ON THE ORIGIN OF IDENTICAL BANDS IN THE SUPERDEFORMED STATES OF ATOMIC NUCLEI*

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Some of the single-nucleon configurations in the rotating nuclear potential appear to be almost insensitive to nuclear rotation. The contributions both to the angular momentum alignment and to the dynamical moment of inertia are almost negligible for these special orbits. It is suggested that the appearance and properties of the identical bands discovered in superdeformed regions depend crucially on the population of such special orbits. Examples of the resulting identity relations between various superdeformed bands in nuclei are discussed.

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The existence of identical-energy gamma lines deexciting the high spin rotational states in some Rare Earth nuclei (and later on in other regions) came in 1989 as an unexpected fascinating discovery [1]. It soon became obvious that such a phenomenon observed in several s-d bands (superdeformed bands) in the presence of a fast rotation and enormously large nuclear distortions consists a great challenge for all those willing to understand it in terms of the existing knowledge of nuclear structure. Since the year 1989 many experiments have brought a rich evidence about the existence of identical bands. The identity relations have been discovered to connect more than just pairs of bands. Sometimes several bands exist in a whole group of different nuclides that are seen to be linked by the identity relations. Numerous papers both experimental and theoretical have been published within recent five years that attempt to elucidate the origin of the phenomenon. Nevertheless, no simple explanation of the remarkable effect of identical bands has been offered. In particular, the appearance of the identity relations in

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some rotational bands and their disappearance in other bands has not been understood in any satisfactory way.

In the present paper we shall suggest a possible explanation of the phenomenon of identical bands (*cf.* Ref. [15]). However, this explanation will be limited to the extreme single-particle model of nucleons moving in an average deformed mean field rotating with a frequency given *a priori*. We realize fully that such an approach is oversimplified since it neglects the many-body effects that go beyond the mean field scheme. Thus for example the polarization of the nuclear core by the existence of the nucleonic orbits, the influence of the pairing correlations on the particle motion, or the changes in the mean field induced by the rotation are entirely neglected in the following discussion although effects of this type may prove to be important. Our aim is mainly to exhibit more clearly the leading role of the single-particle orbits.

Two features seem to be characteristic for the identity relations: a rather high precision of the energy relations (sometimes of the order of 1 keV, or even less) and the appreciable range of their validity sometimes extending over 10, or even more transitions. This implies that the single-nucleon orbit generating the band identical with that in the neighbouring core nucleus should appear as a configuration which is to a large extent insensitive to nuclear rotation. This in turn implies that the contributions

$$i_\nu = -\frac{\partial e_\nu^\omega}{\partial \omega} \tag{1}$$

and

$$\delta_\nu \mathcal{J}^{(2)} = \frac{\partial i_\nu}{\partial \omega}\,, \tag{2}$$

to the alignment (*i.e.* the angular momentum component along the rotation axis coming from the ν -th orbit) and the dynamical moment of inertia, respectively must be exceptionally weak as compared to the analogous contributions coming from other orbits. This implies that the single-particle Routhian e_ν^ω of this orbit must be almost independent of the rotational frequency ω to a very good approximation over an appreciable region of ω. We shall call orbits of this type "special orbits". Their appearance in the nucleonic spectra in a fast rotating nucleus seems to be an essential characteristic feature in the explanation of the origin of identical bands.

In order to investigate the structure of nuclear rotational bands we shall consider the single-particle motion of a nucleon in the highly deformed and fast rotating mean field. This may be achieved by the rather well known procedure of the cranking model. In this domain, however, the nucleonic motion seems to be in addition essentially influenced by the appearance of the special symmetry, namely the pseudospin, or even more restrictive pseudo-SU(3) symmetry (see Refs [11–13, 5] for the pseudospin method in nuclear structure, and Refs [4, 2, 3, 14] for the applications to the relevant problems in nuclear structure). The pseudospin picture which will be used extensively throughout this work is mainly connected with the cranking treatment of the rotation. The cranking Hamiltonian

$$H^{\omega} = H - \omega j_1 , \tag{3}$$

will therefore be employed here together with the pseudospin picture. In this representation angular momentum j_1 will be explicitly treated as a sum of the pseudoorbital angular momentum $\tilde{l}_1$ and pseudospin $\tilde{s}_1$ instead of the usual quantities l_1 and s_1. The great advantage of this picture is that the spin-orbit coupling $(\tilde{l} \cdot \tilde{s})$ is known to be very weak. We shall simply neglect it. As is well known, such an assumption is rather seriously limited since many intruder states come close to the Fermi surface and interact with the normal -parity states. Nevertheless, in the region close to the yrast line it may serve as a convenient first approximation.

