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EFFECTIVE ACTION FOR HIGH–ENERGY SCATTERING IN QUANTUM GRAVITY *, **

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I discuss various approaches to the high-energy scattering in quantum gravity, which are based on perturbation theory. First, the results for the elastic scattering amplitude, obtained within the eikonal approximation, are reviewed. Then, the effective action approach to the gravity in the multi-Regge kinematics is discussed.

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1. Introduction

The present lecture is devoted to studies of quantum gravity in the multi-Regge limit. In this kinematics the energy E involved in a scattering process exceeds significantly the Planck mass M_P

$$E \gg M_P \,, \tag{1.1}$$

and quantum effects become important. Their description requires the knowledge of the consistent theory of quantum gravity. Unfortunately, challenging problem of construction such a theory is unsolved till now. One of the main obstructions is that the quantization of general relativity leads to perturbatively non-renormalizable theory. A consequence of this fact is the general opinion that studies of quantum gravity which are based on perturbation theory are without predictive power.

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It turns out that this view is too pessimistic in case of processes occurring in the multi-Regge kinematics (MRK). In this kinematics, the produced particles which arise in the scattering of two high energy particles fly mainly in the direction of one of the incoming particles. They have very large and strongly ordered longitudinal momenta and small transverse momenta. The calculations of scattering amplitudes show that renormalization effects do not contribute to leading and next-to-leading terms and one can obtain definite predictions. In particular, Lipatov has calculated the inelastic amplitudes corresponding to production of gravitons in MRK and graviton's Regge trajectory [4]. More recently Amati, Ciafaloni and Veneziano have calculated the quantum correction to the classical deflection angle of a graviton in the field of a black hole [2].

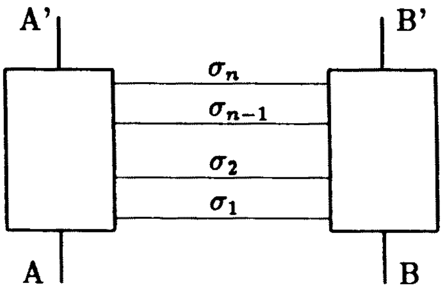

Fig. 1.

Gravity in MRK is a strongly interacting theory. The easiest way to see this is based on the Asimov–Mandelstam (AM) rule [3]. Consider the scattering amplitude describing an arbitrary elastic process in Regge kinematics (see Fig. 1) *i.e.* for

$$s = (p_A + p_B)^2 \to \infty, \quad t = (p_A - p_{A'})^2 \sim \text{constant}. \tag{1.2}$$

If the scattering amplitude has a t-channel intermediate state with n particles carrying spins σ_i, $i = 1, ..., n$, then this t-channel cut leads to a term in the scattering amplitude which is of the order (up to logarithms of s)

$$s^{\sum_{i=1}^{n} \sigma_i - n + 1}. \tag{1.3}$$

It follows from the AM rule that in the case of gravity ($\sigma = 2$) the t-channel intermediate state with n gravitons leads to a contribution which behaves as s^{n+1}, *i.e.* t-channel exchanges involving more gravitons are more important[1]. This means that the effective coupling constant sG, where G is Newton's constant, is very big. As a consequence, to obtain reliable results, one should sum the contributions corresponding to an arbitrary number of gravitons in the t-channel.

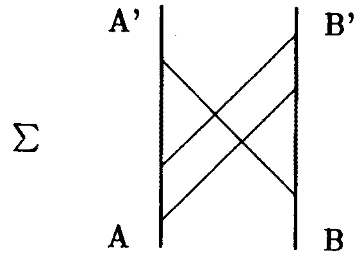

Fig. 2.

In the first part of the lecture I discuss the scattering amplitude obtained in the eikonal approximation. It corresponds to taking into account the contributions to the elastic scattering which correspond to all possible exchanges between incoming particles of non-interacting gravitons (see Fig. 2). According to the AM rule this set of diagrams gives the dominant contributions in each perturbative order. Next, I discuss the limitations of the results obtained within the eikonal approximation and the necessity to look for corrections to these results. The second part of the lecture is devoted to the construction of the effective Lagrangian for gravity in the MRK, which serves as the main tool of the new method proposed by Lipatov [4] which attempts to go beyond the eikonal approximation. This construction is the subject of common work with Kirschner [14].

