is achieved by projecting the Bethe–Salpeter amplitude on a particular (*e.g.*, equal-time or light-front) hyperplane.

In the light-cone approach (see, *e.g.*, [10]), a hadron is described by a set of light-cone wave functions (Fock components) $\Psi^{(N)}(\{x_i, k_{\perp_i}\})$. For mesons, the two-body wave function $\Psi^{(2)}(x, k_\perp)$ can be related to the Bethe–Salpeter amplitude taken in the light-cone gauge and integrated over the minus component of the relative momentum [11]. A bonus of the light-cone approach is that effects due to transverse momenta are unambigously separated from those related to higher Fock components. Hence, if the lowest Fock component gives the dominant contribution, one can hope to construct a reasonable phenomenology based on modelling the two-body LC wave function. On the other hand, if one should perform a summation over all Fock components, the predictive power of the scheme is rather limited, since constructing models for numerous higher Fock components leaves too much freedom for the model builders. This poses a serious problem for phenomenological applications of the standard light-cone formalism. In particular, if one uses the popular BHL set of constraints for the pion LC wave function [12], the $\bar{q}q$ component always contributes less than 50% to the pion form factor value at $Q^2 = 0$, and higher Fock components are absolutely needed to ensure the correct normalization $F_\pi(Q^2 = 0) = 1$.

A possible way out is to introduce an effective two-body wave function (see, *e.g.*, [13]), which includes the low-energy contribution from the higher Fock components, so that one would get $F_\pi(Q^2 = 0) \approx 1$ just from the overlap of these wave functions. One can interpret such a wave function as a wave function for a constituent quark, *i.e.*, a quark dressed by soft gluons. However, for different processes, the higher Fock components can appear with different process-dependent weights, and it is not clear *a priori* whether the effective wave function can be introduced in a universal way. Another point is that while absorbing information about soft gluons into a $\bar{q}q$ wave function may be a good approximation, hard gluons cannot be correctly taken into account in this way. Hence, even using the effective two-body wave function, one should allow for a possibility of having explicit multi-body wave functions. Still, the dominant role of the effective two-body component may take place in such a scheme, since each emission of a hard gluon is suppressed by the QCD coupling constant $\alpha_s/\pi \sim 0.1$, and the contribution of the multi-body components may be relatively small. The problem is that it is unclear how to combine the QCD corrections with the constituent quark picture, because the constituent quark is not a field one readily finds in the original QCD Lagrangian.

Here, using the pion as an example, we will outline a new approach to transverse-momentum effects in exclusive processes. It is based on QCD sum rule ideas. On several examples, we show that results obtained using the quark-hadron duality prescription [14] can be reformulated in terms of a universal effective wave function $\Psi^{\rm LD}(x, k_\perp)$ absorbing information about soft dynamics. The scheme starts with diagrams of ordinary covariant perturbation theory and allows for a systematic inclusion of the radiative corrections in a way totally consistent with the basics of QCD.

2. Handbag diagram and ξ-scaling

A naive idea is that, to take into account effects due to the finite size of the hadrons, one should just write the "parton model" formulas without neglecting intrinsic transverse momentum in hard scattering amplitudes. In doing this, however, one should explicitly specify a field-theoretic approach which is used for such a generalization of the standard parton model. As emphasized in the Introduction, one can choose here between at least two basically different alternatives: standard covariant 4-dimensional formalism or 3-dimensional approaches analogous to the old-fashioned perturbation theory. The bonus of the 4-dimensional approach, in the form of the OPE, is a gauge-invariant and a Lorentz-covariant description of the hadrons in terms of matrix elements of composite operators. However, as argued above, interpretation of the OPE results in terms of transverse degrees of freedom is not self-evident. Moreover, there are some practically important amplitudes which are "protected" from the dynamical $(D^2)^n$-type higher-twist corrections. The most well-known example is given by the classic "handbag" diagram for deep inelastic scattering. As we will see below, in a scalar toy model its contribution contains only target-mass corrections, *i.e.*, it gives no information about finite-size effects. In QCD, the handbag contribution contains a twist-4 operator with extra D^2, but no operators with higher powers of D^2.

2.1. Scalar model

To illustrate the effect in its cleanest form, let us consider the handbag contribution in a model where all fields are scalar (Fig. 1):

$$T(q,p) = -\int \frac{d^4k}{(k+q)^2} F(p,k)\,. \tag{2.1}$$

At large $Q^2 \equiv -q^2$, one can neglect the parton virtuality k^2 in $(k+q)^2 = q^2 + 2(kq) + k^2$ and expand the propagator in powers of $2(kq)/q^2$ to obtain

$$T(q,p) = \int d^4k \sum_{n=0}^{\infty} \frac{(2qk)^n}{Q^{2n}} F(p,k)\,. \tag{2.2}$$

Now, the integral

$$\int k^{\mu_1} \ldots k^{\mu_n} F(p,k) d^4k = A_n p^{\mu_1} \ldots p^{\mu_n} + \text{traces} \tag{2.3}$$

is evidently the matrix element of a local operator with n derivatives.The usual parton density $f(x)$ is introduced by treating the coefficients A_n as its moments [4, 5]:

$$A_n = \int_0^1 x^n f(x) dx\,. \tag{2.4}$$

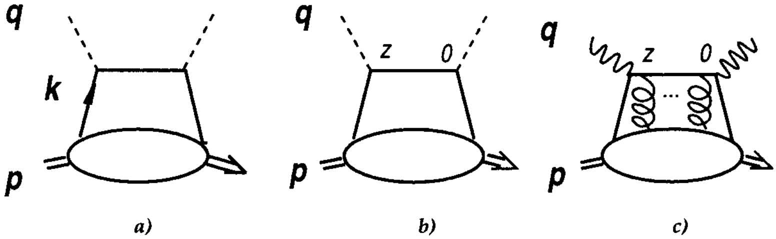

Fig. 1. Handbag diagram. *a)* Momentum representation for scalar model. *b)* Coordinate representation for scalar model. *c)* QCD modification of the quark propagator.

As a result, the amplitude can be written as

$$T(q,p) = \frac{1}{Q^2}\int_0^1 \sum_n^{\infty}\left[\frac{2x(qp)}{Q^2}\right]^n f(x)dx + O(1/Q^4)$$

$$= \int_0^1 f(x)\frac{1}{Q^2-2x(qp)} + O(1/Q^4)\,. \tag{2.5}$$

Taking its imaginary part $W(q,p) \sim \operatorname{Im} T(q,p)$, we get

$$W(q,p) = \int_0^1 f(x)\,\delta(Q^2-2x(qp))\,dx + O(1/Q^4) = \frac{1}{Q^2}x_B f(x_B) + O(1/Q^4)\,, \tag{2.6}$$

where $x_B \equiv Q^2/2(qp)$ is the standard Bjorken variable.

2.2. Power corrections and ξ-scaling

Eq. (2.6) gives the lowest-twist contribution. The power-suppressed terms denoted by $O(1/Q^4)$ are apparently due to the neglected k^2 term in the original propagator. One can expect that supplementing the $(kq)/Q^2$ expansion by the k^2/Q^2 expansion, one can take into account the effects due to nonzero virtuality k^2 by introducing phenomenological functions related to matrix elements like

$$\int k^{\mu_1}\ldots k^{\mu_n}(k^2)^N F(p,k)d^4k \;=\; A_n^{(N)} p^{\mu_1}\ldots p^{\mu_n} + \text{traces}\,. \tag{2.7}$$

Of course, one should be more careful now with the "traces" in this parameterization. The best way to maintain the necessary accuracy is well-known: one should take the traceless part $\{k^{\mu_1}\ldots k^{\mu_n}\}$ of the original tensor $k^{\mu_1}\ldots k^{\mu_n}$. Then, the right-hand-side will also be a traceless tensor, constructed from the 4-vector p^μ.