Once the above assumption is accepted a very simple model of nuclear rotation emerges with the external rotation affects only the orbital nucleonic motion. On the other hand, the nucleonic pseudospin is completely decoupled from rotation. This enables us to describe the whole dynamics in terms of a simple picture of a rotating harmonic oscillator (rho) in the coordinate space. It is well known that in such situation an exact rigorous solution to the cranking Hamiltonian H^{ω} exists (*cf.* Refs [6] and [7]) and may be employed to investigate *explicitly* the single-particle Routhian. The solution has the form of three independent normal modes of the (ho) (= harmonic oscillator) type and we obtain the one-nucleon Routhians as

$$e^{\omega}_{\nu} = (n_1 + \tfrac{1}{2})\omega_1 + (n_2 + \tfrac{1}{2})\Omega_2 + (n_3 + \tfrac{1}{2})\Omega_3 , \tag{4}$$

in case of the rotation about the 1-st axis. Here $\omega_1, \omega_2, \omega_3$ are the three original (ho) frequencies. The two modified (normal) frequencies Ω_2 and Ω_3 are simple functions of ω_2 and ω_3 and rotational frequency ω ([6, 7]). Integers n_1, n_2, and n_3 are the three quantum numbers of the (rho). It seems to be a remarkable result of such a model that whenever the condition

$$n_2 = n_3 \tag{5}$$

is fulfilled the orbit (n_1, n_2, n_3) becomes almost a flat line in the $e^{\omega}_{n_1,n_2,n_3} = f(\omega)$ representation in a rather large interval of ω. Thus for such "special" orbits the condition given by Eq. (1) and Eq. (2) becomes very well satisfied and thus the orbit presents itself as e very good candidate for the configuration underlying the identical bands. This is illustrated in Fig. 1.

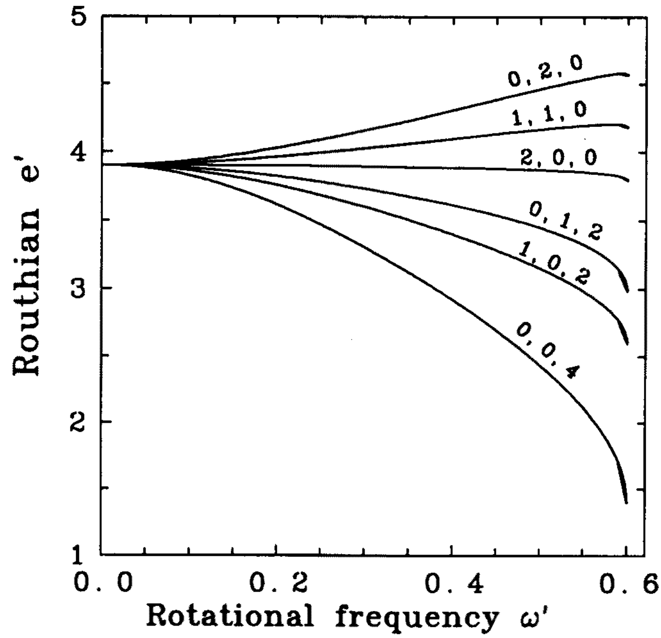

Fig. 1. Single-particle Routhians $e' = e^{\omega}_{\nu}/\omega_0$ (in units of oscillator frequency ω_0) versus rotational frequency $\omega' = \frac{\omega}{\omega_0}$ (in units of ω_0). The only orbits included are those with $N_{sh} = 2n_{\perp} + n_3 = 4$. Quantum numbers n_1, n_2, n_3 are shown at each curve.