2. Gravity in the eikonal approximation

The calculations of eikonal diagrams (Fig. 2) in the case of gravity proceed in a similar way as analogous calculations in QED (compare [5]). Kabat and Ortiz have calculated the leading terms of those diagrams in the case of scalar-scalar scattering by graviton exchanges [6]. The obtained

[1] This result one should confront with the analogous contribution due to gluonic exchanges ($\sigma = 1$) which is of the order s (modulo $\ln s$'s), independently how many gluons are exchanged.

result for the scattering amplitude M is the following

$$M(s,t=-\vec{q}_\perp^{\,2}) = -2is\int d^2x_\perp e^{-i\vec{q}_\perp \vec{x}_\perp}\left[\exp(i4\pi Gs\int\frac{d^2k_\perp}{(2\pi)^2}\frac{e^{-i\vec{k}_\perp\vec{x}_\perp}}{k_\perp^2+\mu^2})-1\right], \tag{2.1}$$

where μ is a graviton mass which serves as an infrared cut-off. Note that in contradistinction to the QED case, the eikonal in Eq. (2.1) contains in the exponent an additional factor s which reflects that the graviton has spin two. Performing the integrals in Eq. (2.1) one gets

$$M(s,t) = \frac{8\pi Gs^2}{-t}\frac{\Gamma(1-iGs)}{\Gamma(1+iGs)}\left(\frac{4\mu^2}{-t}\right)^{-iGs}. \tag{2.2}$$

This result can also be obtained using other methods, which do not refer to Feynman diagrams. t'Hooft has considered [7] the scattering of gravitons as a quantum mechanical problem of one particle moving in the gravitational field of the other particle. This gravitational field has the form of "shock-wave" as described by the Aichelburg–Sexl metric [8]. Amati, Ciafaloni and Veneziano [2] as well as Muzinich and Soldate [9] derived Eq. (2.2) by considering the low energy limit of string amplitudes. Finally, E. and H. Verlinde obtained this result by constructing the effective theory for gravity in Regge kinematics [10]. This effective theory emerges as a result of a natural separation of longitudinal and transverse degrees of freedom in the underlying kinematics.

Although Eq. (2.2) was obtained by summing the leading terms arising in each order of perturbation theory, it needs to be corrected. Let us observe that the product of the last two factors in Eq. (2.2)

$$\frac{\Gamma(1-iGs)}{\Gamma(1+iGs)}\left(\frac{4\mu^2}{-t}\right)^{-iGs} \tag{2.3}$$

is a pure phase. As a consequence the whole Eq. (2.2) is of the order Born term *i.e.*

$$M_{\rm Born}(s,t) = \frac{8\pi Gs^2}{-t}. \tag{2.4}$$

This means that in order to obtain a meaningful result one should correct the above calculations by taking into account non-leading terms which were previously neglected. In particular it is not enough to calculate the eikonal diagrams only up to leading accuracy but higher precision is needed. Moreover, it is necessary to consider also diagrams in which the exchanged t-channel gravitons interact each with other. In the MRK this corresponds to taking into account inelastic diagrams with the production of many gravitons in s-channel. The importance of non-leading terms for the final result is clearly shown up in calculations of quantum correction to the classical deflection angle [2]. For distances close to the Schwarzschild radius the magnitude of the quantum correction is of the order of the classical expression which requires a further improvement of the approximation.

The problem how to go beyond the eikonal approximation in a systematic way is unsolved till now. The methods which work well within the eikonal approximation are difficult to generalize beyond it (for a discussion of these questions see Ref. [11]). The new approach to this problem was proposed by Lipatov in Ref. [4] and is based on the effective action for gravity in the MRK. The effective action involves only those degrees of freedom which are relevant for processes in the underlying kinematics. The analogous effective action to that of Ref. [4] turned out to be a useful tool also in the superstring approach to high-energy gravitational scattering [12].