So, if one decides to keep the k^2 terms, one should supplement this by a re-expansion of the $(kq)^n$-factors over traceless tensors. In fact, the actual problem is simpler than it seems, because a straightforward expansion of the propagator is just in terms of the traceless combinations:

$$-\frac{1}{(q+k)^2}\bigg|_{k<q} = \sum_{n=0}^{\infty} \frac{2^n}{(Q^2)^{n+1}} q^{\mu_1} \dots q^{\mu_n} \{k_{\mu_1} \dots k_{\mu_n}\} . \tag{2.8}$$

Note, that there are no $\{k_{\mu_1} \dots k_{\mu_n}\}(k^2)^N$ terms with $N \neq 0$ in this expansion: the $(k^2)^N$ terms from a naive expansion over powers of (kq) and k^2 are exactly cancelled by $(k^2)^N$ terms from the reexpansion of $(qk)^n$-factors over traceless tensors. In other words, the handbag diagram is insensitive to nonzero-virtuality effects. Introducing the twist-2 distribution function *via*

$$\int \{k^{\mu_1} \dots k^{\mu_n}\} F(p,k) d^4k = \{p^{\mu_1} \dots p^{\mu_n}\} \int_0^1 x^n f(x) dx \tag{2.9}$$

and performing the summation over n by inverting the expansion formula (2.8), we get

$$T(q,p) = \int_0^1 \frac{1}{(q+xp)^2} f(x)\, dx . \tag{2.10}$$

The essential point is that no power-suppressed terms were neglected in this derivation. Hence, we can write $(q+xp)^2 = -Q^2 + 2x(qp) + x^2p^2$ keeping all the terms here and calculate the imaginary part:

$$W(q,p) = \int_0^1 f(x)\, \delta\left(-Q^2 + 2x(qp) + x^2p^2\right) dx = \frac{1}{Q^2} \frac{x_B}{\sqrt{1+\frac{4p^2x_B^2}{Q^2}}} f(\xi) , \tag{2.11}$$

where

$$\xi = \frac{2x_B}{1+\sqrt{1+\frac{4p^2x_B^2}{Q^2}}} \tag{2.12}$$

is the Nachtmann–Georgi–Politzer ξ-variable [15, 4]. Hence, all the power-suppressed contributions contained in the handbag diagram can be interpreted as the target-mass corrections. In particular, the handbag contribution contains no power corrections for a massless target.

2.3. Coordinate representaion

Absence of the higher twist terms as well as the possibility to easily calculate the target-mass dependence of the handbag contribution is directly related to the fact that the propagator of a massless particle has a simple singularity structure. To illustrate this, let us write $T(q,p)$ in the coordinate representation:

$$T(q,p) = \int \frac{d^4z}{z^2} \mathrm{e}^{iqz} \langle p|\phi(0)\phi(z)|p\rangle \, . \tag{2.13}$$

The first term in the z^2-expansion for the matrix element

$$\langle p|\psi(0)\psi(z)|p\rangle = \xi_2(zp) + z^2\xi_4(zp) + (z^2)^2\xi_6(zp) + \ldots \tag{2.14}$$

corresponds to the twist-2 distribution amplitude

$$\chi(zp)\Big|_{z^2=0} = \int_0^1 \varphi(x)\mathrm{e}^{ix(zp)}\, dx\Big|_{z^2=0} , \tag{2.15}$$

while subsequent terms correspond to operators containing an increasing number of ∂^2's. It is straightforward to observe that, while the twist-2 term produces the $1/Q^2$ contribution, the twist-4 term is accompanied by an extra z^2-factor which completely kills the $1/z^2$-singularity of the quark propagator, and the result of the d^4z integration is proportional in this case to $\delta^4(q-xp)$, *i.e.*, this term is invisible for large Q^2. The same is evidently true for all the terms accompanied by higher powers of z^2. This means that the handbag diagram contains only one term: it cannot generate higher powers of $1/Q^2$ which one could interpret as the $(\langle k^2\rangle/Q^2)^n$ expansion.

For spin-1/2 particles, the quark propagator $S^c(z) \sim \gamma_\mu z^\mu/(z^2)^2$ has a stronger singularity for $z^2 = 0$, which is cancelled only by the $O(z^4)$ term in the expansion of the matrix element $\langle p|\bar{q}(0)\gamma_\mu q(z)|p\rangle$. Hence, one may expect that there is a non-vanishing twist-4 contribution corresponding to the $O(z^2)$ term of this expansion, but no higher terms. In a gauge theory, like QCD, one should also take into account the fact that the gluonic field A_ν, in a covariant gauge, has zero twist. As a result, if the gluons have longitudinal polarization, the configurations shown in Fig. 1*c* are not power-suppressed compared to the original handbag contribution. The net result of such gluonic insertions into the quark propagator is a phase factor

$$S^c(z) \to S^c(z) P\exp\left(igz^\nu \int_0^1 A_\nu(tz)dt\right)\{1+O(G)\} , \tag{2.16}$$

where the $O(G)$ term corresponds to insertion of physical gluons. The latter are described by the gluonic field-strength tensor $G_{\alpha\beta}$ and produce $1/Q^2$-suppressed contributions. Thus, including the phase factor, we get the modified QCD handbag contribution

$$T(q,p) \sim \int d^4z \, \exp(-iq_1 z) \, \frac{z^\mu}{(z^2)^2} \times \langle p|\bar{q}(0)\gamma_\mu P \exp\left(igz^\nu \int_0^1 A_\nu(tz)dt \right) q(z)|p\rangle \, . \qquad (2.17)$$

The matrix element

$$\langle p|\bar{q}(0)\gamma_\mu P \exp\left(igz^\nu \int_0^1 A_\nu(tz)dt \right) q(z)|p\rangle \qquad (2.18)$$

can be Taylor-expanded just like $\langle p|\bar{q}(0)\gamma_\mu q(z)|p\rangle$, with the only change $\partial_\nu \to D_\nu$ in the resulting local operators. Thus, the incorporation of gauge invariance does not change our conclusion that the (generalized) handbag diagram cannot generate a tower of the $(1/Q^2)^n$ corrections which one could interpret as the $(\langle k_\perp^2\rangle/Q^2)^n$ or $(\langle k^2\rangle/Q^2)^n$ expansion. The power corrections are produced by the final-state interaction which is described by complicated contributions due to the operators of $\bar{q}G\ldots Gq$-type. At twist 4, the $\bar{q}D^2q$ and $\bar{q}Gq$ terms combined together can be interpreted in terms of functions related to operators of $\bar{q}D_\perp^2 q$ type [1]. However, for twist-6 and higher, the absence of the generic $\bar{q}(D^2)^N q$ contribution stops further progress in this direction. Hence, a simple phenomenological description of higher-twist corrections in terms of something like the transverse-momentum distribution $f(x, k_T)$ is impossible.

3. Exclusive processes: $\gamma^*\gamma^* \to \pi^0$ transition

The transition $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$ of two virtual photons into a neutral pion (Fig. 2a) is the cleanest exclusive process for testing QCD predictions. The relevant form factor $F_{\gamma^*\gamma^*\pi^0}\left(q^2, Q^2\right)$, with $q^2 \equiv -q_1^2, Q^2 \equiv -q_2^2$, can be defined in terms of the pion-to-vacuum matrix element of the product of two electromagnetic currents:

$$4\pi \int d^4x e^{-iq_1 x} \langle \pi, \vec{p}|T\left\{J_\mu(x)\, J_\nu(0)\right\}|0\rangle = i\sqrt{2}\epsilon_{\mu\nu q_1 q_2} F_{\gamma^*\gamma^*\pi^0}\left(q^2, Q^2\right) . \qquad (3.1)$$

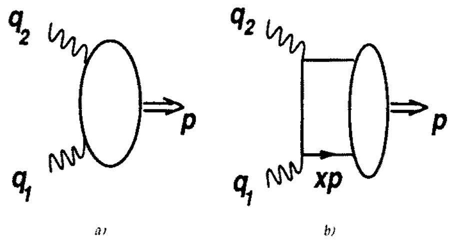

Fig. 2. Form factor of the $\gamma^*\gamma^* \to \pi^0$ transition. *a)* General structure. *b)* Leading-order pQCD term.