Up to now the existence of the pseudospin of the nucleon was entirely ignored in our considerations as decoupled from rotation and thus not affecting the energy. Nevertheless, pseudospin of the nucleon affects the particle angular momentum and thus also its signature. This is due to the fact that the quantisation of angular momentum has to be taken into account in addition to the cranking model. In the case of the one-dimensional rotation considered in our case the quantisation conditions can be only fulfilled in the approximate way. In this case the quantisation reduces to the requirement that the calculated expectation values of the j_1 operator (*i.e.* the angular momentum projection on the rotation axis) are integer (half integer) for the system with even (odd) number of particles. In this way the pseudospin of a nucleon comes into our considerations. We shall see below that the pseudospin degree of freedom decides effectively about the nature of the identity relations that occur between the bands. As is well known there exist three types of identity relations. The best known of them is simply the identity of the gamma-ray energies in the two bands in question. This case is usually referred to as the twin bands (TB). In addition to this simplest relation it may happen that the gamma-ray energies in one band appear to be equal to some averages of the corresponding energies in the second band. Table I illustrates all the three types of identity relations together with some simplest way to classify them.

TABLE I

Description of identity relations. Incremental alignment Δi as defined by Stephens *et al.* [3] $\Delta i = 2(E'_\gamma(I') - E_\gamma(I))/(E_\gamma(I+2) - E_\gamma(I))$ and decoupling parameter defined by Nazarewicz *et al.*, Ref. [2] $E_\gamma(I) = A[2I - 1 + a(-1)^{I+1/2}]$.

Identity type	twin bands (TB)	indirect twin bands (ITB)	coupled bands (CB)
Relation between γ-ray energies	$E'_\gamma(I') = E_\gamma(I)$	$E'_\gamma(I') = {}^1\!/_2[E_\gamma(I) + E_\gamma(I+2)]$	$E'_\gamma(I') = {}^1\!/_4[3E_\gamma(I) + E_\gamma(I+2)]$ or $E'_\gamma(I') = {}^1\!/_4[E_\gamma(I) + 3E_\gamma(I+2)]$
decoupling parameter a	$+1$	-1	0
Incremental alignment Δi	0	$^1\!/_2$	$^3\!/_2$ or $-^1\!/_2$

Before we proceed further let us recall a quantity α called signature exponent. It is related to signature r by a simple relation:

$$r = \mathrm{e}^{-i\pi\alpha} \,. \tag{6}$$

Obviously, $\alpha = 0, -1, {}^1\!/_2$ and $-{}^1\!/_2$ for $r = +1, -1, -i$ and $+i$, respectively. All these four relations are valid as $a = b(\mathrm{mod}2)$ *i.e.* a differs from b by an integer multiple of 2. It is a well known fact that α differs from angular momentum of the corresponding state by an even integer. Now let us analyse what happens when a particle with pseudospin is added to its pseudoorbital part. For the sake of simplicity let us limit our considerations to the case of two s-d bands (B) and (B') differing by one nucleon. The two sequences:

$$\begin{aligned} &I\,,\ I+2\,,\ I+4\,,\ \ldots(B) \\ &I'\,,\ I'+2\,,\ I'+4\,,\ \ldots(B')\,, \end{aligned} \tag{7}$$

correspond to the core nucleus and core plus one valence nucleon, respectively. Let us consider now various particular cases that can occur.

(i) Pseudospin of the valence nucleon aligned with the rotational axis. The natural cranking model relation between the angular momenta in the two bands. In this case the projection of the particle pseudospin on the rotation axis s_1 is a good quantum number. In the same time the signature exponents I and I' in both the bands (B) and (B') obey the natural relation following from the cranking model

$$\alpha' = \alpha + s_1\,, \tag{8}$$

equivalent to

$$I' = I + s_1 \,. \tag{9}$$

In this case the quantisation of signature exponent implies (*cf.* Ref. [15]) the simple equalities between the gamma-ray energies in the two bands (B) and (B')

$$\begin{aligned} E'_{I'} &= E_I \\ E'_{I+2} &= E_{I+2} \\ &\vdots \end{aligned} \tag{10}$$

This is the case of identical bands *sensu stricto* usually referred to as the twin bands (TB).