3. The multi-Regge effective action for gravity

Since gravity is a gauge theory many of its properties have analogs in Yang–Mills theory. This means that some methods which were originally invented for Yang–Mills theory can also be applied in gravity. The effective action approach [4] is an example of such a method. In the case of QCD it provides a technique to go beyond the leading logarithm approximation commonly used in studies of inelastic processes $p_A p_B \to k_0 k_1 ... k_{n+1}$ (see Fig. 3) in the MRK. In this kinematics, the variables $s = (p_A + p_B)^2, s_i = (k_i + k_{i-1})^2, k_i = q_i - q_{i-1}$ satisfy in the following conditions

$$\begin{gathered} s \gg s_i \sim s_j \gg |q_i^2| \sim |q_j^2|, \quad i,j = 0,1,...,n+1, \\ k_{+i} \ll k_{+i-1}, \quad k_{-i} \gg k_{-i+1}, \quad \prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} |\kappa_i^2|, \end{gathered} \tag{3.1}$$

where κ^μ is defined by the Sudakov decomposition

$$k^\mu = \frac{1}{\sqrt{s}}(k_+ p_B^\mu + k_- p_A^\mu) + \kappa^\mu . \tag{3.2}$$

The effective Lagrangian for QCD in MRK was derived by Kirschner, Lipatov and the present author from the original QCD Lagrangian [13]. Recently, Kirschner and the author have shown, that in a similar way also the effective Lagrangian for gravity in MRK can be obtained from the Einstein Lagrangian [14]. Below I discuss the main points of this derivation.

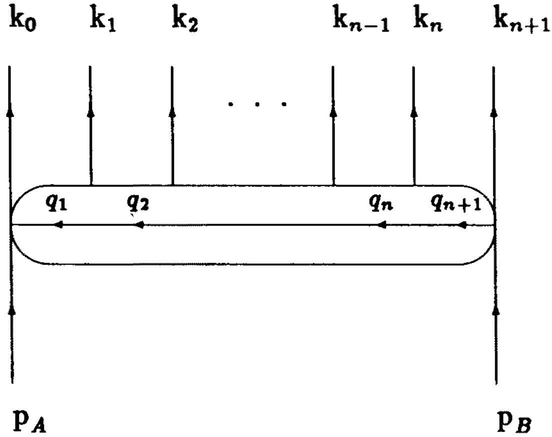

Fig. 3.

The starting point is the Einstein action. Because of the MRK it is natural to perform all derivations in the axial gauge. We choose the light-cone gauge with the momentum of an incoming particle as the gauge vector ($p_B^\mu = \frac{s^{1/2}}{2}(1,0,0,-1)$). The gauge fixing conditions are chosen as

$$g_{--} = g_{-i} = 0, \quad g_{-+} = 2e^{\psi/2}, \tag{3.3}$$

where the light-cone variables are defined as $x_\pm = x_0 \pm x_3$. The physical degrees of freedom can be represented by the two independent matrix elements γ_{11}, γ_{12}, where γ_{ij} is defined by the transverse components of the metric $g_{ij}, i,j = 1,2$

$$g_{ij} = e^{\psi} \ \gamma_{ij}, \qquad \det(\gamma_{ij}) = 1\,. \tag{3.4}$$

Solving the constraints (3.3) one can eliminate g_{++}, g_{+i} and ψ and the Lagrangian will be expressed only in terms of the matrix elements γ_{ij}. We parametrize them as

$$\gamma_{ij} = (e^h)_{ij}, \qquad \mathrm{Sp}\ h = 0\,, \tag{3.5}$$

and we introduce the complex field h defined by the two independent elements of the matrix h_{ij} as

$$h = \frac{1}{\sqrt{2}}(h_{11} - ih_{12})\,. \tag{3.6}$$

Complex notations will also be used for two-dimensional transverse momentum and position vectors ($x = x^1 + ix^2, \partial = \frac{\partial}{\partial x}$) as in [13]. Expanding Einstein's Lagrangian in powers of h and keeping all terms including the quartic in h we arrive at the following starting point of our analysis

$$
\begin{aligned}
\mathcal{L} = & \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)} + \ldots , \\
\mathcal{L}^{(2)} = & -2h^*(\partial_+\partial_- - \partial\partial^*)h , \\
\mathcal{L}^{(3)} = & 2\alpha\{(\partial_- h^*\partial_- h)\partial^{*2}\partial_-^{-2}h + \partial_- h^* h\partial^{*2}\partial_-^{-1}h - 2\partial_- h^*\partial^* h\partial^*\partial_-^{-1}h + \text{c.c.}\} , \\
\mathcal{L}^{(4)} = & 2\alpha^2\{-2|\partial_-^{-2}(\partial_-^3 h^*\partial^*\partial_-^{-1}h - \partial_-^2\partial^* h^* h)|^2 \\
& + |\partial_-^{-2}(\partial_-^2 h^*\partial^* h - \partial_-\partial^* h^*\partial_- h)|^2 \\
& + |\partial_-^{-1}(\partial_- h^*\partial^* h - \partial^* h^*\partial_- h)|^2 - 3|\partial_-^{-1}(\partial_- h^*\partial^* h)|^2 \\
& + 3\partial_-^{-1}(\partial_- h^*\partial_- h)\partial_-^{-1}(\partial h^*\partial^* h) + [\partial_-^{-2}(\partial_- h^*\partial_- h) - h^* h] \\
& \times [\partial h^*\partial^* h + \partial^* h^*\partial h - \partial\partial^*\partial_-^{-1}h^*\partial_- h - \partial_- h^*\partial\partial^*\partial_-^{-1}h]\} . \qquad (3.7)
\end{aligned}
$$