3.1. pQCD results

In the lowest order of perturbative QCD, the asymptotic behaviour of $F_{\gamma^*\gamma^*\pi^0}\left(q^2, Q^2\right)$ can be calculated from a diagram similar to the handbag diagram (see Fig. 2*b*) [10]. The basic change is that one should use now the pion distribution amplitude $\varphi_\pi(x)$ instead of the parton density $f(x)$:

$$F^{pQCD}_{\gamma^*\gamma^*\pi^0}(q^2, Q^2) = \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2 + \bar{x}q^2}dx + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{Q^4}\right). \tag{3.2}$$

Experimentally, the most important situation is when one of the photons is real: $q^2 = 0$. In this case, pQCD predicts that [10]

$$F^{pQCD}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2}dx + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{Q^4}\right). \tag{3.3}$$

The nonperturbative information is accumulated here by the integral

$$I = \int_0^1 \frac{\varphi_\pi(x)}{x}dx\,. \tag{3.4}$$

Its value depends on the shape of the pion distribution amplitude $\varphi_\pi(x)$. In particular, using the asymptotic form [16, 17, 11]

$$\varphi_\pi^{as}(x) = 6f_\pi x(1-x)\,, \tag{3.5}$$

gives the following prediction for the large-Q^2 behaviour [10]:

$$F^{as}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{4\pi f_\pi}{Q^2}\,. \tag{3.6}$$

3.2. Anomaly and BL-interpolation

Of course, the asymptotic $1/Q^2$-dependence cannot be the true behaviour of $F_{\gamma\gamma^*\pi^0}(Q^2)$ in the low-Q^2 region, since the $Q^2 = 0$ limit of $F_{\gamma\gamma^*\pi^0}(Q^2)$ is known to be finite and normalized by the $\pi^0 \to \gamma\gamma$ decay rate. In fact, incorporating PCAC and ABJ anomaly [18], one can calculate $F_{\gamma\gamma^*\pi^0}(0)$ theoretically:

$$F_{\gamma\gamma^*\pi^0}(0) = \frac{1}{\pi f_\pi}\,. \tag{3.7}$$

In pQCD, one can imagine that the transition from the high-Q^2 asymptotics to the low-Q^2 behaviour is reflected by higher twist corrections of $(M^2/Q^2)^n$-type, which may sum up into something like $1/(Q^2 + M^2)$, *i.e.*, some expression finite at $Q^2 = 0$ and behaving like $1/Q^2$ for large Q^2. This idea was originally formulated by Brodsky and Lepage [19] who proposed the interpolation formula

$$F_{\gamma\gamma^*\pi^0}(Q^2) = \frac{1}{\pi f_\pi \left(1 + \frac{Q^2}{4\pi^2 f_\pi^2}\right)}, \tag{3.8}$$

which reproduces both the $Q^2 = 0$ value (3.7) and the high-Q^2 behaviour (3.6) with the normalization corresponding to the asymptotic distribution amplitude (3.5).

The BL-interpolation formula (3.8) has a monopole form

$$F_{\gamma\gamma^*\pi^0}(Q^2) \sim \frac{1}{1 + \frac{Q^2}{s_0}}$$

with the scale $s_0 = 4\pi^2 f_\pi^2 \approx 0.67\,\mathrm{GeV}^2$, which is numerically close to the ρ-meson mass squared: $m_\rho^2 \approx 0.6\,\mathrm{GeV}^2$. So, the BL-interpolation suggests a form similar to that based on the VMD expectation $F_{\gamma\gamma^*\pi^0}(Q^2) \sim 1/(1 + Q^2/m_\rho^2)$. In the VMD-approach, the ρ-meson mass m_ρ serves as a parameter determining the pion charge radius, and one can expect that the tower of $(s_0/Q^2)^N$-corrections suggested by the BL-interpolation formula can be attributed to the intrinsic transverse momentum.

3.3. Light-cone formalism and power corrections

As noted earlier, the relative weight and interpretation of power corrections depends on a particular formalism used for a beyond-the-leading-twist extension of pQCD formulas. In the operator product expansion approach, the lowest-order (in α_s) "handbag" contribution to the $\gamma\gamma^* \to \pi^0$ form factor again has only pion-mass corrections:

$$F_{\gamma\gamma^*\pi^0}^{\text{handbag}}(Q^2) = \frac{4\pi}{3}\int_0^1 \frac{\varphi_\pi(x)}{xQ^2 + x(1-x)m_\pi^2}\,dx\,. \tag{3.9}$$

This means that, in the OPE approach, the ("s_0"$/Q^2)^N$-type corrections can come only from the $\bar{q}G\ldots Gq$ operators, for which simple phenomenology is impossible.

As an alternative to the covariant perturbation theory and OPE, one can use the light-cone (LC) formalism [10], in which effects due to the intrinsic transverse momentum $k_\perp$ are described by the light-cone wave function $\Psi(x,k_\perp)$. The LC formula for the $\gamma\gamma^* \to \pi^0$ form factor looks like

$$(\epsilon_\perp \times q_\perp)F_{\gamma\gamma^*\pi^0}(Q^2) \sim \int \Psi(x,k_\perp)\frac{(\epsilon_\perp \times (xq_\perp - k_\perp))}{(xq_\perp - k_\perp)^2}\,dx\,d^2k_\perp\,, \tag{3.10}$$

where $q_\perp$ is a two-dimensional vector in the transverse plane satisfying $q_\perp^2 = Q^2$ and $\epsilon_\perp$ is a vector orthogonal to $q_\perp$ and also lying in the transverse plane [10]. In the LC formalism, quark and gluon fields are on-shell. However, the invariant mass $\mathcal{M}$ of an intermediate state does not coincide with the pion mass. In particular, for the $\bar{q}q$-component, $\mathcal{M}^2 = (k_\perp^2 + m_q^2)/x(1-x)$. Hence, integrating over $k_\perp$ is equivalent to integrating over the invariant masses (or LC-energies) of intermediate states, with $\Psi(x,k_\perp)$ specifying the probability amplitude for each particular "mass".

Unfortunately, Eq. (3.10) also has little chances of producing the series of the $\langle k_\perp^2\rangle/Q^2$ corrections, because the expansion

$$\frac{1}{(xq_\perp - k_\perp)^2} =$$
$$\frac{1}{x}\sum_0^\infty 2^n\left[\frac{\theta(|k_\perp| < xQ)}{(xQ^2)^{n+1}} + \frac{\theta(|k_\perp| > xQ)}{(k_\perp^2/x)^{n+1}}\right] q_\perp^{\mu_1}\ldots q_\perp^{\mu_n}\{k_\perp^{\mu_1}\ldots k_\perp^{\mu_n}\} \tag{3.11}$$

contains only traceless combinations $\{k_\perp^{\mu_1}\ldots k_\perp^{\mu_n}\}$. Substituting it into Eq. (3.10) and using that the wave function $\Psi(x,k_\perp)$ depends on $k_\perp$ only through $k_\perp^2$, we obtain that all terms of this expansion (proportional to Legendre's polynomials) vanish after the angular integration, except for the $n = 0$ and $n = 1$ terms. As a result, the leading $1/Q^2$ term is corrected only by the term resulting from integration of $\Psi(x,k_\perp)/k_\perp^2$ over the region $|k_\perp| > xQ$: there is no tower of $(\langle k_\perp^2\rangle/Q^2)^N$ power corrections in the handbag contribution. Thus, one is forced again to explain the $(1/Q^2)^N$-corrections by contributions from higher $\bar{q}G\ldots Gq$ Fock components, which unavoidably leads to a complicated phenomenology.

The last but not the least comment is that the LC formalism is based on bound-state equations like $\Psi = K\otimes\Psi$, with K being an "interaction kernel". However, it is not clear whether such an equation has any justification in QCD outside perturbation theory. Moreover, it is known that QCD has a lot of nonperturbative effects: complicated vacuum, quark and gluon condensates, *etc.*, which play a dominant role in determining the properties of QCD bound states. So, it is very desirable to develop a QCD description of hadrons in terms of functions similar to the bound state wave functions $\Psi(x, k_\perp)$, but without assuming existence of bound state equations. Below, we outline our attempt to derive such a description from QCD sum rules and quark-hadron duality.

