THE LINKED DIPOLE CASCADE MODEL*,**

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In this talk I would like to report a set of new developments in the Lund Model for Quark and Gluon Interactions. We feel that by the completion of the Linked Dipole Chain Model (developed together with G. Gustafson and a graduate student J. Samuelsson) *we have a complete description of all kinds of perturbative QCD interactions* (although only a few are reported in my talk) *in terms of dipoles.* This model describes the evolution of the wave function (or rather the square of it as measured by the partonic structure functions) in terms of space-like cascades of connected dipoles with both a mass and a virtuality. In the same way the Lund Dipole Cascade Model, which has been presented before repeatedly, describes the time-like perturbative cascades in terms of the building of dipoles decaying into smaller dipoles until the fragmentation process into hadrons ("the ultimate dipoles") sets in.

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1. Introduction

1.1. Preliminaries on dipoles

In the Lund Model for Quark and Gluon Interactions we have until recently only been able to treat the fragmentation process together with the production of the partonic states relevant to e^+e^--annihilation reactions (although we have by means of the Soft Radiation Model, as implemented in the Monte Carlo simulation program ARIADNE also considered some

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** Work done in Lund together with G. Gustafson and a graduate student J. Samuelsson.

Deep Inelastic Scattering reactions). In this talk I would like to describe a new model, the Linked Dipole Cascade Model (developed together with G. Gustafson and a student J. Samuelson in Lund) which is meant to describe in a very general framework the Deep Inelastic Scattering states.

Before I start with the model I would like to make the comment that with the completion of this model one may say that *all the features of the Lund Model correspond to a description of QCD in terms of (color) dipoles.*

To obtain the partonic states in the "time-like" cascades (which are the ones relevant to e^+e^--annihilation reactions) we have made use of the Dipole Cascade Model (which also is implemented in ARIADNE). In this model the original color $(3\bar{3})$ (quark, q, anti-quark, $\bar{q}$) pair, which is excited out of the vacuum by the annihilation, immediately starts to move apart. In accordance with the dictum in a gauge theory that all charges must "carry" a Coulomb field there will be fluctuations in the production of the field, implying that the charges will start to emit (coherent) gluonic bremsstrahlung, *i.e.* a color 8 gluon, g_1 (index 1 stands for "the first").

At this point there is a major difference between the Abelian gauge theory, QED, and a non-Abelian gauge theory like QCD. The gauge bosons in QED are chargeless so that the current in QED is, besides recoil contributions, the same after the emission. For QCD and any non-Abelian gauge theory the gluons are charged and therefore in principle there is a major change in the current by the emission. But "then a miracle occurs" (as the Europeans would say) or "there is for once a free luncheon" (as the Americans would say). *Instead of a complex charge configuration one obtains just a splitting of the original dipole into two for non-Abelian gauge theories.* Thus in QCD the original color $(3\bar{3})$-dipole is partitioned into two *independent* dipoles, one between the (qg_1) and one between the $(g_1\bar{q})$ (although the corrections to this statement might have been of the order of $1/N_c^2$, *i.e.* roughly 10%, we have shown that the approximation is better than 99%).

In the next step each of the two independent dipoles may emit a new gluon, thereby producing three independent dipoles *etc.* The masses of the dipoles quickly decrease and therefore also the "hardness", as measured in terms of *e.g.* the transverse momentum, $k_\perp$, of the gluon emissions. For values of $k_\perp$ smaller than a cutoff $k_{\perp c}$ the fragmentation process starts and the final state hadrons appear.

Note that these hadrons again correspond to dipoles, although this time colorless, *i.e.* singlet parts of the decaying QCD force field. The fact that the $(q\bar{q})$-pairs in this decay process in the Lund String Fragmentation Model stem from different space-time (or energy-momentum space) points have some interesting consequences for *e.g.* Bose–Einstein correlations but these features are outside my talk of today.

We have also shown that this decay process is independent of the cutoff, *i.e.* that you may chose $k_{\perp c}$ anywhere between $\simeq$ 7 GeV and just above $\Lambda_{\rm QCD}$ and still obtain the same hadronic distributions (with explainable small changes of the fragmentation parameters). Thus the whole process is *infra-red stable.*

The Linked Dipole Model then corresponds to the statement that in case *you would go into the hadrons using a well-defined probe, then once again you recover color dipoles.* This reminds me of a statement by one of the really good experimentalists I have had the opportunity to work with, Guy von Dardel. He said that "You may take an elephant and as a physicist cut it into slices, then you find that each slice contains molecules, and the molecules contains atoms, the atoms contains nucleons but how is it, does not the nucleons contain new elephants?!"