(ii) Pseudospin of the valence nucleon aligned. The unnatural cranking model relations between angular momenta in the two bands (B) and (B'). Here again s_1 is a good quantum number as in case *(i)*. However, relation between α' and α is different

$$\alpha' = \alpha - s_1 \,, \tag{11}$$

equivalent to

$$I' = I - s_1 \,. \tag{12}$$

This relation does not follow from the simple cranking model in the case of a pure single-particle picture [15]. It looks as if the new band I', $I'+2$, $I'+4$, ... were formed by adding a single nucleon with the simultaneous change of the core band I, $I+2$, $I+4$, ... into a signature partner of band (B) *i.e.* the new band with a spin sequence $I+1$, $I+3$, $I+5$, ... For the moment there seems to be no experimental evidence for the existence such signature partners in general. Nevertheless, it seems necessary to consider such "unnatural" mechanisms since it seems to be the only way to explain the existence of the so called indirect twin bands (ITB) which do exist really (*cf.* Table I). The ITB identity relation means that the gamma-ray energies $E'_{I'}$ in band (B') are related to those in band (B) by the relation of the *arithmetic averages*:

$$E'_{I'} = \tfrac{1}{2}(E_I + E_{I+2}) \,. \tag{13}$$

Finally, it may happen that the pseudospin of a single valence nucleon is aligned with the direction of nuclear deformation symmetry axis. In this case the 3-rd component s_3 of the pseudospin could be a good quantum number but in the presence of nuclear rotation it is rather a state of good signature (*i.e.* a sum or difference of two states with $s_3 = {}^1\!/_2$ and $s_3 = -{}^1\!/_2$)

that come into account. In this case the detailed geometric consideration of angular momenta lead to the conclusion [15] that the gamma-ray energies in band (B') are simply the weighted averages of the gamma-ray energies in band (B) with coefficients ($^1/_4$ and $^3/_4$), or ($^3/_4$ and $^1/_4$) depending on the signature (*cf.* Table I). This is the so called case of the "coupled bands", (CB).

All the three cases presented above were primarily discussed by Nazarewicz *et al.* (Ref. [2]) in terms of the rotor plus particle model and by Stephens *et al.* (Ref. [3]) in terms of the concept of the incremental alignment. Table I summarizes all the three cases discussed above as already mentioned.

TABLE II

Correspondence in the wave functions. First column lists the quantum number $N_{\rm shell}$ labeling the deformed shells. In our case (for the 2:1 shape) $N_{\rm shell} = 2n_\perp + n_3$. Second column lists all the (rho) states for $N_{\rm shell} = 0$ to 6. An asterisk $*$ denotes a special configuration. Third column lists the nonrotating deformed (ho) state in the pseudospin picture while fourth column — in the ordinary (ho) representation.

$N_{\rm shell}$	$(n_1\, n_2\, n_3)$	$\tilde{N} n_3 \tilde{\Lambda} \Omega\rangle$	$[N n_3 \Lambda \Omega\rangle$
1	2	3	4
0	$*$ (0, 0, 0)	$[000\ ^1/_2\rangle$	$[101\ ^1/_2\rangle$
1	(0, 0, 1)	$[110\ ^1/_2\rangle$	$[211\ ^1/_2\rangle$
2	(0, 1, 0)	$[101\ ^3/_2\rangle$	$[202\ ^3/_2\rangle$
	$*$ (1, 0, 0)	$[101\ ^1/_2\rangle$	$[200\ ^1/_2\rangle$
	(0, 0, 2)	$[220\ ^1/_2\rangle$	$[321\ ^1/_2\rangle$
3	$*$ (0, 1, 1)	$[211\ ^3/_2\rangle$	$[312\ ^3/_2\rangle$
	(1, 0, 1)	$[211\ ^1/_2\rangle$	$[310\ ^1/_2\rangle$
	(0, 0, 3)	$[330\ ^1/_2\rangle$	$[431\ ^1/_2\rangle$
4	(0, 2, 0)	$[202\ ^5/_2\rangle$	$[303\ ^5/_2\rangle$
	(1, 1, 0)	$[202\ ^3/_2\rangle$	$[301\ ^3/_2\rangle$
	$*$ (2, 0, 0)	$[200\ ^1/_2\rangle$	$[301\ ^1/_2\rangle$
	(0, 1, 2)	$[321\ ^3/_2\rangle$	$[422\ ^3/_2\rangle$
	(1, 0, 2)	$[321\ ^1/_2\rangle$	$[420\ ^1/_2\rangle$
	(0, 0, 4)	$[440\ ^1/_2\rangle$	$[541\ ^1/_2\rangle$