4. Basics of quark–hadron duality

4.1. Outline of the QCD sum rule calculation of f_π

The basic idea of the QCD sum rule approach [20] is the quark-hadron duality, *i.e.*, the possibility to describe one and the same object in terms of either quarks or hadrons. To calculate f_π, we consider the $p_\mu p_\nu$-part of the correlator of two axial currents:

$$\begin{aligned}\Pi^{\mu\nu}(p) &= i\int e^{ipx}\langle 0|T\,(j^+_{5\mu}(x)\,j^-_{5\nu}(0)\,)|\,0\rangle\, d^4x \\ &= p_\mu p_\nu \Pi_2(p^2) - g_{\mu\nu}\Pi_1(p^2)\,. \end{aligned} \tag{4.1}$$

The dispersion relation

$$\Pi_2(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho(s)}{s-p^2}ds + \text{“subtractions”} \tag{4.2}$$

represents $\Pi_2(p^2)$ as an integral over hadronic spectrum with the spectral density $\rho^{\text{hadron}}(s)$ determined by projections

$$\langle 0|j_{5\mu}(0)|\pi; P\rangle = if_\pi P_\mu\,, \tag{4.3}$$

etc., of the axial current onto hadronic states

$$\rho^{\text{hadron}}(s) = \pi f_\pi^2\delta(s-m_\pi^2) + \pi f_{A_1}^2\delta(s-m_{A_1}^2) + \text{“higher states”} \tag{4.4}$$

($f_\pi^{\rm exp} \approx 130.7\,\text{MeV}$ in our normalization). On the other hand, when the probing virtuality p^2 is negative and large, one can use the operator product expansion

$$\Pi_2(p^2) = \Pi_2^{\rm pert}(p^2) + \frac{A}{p^4}\langle\alpha_s GG\rangle + \frac{B}{p^6}\alpha_s\langle\bar{q}q\rangle^2 + \ldots\,, \tag{4.5}$$

where $\Pi_2^{\rm pert}(p^2) \equiv \Pi_2^{\rm quark}(p^2)$ is the perturbative version of $\Pi_2(p^2)$ given by a sum of pQCD Feynman diagrams while the condensate terms $\langle GG\rangle$, $\langle\bar{q}q\rangle$, *etc.*, (with perturbatively calculable coefficients A, B, *etc.)* describe the nontrivial structure of the QCD vacuum.

For the quark amplitude $\Pi_2^{\rm quark}(p^2)$, one can also write down the dispersion relation (4.2), with $\rho(s)$ substituted by its perturbative analogue $\rho^{\rm quark}(s)$:

$$\rho^{\rm quark}(s) = \frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi} + \ldots\right) \tag{4.6}$$

(we neglect quark masses). Hence, for large $-p^2$, one can write

$$\frac{1}{\pi}\int_0^\infty \frac{\rho^{\rm hadron}(s) - \rho^{\rm quark}(s)}{s - p^2}\,ds = \frac{A}{p^4}\langle\alpha_s GG\rangle + \frac{B}{p^6}\alpha_s\langle\bar{q}q\rangle^2 + \ldots\,. \tag{4.7}$$

This expression essentially states that the condensate terms describe the difference between the quark and hadron spectra. At this point, using the known values of the condensates, one can try to construct a model for the hadronic spectrum.

In the axial-current channel, one has an infinitely narrow pion peak $\rho_\pi = \pi f_\pi^2\delta(s - m_\pi^2)$, a rather wide peak at $s \approx 1.7\,\text{GeV}^2$ corresponding to A_1 and then "continuum" at higher energies. The simplest model is to treat A_1 also as a part of the continuum, *i.e.*, to use the model

$$\rho^{\rm hadron}(s) \approx \pi f_\pi^2\delta(s - m_\pi^2) + \rho^{\rm quark}(s)\,\theta(s \geq s_0)\,, \tag{4.8}$$

in which all the higher resonances including the A_1 are approximated by the quark spectral density starting at some effective threshold s_0. Neglecting the pion mass and requiring the best agreement between the two sides of the resulting sum rule

$$\frac{f_\pi^2}{p^2} = \frac{1}{\pi}\int_0^{s_0} \frac{\rho^{\rm quark}(s)}{s - p^2}\,ds + \frac{A}{p^4}\alpha_s\langle GG\rangle + \frac{B}{p^6}\alpha_s\langle\bar{q}q\rangle^2 + \ldots \tag{4.9}$$

in the region of large p^2, we can fit the remaining parameters f_π and s_0 which specify the model spectrum. In practice, the more convenient SVZ-borelized version of this sum rule (multiplied by M^2)

$$f_\pi^2 = \frac{1}{\pi}\int_0^{s_0} \rho^{\text{quark}}(s)e^{-s/M^2}ds + \frac{\alpha_s\langle GG\rangle}{12\pi M^2} + \frac{176}{81}\frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{M^4} + \ldots, \quad (4.10)$$

is used for actual fitting. Using the standard values for the condensates $\langle GG\rangle$, $\langle\bar{q}q\rangle^2$, we adjust s_0 to get an (almost) constant result for the rhs of Eq. (4.10) starting with the minimal possible value of the SVZ-Borel parameter M^2. The magnitude of f_π extracted in this way, is very close to its experimental value $f_\pi^{\text{exp}} \approx 130\,\text{MeV}$.

4.2. Local duality

Of course, changing the values of the condensates, one would get the best stability for a different magnitude of the effective threshold s_0, and the resulting value of f_π would also change. There exist an evident correlation between the values of f_π and s_0 since, in the $M^2 \to \infty$ limit, the sum rule reduces to the local duality relation

$$f_\pi^2 = \frac{1}{\pi}\int_0^{s_0} \rho^{\text{quark}}(s)\,ds\,. \quad (4.11)$$

Thus, the local quark-hadron duality relation states that, despite their absolutely different form, the two densities $\rho^{\text{quark}}(s)$ and $\rho^{\text{hadron}}(s)$ give the same result if one integrates them over the *appropriate* duality interval s_0. The role of the condensates was to determine the size of the duality interval s_0, but after it was fixed, one can write down the relation (4.11) which does not involve the condensates.

Using the explicit lowest-order expression $\rho_0^{\text{quark}}(s) = 1/4\pi$, we get

$$s_0 = 4\pi^2 f_\pi^2\,.$$

Notice that $s_0 = 4\pi^2 f_\pi^2$ is exactly the combination which appeared in the Brodsky-Lepage interpolation formula (3.8). Numerically, $4\pi^2 f_\pi^2 \approx 0.67\,\text{GeV}^2$, *i.e.*, the pion duality interval is very close to the ρ-meson mass: $m_\rho^2 \approx 0.6\,\text{GeV}^2$. In fact, in the next-to-leading order

$$\rho_{\text{NLO}}^{\text{quark}}(s) = \frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\,. \quad (4.13)$$

So, using $\alpha_s/\pi \approx 0.1$, one gets s_0 practically coinciding with m_ρ^2. For the form factors, this leads to results close to the VMD expectations, even though no explicit reference to the existence of the ρ-meson is made.

4.3. Local duality and pion wave function

In the lowest order, the perturbative spectral density is given by the Cutkosky-cut quark loop integral (see Fig. 3*a*)

$$\rho^{\text{quark}}(s) = \frac{3}{2\pi^2}\int \frac{k_+}{p_+}\left(1-\frac{k_+}{p_+}\right)\delta^{(+)}\left(k^2\right)\delta^{(+)}\left((p-k)^2\right)d^4k\,, \quad (4.14)$$

where $s \equiv p^2$. Introducing the light-cone variables for p and k:

$$p = \left\{p_+ \equiv P, p_- = \frac{s}{P}, p_\perp = 0\right\}\ ; \quad k = \{k_+ \equiv xP, k_-, k_\perp\}$$

and integrating over k_-, we get

$$\rho^{\text{quark}}(s) = \frac{3}{2\pi^2}\int_0^1 dx \int \delta\left(s - \frac{k_\perp^2}{x\bar{x}}\right) d^2k_\perp\,. \quad (4.15)$$

The delta-function here expresses the fact that, since we are working in the 4-dimensional formalism, the light-cone combination $k_\perp^2/x\bar{x}$ coincides with s, the square of the external momentum p.

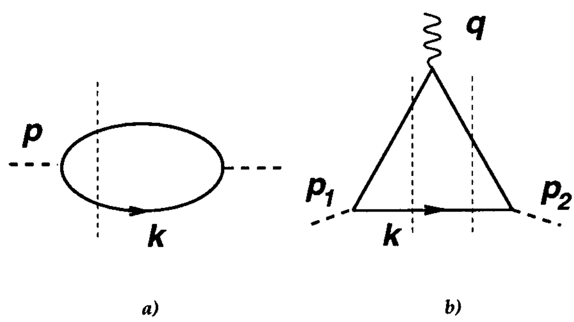

Fig. 3. Leading-order contributions for *a)* two-point spectral density $\rho(s)$ (4.14) and *b)* three-point spectral density $\rho(s_1, s_2, Q^2)$ (5.8). The narrow dashed lines indicate Cutkosky cuts.