What I mean is that the wave-functions of the hadrons (or rather the absolute squares, *i.e.* what in Deep Inelastic Scattering reactions are named "the flux" of the partons or the structure functions) can once again be described in dipole language. This description of the large frequency parts of the wave-functions (usually called "going into larger virtualities") is in general known as "space-like" perturbative QCD cascades.

Thus we find that *the "effective" correspondence to the QED "point-charges" is in the non-Abelian gauge theories everywhere dipoles*, *i.e.* extended objects (this is also a major reason why non-Abelian field theories exhibit "transparency" to small wave-length probes, *i.e.* "asymptotic freedom"). I would like to remind you that in the only completely solvable confined model we know of, the Schwinger Model, *i.e.* $1+1$-dimensional QED, the fermion fields together with the (Abelian) gauge field combine into a free scalar field theory, corresponding to a dipole density. The quanta combine into stable states containing a "piece of field" (note that the potential in $1+1$-dimensional QED is linear, *i.e.* the force field is a constant just as in the Lund string description of the QCD force field) and a fermion-antifermion pair, which evidently corresponds in one space dimension to a dipole.

1.2. The description of the cross-sections in Deep Inelastic Scattering

A major problem in Deep Inelastic Scattering (DIS) is to calculate the flux-factors of the available partons, *i.e.* the structure functions of the participating hadron. In a perturbative treatment of QCD this means the summation over a large amount of Feynman graphs relating the constituents, which occur on a certain "initial" scale, to those of the "final" scale, determined by the properties of the probe.

It is not necessary to know all the radiation in the states in order to be able to calculate (this change in) the flux factors. It is perfectly feasible to subdivide the emissions into two sets, "the main emissions", which we conventionally will refer to as the Initial State Bremsstrahlung (ISB) set, and "the remainder" (the Final State Bremsstrahlung, FSB), which is possible to emit in case the ISB is already provided. (In the calculations of the cross-sections the FSB is treated as the contributions from the radiative corrections to the main ISB emissions).

It is possible to make any choice of the ISB set for a particular configuration as long as the FSB can be emitted from this set *in accordance with the coherence conditions of QCD.* (It is, of course, useful to chose the ISB so that the later FSB emissions do not disturb the original configurations very much by recoils and kinematics). For a particular choice of the ISB it is, however, necessary to calculate the corresponding radiative corrections to these real emissions. In the perturbative QCD cascades the results of such calculations exponentiate (at least for the leading contributions, *e.g.* in the Leading Log Approximation, LLA) and are known as the corresponding Sudakov factors. It should be recognised, however, that if one changes the ISB set, *e.g.* by including some of the "earlier" ISB gluons into the FSB set, then it is also necessary to change the corresponding radiative corrections.

The Sudakov factors obtained by the exponentiation of the radiative corrections can often be interpreted in a probabilistic language. Then they correspond to the probability to not emit anything inside the phase space regions where it should have been dynamically allowed. The perturbative QCD cascades are in general formulated as stochastic processes. There is then an (inclusive) density of emissions in phase space dn, which will lead to an inclusive probability $d\mathcal{P}$ to emit a particular set of partons (the coherence conditions in the QCD radiation are taken into account by introducing an ordering of the process). The semi-inclusive probability to emit just those partons and to emit nothing inside the remaining region, Ω, (allowed by coherence, *i.e.* the ordering, and energy-momentum conservation) is then given by the formula

$$d\mathcal{P} \exp\left(-\int_{\Omega} dn\right) . \tag{1}$$

The exponential factor is just the Sudakov factor, whether it is obtained by calculating the radiative corrections or by general probability arguments. The main point is, of course, that the radiative corrections to a given process often contains the same (inclusive) emission density dn as the "real" emissions.

In conclusion, although the main emission probability at any bremsstrahlung vertex is the same, it is necessary to "correct" it with the proper Sudakov, depending upon the choice of the ISB gluons. In this way the resulting sum over all the contributions corresponding to the structure functions will be the same.