TABLE II continued

1	2	3	4
5	(0, 2, 1)	[312 5/2⟩	[413 5/2⟩
	* (1, 1, 1)	[312 3/2⟩	[411 3/2⟩
	(2, 0, 1)	[310 1/2⟩	[411 1/2⟩
	(0, 1, 3)	[431 3/2⟩	[532 3/2⟩
	(1, 0, 3)	[431 1/2⟩	[530 1/2⟩
	(0, 0, 5)	[550 1/2⟩	[651 1/2⟩
6	(0, 3, 0)	[303 7/2⟩	[404 7/2⟩
	(1, 2, 0)	[303 5/2⟩	[402 5/2⟩
	(2, 1, 0)	[301 3/2⟩	[402 3/2⟩
	* (3, 0, 0)	[301 1/2⟩	[400 1/2⟩
	* (0, 2, 2)	[422 5/2⟩	[523 5/2⟩
	(1, 1, 2)	[422 3/2⟩	[521 3/2⟩
	(2, 0, 2)	[420 1/2⟩	[521 1/2⟩
	(0, 1, 4)	[541 3/2⟩	[642 3/2⟩
	(1, 0, 4)	[541 1/2⟩	[640 1/2⟩
	(0, 0, 6)	[660 1/2⟩	[761 1/2⟩

In order to memorize better the three different identity relations described above and listed in Table II we suggest to use three different ways of denoting them in the graphical representations (see Figures 2 to 5 below). Thus we shall use a solid line (with no arrow) for (TB), a dashed line for the (ITB) (no arrow) and a solid line with an arrow for the (CB). The direction indicated by the arrow from (B) to (B') corresponds to the weighted average:

$$E'_{I"} = \tfrac{3}{4}E_I + \tfrac{1}{4}E_{I+2}\,, \tag{14}$$

where $E'_{I'}$ are gamma lines in band (B') while E_I are gamma lines in band (B). For the weighted average with coefficients $1/4$ and $3/4$ the direction of the arrow is opposite.

Let us discuss now some examples of the identical bands observed in various experiments. The most typical case is that of the twin bands formed by the yrast band in the ^{152}Dy nucleus and the excited band (b2) in the neighbouring nucleus ^{151}Tb. This is actually the first pair of identical bands observed in an experiment [1]. The odd-A nucleus ^{151}Tb may be regarded as a one-proton hole state in ^{152}Dy. Since the two bands are twin bands one

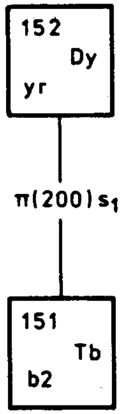

Fig. 2. Graphical illustrations of the identical bands in ^{151}Tb and ^{152}Dy. Solid line connecting the two nuclides denotes twin bands (TB), *cf.* Table I. Assignment on the line indicates the suggested configuration of the odd-proton hole.

may expect that the hole state is characterized by a special configuration. In this region of proton numbers the configuration $(n_1, n_2, n_3) = \pi(2,0,0)$ is the only available close to the Fermi surface. Thus we assign the configuration $\pi(n_1, n_2, n_3) = \pi(2,0,0)s_1$ as the one missing in ^{151}Tb relative to the core ^{152}Dy (Fig. 2). The quasispin projection on the first axis s_1 may be equal either $+^1/_2$, or $-^1/_2$.