Substituting now $\rho^{\text{quark}}(s)$ into the local duality formula, we obtain

$$f_\pi^2 = \frac{3}{2\pi^3}\int_0^1 dx \int \theta\left(k_\perp^2 \leq x\bar{x}s_0\right) d^2k_\perp\,. \quad (4.16)$$

This representation has the structure similar to the expression for f_π in the light-cone formalism [10]

$$f_\pi = \sqrt{6} \int_0^1 dx \int \Psi(x, k_\perp) \frac{d^2 k_\perp}{8\pi^3}, \tag{4.17}$$

where $\Psi(x, k_\perp)$ is the $\bar{q}q$-component of the pion light-cone wave function. To cast the local duality result (4.16) into the form of Eq. (4.17), we introduce the "local duality" wave function for the pion:

$$\Psi^{\rm LD}(x, k_\perp) = \frac{2\sqrt{6}}{f_\pi} \theta(k_\perp^2 \leq x\bar{x}s_0). \tag{4.18}$$

The specific form dictated by the local duality implies that $\Psi^{\rm LD}(x, k_\perp)$ simply imposes a sharp cut-off at $k_\perp^2 x\bar{x} = s_0$.

It should be emphasized that in Eq. (4.17) we are integrating over $k_\perp$, *i.e.*, the combination $k_\perp^2 x\bar{x}$ no longer coincides with the mass^2 of the external particle (which is m_π^2 in our case). This is precisely the feature of 3-dimensional bound-state formalisms: internal particles are on mass shell $k^2 = m_q^2$, but off the energy shell: $k_\perp^2 x\bar{x} \neq m_\pi^2$. In what follows, we show, on less trivial examples, that the local duality prescription allows one to get results reminiscent of 3-dimensional formalisms, though all the actual calculations are performed in the standard 4-dimensional perturbation theory.

5. Pion electromagnetic form factor

5.1. Sum rule

To demonstrate that the function $\Psi^{\rm LD}(x, k_\perp)$ really has the properties one expects from the pion wave function, let us consider the quark-hadron duality in a more complicated context of the pion electromagnetic form factor $F_\pi(Q^2)$. It is defined by

$$\langle p_2 | J^\mu(0) | p_1 \rangle = (p_1^\mu + p_2^\mu) F_\pi(Q^2), \tag{5.1}$$

where $Q^2 = -(p_2 - p_1)^2$. To apply the QCD sum rule technique, we should consider in this case the correlator [21, 14]

$$T_{\alpha\beta}^\mu(p_1, p_2) = i \int e^{-ip_1x + ip_2y} \langle 0 | T\{j_\beta(y) J^\mu(0) j_\alpha^+(x)\} | 0 \rangle d^4x d^4y \tag{5.2}$$

of two axial currents j_α^+, j_β and one electromagnetic current J^μ. The pion EM form factor can be extracted from the invariant amplitude $T(p_1^2, p_2^2, Q^2)$ corresponding to the structure $P_\alpha P_\beta P^\mu$, where $P = (p_1 + p_2)/2$.

The obvious complication now is that we have two channels to be "hadronized", since the pion is present both in the initial and final states. This necessitates the use of the double dispersion relation

$$T(p_1^2, p_2^2, q^2) = \frac{1}{\pi^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \, \frac{\rho(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + \text{“subtractions”} \quad (5.3)$$

involving the double spectral density $\rho(s_1, s_2, Q^2)$. Its hadronic version $\rho^{\text{hadron}}(s_1, s_2, Q^2)$ contains the term corresponding to the pion form factor

$$\rho_{\pi\pi}(s_1, s_2, Q^2) = \pi^2 f_\pi^2 F_\pi(Q^2) \delta(s_1 - m_\pi^2) \delta(s_2 - m_\pi^2) \quad (5.4)$$

and the contributions corresponding to transitions between the pion and higher resonances, and also the terms related to elastic and transition form factors of the higher resonances. To construct the two-dimensional analog of the "lowest state plus continuum" ansatz, we will treat all the contributions, except for the $\rho_{\pi\pi}$, as "continuum", *i.e.*, we will model $\rho^{\text{hadron}}(s_1, s_2, Q^2)$ by the $\rho^{\text{quark}}(s_1, s_2, Q^2)$ outside the square $(0, s_0) \times (0, s_0)$:

$$\begin{aligned} \rho(s_1, s_2, Q^2) &= \rho_{\pi\pi}(s_1, s_2, Q^2) \\ &\quad + (1 - \theta(s_1 < s_0)\theta(s_2 < s_0))\, \rho^{\text{pert.}}(s_1, s_2, Q^2) \,. \end{aligned} \quad (5.5)$$

The SVZ-borelized sum rule (with $M_1^2 = M_2^2 \equiv M^2$) for the pion form factor then has the form [21, 14]

$$\begin{aligned} f_\pi^2 F_\pi(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \, \rho^{\text{quark}}(s_1, s_2, Q^2) \exp\left(-\frac{s_1 + s_2}{M^2}\right) \\ + \frac{\alpha_s \langle GG \rangle}{12\pi M^2} + \frac{16}{81} \frac{\pi \alpha_s \langle \bar{q} q \rangle^2}{M^4} \left(13 + \frac{2Q^2}{M^2}\right) \end{aligned} \quad (5.6)$$

(the pion mass was neglected as usual).

5.2. *Local duality*

In the large-M^2 limit, this gives the local duality relation [14]

$$f_\pi^2 F_\pi^{\text{LD}}(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \, \rho^{\text{quark}}(s_1, s_2, Q^2) \,. \quad (5.7)$$

Again, the perturbative spectral density $\rho^{\text{quark}}(s_1, s_2, Q^2)$ corresponding to the triangle diagram Fig. 3*b* can be easily calculated using the Cutkosky rules and light-cone variables in the frame where the initial momentum p_1 has no transverse components $p_1 = \{p_1^+ = P, p_1^- = s_1/P, 0_\perp\}$, while the momentum transfer $q \equiv p_2 - p_1$ has no "plus" component: $p_2 = \{P, (s_2 + Q_\perp^2)/P, Q_\perp\}$:

$$\rho^{\text{quark}}(s_1, s_2, Q^2) = \frac{3}{2\pi} \int_0^1 dx \int d^2k_\perp \, \delta\left(s_1 - \frac{k_\perp^2}{x\bar{x}}\right) \delta\left(s_2 - \frac{(k_\perp + xq)^2}{x\bar{x}}\right). \tag{5.8}$$

Here, x is the fraction of the total "plus" light-cone momentum carried by the quark absorbing the momentum transfer from the virtual photon and $k_\perp$ is its transverse momentum.

Substituting $\rho^{\text{quark}}(s_1, s_2, Q^2)$ into the local duality relation, we get the light-cone formula for the pion form factor

$$F_\pi^{\text{LD}}(Q^2) = \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \Psi^{\text{LD}}(x, k_\perp) \Psi^{\text{LD}}(x, k_\perp + xq), \tag{5.9}$$

where $\Psi^{\text{LD}}(x, k_\perp)$ is exactly the local duality (4.18) wave function introduced in the previous section.

At $Q^2 = 0$, the LC formula reduces to the integral

$$P = \int \frac{dx \, d^2k_\perp}{16\pi^3} |\Psi(x, k_\perp)|^2 \tag{5.10}$$

which can be interpreted as the probability to find the pion in the state described by the wave function $\Psi(x, k_\perp)$. Using the explicit form of $\Psi^{\text{LD}}(x, k_\perp)$, we immediately get

$$P = \frac{s_0}{4\pi^2 f_\pi^2},$$

which reduces to 1 if we take the lowest-order value $s_0 = 4\pi^2 f_\pi^2$ for the duality interval. Hence, $P^{(0)} = 1$, *i.e.*, the probability to find the pion in the state described by $\Psi^{\text{LD}}(x, k_\perp)$ is 100%. In other words, $\Psi^{\text{LD}}(x, k_\perp)$ may be treated as an effective wave function absorbing the low-energy information about all Fock components.

It should be emphasized, however, that if one uses the next-to-leading order value

$$s_0^{(1)} = \frac{4\pi^2 f_\pi^2}{1 + \frac{\alpha_s}{\pi}},$$

for the duality interval, the probability integral P will be smaller than 1 ($P^{(1)} \approx 0.9$ for $\alpha_s \approx 0.3$). This is a direct manifestation that the local duality prescription explicitly produces contributions which can be interpreted as hard parts of the higher Fock components like $\bar{q}Gq$, *etc.* (see Fig. 4 and Fig. 5 below). However, the total probability to find the pion in such a higher Fock state is rather small: it is suppressed by the factor $\alpha_s/\pi \approx 0.1$.

