THE $\Delta I = 4$ BIFURCATION IN SUPERDEFORMED BANDS*

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The origin of the $\Delta I = 4$ staggering effect which has recently been observed in several superdeformed bands is discussed. The Hamamoto and Mottelson model, which is based on a phenomenological parametrization of the hamiltonian as a quartic function of angular momentum, is analyzed. A stability of the description with respect to the C_4-symmetry breaking terms is studied.

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Recently, a puzzling new phenomenon has been observed in few superdeformed rotational sequences. Namely, dynamical moments of inertia of several bands exhibit small regular variations (staggering) as functions of angular momentum. The effect consists in systematic energy displacements of rotational states, which are alternately pushed up and down along the rotational band. Thus the energy levels are separated into two sequences with the spin values I, $I+4$, $I+8$, ... and $I+2$, $I+6$, $I+10$, ..., respectively. One should mention that the effect is of the order of 50 eV,

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and since the excitation energies of superdeformed bands are of the order of 20 MeV, the observed shifts correspond to a 10^{-6} perturbation of the energy levels.

This phenomenon was first observed in the superdeformed yrast band of ^{149}Gd [1]. To date the staggering has also been found in three bands of ^{194}Hg [2] and in one band of ^{153}Dy [3]. The characteristic feature is that the bifurcation into two $\Delta I = 4$ families appears at a certain (high) rotational frequency, the amplitude of the oscillations increases with spin, and, sometimes, the change of phase in the oscillatory pattern is observed.

The bifurcation of the rotational band into the two families of states can be associated with the existence of a higher-order symmetry in superdeformed states. The symmetry results in the appearance of a new quantum number distinguishing the two branches. Since the period of the oscillations is $\Delta I = 4$, a possible explanation can be based on the four-fold rotational symmetry [1]. The possible origin of this symmetry could be the coupling between the rotational motion and the hexadecapole vibration. We expect that for high angular momenta the hexadecapole phonon becomes aligned along the rotation axis and gives rise to a small hexadecapole perturbation of the average field. The above mechanism was suggested by Peker *et al.* [4] to explain a similar effect observed at low spins in ^{236}U, ^{238}U, and ^{218}Ra nuclei.

In order to study a possibility that the hexadecapole correlations are responsible for the staggering in the moment of inertia, we analyzed the exact solutions of the single-j shell system with identical particles interacting via quadrupole-quadrupole and hexadecapole-hexadecapole interactions [5]. The Hamiltonian for this simple model can be written in the following form:

$$\hat{H} = -\chi_2 \hat{Q}_2 \cdot \hat{Q}_2 - \chi_4 \hat{Q}_4 \cdot \hat{Q}_4 , \qquad (1)$$

where $\hat{Q}_\lambda \cdot \hat{Q}_\lambda = \sum_\mu (-)^\mu \hat{Q}_{\lambda\mu} \hat{Q}_{\lambda-\mu}$ and $\hat{Q}_{\lambda\mu}$ for $\lambda = 2$ and 4 denotes the quadrupole and hexadecapole operator, respectively. Such a model has two properties which are of great importance in this case: it fully takes into account quantum dynamics and yields states with good angular momentum.

We analyzed collective quadrupole bands for $j = {}^{15}/_2$ and $j = {}^{21}/_2$ and different particle numbers. We found that the staggering effect (see Fig. 1) appears in yrast bands when the hexadecapole coupling constant χ_4 is larger than a certain critical value $\chi_4^{\rm crit}$. This shows that the hexadecapole correlations may indeed be responsible for the $\Delta I{=}4$ staggering.

On a phenomenological level, effects of the hexadecapole deformation can be simulated by using an intrinsic Hamiltonian which is a quartic function of angular momentum. Such an operator can in principle have the same spatial four-fold symmetry as the microscopic mean field with hexadecapole distortions. Recently such an approach has been proposed by Hamamoto