In Ref. [1] we have presented a generalization and simplification of the model for the states in DIS, which has been developed by Ciafaloni, Catani, Marchesini and Fiorani (the CCMF Model) [2]. In the Linked Dipole Chain (LDC) Model there is a (small) change in the choice of the ISB set of gluons, as compared to the CCMF Model. While the set chosen in the CCMF Model can be described as the most general one possible, if one includes the QCD coherence conditions ("angular ordering") and if one requests energy–momentum conservation at each emission vertex, the set chosen in the LDC Model is restricted to a (large) subset.

In this way we obtain in the LDC Model [1] that

I The Final State Bremsstrahlung (FSB) can be treated as the emission from a set of color dipoles, spanned by the chosen gluons in the ISB set. Therefore the FSB can be treated directly by means of the Lund Dipole Cascade Model (the DCM), which is implemented in the Monte Carlo simulation program ARIADNE.

II The probabilities (in particular the radiative corrections) for the ISB set of states chosen in the LDC Model are simpler than the results for the CCMF Model. The stochastical process obtained is further explicitly "local" (Markovian) and symmetric with respect to emissions from the hadron end and the probe end. In this way the predictions of the LDC can be easily implemented into Monte Carlo simulation programs to study the particular ISB sets of the model.

III It is possible to incorporate into the formalism both the "ordinary" (perturbative) QCD parton interactions, the Boson-Gluon Fussion interactions and also the resolved (virtual) probe structure functions, including Rutherford interactions between the probe- and the hadron-ends. There is consequently in the LDC no cutoff (besides energy-momentum conservation) necessary for large transverse momenta in the ISB gluon emissions at the same time as the gluonic bremsstrahlung in a well-defined way also cuts off small transverse momentum (Rutherford) scatterings.

Consequently the LDC Model and the DCM constitute a general method to treat all perturbative QCD interactions along the same lines, in terms of color dipole emissions. This approach directly fits into the final state hadronisation model, *i.e.* the Lund String Fragmentation Model (as implemented in the Monte Carlo simulation program JETSET)

The results are for both the LDC and the CCMF Models valid in the Leading Log Approximation (LLA) [3]. Further both models interpolate betwen the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) mechanism, which is relevant for large Q^2- and medium to small x_B-values and the BFKL (Balitsky–Fadin–Kuraev–Lipatov) mechanism, expected to be valid for medium Q^2 and very small x_B. We consider some properties of the models in Section 2.

The intention of this talk is to present some general properties of the two models and in particular what is possible to do in the LDC Model. We note that it is possible to consider the results both for a fixed and a running coupling in the LDC Model. As of now we have results on the following properties, although I will only mention them here and refer to a forthcoming paper [4]:

A The behavior of the structure functions for different (x_B, Q^2) (to be precise we treat in Ref. [4] in accordance with the LLA only the case when the color emission lines contains gluon emission from an incoming gluon at $x_{B0} = 1$ at a (small) initial virtuality Q_0^2 up to the probe at (x_B, Q^2). The methods will be extended to the other possible emission processes in the near future). We consider parametrizations of both the DGLAP type, $\propto \exp C\sqrt{\log(1/x_B)}$, and of the BFKL type $\propto x^{-\lambda}$ with C and λ possibly Q^2 dependent.

B The average "road" in the available phase space from (x_{B0}, Q_0^2) to (x_B, Q^2), together with the regions containing roads at most one standard deviation from the mean. At the same time we consider the density of the ISB gluon emission along the roads. We find that besides a region "in the beginning" (about $3-4$ units in rapidity or $\log(1/x)$), which depends upon the starting-out conditions for the "undisturbed" hadronic wave-function constituents, there is a region in which the main road stays close to small $k_\perp$. Finally there is a region with a size depending upon the value of Q^2, *i.e.* the properties of the probe. The latter two regions correspond to contributions expected from a BFKL and a DGLAP treatment of the structure functions, respectively. The larger the total $\log(1/x)$ region is for a given $\log(Q^2)$, the more BFKL and vice versa for the DGLAP mechanism.