Though the probability integral P differs from 1 beyond the leading order, the relation $F_\pi^{\rm LD}(0) = 1$ holds to all orders of perturbation theory. The reason is that, at any order, there exists the Ward identity relation between the 3-point function $T^\mu_{\alpha\beta}(p,p)$ and the 2-point function $\Pi_{\alpha\beta}(p)$: $T^\mu_{\alpha\beta}(p,p) = -\partial \Pi_{\alpha\beta}(p)/\partial p_\mu$. As a result, the 3-point spectral density $\rho^{\rm quark}(s_1,s_2,Q^2)$ reduces to the 2-point spectral density $\rho^{\rm quark}(s)$:

$$\rho^{\rm quark}(s_1,s_2,Q^2=0) = \pi\delta(s_1-s_2)\rho^{\rm quark}(s_1)\,. \tag{5.11}$$

Hence, for $Q^2 = 0$, the duality integral for the pion form factor automatically reduces to a one-dimensional integral over s_1 (or s_2), and the duality integral (5.7) for $f_\pi^2 F_\pi(0)$ would coincide with that for f_π^2 (4.11), provided that the sides of the duality square for the three-point function are exactly equal to the duality interval s_0 for the two-point function (the latter not necessarily being equal to the lowest-order value $s_0^{\rm LO} = 4\pi^2 f_\pi^2$). As a result, the local duality prescription gives $F_\pi^{\rm LD}(0) = 1$ to all orders of perturbation theory. In the lowest order, it also gives $P^{(0)} = 1$.

One should not overestimate the accuracy of the local duality results in the region of small Q^2. Though $F_\pi^{\rm LD}(Q^2)$ dictates the values rather close in magnitude to the VMD curve $F_\pi^{\rm VMD}(Q^2)$ or any other fit to data, the LD-formula

$$F_\pi^{LD}(Q^2) = 1 - \frac{1+\frac{6s_0}{Q^2}}{\left(1+\frac{4s_0}{Q^2}\right)^{3/2}}\,, \tag{5.12}$$

gives infinite slope at $Q^2 = 0$, and one should not use it for calculating the derivatives of $F_\pi(Q^2)$ below $Q^2 \sim s_0$. As emphasized above, we obtained the correct value for $F_\pi(0)$ only because this value was protected by the Ward identity. It is well known that the 3-point function in the small-Q^2 region has more complicated quark-hadron duality properties which require a separate study. For the same reason, the local duality fails to produce reasonable valence parton densities for the pion.

6. Quark-hadron duality for the $F_{\gamma^*\gamma^*\pi^0}(Q^2)$ form factor

6.1. Basics

Within the QCD sum rule approach, one can extract information about the $\gamma^*\gamma^* \to \pi^0$ form factor from the three-point correlation function ([22]):

$$\mathcal{F}_{\alpha\mu\nu}(q_1, q_2) = \frac{4\pi}{i\sqrt{2}} \int d^4x\, d^4y e^{-iq_1x - iq_2y} \langle 0|T\left\{J_\mu(x)\, J_\nu(y)\, j_{5\alpha}(0)\right\}|0\rangle \tag{6.1}$$

calculated in the region where all the virtualities $q_1^2 \equiv -q^2, q_2^2 \equiv -Q^2$ and $p^2 = (q_1 + q_2)^2$ are spacelike.

To study the form factor $F_{\gamma^*\gamma^*\pi^0}(q^2, Q^2)$, one should consider the invariant amplitude $F\left(p^2, q^2, Q^2\right)$ corresponding to the tensor structure $\epsilon_{\mu\nu q_1 q_2} p_\alpha$. The dispersion relation for the three-point amplitude

$$F\left(p^2, q^2, Q^2\right) = \frac{1}{\pi} \int_0^\infty \frac{\rho\left(s, q^2, Q^2\right)}{s - p^2}\, ds + \text{subtractions} \tag{6.2}$$

specifies the relevant spectral density $\rho\left(s, q^2, Q^2\right)$. Its hadronic version

$$\rho^{\text{hadron}}\left(s, q_1^2, q_2^2\right) = \pi f_\pi F_{\gamma^*\gamma^*\pi^0}(q^2, Q^2)\delta(s - m_\pi^2) + \text{“higher states”} \tag{6.3}$$

contains the term with the form factor we are interested in. The relevant perturbative spectral density $\rho^{\text{quark}}(s, q^2, Q^2)$, in the lowest order, is given by the integral representation

$$\rho^{\text{quark}}(s, q^2, Q^2) = \\ 2\int_0^1 \delta\left(s - \frac{q^2 x_1 x_3 + Q^2 x_2 x_3}{x_1 x_2}\right) \delta\left(1 - \sum_{i=1}^{3} x_i\right) dx_1 dx_2 dx_3 \tag{6.4}$$

in terms of the Feynman parameters for the one-loop triangle diagram. Scaling the integration variables: $x_1 + x_2 = y$, $x_2 = xy$, $x_1 = (1 - x)y \equiv \bar{x}y$ and taking trivial integrals over x_3 and y, we get

$$\rho^{\text{quark}}(s, q^2, Q^2) = 2\int_0^1 \frac{x\bar{x}(xQ^2 + \bar{x}q^2)^2}{[sx\bar{x} + xQ^2 + \bar{x}q^2]^3}\, dx\, . \tag{6.4}$$

It can be shown that the variable x here is the light-cone fraction of the pion momentum p carried by one of the quarks.

6.2. Local duality

Incorporating the local duality, we obtain

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2) = \frac{1}{\pi f_\pi}\int\limits_0^{s_0} \rho^{\rm quark}(s,q^2,Q^2)$$

$$= \frac{2}{\pi f_\pi}\int\limits_0^1 dx \int\limits_0^{s_0} ds \frac{x\bar{x}(xQ^2+\bar{x}q^2)^2}{[sx\bar{x}+xQ^2+\bar{x}q^2]^3}\,. \qquad (6.6)$$

Substituting the variable s (the mass^2 of the $\bar{q}q$ pair) by the light-cone combination $k_\perp^2/x\bar{x}$, we get $F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2)$ as an integral over the longitudinal momentum fraction x and the transverse momentum $k_\perp$:

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2) = \frac{2}{\pi^2 f_\pi}\int\limits_0^1 dx \int d^2k_\perp\, \theta(k_\perp^2 \le x\bar{x}s_0)\frac{(xQ^2+\bar{x}q^2)^2}{(xQ^2+k_\perp^2)^3}\,. \quad (6.7)$$

Finally, introducing the effective wave function $\Psi^{\rm LD}(x,k_\perp)$ given by (4.18) we can write $F^{\rm LD}\left(Q^2\right)$ in the "light-cone" form:

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2) = \frac{1}{\pi^2\sqrt{6}}\int\limits_0^1 dx \int d^2k_\perp\, \Psi^{\rm LD}(x,k_\perp)\frac{(xQ^2+\bar{x}q^2)^2}{(xQ^2+\bar{x}q^2+k_\perp^2)^3}\,. \qquad (6.8)$$

It is instructive to analyze this expression in some particular limits.

6.3. Limits

1. Both photons are real. When both q^2 and Q^2 are small, we can use the fact that

$$\frac{\mu^4}{(\mu^2+k_\perp^2)^3} \to \frac{1}{2}\,\delta(k_\perp^2) \qquad (6.9)$$

in the $\mu^2 \to 0$ limit, to obtain (*cf.* [12]) that the $\pi^0 \to \gamma\gamma$ decay rate is determined by the magnitude of the pion wave function at zero transverse momentum:

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(0,0) = \frac{1}{2\pi\sqrt{6}}\int\limits_0^1 \Psi^{\rm LD}(x,k_\perp=0)\,dx\,. \qquad (6.10)$$

Using the explicit form of $\Psi^{\rm LD}(x,k_\perp)$ (4.18), we obtain

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(0,0) = \frac{1}{\pi f_\pi}\,, \tag{6.11}$$

which is exactly the value (3.7) dictated by the axial anomaly.

2. pQCD limit. Assuming that both q^2 and Q^2 are so large that the $k_\perp^2$-term can be neglected, we get the expression

$$F^{\rm LD}_{\gamma^*\gamma^*\pi^0}(q^2,Q^2) = \frac{1}{\pi^2\sqrt{6}}\int\limits_0^1 \frac{dx}{xQ^2+\bar{x}q^2}\int d^2k_\perp\,\Psi^{\rm LD}(x,k_\perp) + O\left(\frac{1}{Q^4}\right). \tag{6.12}$$

Identifying the wave function integrated over the transverse momentum with the pion distribution amplitude

$$\varphi_\pi^{\rm LD}(x) = \frac{\sqrt{6}}{(2\pi)^3}\int \Psi^{\rm LD}(x,k_\perp)\,d^2k_\perp\,, \tag{6.13}$$

we arrive at the lowest-order pQCD formula

$$F^{pQCD}_{\gamma^*\gamma^*\pi^0}(Q^2) = \frac{4\pi}{3}\int\limits_0^1 \frac{\varphi_\pi(x)}{xQ^2+\bar{x}q^2}dx + O\left(\frac{1}{Q^4}\right) \tag{6.14}$$

for the large-virtuality behaviour of the $\gamma^*\gamma^* \to \pi^0$ transition form factor.