The assignments shown in Fig. 2 (and in Figs 3 to 5 as well) are based on the (rho) wave functions (n_1, n_2, n_3) described above. In order to clarify better their physical meaning it is desirable to expand them in terms of the familiar representation $|Nn_3\Lambda\Omega\rangle$ of the asymptotic nonrotating harmonic oscillator. In fact, such an expansion is not straightforward since the angular momentum operator j_1 entering the cranking model formula (3) couples all the states $|\tilde{N}n_3\tilde{\Lambda}\Omega\rangle$ in the pseudospin picture so that expansion of any (rho) state (n_1, n_2, n_3) into the states $|\tilde{N}n_3\tilde{\Lambda}\Omega\rangle$ is infinite. Nevertheless for slow rotation a certain correspondence between the two representations can be established approximately. This is illustrated in Table II which lists all the lowest states of the (rho) together with those of the asymptotic representation of the nonrotating deformed (ho). In this way the states (n_1, n_2, n_3) are related with of the asymptotic (ho) representation in the pseudospin picture. The approximate correspondence between the states in the two representations indicates roughly that the state (n_1, n_2, n_3) with $\tilde{N} = n_1 + n_2 + n_3$ and $\tilde{\Lambda} = (\tilde{N} - n_3), (\tilde{N} - n_3) - 2, \ldots$ while the quantum number n_3 remains the same in both the representations. The corresponding mixtures in the wave functions are indicated by curly braces between columns 2 and 3 in Table II. The second step is to transform the pseudospin deformed (ho) representation into the usual deformed (ho) representation $|Nn_3\Lambda\Omega\rangle$. The corresponding procedure is well known (*cf.* Ref. [13], or [5]):

$$|\tilde{N}n_3\tilde{\Lambda}\Omega\rangle \longrightarrow |N+1, n_3\Lambda\Omega\rangle\,, \tag{15}$$

where $\Lambda = \tilde{\Lambda} \pm 1$ for $\Omega = \tilde{\Lambda} \pm (^1/_2)$

Thus for example the state $(n_1, n_2, n_3)s_1 = (2,0,0)s_1$ goes into the mixture of states $|\tilde{2}0\tilde{2}\,^5/_2\rangle$, $|\tilde{2}0\tilde{2}\,^3/_2\rangle$ and $|\tilde{2}0\tilde{0}\,^1/_2\rangle$ in the pseudospin picture which gives a set $|303\,^5/_2\rangle$, $|301\,^3/_2\rangle$ and $|301\,^1/_2\rangle$ in the ordinary standard (ho) representation. One can see immediately that the state $|301\,^1/_2\rangle$ suggested originally as a basic configuration for the identical bands in this case exist as a component of the special configuration $(2,0,0)s_1$.

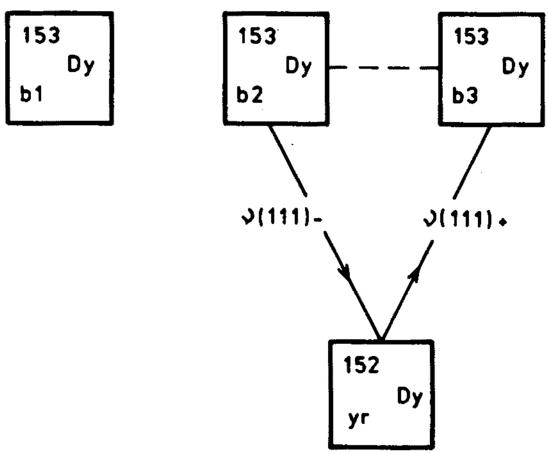

Fig. 3. Graphical illustration of relations between the three s-d bands in ^{153}Dy and ^{152}Dy. Solid line with arrows indicates the coupled bands (CB), while dashed line denotes the indirect twin bands (ITB), *cf.* Table I.