C The contributions to the structure functions from the different channels mentioned above for different (x_B, Q^2). In particular we investigate in some detail the regions where the DGLAP mechanism dominates, where the Boson-Gluon Fussion will provide essential contributions and where the (virtual) probe-hadron interactions contains large Rutherford contributions. In this way we exhibit the regions inside which a cutoff in the ISB gluon transverse momenta, $q_\perp^2$, by *e.g.* $q_\perp^2 < Q^2$ will be dangerous according to the LDC.

2. The LDC and the CCMF models

2.2. Some properties of the CCMF Model

In order to describe the choice of the ISB set of gluon emissions in the CCMF Model we consider the fan diagram shown in Fig. 1. This does not correspond to a single Feynman diagram but rather to a collection of such diagrams. Besides the set $\{q_j\}$ of emissions (which will be the ISB) along the main line from the incoming parton, denoted by the energy-momentum vector P, to the probe, with the vector q, there is another set, which is not noted out, $\{C_j\}$, (the corresponding FSB) but occurs as emissions "in between" the q_j and q_{j+1}. There is further a set of "connector vectors", $\{k_j\}$, between the q_j-emissions. Energy-momentum conservation is imposed for each q_j-emission (justified by chosing the q_j as more "energetic", *cf.* below, than the C_j-gluons):

$$k_j = P - \sum_{m=1}^{j} q_m \quad i.e. \quad k_j = k_{j-1} - q_j \,. \tag{2}$$

All emissions are ordered in rapidity, (which due to the relation between angle and rapidity means *strong angular ordering* along the chain). The CCMF then pick the ISB $\{q_j\}$ set from the set of all emissions as those which are not followed (in the rapidity ordering variable) by another one with a larger light-cone energy-momentum $q_+(= q_0 + q_\ell)$, *i.e.* in this way the q_j has larger "energy" than the rest. (The ℓ-direction stands for longitudinal and I will in this talk not specify it further).

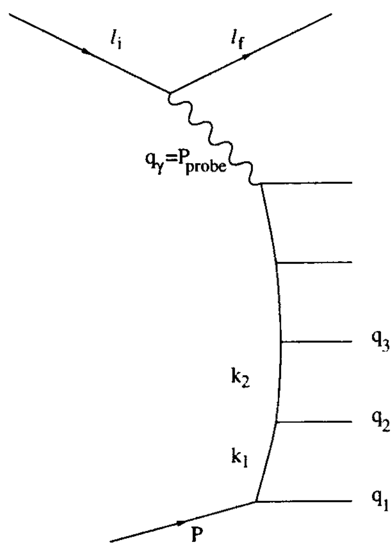

Fig. 1. A DIS fan diagram

To be more precise CCMF introduce the variables $(z_j, \vec{q}_{\perp j})$ with $k_{+j} = z_j k_{+(j-1)}$ and $\vec{k}_{\perp j} = \vec{k}_{\perp(j-1)} - \vec{q}_{\perp j}$. In the LLA the CCMF choice for the

q_+ will imply $q_{+j} \ll q_{+(j-1)}$ which corresponds to an approximation of the splitting function $P(z) \simeq 1/z$ and the assumption that z is small enough so that $(1-z) \simeq 1$. Further the gluons in the sets $\{C_j\}$ are in accordance with the LLA, treated as soft enough so that the q-vectors can be taken on-shell and massless.

The corresponding propagator vectors k_j are, however, spacelike with the values $k_{+(j-1)} = q_{+j}/(1-z_j) \simeq q_{+j}$. The transverse momentum of the propagators $k_{\perp j}$ are dominated by the $q_\perp$-emissions in the neighbourhood (for a detailed discussion *cf.* [1]). One major kinematical constraint is that

$$k_{\perp j}^2 > z_j q_{\perp j}^2 \,. \tag{3}$$

If this condition is not fulfilled then the "virtuality" of the propagator will in the LLA fulfil $|k^2| \gg k_\perp^2$, which implies strong suppression. Each step in the emission chain will in the CCMF Model be described by the weight

$$\bar{\alpha} \frac{dz_j}{z_j} \frac{dq_{\perp j}^2}{q_{\perp j}^2} \Delta_{\mathrm{ne}}(z_j, k_{\perp j}, q_{\perp j}) \,. \tag{4}$$