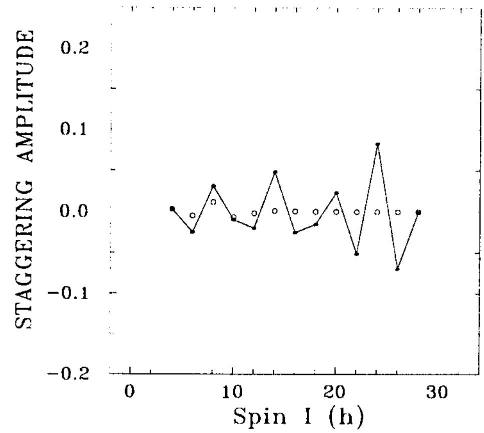

Fig. 1. Staggering of transition energies in the yrast band obtained for 8 particles moving in the single-j shell for $j = 15/2$. Open circles denote results for pure quadrupole interaction, χ_4=0. Full circles represent results obtained for the system with perturbation of the hexadecapole type for $\chi_4/\chi_2 = 0.4$. The formula used to derive the staggering amplitude is given in Ref. [1].

and Mottelson [6]. They considered a rotational Hamiltonian with an explicit four-fold symmetry around the z-axis (perpendicular to the rotational axis) which has the form:

$$\hat{H} = A\hat{I}_z^2 + B_1(\hat{I}_x^2 - \hat{I}_y^2)^2 + B_2(\hat{I}_x^2 + \hat{I}_y^2)^2 \,. \tag{2}$$

The four lowest states form then a quartet of states which can be labeled by the quantum number associated with the C_4 symmetry. The authors found that for a certain ratio of parameters A/B_1 and for $B_2 = 0$ the staggering appears as a result of the tunneling between the four degenerate different minima in the total energy surface of Hamiltonian (2).

In the present study we address the question whether the staggering occurs also when the C_4 term in the Hamiltonian is a small perturbation on the top of the large C_2-invariant term (*i.e.*, term which possesses a two-fold symmetry). We consider a modified Hamiltonian of the form

$$\hat{H} = A\hat{I}_z^2 + D_1 I(I+1)(\hat{I}_x^2 - \hat{I}_y^2) + B_1(\hat{I}_x^2 - \hat{I}_y^2)^2 \,. \tag{3}$$

We neglect terms proportional to $(\hat{I}_x^2 + \hat{I}_y^2)$ and $(\hat{I}_x^2 + \hat{I}_y^2)^2$ as they lead only to a renormalization of the coefficient A. The additional term in (3) has the explicit two-fold symmetry and henceforth breaks the C_4 symmetry when $D_1 \neq 0$. We put parameters A and B_1 to be equal 90 and 1, respectively. For these values Hamamoto and Mottelson [6] found the staggering pattern present. Having the parameters A and B_1 fixed we changed the coefficient

D_1 (see Fig. 2). One can see that even for relatively high values of D_1 the staggering effect is still present. Although the C_4 symmetry is broken in this case, and the corresponding quantum number has only an approximate meaning, one is still able to use this label to distinguish states of the lowest quartet. Finally, for the chosen set of parameters A and B_1 the staggering vanishes at $D_1 \approx 1.4$. In this limit, the system rotates around the y-axis (see Fig. 3). This is a consequence of the appearance of the two well-defined minima in the total energy surface (see Fig. 4). The quartet of the lowest states separates then into two families of degenerate pairs labeled by the quantum number of C_2.

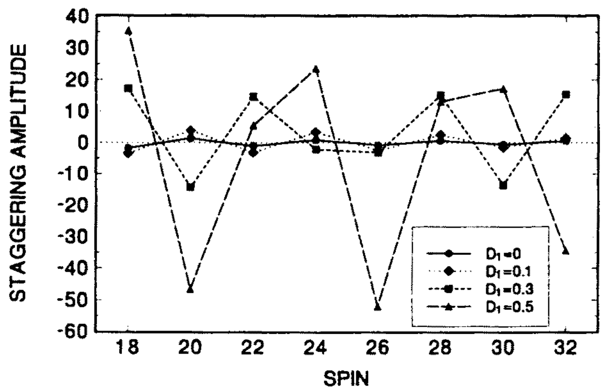

Fig. 2. Staggering amplitudes calculated for the band obtained by using Hamiltonian (3) with four different values of the coefficient D_1.