In writing down Eq. (3.7) we included a factor $(8\pi G)^{1/2}$ into the definition of h; $\alpha = (4\pi G)^{1/2}$, where G is Newton's constant. Moreover, we assume for simplicity of notation that the differential operators act only on the nearest fields.

The fields in Eq. (3.7) as it stays contain all modes. So our aim is to eliminate those degrees of freedom which are not present in MRK. Let us first separate the fields modes according to the kinematics (3.1)

$$
h \to h_1 + h + h_t , \qquad (3.8)
$$

where h_1, h and h_t contain the modes of the following momentum ranges

$$
\begin{aligned}
h_1 &: |k_+k_-| \gg |\kappa|^2 \sim |q|^2 , \\
h &: |k_+k_- - |\kappa|^2| \sim |q|^2 , \\
h_t &: |k_+k_-| \ll |\kappa|^2 \sim |q|^2 . \qquad (3.9)
\end{aligned}
$$

The field h_t describes exchanged particles in t-channel whereas field h corresponds to the scattered particles. The "heavy" modes h_1 describe highly virtual particles and these have to be integrated out.

Consider first the kinetic term of the Lagrangian (3.7). With the separation (3.8) it decomposes as

$$
\mathcal{L}^{(2)} = -2h_1^*(\partial_+\partial_- - \partial\partial^*)h_1 - 2h^*(\partial_+\partial_- - \partial\partial^*)h + 2h_t^*\partial\partial^* h_t . \qquad (3.10)
$$

Although in the first term of (3.10) the longitudinal part of the d'Alembert operator dominates, for the heavy modes also corrections proportional to $|\kappa|^2/k_+k_-$ has to be taken into account.

The part of the triple interaction vertices $\mathcal{L}^{(3)}$ (3.7) which leads to large contributions is obtained when the inverse of ∂_- acts on the field with the smallest momentum component k_-. In order to isolate this part $\mathcal{L}^{(3+)}$ we separate the modes in $\mathcal{L}^{(3)}$ by making the substitution

$$h \longrightarrow \tilde{h} + \tilde{h}_t \,.$$

Here, $\tilde{h}$ denotes the field with all modes *i.e.* those of h, h_t and h_1 whereas $\tilde{h}_t$ describes the fields carrying those modes of h and h_t whose momentum component k_- is, due to the MRK, much smaller than the ones in $\tilde{h}$. We obtain

$$\mathcal{L}^{(3+)} = 2\alpha\{\tilde{T}_{--}\tilde{\mathcal{A}}_{++} - \tilde{J}_-\tilde{\mathcal{A}}'_+ - \tilde{T}^*_-\tilde{\mathcal{A}}_+ - \tilde{T}_-\tilde{\mathcal{A}}^*_+\}\,, \tag{3.11}$$

where the currents depending on the fields $\tilde{h}$ are defined by

$$\begin{aligned} \tilde{T}_{--} &= \partial_-\tilde{h}^*\partial_-\tilde{h}\,, & \tilde{T}^*_- &= \tfrac{1}{2}(\partial_-\tilde{h}^*\partial^*\tilde{h} + \partial^*\tilde{h}^*\partial_-\tilde{h})\,, \\ \tilde{T}_- &= (\tilde{T}^*_-)^*\,, & \tilde{J}_- &= i(\tilde{h}^*\overleftrightarrow{\partial}_-\tilde{h})\,, \end{aligned} \tag{3.12}$$

and those depending on the fields $\tilde{h}_t$ are given by formulae

$$\begin{aligned} \tilde{\mathcal{A}}_{++} &= \partial_-^{-2}(\partial^{*2}\tilde{h}_t + \partial^2\tilde{h}^*_t), & \tilde{\mathcal{A}}'_+ &= -i\partial_-^{-1}(\partial^{*2}\tilde{h}_t - \partial^2\tilde{h}^*_t)\,, \\ \tilde{\mathcal{A}}_+ &= 2\partial_-^{-1}\partial^*\tilde{h}_t, & \tilde{\mathcal{A}}^*_+ &= 2\partial_-^{-1}\partial\tilde{h}^*_t\,. \end{aligned} \tag{3.13}$$