Here $\bar{\alpha}$ is the effective coupling (including color factors) and Δ_{ne} is the so-called "non-eikonal formfactor" with

$$\Delta_{\mathrm{ne}}(z_j, k_{\perp j}, q_{\perp j}) = \exp\left(-\bar{\alpha} \log\left(\frac{1}{z_j}\right) \log\left(\frac{k_{\perp j}^2}{z_j q_{\perp j}^2}\right)\right) \,. \tag{5}$$

The major result in the CCMF Model is this non-eikonal formfactor, corresponding to the radiative corrections for the choice of the ISB set defined above. We note in particular that due to the properties of the non-eikonal formfactor small values of z_j and $q_{\perp j}$ in Eq. (4) are effectively cut-off in case we assume $k_\perp$ is finite.

2.2. The properties of the LDC model

In Ref. [1] the results in Eqs (4) and (5) are analysed and reformulated. It is shown that the non-eikonal formfactor actually can be considered as an "ordinary" Sudakov formfactor, in the probability sense we described in Section 1. The negative exponent represents, besides the $\bar{\alpha}$-factor, a region excluded for gluon emission by the particular choice of the ISB $\{q_j\}$ in the CCMF Model.

In the LDC Model there are a restriction of the $\{q_j\}$ states into those in which

$$q_{\perp j} = \max(k_{\perp j}, k_{\perp (j-1)}) \,. \tag{6}$$

In this way we obtain in the LDC *that the weights in the q_j emissions are given by Eq. (4) but with the non-eikonal formfactor $\Delta_{\rm ne}$ exchanged for* 1. What is shown in Ref. [1] is that *if we include into the FSB the gluons which in the CCMF belong to the IBS-set, but do not fulfil Eq. (6), then the sum over the states containing such gluons, with the appropriate non-eikonal weights, will provide just the factor* 1.

This evidently corresponds to a major simplification and the weight in the LDC can be written solely in terms of the connectors $\{k_j\}$ as (writing $\log(k_\perp^2) = \kappa$)

$$\bar{\alpha}\frac{dz_j}{z_j}\frac{dk_{\perp j}^2}{k_{\perp j}^2}\frac{k_{\perp j}^2}{\max(k_{\perp j}^2, k_{\perp(j-1)}^2)} = \begin{cases} \bar{\alpha} d\ln\left(\frac{1}{z_j}\right) d\kappa_j & \text{if } \kappa_j > \kappa_{j-1} \\ \bar{\alpha} d\ln\left(\frac{1}{z_j}\right) d\kappa_j \exp(\kappa_j - \kappa_{j-1}) & \text{otherwise} \end{cases} \tag{7}$$

(the last factor stems from the requirement in Eq. (6) and obviously corresponds to a "local" statement, *i.e.* a comparison of the sizes of two neighboring connector vectors).

Before I continue I will make a few comments on the time-like dipole cascades. We note that for a dipole bremsstrahlung emission (from *e.g.* a gluon-gluon (g_0, g_0') state) of a gluon g_1 with transverse momentum $q_{\perp 1}$ and rapidity y_1 the inclusive density dn_1 is in the LLA

$$dn_1((g_0, g_0') \to (g_0, g_1, g_0')) \equiv dn_1((g_0, g_1, g_0')) = \bar{\alpha} dy_1 \frac{dq_{\perp 1}^2}{q_{\perp 1}^2}. \tag{8}$$

The phase space for such an emission from a dipole with mass W is in the dipole cms:

$$q_\perp \cosh(y) \leq \frac{W}{2}. \tag{9}$$

This relation can be conveniently approximated, *cf.* Fig. 2, in the $(y, \log(q_\perp^2))$-plane as the inside of a triangle

$$|y| \leq (L - \kappa)/2 \quad \text{with} \quad (L, \kappa) = (\log(W^2), \log(q_\perp^2)). \tag{10}$$

We note that in case we consider the corresponding inclusive density to emit two gluons at $(q_{\perp j}, y_j)$, $j = 1, 2$, then the density is factorizable, so that (this is shown to be good to about 99% all over phase space in Ref. [5])

$$dn_2((g_0, g_0') \to (g_0, g_1, g_2, g_0')) = dn_1((g_0, g_1, g_0'))(dn_1((g_0, g_2, g_1)) + dn_1((g_1, g_2, g_0')), \tag{11}$$