3. One real photon. A very simple result for $\rho^{\rm quark}(s,q^2,Q^2)$ appears when $q^2=0$:

$$\rho^{\rm quark}(s,q^2=0,\,Q^2) = \frac{Q^2}{(s+Q^2)^2}\,. \tag{6.15}$$

This formula explicitly shows that if Q^2 also tends to zero, the spectral density $\rho^{\rm quark}(s,q^2=0,Q^2)$ becomes narrower and higher, approaching $\delta(s)$ in the $Q^2\to 0$ limit (*cf.* [23]). Thus, the perturbative triangle diagram dictates that two real photons can produce only a single massless pseudoscalar state: there are no other states in the spectrum of final hadrons. As Q^2 increases, the spectral function broadens, *i.e.*, higher states can also be produced. Assuming the local duality, we obtain:

$$f_\pi F^{\rm LD}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{1}{\pi}\int\limits_0^{s_0}\rho^{\rm quark}(s,0,Q^2)\,ds = \frac{1}{\pi(1+\frac{Q^2}{s_0})}\,. \tag{6.16}$$

For large Q^2, this gives

$$F^{as}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{4\pi f_\pi}{Q^2} + O\left(\frac{1}{Q^4}\right) . \tag{6.17}$$

This result can also be obtained from the $q^2 = 0$ version

$$F^{pQCD}_{\gamma\gamma^*\pi^0}(Q^2) = \frac{4\pi}{3} \int_0^1 \frac{\varphi_\pi(x)}{xQ^2} dx + O\left(\frac{1}{Q^4}\right) , \tag{6.18}$$

of the pQCD formula (6.14), if we will use there the asymptotic form of the pion distribution amplitude

$$\varphi_\pi^{\rm LD}(x) = 6 f_\pi x(1-x) \tag{6.19}$$

produced by the local duality prescription.

In other words, the local duality formula (6.16) exactly reproduces the Brodsky–Lepage interpolation (3.8) between the $Q^2 = 0$ value $1/\pi f_\pi$ fixed by the ABJ anomaly and the leading large-Q^2 term $4\pi f_\pi/Q^2$ calculated for the asymptotic form of the pion distribution amplitude.

7. Normalization properties of the effective wave function

7.1. Momentum representation

By construction, $\Psi^{\rm LD}(x, k_\perp)$ satisfies the standard constraint

$$\int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \Psi^{\rm LD}(x, k_\perp) = \frac{f_\pi}{2\sqrt{6}} \tag{7.1}$$

imposed on the two-body Fock component of the pion light-cone wave function by the correspondence with the $\pi \to \mu\nu$ rate.

Furthermore, the x-integral of $\Psi^{\rm LD}(x, k_\perp)$ at zero transverse momentum

$$\int_0^1 dx\, \Psi^{\rm LD}(x, k_\perp = 0) = \frac{2\sqrt{6}}{f_\pi} \tag{7.2}$$

has the right magnitude to produce the correct value $F_{\gamma^*\gamma^*\pi^0}(0,0) = 1/\pi f_\pi$ imposed by the $\pi^0 \to \gamma\gamma$ rate. Note, that this value is by a factor of 2 larger than the constraint imposed in [12] on the quark-antiquark component of the LC pion wave function. The difference can be traced to the claim, made in [12], that the $\bar{q}q$-component of their pion wave function gives only a half of the $\pi^0 \to \gamma\gamma$ decay amplitude. The other half, it was argued, should be attributed to the $\bar{q}q\gamma$-component of the pion wave function. Within our approach, an analogue of the $\bar{q}q\gamma$-component appears only in the first order in α_s. At the leading order, there is only one term, which, I repeat, correctly reproduces the $Q^2 = 0$ value of $F_{\gamma^*\gamma^*\pi^0}(0, Q^2)$.

7.2. Probability integral

The integral

$$P = \int \frac{dx\, d^2k_\perp}{16\pi^3} |\Psi(x, k_\perp)|^2. \tag{7.3}$$

gives the probability to find the pion in the state described by the wave function $\Psi(x, k_\perp)$. As discussed earlier, substituting the local duality wave function $\Psi^{\rm LD}(x, k_\perp)$ into this relation, one would get $P = s_0/(4\pi^2 f_\pi^2) = 1$. At the lowest order in α_s, the local duality wave function $\Psi^{\rm LD}(x, k_\perp)$, in this sense, describes 100% of the pion content, *i.e.*, $\Psi^{\rm LD}(x, k_\perp)$ can be understood as an effective wave function dual to all Fock components of the pion light-cone wave function.

It is worth noting here that, in our approach, $P \le 1$ for any wave function $\Psi(x, k_\perp)$ which

a) depends only on the combination $k_\perp^2/x\bar{x}$: $\Psi(x, k_\perp) = f(k_\perp^2/x\bar{x})$,
b) monotonically decreases with the increase of $k_\perp^2/x\bar{x}$,
c) never becomes negative and
d) satisfies the constraints (7.1) and (7.2).

The upper limit for P is reached when $\Psi(x, k_\perp)$ assumes the steplike form (4.18) dictated by the local duality. The requirement that the (generalized) valence content of the pion should not exceed 100% is not unreasonable. Furthermore, I fail to see why, in a particular model, this probability cannot reach 100%. However, if instead of our constraint (7.2) one apllies that proposed in [12], the upper limit for P, under the same conditions *a)*–*d)*, is 0.5, *i.e.*, one should mandatorily require that at least 50% of the pion content must always be related to non-valence components.

7.3. Impact parameter representation

Defining the impact parameter $b_\perp$ as the variable which is Fourier-conjugate to the transverse momentum $k_\perp$:

$$\Psi(x, k_\perp) = \int e^{ik_\perp b_\perp} \widetilde{\Psi}(x, b_\perp) d^2 b_\perp\,, \tag{7.4}$$

we can write down the normalization conditions for the $b_\perp$-space wave function $\widetilde{\Psi}(x, b_\perp)$. Eq. (7.1), following from the requirement that $\pi \to \mu\nu$ rate is specified by f_π, gives the magnitude of $\widetilde{\Psi}^{\rm LD}(x, b_\perp)$ at the origin:

$$\int_0^1 dx\, \widetilde{\Psi}^{\rm LD}(x, b_\perp = 0) = \frac{2\pi f_\pi}{\sqrt{6}}, \tag{7.5}$$

and Eq. (7.2), following from the requirement that $\pi^0 \to \gamma\gamma$ rate is given by axial anomaly, specifies its integral over the *whole* $b_\perp$-plane:

$$\int_0^1 dx \int \widetilde{\Psi}^{\rm LD}(x, b_\perp)\, d^2 b_\perp = \frac{2\sqrt{6}}{f_\pi}. \tag{7.6}$$

There is a widespread opinion that the axial anomaly is a purely short-distance phenomenon produced by ultraviolet divergences. However, the constraint (7.6) involving integration over all impact parameters clearly shows that the axial anomaly is deeply related to the long-distance physics as well. In particular, calculating the spectral density $\rho(s, q^2, Q^2)$ exhibiting the anomaly behaviour in the $q^2, Q^2 \to 0$ limit, we never faced any ultraviolet divergences (*cf.* [23]).

For reference purposes, we also give the impact-parameter version of the formula for the pion electromagnetic form factor:

$$F_\pi^{\rm LD}(Q^2) = \frac{1}{4\pi} \int_0^1 dx \int e^{ix(Q_\perp b_\perp)} |\widetilde{\Psi}^{\rm LD}(x, b_\perp)|^2\, d^2 b_\perp \tag{7.7}$$

and the $b_\perp$-space form of our effective wave function:

$$\widetilde{\Psi}^{\rm LD}(x, b_\perp) = \frac{\sqrt{6}}{\pi f_\pi b_\perp} \sqrt{x\bar{x}s_0}\, J_1(b_\perp \sqrt{x\bar{x}s_0}), \tag{7.8}$$

where $J_1(z)$ is the Bessel function.