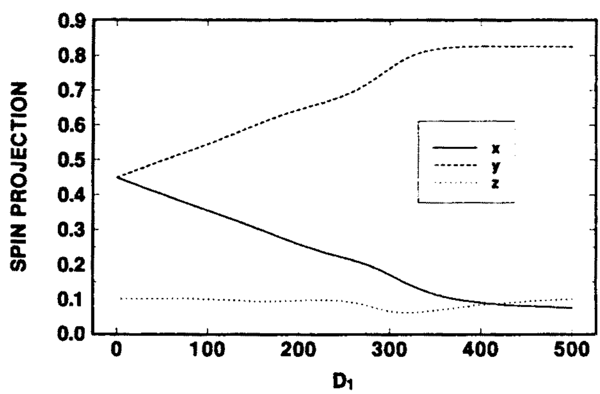

Fig. 3. Average values of components of the angular momentum, $\langle \hat{I}_x^2 \rangle / I(I+1)$, $\langle \hat{I}_y^2 \rangle / I(I+1)$, and $\langle \hat{I}_z^2 \rangle / I(I+1)$, as functions of D_1, calculated at $I = 16$ for the same state as in Fig. 2.

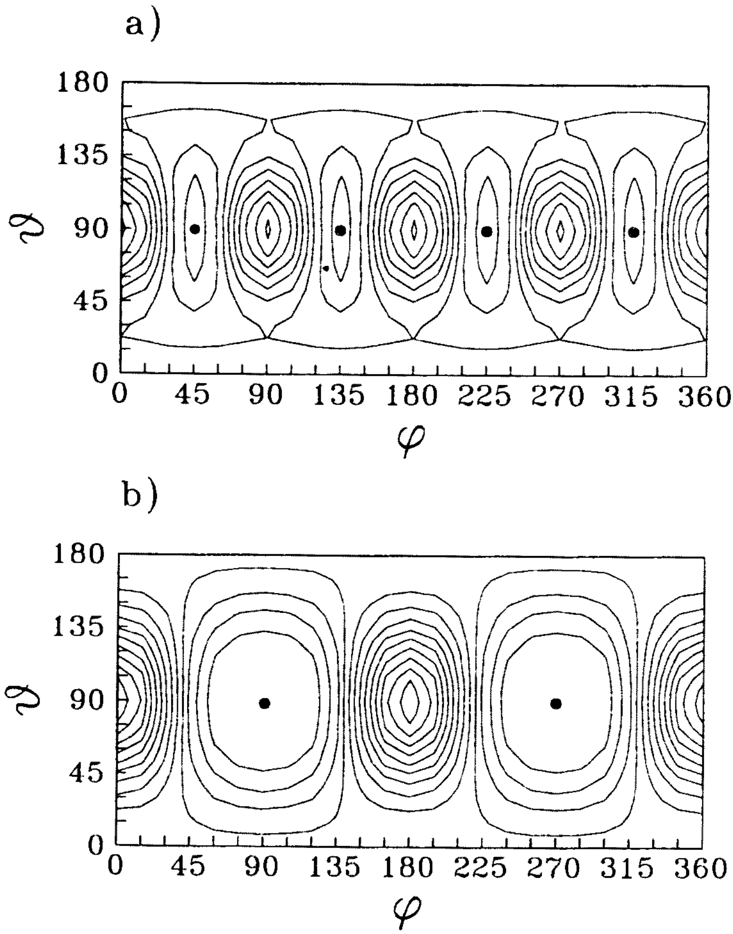

Fig. 4. Total energy surfaces for Hamiltonian (3) and for $D_1 = 0$ (a) and $D_1 = 1.47$ (b). Spherical angles Φ and Θ describe the orientation of the angular momentum treated as a classical variable. Minima in the plots are denoted by dots. Orientations parallel to the x, y, and z axes correspond to $\Phi = 0°$, $\Theta = 90°$, to $\Phi = 90°$, $\Theta = 90°$, and to $\Theta = 0°$, respectively. Calculations were performed for $I = 16$.

The above results suggest that even a small C_4-type perturbation can give rise to a staggering effect. This may suggest that one can search for a microscopic justification of the four-fold symmetry with respect to the rotational axis, where the C_2 term dominates. One should mention that according to recent cranked-Nilsson calculations [7] there is no evidence for a strong four-fold symmetry breaking with respect to the axial symmetry axis.

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