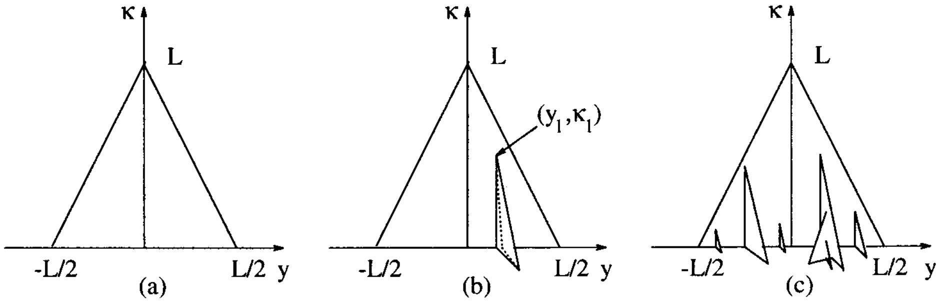

Fig. 2. The phase-space available for gluon emission from a dipole is a triangle in the (y, κ)-plane (a); in case one gluon is emitted at (y_1, κ_1) then the phase-space for a second (softer) gluon is the the area of the folded surface (below κ_1) (b); each emitted gluon increases the phase-space for softer gluons and is represented by this many-facetted surface (c).

in case $q_{\perp 1} > q_{\perp 2}$ (or else the indices have to be exchanged in Eq. (11)). Thus "the original dipole" with (the basement) "length" in rapidity L, *cf.* Fig. 2, is subdivided into two with the corresponding lengths $\pm y_1 + (L + \kappa_1)/2$. Each dipole can independently emit the gluon indexed 2 as shown in Fig. 2. The requirement that $\kappa_2 < \kappa_1$ means that the two new dipole triangles are cut-off at the "height" κ_1. In this way we may define the notion of "virtuality" (as $\log(k_{\perp 1}^2) = \kappa_1$) for the newly produced dipoles.

In the Lund Dipole Cascade Model (DCM) this time-like cascade process is then continued in an obvious way into more and more gluon emissions with smaller and smaller dipoles. The main point is the ordering in $\kappa = \log(k_\perp^2)$ (which is the correspondence to the strong angular ordering, *i.e.* to the coherence conditions in the QCD radiation). This implies that any "new" emission at κ_j in a dipole is limited by the earlier "virtuality" of the dipole, ie $\kappa_{j-1} > \kappa_j$.

We also note that in the "geometry" of the triangular phase space, all points with a fixed value of $q_+ = q_{+j}$ ($q_- = q_{-j}$) are placed along lines parallel to the triangular side(s) because $\log(q_{\pm j}) = \pm y_j + \kappa_j/2$. In this way an on-shell (massless) gluon emission can be described either in terms of (q_{+j}, q_{-j}) or (what amounts to the crossing point of the two (logarithmic) "light-cone-lines") in terms of the coordinates (y_j, κ_j).

In order to interprete the results of the LDC and CCMF Models into the dipole phase space we note that there is in the fan diagram in Fig. 1 in principle a large color "dipole" spanned between the incoming parton P (which we parametrise to be on the mass-shell with the positive light-cone energy-momentum P_+) and the parton "kicked out" by the probe q (the probe is not on the mass-shell, but provides the negative light-cone energy-momentum Q_- to the parton), *cf.* Fig. 3. The essential ISB emissions, $\{q_j\}$, as well as the connector vectors, $\{k_j\}$, can be exhibited in this phase space triangle in the following way for the LDC:

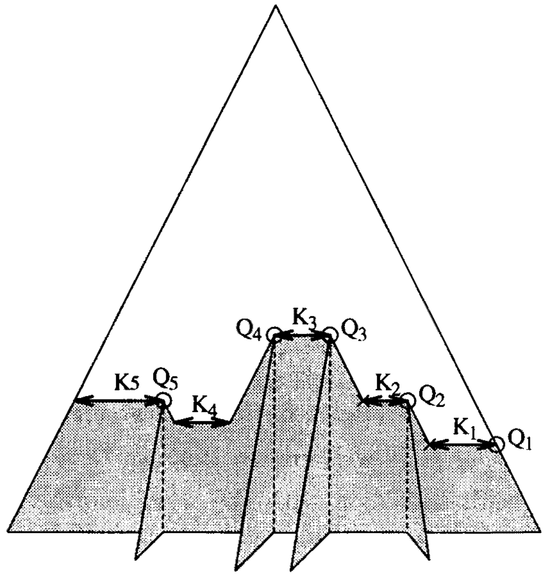

Fig. 3. A gluon-chain in the LDC Model described in the triangular phase-space. The extended folds correspond to on-the-mass-shell gluons and the arrows to the connector propagators. The front(back) line corresponds to $\log(P_+)$ ($\log(Q_-)$), respectively.