Fig. 3 gives an example of the coupled bands, (CB) [9]. Two out of the three s-d bands b1,b2, b3 in ^{153}Dy are in identity relations with the ^{152}Dy core. Gamma-ray energies in bands (b3) and (b2) turn out to be weighted averages of those in the ^{152}Dy yrast band with coefficients $(^3/_4, ^1/_4)$-arrow up and $(^1/_4, ^3/_4)$-arrow down , respectively. The only neutron configuration that is close to the Fermi surface seems to be $(1,1,1)\alpha$ with signature exponent $\alpha = +^1/_2$ and $\alpha = -^1/_2$, respectively. It can be seen from Table II that such a state is roughly a mixture of states $|413^5/_2\rangle$, $|411^3/_2\rangle$ and $|411^1/_2\rangle$. Let us mention that some different assignments were suggested (see *e.g.* Refs [2], or [16]) in terms of the high-K orbits such as $|514^9/_2\rangle$. It is not certain, however, that such orbits as not derived from the special configurations can really fulfill *both* equations (1) and (2). Especially former Eq. (1) may prove to be more restrictive in this case. Thus if really the high-K configurations of this type underlie the coupled bands it may mean that the extreme single-particle approach adopted in this paper requires a further modification or revision. It is interesting to note that the first s-d band (b1) which is most probably the yrast one is not connected by any identity relations either to the ^{152}Dy core nucleus, or to any other of the s-d bands appearing in Fig. 3. It originates most probably from an orbit of completely different type (*cf. e.g.* Ref. [16]).

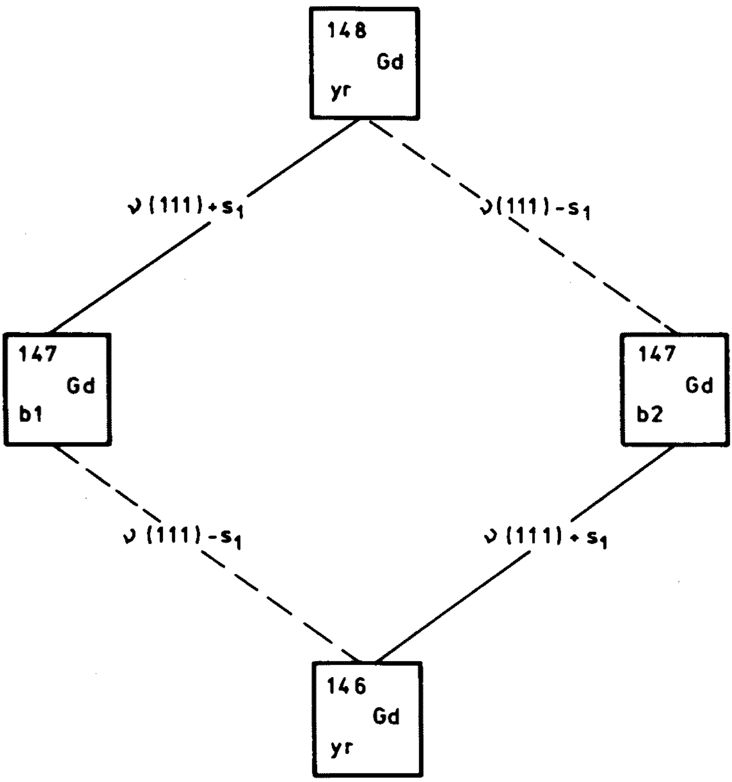

Fig. 4. Graphical illustration of the four s-d bands in three nuclides: ^{146}Gd, ^{147}Gd and ^{148}Gd. For further explanations see captions to Figures 2 and 3.

Fig. 4 illustrates an interesting case of four s-d bands occurring in the three isotopes of Gd. The two s-d bands in ^{147}Gd are connected with the yrast bands in both ^{146}Gd and ^{148}Gd by mutual relations of the (TB) and (ITB) type as follows from the experiment [10]. We suggest again the orbit