We want to remark that the form in which Eq. (3.11) is written down emphasizes the underlying MRK: the currents (staying more) to the left carry larger momentum components k_- than the ones (staying) more to the right (see Eq. (3.1)). In the following formulae a similar ordering of longitudinal momenta will also be assumed. Let us also observe that among the currents (3.12) the light-cone component of the energy-momentum tensor $\tilde{T}_{--}$ is of the order k_-^2 and provides the dominant contribution.

Using equations (3.11), (3.12), and (3.13) one can integrate over the heavy modes h_1 by means of the saddle point method. To do so one needs the equations of motion for those fields in the first perturbative order. The kinetic part of Lagrangian describing the heavy modes h_1 is given by the kinetic term (3.10). The interaction vertices are obtained from the Lagrangian $\mathcal{L}^{(3+)}$ (3.11) by means of the substitution

$$\tilde{h} \longrightarrow h_1 + \tilde{h}\,, \tag{3.14}$$

and by keeping only terms linear in h_1. For simplicity of notation we still use the same symbol $\tilde{h}$ even if this field does not contain heavy modes any more. We obtain the following equation of motion for the modes $h_1^{(0)}$

$$\begin{aligned} (\partial_+\partial_- - \partial\partial^*)h_1^{(0)} = -\alpha\{&\partial_-(\partial_-\tilde{h}\tilde{\mathcal{A}}_{++}) + \tfrac{1}{2}\partial_-\tilde{h}(\partial^*\tilde{\mathcal{A}}_+) \\ &- \tfrac{1}{2}\partial_-\tilde{h}\;\partial\tilde{\mathcal{A}}^*_+ - \partial_-(\partial^*\tilde{h}\tilde{\mathcal{A}}_+) - \partial(\partial_-\tilde{h}\tilde{\mathcal{A}}^*_+)\}\,. \end{aligned} \tag{3.15}$$

The solution $h_1^{(0)}$ of the equation of motion is obtained by the formal inversion of the d'Alembert operator. This has to be done with some care in order to determine correctly not only the leading term but also the next-to-leading term in the MRK. We write

$$\frac{1}{\partial_+\partial_- - \partial\partial^*} \approx \frac{1}{\partial_+\partial_-} + \frac{\partial\partial^*}{(\partial_+\partial_-)^2}\,. \tag{3.16}$$

Although for the modes h_1 the first term in (3.16) dominates we also keep the contribution given by the second term in (3.16) acting on the first term on the r.h.s. of Eq.(3.15). Moreover, in writing down the solution of the equation of motion we take into account that the momentum component k_- of $\tilde{A}$ is much smaller than the one of $\tilde{h}$ whereas the situation is inverse if we consider momentum components k_+

$$\begin{aligned} h_1^{(0)} \simeq &- \alpha\{\partial_-\tilde{h}\partial_+^{-1}\tilde{A}_{++} - (\partial_+\partial_-\tilde{h})\partial_+^{-2}\tilde{A}_{++} + \partial\partial^*(\tilde{h}\partial_+^{-2}\tilde{A}_{++}) \\ &+ \tfrac{1}{2}\tilde{h}\partial_+^{-1}(\partial^*\tilde{A}_+ - \partial\tilde{A}_+^*) - \partial^*\tilde{h}\partial_+^{-1}\tilde{A}_+ - \partial\tilde{h}\partial_+^{-1}\tilde{A}_+^* - \tilde{h}\partial_+^{-1}\partial\tilde{A}_+^*\}\,. \end{aligned} \tag{3.17}$$

The result of the integration over heavy modes $\mathcal{L}^{(1)}$ is given by the kinetic term with opposite sign for fields $h_1^{(0)}$ (3.17)

$$\mathcal{L}^{(1)} = 2h_1^{(0)*}(\partial_+\partial_- - \partial\partial^*)h_1^{(0)}\,. \tag{3.18}$$