LDCa. the q_j-vectors are onshell and massless and are therefore shown as extended triangular folds, just as we describe the emissions of the gluons in the time-like dipole cascade above

LDCb. the connector vectors k_j are not on the mass-shell, but as we have mentioned before, the component $k_{+j} = q_{+(j+1)}$ and (due to the necessary symmetry between the hadron and probe end) the component $k_{-j} = q_{-j}$. Finally the requirement in Eq. (6) means that the transverse momentum $\log(k_{\perp j}^2)$ is fixed by the q_j emission (or vice versa). Therefore the k_j can be described by an extended line between the $q_{+(j+1)}$ and the q_{-j} at the appropriate height $\log(k_{\perp j}^2)$. (Note that in this way, cf also Eq. (3), the "true" virtuality $-k_j^2 \simeq k_{\perp j}^2$)

Thus we obtain the picture in Fig. 3 with a set of connected ISB dipoles spanned between the adjacent $\{q_j\}$ with the virtualities given by the corresponding connectors $\{\log(-k_j^2) \simeq \log(k_{\perp j}^2)\}$. The situation is obviously symmetric between the hadron and probe ends so that in Eq. 7 we may interprete the splitting function z-pole either as the positive light-cone fraction $z = k_{+j}/k_{+(j-1}(= q_{+(j+1)}/q_{+j})$ or the negative light-cone fraction $z \equiv z_- = k_{-j}/k_{-(j+1)}(= q_{-j}/q_{-(j+1)})$ (in case we would describe the process instead from the probe end).

In the CCMF Model more gluons are included in the essential ISB set. In particular when the requirement in Eq. (6) is lifted the model no longer provides complete dipoles but also "parts of dipoles" as described in Ref. [1]. We note one very important requirement, however. *In the FSB of the LDC Model (and this is the same for the CCMF Model) there can be no emission between the* q_j *and* q_{j+1} *with transverse momenta exceeding the virtuality of the connector* $-k_j^2 \simeq k_{\perp j}^2$.

The dynamical reason is that the gluon currents from the pair (q_j, q_{j+1}) due to the existence of the connector k_j "starts out" from two different space-time points. Therefore there is an "effective" (due to Lorentz contraction essentially transverse) distance between these emission points equal to $b \simeq 1/\sqrt{-k_j^2} \simeq 1/k_{\perp j}$. It is well-known that an "antena" of the size b is not well-suited to emit radiation with the wave-length $\lambda < b$ (for smaller wave-length, *i.e.* larger transverse momenta, there will in general be a form-factor suppression). *The corresponding major result from the valiant CCMF Model calculations is that the dipole produced between* q_j *and* q_{j+1} *can emit no FSB bremsstrahlung with transverse momentum larger than the virtuality* $-k_j^2 \simeq k_{\perp j}^2$. There is a cancellation between the virtual and real corrections for the emission region above the transverse momentum $k_{\perp j}$ in between the gluons q_j and q_{j+1}.

2.3. The different channels in Deep Inelastic Scattering

I will end this section with a subdivision into three different channels of the contributions from the dipole chains in the LDC. Firstly, we note that the Bjorken x_B-variable is in this language $x_B = Q^2/2(pq) = Q_+Q_-/P_+Q_- = Q_+/P_+$. We have shown in Fig. 4 both a description of the variables $\log(1/x_B)$ and $\log(Q^2)$, (note that the latter one occurs both as a rapidity region in the "big" dipole phase space triangle and as a measure of virtuality $\kappa_Q = \log(Q^2)$).