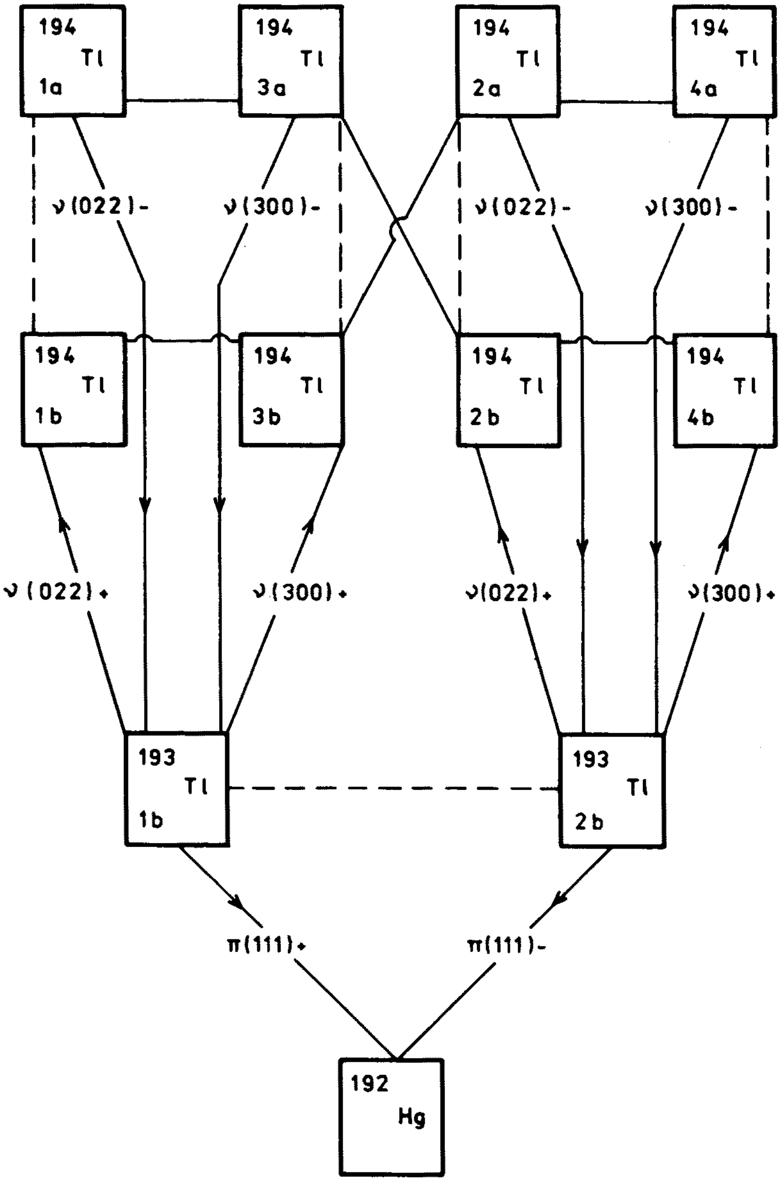

Fig. 5. Graphical illustration of the relations between the s-d bands in ^{194}Tl and ^{193}Tl as well as their connection with ^{192}Hg. For further explanations see captions to Figures 2 and 3.

(n_1, n_2, n_3) as the only one special orbit which is available in the vicinity. The left and right chain visible in Fig. 3 correspond to different order of filling the states with different signs of the pseudospin component $s_1 = +^1/_2$ first and $-^1/_2$ next, or vice versa. A closer look at the relations connecting the signature exponents α and α'' in ^{146}Gd and ^{148}Gd, respectively leads to the conclusion that the corresponding angular momenta I and I'' must obey [15] the following relation:

$$I'' = I + 1(\text{mod}2)\,. \tag{16}$$

Fig. 5 illustrates a slightly more complex scheme of the s-d bands in two isotopes of Tl, namely ^{194}Tl and ^{193}Tl linked also to the nuclide ^{192}Hg. Experiment [17] gives only six s-d bands in ^{194}Tl namely the bands 1a,2a,3a,1b,2b and 3b while our theoretical assignments predict the existence of two more bands, say 4a and 4b (although it is by no means certain that the labels given in experiment correspond exactly to those in the prediction). The eight s-d band predicted the theoretical model come from the fact that in this region of neutron numbers there exist two special orbits, namely the (0,2,2) and (3,0,0) thus the (CB) relations may generate eight possible configurations with signature exponents α equal $+^1/_2$, or $-^1/_2$ out of the two s-d bands in the ^{193}Tl nucleus as can be seen from Fig. 5. For the moment it is not clear whether the eight bands do really exist in nature in this case, or else for some unknown reasons the two additional s-d bands should be ruled out of the theory.

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