8. Higher-order corrections

Calculating the spectral densities $\rho^{\rm quark}(s, \ldots)$ to higher orders in α_s, we can study effects due to gluon radiation. Depending on the position of Cutkosky cuts, one can interpret, *e.g.*, the next-to-leading order contributions either as corrections to the two-body $\bar{q}q$ effective wave function (Fig. 4a, b) or as three-body $\bar{q}Gq$ Fock components (Fig. 4c, d).

In practice, even the lowest $O(\alpha_s)$ correction requires a two-loop calculation, which is rather involved, especially for three-point functions. For the two-point function, the correction is known [20]:

$$\rho_{\rm NLO}^{\rm quark}(s) \equiv \rho_0^{\rm quark}(s) + \rho_1^{\rm quark}(s) = \frac{1}{4\pi}\left(1+\frac{\alpha_s}{\pi}\right). \qquad (8.1)$$

According to the Ward identity (5.11), this result can be used in order to get the $Q^2 = 0$ value of the $O(\alpha_s)$ contribution to the spectral density $\rho^{\rm quark}(s_1, s_2, Q^2)$ related to the pion electromagnetic form factor. As a result, the $O(\alpha_s)$-correction to the pion form factor for $Q^2 = 0$ is given by

$$\delta F_\pi^{(\alpha_s)}(Q^2 = 0) = \frac{\alpha_s(s_0)}{\pi}. \qquad (8.2)$$

The duality interval s_0, in this case, is a natural (and the only possible) scale for the running coupling constant.

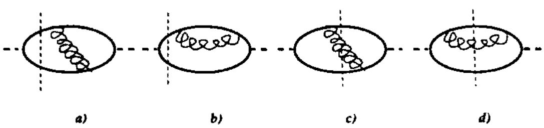

Fig. 4. Some two-loop contributions to the two-point spectral function $\rho(s)$ (8.1). *a,b)* Contributions corresponding to $O(\alpha_s)$ corrections to the two-body $\bar{q}q$ effective wave function. *c,d)* Contributions corresponding to presence of hard three-body $\bar{q}Gq$-components both in the initial and final states.

Another important piece of information can be obtained for large Q^2. In this limit, in contrast to the one-loop term $\rho_0^{\rm quark}(s_1, s_2, Q^2)$, which decreases like $1/Q^4$, the two-loop contribution $\rho_1^{\rm quark}(s_1, s_2, Q^2)$ contains a term (Fig. 5*c*) which behaves like $1/Q^2$:

$$\rho_1^{\rm quark}(s_1, s_2, Q^2) = \frac{8\pi\alpha_s}{Q^2}\rho_0^{\rm quark}(s_1)\rho_0^{\rm quark}(s_2) + O\left(\frac{1}{Q^4}\right), \qquad (8.3)$$

where $\rho_0^{\rm quark}(s_1)$ and $\rho_0^{\rm quark}(s_2)$ are the lowest-order two-point function spectral densities (see Eq. (8.1)). This behaviour agrees with the pQCD factorization theorem [16, 10] and quark counting rules. Substituting the asymptotic expression for $\rho_1^{\rm quark}(s_1, s_2, Q^2)$ into the local duality relation (5.7), we get the large-Q^2 behaviour of $\delta F_\pi^{(\alpha_s)}(Q^2)$:

$$\delta F_\pi^{(\alpha_s)}(Q^2) = \frac{\alpha_s(s_0)}{\pi}\left(\frac{2s_0}{Q^2}\right). \qquad (8.4)$$

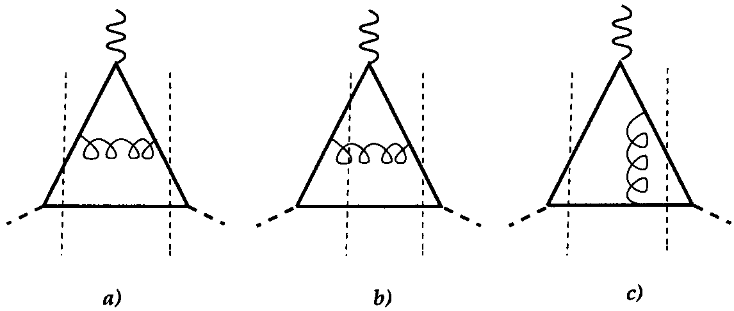

Fig. 5. Some two-loop contributions for the spectral function $\rho(s_1, s_2, Q^2)$. *a)* Correction to the electromagnetic vertex. *b)* Contribution corresponding to the three-body $\bar{q}Gq$-component in the initial state and two-body $\bar{q}q$-component in the final state. *c)* Term producing the $O(1/Q^2)$ contribution.

This result corresponds to the pQCD formula for the one-gluon-exchange contribution to the pion form factor [6, 16, 11]

$$F_\pi^{\mathrm{pQCD}}(Q^2) = \int_0^1 dx \int_0^1 dy\, \varphi_\pi(x)\, \varphi_\pi(y) \frac{8\pi\alpha_s}{9xyQ^2}, \tag{8.5}$$

if one uses the "asymptotic" distribution amplitude $\varphi_\pi^{as}(x) = 6f_\pi x(1-x)$ dictated by the local duality. Now, by analogy with the Brodsky–Lepage interpolation, we can construct a model for $\delta F_\pi^{(\alpha_s)}(Q^2)$ based on the simplest interpolation between its $Q^2 = 0$ value and large-Q^2 asymptotics:

$$\delta F_\pi^{(\alpha_s)}(Q^2) = \left(\frac{\alpha_s}{\pi}\right) \frac{1}{1 + \frac{Q^2}{2s_0}}. \tag{8.6}$$

Combining the $O(1)$ and $O(\alpha_s)$ terms, we get the next-to-leading order LD-model for the pion form factor

$$F_\pi^{\mathrm{LD}}(Q^2) = \frac{F_\pi^{\mathrm{LD}(0)}(Q^2) + \delta F_\pi^{(\alpha_s)}(Q^2)}{1 + \frac{\alpha_s}{\pi}}. \tag{8.7}$$

where $F_\pi^{\mathrm{LD}(0)}(Q^2)$ is the lowest-order result given by Eq. (5.12). For α_s/π one can take a constant value $\alpha_s/\pi = \alpha_s(s_0)/\pi \approx 0.1$ though, for truly asymptotic Q^2, the scale of α_s should have a Q^2-dependent component. The curve based on Eq. (8.7) is in good agreement with existing data.

9. Conclusions

Our main goal here was to demonstrate that the results of the approach based on local quark-hadron duality and QCD sum rules can be reformulated in terms of the effective wave function $\Psi^{\rm LD}(x, k_\perp)$ describing both longitudinal and transverse momentum distribution of quarks inside the pion. This approach has the following features:
1) It is directly related to the QCD Lagrangian, and all the calculations are based on Feynman diagrams of standard 4-dimensional perturbation theory.
2) As a result, the approach is fully compatible with high-Q^2 pQCD calculations and other QCD constraints, *e.g.*, those imposed by the axial anomaly.
3) Radiative (higher-order in α_s) corrections can be added in a regular way, through a well-defined procedure.
4) There is no need for a special procedure separating soft *vs.* hard terms. In a sense, they are separated automatically by the duality interval parameter s_0. The "hard" terms have a natural subasymptotic modification in the low-Q^2 region.
5) The bulk (soft) part of the higher-twist effects is described by an effective 2-body wave function $\Psi^{\rm LD}(x, k_\perp)$ rather than by increasingly complex wave functions of higher Fock components.
6) In this approach, the contributions which can be interpreted as effective wave functions for the higher Fock components are small because they are suppressed by powers of $\alpha_s(s_0)/\pi$. Hence, the effective valence component dominates and, in this respect, this approach resembles the constituent quark model. However, there is no need to introduce constituent quark masses. The scale responsible for the IR cut-offs is set by the duality interval s_0.
7) The effective wave functions are introduced in this approach without any appeal to the existence of bound state equations.

Local quark-hadron duality was also applied to nucleon form factors [24] and to $\gamma p \to \Delta$ transition form factors [25]. The results of these studies can be used to develop a similar formalism for the baryons. Another possible development is to substitute the steplike effective wave functions $\Psi^{\rm LD}(x, k_\perp)$ by smooth functions, but without violating the constraints (7.1), (7.2) and $P^{(0)} = 1$.

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