In Fig. 4 we have shown three possible "chain-roads" and they correspond to, respectively,

i) an "ordinary" quark-parton model interaction, in which the largest virtuality along the chain is given by Q^2. The chain may end after n emissions anywhere along the line $\kappa_n < \log(Q^2) \equiv L_Q$ with a parton with $\prod^n z_j = x_B$. In the Lorentz frame, defined as the (scattered)

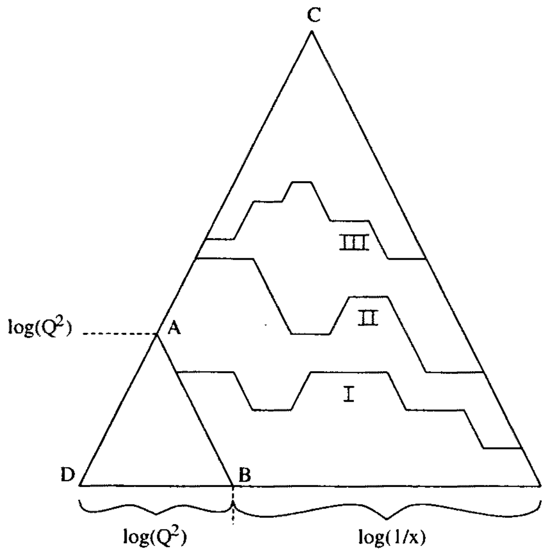

Fig. 4. The different kinds of gluon chains

lepton-hadron cms (*cf.* for a description in other frames Ref. [1]) the parton will (after the interaction with the probe) end up at the point, denoted A at $(\log(1/x_B), \kappa = L_Q)$.

ii) a Boson-Gluon Fussion event, in which the (final) splitting transverse momentum variable exceeds the probe virtuality $\kappa_n (\simeq \log(q^2_{\perp n})) > L_Q$. Then the final parton ends on the line $\log(Q_-)$ above the value L_Q, *i.e.* for $\kappa > L_Q$, due to this final (large virtuality) splitting. According to the way the structure function is defined we obtain from the weight factors of Eq. (7) an extra "going-down" factor $Q^2/k^2_{\perp n}$.

iii) a Rutherford scattering between the probe remnant, which, although it starts out at the probe virtuality L_Q, splits up into a set of emissions $L_Q < \kappa_n < \kappa_{n-1} \ldots < \kappa_{\max}$ to "meet" the corresponding hadron remnant chain with $\kappa_1 < \kappa_2 \ldots < \kappa_{\max}$. In accordance with Eq. (7) we will obtain in this case a factor (in the way we have defined it $\kappa_{\max} = \log(k^2_{\perp\max})$, it is the only maximum in transverse momentum along the chain)

$$\prod_1^n d\kappa_j \frac{Q^2 k^2_{\perp n} k^2_{\perp(n-1)} \cdots}{k^2_{\perp n} k^2_{\perp(n-1)} \cdots k^2_{\perp\max}} = \prod_1^n d\kappa_j \frac{Q^2}{k^2_{\perp\max}} . \qquad (12)$$

We note that although the factor is described in the language of going from the hadron to the probe end, the result would be the same in case we would have used the opposite direction. We also note that this is a direct generalization of the going-down factor obtained in the case *ii)* described above.

The main point is, however, that in Eq. (12) we obtain a factor $d\kappa_j$ for all bremsstrahlung gluons but the factor $dk^2_{\perp\max}/k^4_{\perp\max}$ for the largest transverse momentum, which is characteristic for Rutherford scattering with $k_{\perp\mathrm{Ruth}} = k_{\perp\max}$. In Ref. [1] it is shown that the correct Rutherford scattering behavior occurs both when the propagator corresponds to a gluonic one and when it is a quark(anti-quark) exchange.

Note that in this way (as we have mentioned before, [6, 1]) *the major momentum transfer in a chain diagram always corresponds to a Rutherford scattering* with (coherent) bremsstrahlung contributions describing both the "going-up" and "going-down" sides, but all of them with $k_{\perp\mathrm{brems}} < k_{\perp\mathrm{Ruth}}$. *This means that the (largest) transverse momentum of the bremsstrahlung gluons always provides a lowest cutoff of a (possible) Rutherford scattering.*

The results of this approach will be presented in a set of future publications. I would like to end by saying that I have very much enjoyed my stay at the Zakopane meeting and that I would like to thank the organizers for providing a lot of good vibes, in particular a lot of possibilities for discussions.

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