Calculating $\mathcal{L}^{(1)}$ (3.18) we observe the cancellation of the dominant terms involving third powers of the longitudinal momenta of the $\tilde{h}$ fields. This corresponds to the fact that in gravity there is no dipol radiation. Therefore, the result for $\mathcal{L}^{(1)}$ involves squares of longitudinal momenta of $\tilde{h}$ and it can be written as

$$\begin{aligned} \mathcal{L}^{(1)} =&\alpha^2\tilde{T}_{--}\tilde{\mathcal{I}}_{++}\,, \\ \tilde{\mathcal{I}}_{++} =& - (\partial_+^{-1}\tilde{A}_{++}\overleftrightarrow{\partial}_-\tilde{A}_{++}) \\ &+ \{\partial(\partial_+^{-1}\tilde{A}_{++}\tilde{A}_+^* - \tilde{A}_{++}\partial_+^{-1}\tilde{A}_+^*) + \partial\tilde{A}_{++}\partial_+^{-2}\partial^*\tilde{A}_{++} + \text{c.c.}\}\,. \end{aligned} \tag{3.19}$$

Let us now observe that, although we integrated out over heavy fields propagating in the s-channel, Eq. (3.19) is factorized in the t-channel. This result was expected, but in the actual calculations it emerges only after the cancellation of many terms with different structure than those in formula (3.19).

The form of $\mathcal{L}^{(1)}$ also suggests that this result can be obtained from a Lagrangian which contains:
(a) the kinetic term for t-channel fields h_t from Eq. (3.10),
(b) the leading triple vertex from Eq. (3.11)

$$\mathcal{L}^{(3+)}_{\text{leading}} = 2\alpha\tilde{T}_{--}\mathcal{A}_{++}$$

supplemented by Eqs (3.12), (3.13) in which we restrict ourselves to t-channel fields only,
(c) the new induced vertex $\mathcal{L}^{(1)}_{\text{ind}}$ which is given by formula

$$\mathcal{L}^{(1)}_{\text{ind}} = -\alpha\partial\partial^*\mathcal{A}_{--}\tilde{\mathcal{I}}_{++}\,. \tag{3.21}$$

The newly introduced t-channel field $\mathcal{A}_{--}$ in Eq. (3.21) is defined as

$$\mathcal{A}_{--} = \tfrac{1}{2}\partial_-^2(\partial\partial^*)^{-2}(\partial^{*2}h_t + \partial^2 h_t^*) = \tfrac{1}{2}\partial_-^4(\partial\partial^*)^{-2}\mathcal{A}_{++}\,. \tag{3.22}$$

From the vertices (3.20) and (3.21) one can read off the forms of the effective vertices describing scattering of gravitons off $\mathcal{A}_{++}$ fields in t-channel as well as the production of graviton from t-channel fields. Let us consider the leading vertex (3.20) in which we restrict the modes of both fields appearing in $\tilde{T}_{--}$ (see Eq. (3.12) to the modes of h fields only. This leads to the effective scattering vertex of gravitons off the $\mathcal{A}_{++}$ field

$$\begin{aligned}\mathcal{L}^{(s+)} &= 2\alpha T_{--}\mathcal{A}_{++}\\ T_{--} &= \partial_- h^*\partial_- h\,.\end{aligned} \tag{3.23}$$

The vertex (3.20) gives also a contribution $\mathcal{L}^{(3-+)}$ to the production vertex. It is obtained by restricting the modes of one field $\tilde{h}$ to the modes of the h field and keeping in the second field $\tilde{h}$ only the modes of the h_t field. One gets

$$\mathcal{L}^{(3-+)} = -2\alpha\{\partial^{*2}(\mathcal{A}_{--} - i\partial_-\mathcal{A}'_-)h + \text{c.c.}\}\mathcal{A}_{++}\,, \tag{3.24}$$

where

$$\mathcal{A}'_- = -\tfrac{i}{2}\partial_-(\partial\partial^*)^{-2}(\partial^{*2}h_t - \partial^2 h_t^*)\,. \tag{3.25}$$

As we are only interested in the leading order contribution to the effective vertex the term involving the field $\mathcal{A}'_-$ in Eq. (3.24) is neglected.

Also the induced vertex (3.21) contributes to graviton production. In this case one of the fields in $\tilde{\mathcal{I}}_{++}$ carries the modes h ($\mathcal{A}^{(s)}$) and the other the modes h_t ($\mathcal{A}$). The contributions where the modes h_t are in $\partial_- \mathcal{A}_{++}, \partial_+^{-1} \mathcal{A}_{++}$ or in $\mathcal{A}_+, \mathcal{A}_+^*$ are small. Thus we have

$$\mathcal{L}_{\text{ind}}^{(1-+)} = -2\alpha\partial\partial^* \mathcal{A}_{--}[\partial_- \partial_+^{-1} \mathcal{A}_{++}^{(s)} \mathcal{A}_{++} + \{-\partial(\partial_+^{-1} \mathcal{A}_+^{(s)*} \mathcal{A}_{++}) + \partial_+^{-2} \partial^* \mathcal{A}_{++}^{(s)} \partial \mathcal{A}_{++} + \text{c.c.}\}] . \quad (3.26)$$

Taking into account that $\partial_+ \partial_- \approx \partial\partial^*$ on the h fields, one can rewrite Eq. (3.26) as

$$\mathcal{L}_{\text{ind}}^{(1-+)} = -2\alpha\partial\partial^* \mathcal{A}_{--}[\partial^* \partial^{-2} h \partial \mathcal{A}_{++} - \partial^*(\partial^{-1} h \mathcal{A}_{++}) + \text{c.c.}] . \quad (3.27)$$

The sum of this expression together with Eq. (3.24) leads to the following effective graviton production vertex

$$\mathcal{L}^{(-+)} = -2\alpha(\partial^{*2} \mathcal{A}_{--} \partial^2 \mathcal{A}_{++} - \partial\partial^* \mathcal{A}_{--} \partial\partial^* \mathcal{A}_{++}) \partial^{-2} h + \text{c.c.} . \quad (3.28)$$

Let us note that the production vertex (3.28) contains the non-local expression $\partial^{-2} h$. As our aim is to have an effective Lagrangian which is local we introduce a new s-channel field ϕ defined as

$$\phi = -\frac{1}{\partial^{-2}} h \quad (3.29)$$

(compare [13]). In this way we obtain the following kinetic term for the s-channel fields

$$\mathcal{L}_s^{(2)} = -2\phi^* (\partial_+ \partial_- - \partial\partial^*) \partial^2 \partial^{*2} \phi , \quad (3.30)$$

the scattering vertex off $\mathcal{A}_{++}$ fields

$$\mathcal{L}^{(s+)} = 2\alpha\partial_- \partial^{*2} \phi^* \partial_- \partial^2 \phi \mathcal{A}_{++} , \quad (3.31)$$

and the production vertex

$$\mathcal{L}^{(-+)} = 2\alpha(\partial^{*2} \mathcal{A}_{--} \partial^2 \mathcal{A}_{++} - \partial\partial^* \mathcal{A}_{--} \partial\partial^* \mathcal{A}_{++}) \phi + \text{c.c.} . \quad (3.32)$$

In order to complete the derivation of the effective Lagrangian we have to determine the vertex $\mathcal{L}^{(3-)}$ describing graviton scattering off the field $\mathcal{A}_{--}$. The easiest way to obtain the form of this vertex is to use parity invariance of the theory. This invariance implies that $\mathcal{L}^{(3-)}$ is obtained from $\mathcal{L}^{(3+)}$ (Eq. (3.31)) by the simultaneous exchange of $+ \leftrightarrow -$ and $\phi \leftrightarrow \phi^*$

$$\mathcal{L}^{(s-)} = 2\alpha \mathcal{A}_{--} \partial_+ \partial^{*2} \phi \partial_+ \partial^2 \phi^* . \quad (3.33)$$

However this way to derive $\mathcal{L}^{(s-)}$ is not satisfactory. The reason is that our derivation of the effective Lagrangian is performed in the peculiar axial gauge (3.3) which breaks the $+ \leftrightarrow -$ symmetry of the whole procedure. Of course the final result does not depend on this choice of gauge but expressions in the intermediate steps are gauge dependent. Therefore, to check the consistency of the method it is necessary to reproduce the vertex $\mathcal{L}^{(s-)}$ (3.33) by careful collecting all ingredients which contribute to it. It turns out that the scattering vertex $\mathcal{L}^{(s+)}$ (3.31) is obtained in a rather straightforward way whereas many terms contribute to the vertex $\mathcal{L}^{(s-)}$. This is due to our gauge choice (3.3). Since this part of our derivation is rather technical and tedious I shall not discuss it here in details (see Ref. [14]). Instead I only want to present the main idea. It consists in studying the MRK contributions from all quartic terms. One source of such terms is the quartic part in the original Lagrangian (3.7). The second part is obtained as a result of the integration over heavy modes. It is given by Eq. (3.19) in which all fields carry modes of h fields, supplemented by the MRK-condition that the fields staying more to the left have larger k_- momentum components. The third part of the quartic terms is obtained as a result of exchanging a h_t field between the leading part of the vertex $\mathcal{L}^{(3+)}$ — this vertex involves two h fields with large k_- momentum components — and the whole vertex $\mathcal{L}^{(3)}$ (3.7) — this vertex involves one field h_t and the remaining fields of h type. The momentum components k_- of the fields entering the vertex $\mathcal{L}^{(3)}$ are all of the same order, but they are much smaller than those in the vertex $\mathcal{L}^{(3+)}$. After analogous factorization as in Eqs (3.19), (3.20) and (3.21), the sum of these three contributions leads to an expression from which one gets precisely the scattering vertex $\mathcal{L}^{(3-)}$ in Eq. (3.33). The consistency of the calculations also requires that the t-channel fields $\mathcal{A}_{++}$ and $\mathcal{A}_{--}$ have to be treated as independent fields, despite the fact that they were originally defined by Eqs (3.13) and (3.22) in terms of the same field h_t. This requirement leads to the kinetic term $\mathcal{L}_t^{(2)}$ for those fields

$$\mathcal{L}_t^{(2)} = 2\mathcal{A}_{++}\partial\partial^*\mathcal{A}_{--}\,, \tag{3.34}$$

which differs by a factor 2 from the expression obtained by formal substituting definitions (3.13) and (3.22) into Eq. (3.10). This completes the derivation of the effective Lagrangian $\mathcal{L}^{(\mathrm{eff})}$ for gravity in the MRK. It is given by the sum of the terms

$$\mathcal{L}^{(\mathrm{eff})} = \mathcal{L}_s^{(2)} + \mathcal{L}_t^{(2)} + \mathcal{L}^{(3+)} + \mathcal{L}^{(3-)} + \mathcal{L}^{(-+)}\,, \tag{3.35}$$

where the elements of the sum are given by Eqs (3.30), (3.34), (3.31), (3.33), and (3.32). Let us also emphasize that it would very desirable to have a derivation of the effective Lagrangian (3.35) which is not based on some peculiar choice of gauge. In this case one could perhaps avoid some technical complications encountered in the present method.

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REFERENCES

[1] L.N. Lipatov, *Sov. Phys. JETP* **55**, 582 (1982); L.N. Lipatov, *Phys. Lett.* **116B**, 41 (1982).
[2] D. Amati, M. Ciafaloni, G. Veneziano, *Phys. Lett.* **197B**, 81 (1987); *Int. J. Mod. Phys.* **A3**, 1615 (1988); *Phys. Lett.* **B216**, 41 (1989).
[3] Ya.I. Asimov, *Zh. Eksp. Teor. Fiz.* **43**, 2321 (1962); S. Mandelstam, *Nuovo Cim.* **30**, 1127 (1963); V.N. Gribov, I.Ja. Pomeranchuk, K.A. Ter-Martirosyan, *Yad. Fiz.* **2**, 361 (1065).
[4] L.N. Lipatov, *Nucl. Phys.* **B365**, 614 (1991).
[5] H.M. Fried, *Functional Methods and Models in Quantum Field Theory*, The MIT (1972).
[6] D. Kabat, M. Ortiz, *Nucl. Phys.* **B388**, 570 (1992).
[7] G. t'Hooft, *Phys. Lett.* **198B**, 61 (1987).
[8] P.C. Aichelburg, R.U. Sexl, *Gen. Relativ. Gravitation* **2**, 303 (1971).
[9] I. Muzinich, M. Soldate, *Phys. Rev.* **D37**, 353 (1988).
[10] E. and H. Verlinde, *Nucl. Phys.* **B371**, 246 (1992).
[11] D. Amati, M. Ciafaloni, G. Veneziano, *Phys. Lett.* **B289**, 87 (1992).
[12] D. Amati, M. Ciafaloni, G. Veneziano, *Nucl. Phys.* **B403**, 707 (1993).
[13] R. Kirschner, L.N. Lipatov, L. Szymanowski, *Nucl. Phys.* **B425**, 579 (1994); *Phys. Rev.* **D51**, 838 (1995).
[14] R. Kirschner, L. Szymanowski, *Phys. Rev.* **D52**, 2333 (1995).